Mawuli Segnon, Chi Keung Lau, Bernd Wilfling* and Rangan Gupta

Are multifractal processes suited to forecasting electricity price volatility? Evidence from Australian intraday data

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Abstract: We analyze Australian electricity price returns and find that they exhibit volatility clustering, long memory, structural breaks, and multifractality. Consequently, we let the return mean equation follow two alternative specifications, namely (i) a smooth transition autoregressive fractionally integrated moving average (STARFIMA) process, and (ii) a Markov-switching autoregressive fractionally integrated moving average (MSARFIMA) process. We specify volatility dynamics via a set of (i) short- and long-memory GARCH-type processes, (ii) Markov-switching (MS) GARCH-type processes, and (iii) a Markov-switching multifractal (MSM) process. Based on equal and superior predictive ability tests (using MSE and MAE loss functions), we compare the out-of-sample relative forecasting performance of the models. We find that the (multifractal) MSM volatility model keeps up with the conventional GARCH- and MSGARCH-type specifications. In particular, the MSM model outperforms the alternative specifications, when using the daily squared return as a proxy for latent volatility.

Keywords: electricity price volatility; GARCH-type processes; Markov-switching processes; multifractal modeling; volatility forecasting.

JEL classification: C22; C52; C53.

1 Introduction

The deregulation of electricity markets and the rapid development of related financial products have unleashed an enormous interest in establishing econometric models that appropriately reflect the unique characteristics and dynamics of electricity price behavior. In particular, forecasting electricity price volatility has become a major task for analysts, energy companies and investors, due to its dominant role in power derivative pricing, hedging, and risk allocation. Volatile electricity prices also increase uncertainty as to generators' revenue and suppliers' costs. For industry regulators, high electricity price volatility is a serious issue, due to their concern that this might indicate, at least partly, the outcome of regulatory malfunctioning with deficiencies in market design, and/or the exertion of market power. As a result, a plethora of alternative econometric models have been proposed in the literature with the aim of appropriately modeling price means and variances, and producing accurate volatility forecasts (We provide an overview in Section 2).

In this paper, we apply an additional class of processes for modeling and forecasting electricity price volatility, which originally stem from the analysis of turbulent flows (Mandelbrot 1974). Their adaptation to

***Corresponding author: Bernd Wilfling**, Westfälische Wilhelms-Universität, Münster, Germany, E-mail: bernd.wilfling@wiwi.uni-muenster.de
Mawuli Segnon, Westfälische Wilhelms-Universität, Münster, Germany, E-mail: segnon@uni-muenster.de
Chi Keung Lau, University of Huddersfield, Huddersfield, UK, E-mail: c.lau@hud.ac.uk
Rangan Gupta, University of Pretoria, Pretoria, South Africa, E-mail: rangan.gupta@up.ac.za

finance started with the work of Mandelbrot, Fisher, and Calvet (1997), which led to the first generation of multifractal models. Calvet and Fisher (2001, 2004) then introduced the second model generation, by specifying their Poisson multifractal model and its discretized version, the Markov-switching multifractal process.[1] To date, multifractal processes have been applied successfully in the volatility modeling and forecasting of different asset prices, including exchange rates (Calvet and Fisher 2004; Lux 2008), stock prices (Lux 2008) and commodity prices (Lux, Segnon, and Gupta 2016; Segnon, Lux, and Gupta 2017; Wang, Wu, and Yang 2016). Overall, these processes appear to be robust tools for capturing well-documented stylized facts of financial and commodity market volatility, and are therefore natural candidates for producing accurate volatility forecasts in electricity markets.

The main objective of this study is to investigate whether the multifractal framework—which accommodates both long-term persistence in the volatility process, and structural breaks through regime-switching—(i) constitutes an appropriate tool for analyzing price volatility in deregulated electricity markets, and (ii) provides a comprehensive approach that outperforms alternative volatility models in terms of forecast-accuracy (Aggarwal, Saini, and Kumar 2009).[2] To this end, we use (i) the smooth transition fractionally integrated autoregressive moving average (STARFIMA) specification (Hillebrand and Medeiros 2016), and (ii) the Markov-switching autoregressive fractionally integrated moving average (MSARFIMA) specification (Tsay and Härdle 2009) to model the mean equation of electricity price returns. For volatility modeling, we rely on (i) a Markov-switching multifractal (MSM) process, and (ii) a set of (Markov-switching) GARCH-type processes.

To the best of our knowledge, this study is the first to combine the STARFIMA and MSM processes in a consolidated econometric framework, which we apply to Australian data, for reasons that are explained below. In contrast to previous studies, we do not confine ourselves to forecasting electricity price volatility only for the Australian New South Wales, but provide an extended empirical analysis covering all five Australian regions. Overall, our analysis proceeds in two steps. (i) We investigate (multi-)fractal structures of Australian electricity price returns (by computing the multifractal spectrum) via the so-called Multifractal Detrended Fluctuation Analysis (MDFA), as proposed by Kantelhardt et al. (2002). (ii) We conduct a forecasting investigation in order to compare the volatility forecasting performance of our combined STARFIMA-MSM model with the performance of several other combinations, in each of which the mean equation still follows the STARFIMA or MSARFIMA process, but with the volatility equation being governed by alternative (conventional and Markov-switching) GARCH-type processes.

Our analysis yields two major findings. (i) Daily Australian electricity price returns exhibit multifractal structures, structural breaks and long memory. These empirical findings provide a clear-cut motivation for using multifractal and Markov-switching processes in the modeling of electricity-price dynamics. (ii) Incorporating multifractal structures into the volatility equation of the data-generating electricity-price process yields volatility forecasts that match those of alternative conventional specifications, when using realized variances as a proxy for latent volatility. However, when using the (daily) squared returns as the volatility proxy (what is frequently encountered in the literature), the MSM volatility model outperforms the alternative specifications (in terms of superior predictive ability).

The remainder of the paper is organized as follows. In Section 2, we motivate our interest in Australian electricity markets and provide an overview of alternative forecasting models of electricity price dynamics. In Section 3, we analyze electricity price changes and present the results of the Multifractal Detrended Fluctuation Analysis. Section 4 presents our econometric modeling framework. In Section 5, we present the forecasting procedures and our results. Section 6 concludes.

1 For details on the genesis of multifractal models and applications in finance, see Lux and Segnon (2018).

2 In a related context, Nowotarski et al. (2014) and Nowotarski and Weron (2015) consider forecast combinations of several individual volatility models. In this paper, we do not pursue this approach any further.

2 Preliminaries and literature review

2.1 Characteristics of Australian electricity markets

The literature has extensively explored the price dynamics in the Australian electricity markets, due to its unique characteristics in terms of the scale of the power system and the source of electricity generation. The National Electricity Market (NEM) is the world's largest interconnected power system, running for more than 5000 km from North Queensland to Tasmania and central South Australia. In view of such a large interconnected power system, price volatility is high and uncertainty occurs for several reasons.

First, some generators do not follow the dispatch instructions issued by the Australian Energy Market Operator (AEMO) as a way to increase their revenue at the expense of power-system safety and price uncertainty for end-consumers.[3] Price also varies due to supply issues such as plant outages or constraints in the transmission network that limit transport capabilities. Another unique characteristic of NEM is its heavy dependence on coal fired generators. As the highest dependency compared to other developed countries, 84 percent of electricity was derived from coal in 1998 and 61 percent in 2014. The adoption of a carbon tax policy, implemented between July 2012 and July 2014, increased the cost of power production from coal.

Another policy uncertainty was induced by the introduction of a greenhouse gas emissions trading scheme by 2010. The legislation was initially passed by the House of Representatives, but rejected by the Senate. Apergis and Lau (2015) provide evidence that the Australian electricity market is a volatile one with a high degree of market power exercised by various generators, and one source of volatility stems from the failure to introduce a greenhouse gas emissions trading scheme in 2010. Therefore, it is important to forecast electricity price volatility in NEM, as its occurrence could be caused by demand shocks (e.g., extreme weather conditions), supply disruption (e.g., outages of transmission lines), or political uncertainty (e.g., rejection of greenhouse gas emissions trading scheme).

2.2 Forecasting electricity price dynamics: a literature review

To date, a multitude of models for forecasting electricity price volatility have been proposed in the literature, which we can roughly divide into two groups. The first is based on non-traditional methodologies, like artificial intelligence and hybrid approaches, including (fuzzy) neural networks, fuzzy regression, cascaded architecture of neural networks and committee machines (e.g. Amjady 2006; Amjady 2012; Amjady and Hemmati 2009; Catalão et al. 2007; Vahidinasab and Kazemi 2008).

The second group of studies, which are more relevant to our paper, include (i) traditional autoregressive time series models (Contreras et al. 2003; Garcia-Martos, Rodriguez, and Sanchez 2007; Kristiansen 2012), (ii) generalized autoregressive conditional heteroscedasticity (GARCH-type) models (Chan and Gray 2006; Cifter 2013; Garcia et al. 2005; Gianfreda 2010; Koopman, Ooms, and Carnero 2007), (iii) jump-diffusion models (Chan, Gray, and Campen 2008; Huisman and Mahieu 2003), (iv) autoregressive conditional hazard models (Christensen, Hurn, and Lindsay 2012), and most recently, (v) multivariate models (Raviv, Bouwman, and van Dijk 2015). Several authors apply alternative machine learning techniques, such as support vector regressions and neural networks, in order to model nonlinear behavior in high-frequency electricity price returns (Lago, De Ridder, and De Schutter 2018; Wu and Shahidehpour 2010). Yet others combine traditional models with fundamentals (Huurman, Ravazzolo, and Zhou 2012; Karakatsani and Bunn 2008), use high-frequency data to forecast price volatility (Gianfreda and Grossi 2012; Higgs and Worthington 2005; Haugom and Ullrich 2012;

3 In July 2014, an electricity generator Snowy Hydro failed to comply with dispatch instructions issued by the Australian Energy Market Operator (AEMO), and the Australian Energy Regulator (AER) instigated proceedings against the company. Snowy Hydro paid total penalties of $400,000, because of the potential hazard to public safety and material risk of damaging equipment.

Haugom et al. 2011), and study the link between fundamentals, other energy markets and electricity price volatility (Gianfreda 2010; Goss 2006; Jonsson and Dahl 2016).[4]

Although the literature has expressed a strong interest in studying the forecasting performance of electricity spot prices (Hong 2015; Maciejowska, Nowotarski, and Weron 2016; Nowotarski and Weron 2015; Nowotarski et al. 2014; Weron 2014), only a few empirical studies have focused on forecasting realized volatility in world-wide electricity markets. Nowotarski et al. (2014) perform a backtesting analysis using seven averaging and one selection scheme on day-ahead electricity prices in three major European and US markets. While the authors find that averaging forecasting techniques outperforms the counterpart models under normal market conditions, it fails to outperform an individual model in a more volatile environment or in the presence of price spikes. According to Clements, Herrera, and Hurn (2015), there is an ample early literature dealing with price spikes, when forecasting spot electricity prices with various modeling approaches, including thresholds, Bernoulli and Poisson jump processes, heavy-tailed error processes, Markov-switching, and diffusion models with time-varying intensity parameters. Examples include Misiorek et al. (2006); Knittel and Roberts (2005); Swider and Weber (2007); Higgs and Worthington (2008).

Another strand of the recent literature focuses on forecasting the probability of such spike events, instead of simply forecasting the level of spot prices. These authors apply a multivariate point process to model the occurrence and size of extreme price events, and conclude that physical infrastructure is the main influential factor in determining the transmission of price spikes in interconnected regions of the Australian electricity market (Clements, Herrera, and Hurn 2015). Their econometric approaches also lead to improved forecast indicators, such as forecasts and estimates of risk measures (for example value-at-risk and expected shortfall).

Qu et al. (2016) note that most of the early research uses the generalized autoregressive conditional heteroscedastic (GARCH) model introduced by Bollerslev (1986), and related models for estimating and forecasting electricity price volatility. The authors use a group of logistic smooth transition heterogeneous autoregressive (LSTHAR) models to forecast realized volatility in the Australian New South Wales (NSW) electricity market. Their modeling approaches enable simultaneously capturing long-memory behavior, as well as sign and size asymmetries, and provide improved volatility forecasts.

3 Preparatory data analysis

Currently, there are five Australian states — Queensland (QLD), New South Wales (NSW), Victoria (VIC), South Australia (SA) and Tasmania (TAS) — operating via a nationally interconnected grid. Since December 1998, the Australian Energy Market Operator (AEMO) has been responsible for operating Australia’s electricity markets and power systems, and the main domestic network is known as the National Electricity Market (NEM). In 1998, NEM started operating as a wholesale market for the supply of electricity to retailers and end-users in Queensland, New South Wales, Victoria and South Australia, whereas Tasmania joined the NEM only in 2005.[5]

Exchange between electricity producers and retailers is facilitated through a spot and future market operated by the Australian Energy Market Operator, in which the output from all generators is aggregated and scheduled to meet the demand of end-use customers. In our analysis, we use 5-min intraday data, and the electricity dispatched price was obtained from AEMO with prices quoted in Australian dollars per MW hour (MWh).[6] In each 24-h period, there are 288 trading intervals, and we transform our 1087776 intraday observations, covering the time period from 1 January 2006 00:00 until 4 May 2016 23:55 into 3777 daily prices, by averaging intraday prices. The spot price is the unit price received by generators, by selling electricity to the pool, where the output from all generators is aggregated and scheduled to meet demand.

4 For a detailed review of the various methodologies, see Möst and Keles (2010), Zareipour (2012) and Weron (2014).

5 The NEM operates the world’s longest interconnected power system with a distance of around 5000 km. The annual turnover of electricity traded is more than $10 billion, so as to meet the demand of more than eight million end-user consummers.

6 The data can be downloaded from http://www.aemo.com.au/Electricity/National-Electricity-Market-NEM/Data-dashboard.

In a first step, we compute the (daily) realized variance (RV) using 5-min (intraday) data. For $t = 0, 1, 2, \ldots$, we denote the daily logarithmic dispatch prices by p_t. To formally establish the realized variance at date t, we further partition the daily log dispatch electricity price process $\{p_t\}$, by observing $n + 1$ equidistantly spaced (log) intraday prices $p_{t:0}, p_{t:1}, \ldots, p_{t:n}$ (where $p_{t:0} = p_{t-1:n}$), and then define the realized variance at date t as

$$\mathrm{RV}_t = \sum_{i=1}^{n} \left(p_{t:i} - p_{t:i-1}\right)^2. \tag{1}$$

The stochastic properties of the realized variance RV_t from Eq. (1), in particular its use as a consistent estimator of the so-called integrated variance from continuous-time (jump) diffusion models of the log electricity price, have been discussed extensively in the literature (see McAleer and Medeiros 2008, for an in-depth overview). However, in our subsequent empirical analysis, we consider not only RV_t, but also daily squared returns as proxies of the true (but in practice unobservable) daily volatility, and use both quantities for assessing the accuracy of our volatility forecasts in Section 5.[7] Our use of the daily squared returns is motivated by our loss functions, applied in the subsequent forecast comparisons (see Awartani and Corradi 2005).

Figure 1 displays the daily electricity price returns (defined as the difference between two successive daily log prices, $x_t = p_t - p_{t-1}$) and the daily realized variances RV_t. Figure 2 depicts the daily squared returns along with their autocorrelation functions for the five Australian states. Descriptive statistics of the time series for all five states are reported in Table 1. We observe negatively skewed electricity price returns in New South Wales, Queensland, Tasmania and Victoria, and positively skewed price returns in South Australia. All return series exhibit excess kurtosis, thus conflicting with the normal distribution. Table 1 also displays the tail index of the price returns, which we computed via the Hill estimator.[8] All tail indices range between 1.7 and 2.5, indicating that the returns series for all five states exhibit heavy tails with (i) finite means, but infinite variances for New South Wales, and Victoria (tail indices between 1 and 2), and (ii) finite means and finite variances for Queensland, South Australia and Tasmania (tail indices larger than 2). In line with these latter findings, the Jarque-Bera test (JB in Table 1) rejects the null hypothesis of normally distributed price returns for all five Australian states.

In order to analyze long memory properties in the data, we use the Hurst index, computed via the detrended fluctuation analysis (DFA) described in Weron (2002). For all five states, the Hurst index values obtained for the price returns (Hurst index^1 in Table 1) are close to 0, indicating strongly anti-persistent (or mean-reverting) dynamics. We note that this anti-persistence in Australian electricity-return data contrasts with the price-return dynamics frequently observed for other commodities (where Hurst exponents are typically close to 0.5). For the realized variances and squared returns, we obtain Hurst exponents (Hurst index^2, Hurst index^3 in Table 1) substantially larger than 0.5, indicating the presence of long memory in return volatility.

The Ljung-Box Q-tests out to lag 5 (Q(5) in Table 1) reject the null hypothesis of no autocorrelation in the electricity price returns and the Engle (1982) tests for heteroscedasticity (ARCH-tests at lag 1) indicate significant ARCH effects in the returns for all five states. The Phillips-Perron tests (PP and PP*) in Table 1 reject the null hypothesis of a unit root for all states at the 1% level.

In order to test for structural breaks in the (unconditional) variance processes of the price returns, we apply the modified iterated cumulative squares (ICSS) algorithm established by Sansó, Arragó, and Carrion (2004). As reported in Table 2, the algortihm detects (at the 5% level) two breakpoints in the return variances for New South Wales, 5 for Queensland, one for South Australia and Victoria, and two breakpoints for Tasmania. The dates at which the breakpoints occur, are highlighted in Table 2. These breakpoints justify the use of Markov-switching processes, and will become relevant in our forecasting analysis in Section 5.

Finally, we use the multifractal detrended fluctuation analysis (MDFA), as established by Kantelhardt et al. (2002), to (i) detect long-range dependencies, and (ii) to determine (multi-)fractal scaling properties in

7 The concept of approximating unobservable daily volatility by the realized variance, obtained from intraday data, was proposed by Andersen and Bollerslev (1998). See also Marcucci (2005) and Reher and Wilfling (2016), who recently use this approach in the context of volatility forecasting.

8 We refer the reader to Resnick and Stărică (1995, 1997, 1998) for the computation of Hill's estimator, when using dependent data.

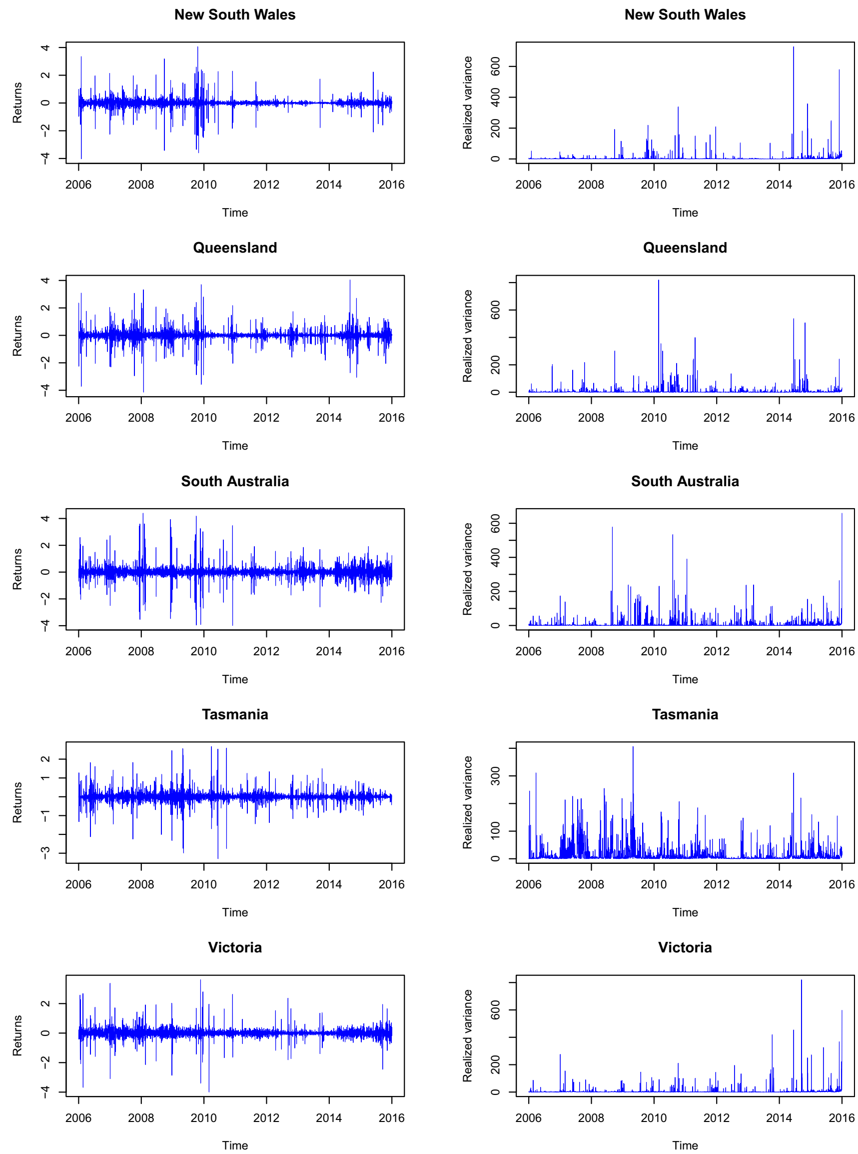

Figure 1: Daily electricity price returns (left panels) and daily realized variances (right panels).

Australian electricity price returns.[9] The importance of the MDFA is twofold. First, it constitutes a robust approach to estimating the multifractal spectrum, which describes the distribution of the Hölder exponents. Second, the MDFA allows us to effectively discriminate between (i) multifractality (stemming from a broad range of Hölder exponents), and (ii) monofractality (stemming from a narrow range of Hölder exponents). While the Hölder exponents (denoted by α) represent local scaling rates governing the time series, the Hurst exponent (denoted by H) quantifies the global scaling property of the series. Figure 3 displays the relationship between the (generalized) Hurst exponent and the q-order moments. For all five Australian states, we observe a strong dependency, where for positive (negative) values of q, the Hurst exponent $H(q)$ describes the scaling behavior of the segments with large (small) fluctuations. Figure 4 displays the multifractal spectra, $f(\alpha)$, for the 5 Australian states, each of which resembles a "large arc". This characteristic is in contrast with the "small arc"-spectra that are typically observed for monofractal time series.

9 See Kantelhardt et al. 2002 for a detailed description of the MDFA, and Thompson and Wilson (2016) for a robust and computationally efficient software implementation of the methodology. We refer to the technical definition of multifractality from Lux and Segnon (2018).

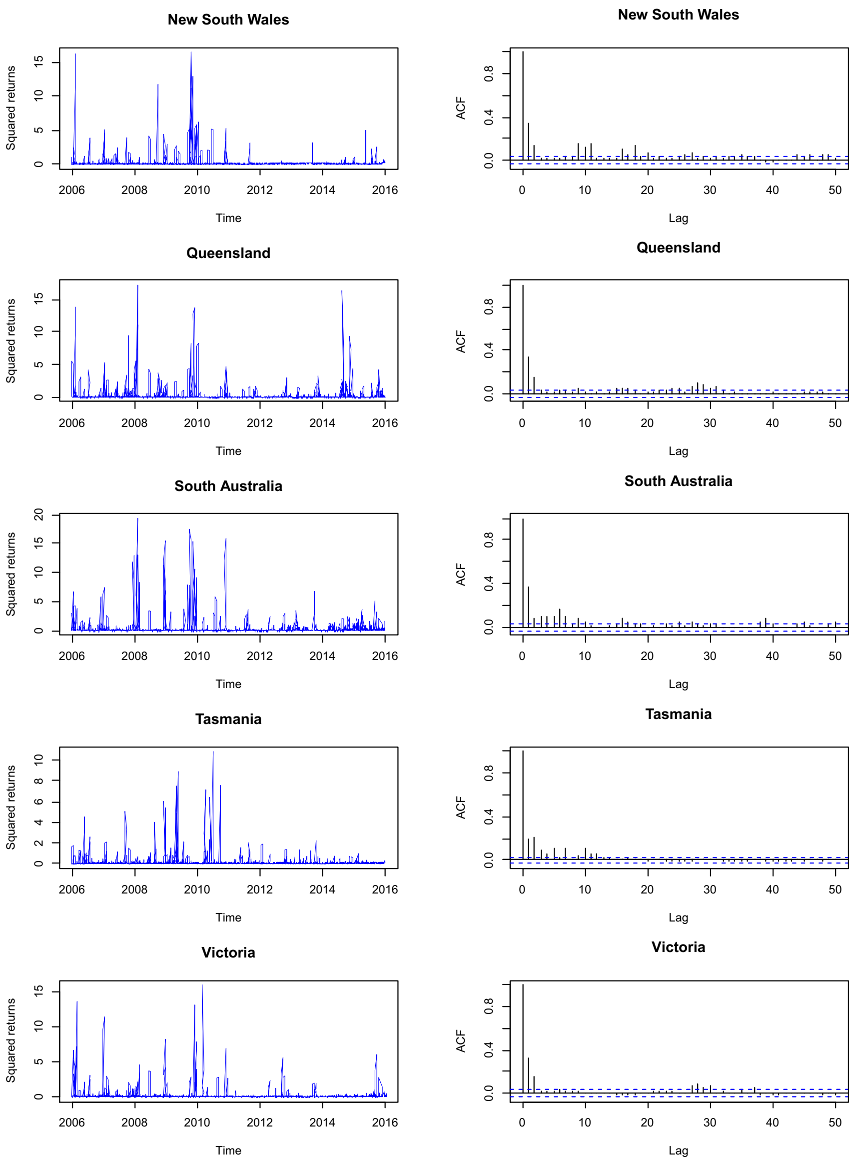

Figure 2: Daily squared price returns (left panels) and autocorrelation functions (right panels).

Overall, our preparatory analysis yields the following major findings. First, we observe that structural breaks and long memory are salient features of Australian electricity price returns. In line with Haldrup and Nielsen (2006), a plausible theoretical specification of the return process could be a regime-switching long-memory model. Second, we also find conditional heteroscedasticity and indication of multifractality. In all, this extended set of properties suggests modeling the returns as a Markov-switching GARCH-type process with long-memory and multifractal components.

4 Econometric modeling

For modeling the Australian electricity price returns $\{x_t\}$, we consider (i) two classes of autoregressive fractionally integrated moving average (ARFIMA) processes, and (ii) a broad set of volatility specifications. In the formal representation of the processes, we denote the conventional lag operator by L.

Table 1: Descriptive statistics for seasonally adjusted electricity price returns.

	NSW	QLD	SA	TAS	VIC
Nb. of observations	3776	3776	3776	3776	3776
Minimum	−4.031	−4.143	−3.984	−3.296	−3.995
Maximum	4.064	4.041	4.397	2.675	3.616
Mean	2.290E-18	2.865E-17	3.470E-18	3.680E-18	3.660E-18
Std	0.335	0.406	0.468	0.289	0.335
Skewness	−0.486	−0.205	0.343	−0.362	−0.166
Kurtosis	42.941	26.590	25.068	31.467	36.319
Tail index	1.908	2.475	2.056	2.463	1.773
JB	2.511E+5	8.758E+4	7.669E+4	1.276E+5	1.747E+5
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Hurst index[1]	0.157	0.143	0.115	0.174	0.134
Hurst index[2]	0.823	0.853	0.745	0.850	0.755
Hurst index[3]	0.862	0.834	0.848	0.772	0.781
Q(5)	329.336	343.447	382.259	324.509	341.243
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ARCH-test(1)	459.110	429.715	526.373	153.795	382.253
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
PP	−75.075	−77.579	−77.645	−76.961	−72.385
PP*	−75.085	−77.590	−77.656	−76.971	−72.385

The Hurst index[1] denotes the Hurst values for electricity price returns, Hurst index[2] for the realized variances, and Hurst index[3] for daily squared returns. *p*-values are in parentheses. PP and PP* are the Phillips-Perron adjusted *t*-statistics of the lagged dependent variable in a regression with (i) intercept and time trend, and (ii) intercept only. The respective critical values at the 1% level are −3.966 and −3.436. The five Australian states are abbreviated as NSW, New South Wales; QLD, Queensland; SA, South Australia; TAS, Tasmania; VIC, Victoria.

Table 2: Structural breaks in the (unconditional) variance processes of electricity price returns.

State	No. of break points	Date (dd/mm/yyyy)	Standard deviation
New South Wales	2	01/01/2006–**01/11/2009**	0.374
		02/11/2009–**22/02/2010**	1.080
		23/02/2010–04/05/2016	0.208
Queensland	5	01/01/2006–**15/02/2010**	0.501
		16/02/2010–**13/11/2014**	0.251
		14/11/2014–**20/03/2015**	0.834
		21/03/2015–**02/04/2015**	0.257
		03/04/2015–**28/03/2016**	0.637
		29/03/2016–04/05/2016	0.637
South Australia	1	01/01/2006–**10/02/2010**	0.574
		11/02/2010–04/05/2016	0.382
Tasmania	2	01/01/2006–**01/11/2009**	0.354
		02/11/2009–**22/02/2010**	0.150
		23/02/2010–04/05/2016	0.247
Victoria	1	01/01/2006–**22/04/2010**	0.430
		23/04/2010–04/05/2016	0.246

The bold dates represent the structural breakpoints, obtained from the modified iterated cumulated squares algorithm suggested by Sansó, Arragó, and Carrion (2004).

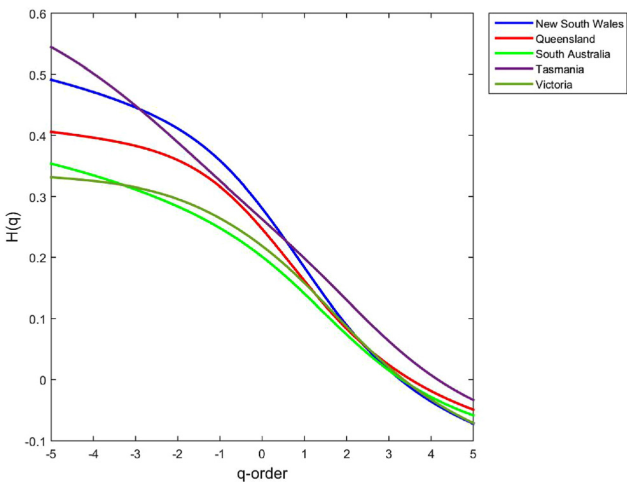

Figure 3: Hurst exponent $H(q)$ and q-order moments.

4.1 Mean-equation modeling

1. *Smooth transition ARFIMA (STARFIMA) model:*

The STARFIMA(p,d,q) process can be expressed as

$$\Phi_{z_t;\eta}(\mathrm{L})(1-\mathrm{L})^d x_t = \theta(\mathrm{L})\varepsilon_t. \tag{2}$$

In Eq. (2), $\varepsilon_t|\Omega_{t-1} \sim N(0,\sigma_t^2)$, where Ω_{t-1} is the σ-field generated by the past information set $\{x_{t-1}, x_{t-2}, \ldots, \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots\}$. While $\theta(\mathrm{L}) = 1 + \theta_1\mathrm{L} + \cdots + \theta_q\mathrm{L}^q$ denotes a regular lag polynomial of order q, the left-hand-side lag polynomial is defined as $\Phi_{z_t;\eta}(\mathrm{L}) = 1 - \phi_1(z_t;\eta_1)\mathrm{L} - \cdots - \phi_p(z_t;\eta_p)\mathrm{L}^p$, with coefficients $\phi_i(z_t,\eta_i) = \phi_{i0} + \phi_{i1}G(z_t,\tau,c)$ for $i = 1,\ldots,p$, that are nonlinear functions to be defined. z_t is the transition variable, c the location parameter, τ the transition parameter, and $\eta_i = (\phi_{i0},\phi_{i1},\tau,c)'$ a vector of parameters. For $d \in (-0.5, 0.5)$, $(1-\mathrm{L})^d$ is the fractional differencing operator given by

$$(1-\mathrm{L})^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)\mathrm{L}^k}{\Gamma(-d)\Gamma(k+1)}, \tag{3}$$

where $\Gamma(\cdot)$ is the gamma function. The transition function $G(z_t, \tau, c)$ is defined as the logistic function

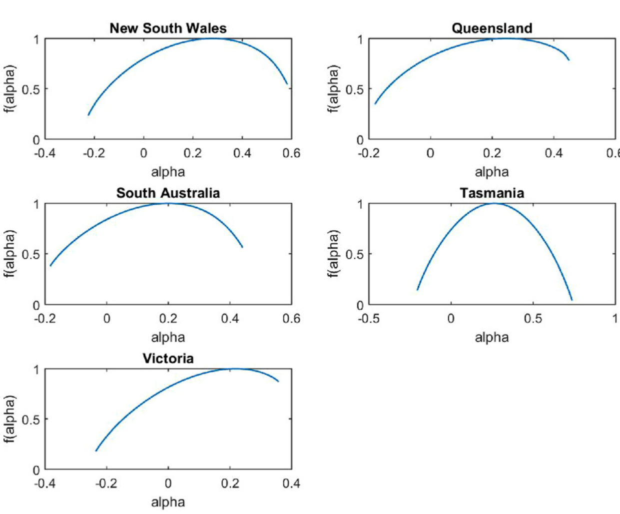

Figure 4: Multifractal spectra ($f(\alpha)$) and Hölder exponents (α).

$$G(z_t, \tau, c) = \frac{1}{1 + \exp(-\tau(z_t - c))}, \tau > 0. \tag{4}$$

Remark: *For $d = 0$, $q = 0$, the* STARFIMA*(p,d,q) model reduces to the STAR(p) model. The logistic function has the following properties: (i)* $\lim_{z_t \to -\infty} G(z_t, \tau, c) = 0$, *(ii)* $\lim_{z_t \to \infty} G(z_t, \tau, c) = 1$, *(iii)* $G(z_t, 0, c) = 1/2$, *(iv)* $\lim_{\tau \to -\infty} G(z_t, \tau, c) = 0$, *(v)* $\lim_{\tau \to \infty} G(z_t, \tau, c) = 1$. *The transition parameter τ is assumed to be positive (for identification purposes) and characterizes the speed of transition between extreme regimes.*

2. *Markov-switching ARFIMA (MSARFIMA) model:*

The MSARFIMA(p,d,q) process, proposed by Tsay and Härdle (2009), treats the concepts of 'Markov-switching' and 'long memory' in a unified framework. We state it as

$$x_t = \mu_{\Delta_t} I\{t \geq 1\} + (1 - \mathrm{L})^{-d_{\Delta_t}} \phi(\mathrm{L})^{-1} \theta(\mathrm{L}) \varepsilon_t I(t \geq 1). \tag{5}$$

In Eq. (5), the lag polynomials are $\phi(\mathrm{L}) = 1 - \sum_{i=1}^{p} \phi_i$, $\theta(\mathrm{L}) = 1 + \sum_{i=1}^{q} \theta_i$, and $I(\cdot)$ is the indicator function. Δ_t denotes the Markov-chain regime-indicator with state space $S = \{1, 2\}$, and transition probability matrix $\mathbf{P} = (p_{ij}) = (\Pr(\Delta_t = j | \Delta_{t-1} = i))$, $i, j = 1, 2$, with $\sum_{j=1}^{2} p_{ij} = 1$ for $i = 1, 2$. Finally, $\varepsilon_t \sim N(0, \sigma_t^2)$.

Remark: *It is assumed that Δ_t is independent of ε_s for all t, s. The indicator function permits us to truncate the influence of (distantly) past observations of ε_t on x_t, since d_{Δ_t} may be greater than or equal to 0.5.*

4.2 Volatility-equation modeling

It remains to specify the innovation process $\{\varepsilon_t\}$ in Eqs. (2) and (5), for which we assume the general form

$$\varepsilon_t = u_t \sigma_t, \tag{6}$$

where $u_t \overset{\text{i.i.d.}}{\sim} N(0, 1)$. We consider the following volatility models for capturing the time-varying dynamics of σ_t:

1. *Markov-switching multifractal (MSM) model:*

The MSM model assumes that the volatility dynamics are driven by the hidden Markov-chain vector $\mathbf{M}_t$, consisting of the k independent random volatility components (called multipliers) $M_t^{(1)}, M_t^{(2)}, \ldots, M_t^{(k)}$. The volatility equation is

$$\sigma_t = \overline{\sigma} \sqrt{\prod_{j=1}^{k} M_t^{(j)}}, \tag{7}$$

with $\overline{\sigma}$ denoting the scaling factor. The dynamics governing the k multipliers characterize the multifractal structure. At date t, each multiplier $M_t^{(j)}$ is drawn from the base distribution F (to be specified) which has positive support and unit expectation. Depending on its rank within the hierarchy of multipliers, $M_t^{(j)}$ changes from one period to the next with probability γ_j (and remains unchanged w.p. $1-\gamma_j$), thus providing a spectrum of low and high frequencies of multiplier renewal.

The k transition probabilities are specified as

$$\gamma_j = 2^{j-k}, \quad j = 1, \ldots, k, \tag{8}$$

and the transition matrix related to the jth multiplier is given by

$$\mathbf{P}_j = \begin{pmatrix} 1 - 0.5\gamma_j & 0.5\gamma_j \\ 0.5\gamma_j & 1 - 0.5\gamma_j \end{pmatrix}.$$

In the event of a change, each multiplier, $M_t^{(j)}$ is drawn from a (two-point) distribution with support $\{m_0, 2-m_0\}$, $1 < m_0 < 2$, and equal point probability 0.5, implying the unconditional expectation $\mathbb{E}(M_t^j) = 1$. The transition matrix of the vector $\mathbf{M}_t = (M_t^{(1)}, \ldots, M_t^{(k)})'$ becomes the $2^k \times 2^k$ matrix $\mathbf{P} = \mathbf{P}_1 \otimes \mathbf{P}_2 \otimes \cdots \otimes \mathbf{P}_k$ ($\otimes$ is the Kronecker product). Letting each multiplier follow this translated binomial base distribution F, we obtain the finite support $\Gamma \equiv \{m_0, 2-m_0\}^k$ for $\mathbf{M}_t$, which renders maximum likelihood estimation feasible.[10]. We finally note that Calvet and Fisher (2004) provide details on the computation of various relevant conditional distributions of $\mathbf{M}_t$.

2. *Short-memory GARCH-type models:*

Hentschel (1995) proposes a comprehensive family of short-memory GARCH-type models via the Box and Cox (1964) transformation of the conditional standard deviation,

$$\frac{\sigma_t^\delta}{\delta} = \omega + \alpha\sigma_{t-1}^\delta f^\nu(u_t) + \beta\frac{\sigma_{t-1}^\delta - 1}{\delta}. \tag{9}$$

In Eq. (9), $\omega > 0$, $\alpha > 0$, $\beta > 0$ are parameters, and u_t is a shock term with zero mean and unit variance. For the parameters b, c, the absolute value function $f(\cdot)$ is defined as $f(u_t) = |u_t - b| - c(u_t - b)$. Its ν-power, $f^\nu(\cdot)$, controls for the impact of the shock u_t on the Box-Cox transformed conditional standard deviation σ_t. The parameter δ determines the curvature of the Box-Cox transformation.[11]

Hentschel's (1995) volatility Eq. (9) includes, among others, the GARCH-type specifications of Bollerslev (1986) (standard GARCH), Glosten, Jagannathan, and Runkle (1993) (GJR-GARCH), Nelson (1991) (EGARCH), and Ding, Granger, and Engle (1993) (APARCH). See Ling and McAleer (2002a, b) for moment conditions in GARCH and APARCH models, Bougerol and Picard (1992) for stationarity conditions of GARCH processes, and Lux, Segnon, and Gupta (2016) for volatility forecasting formulas in GARCH(1,1)-type models. The following parameter restrictions in Eq. (9) lead to the above-mentioned submodels:

- (standard) GARCH: $\delta = 2$, $\nu = 2$, $b = 0$, $c = 0$.
- GJR-GARCH: $\delta = 2$, $\nu = 2$, $b = 0$, $c =$ free.
- EGARCH: $\delta = 0$, $\nu = 1$, $b = 0$, $c =$ free.[12]
- APARCH: $\delta =$ free, $\nu = \delta$, $b = 0$, $|c| \leq 1$.

3. *Fractionally integrated GARCH (FIGARCH) model:*

Via fractional differences in the GARCH process, Baillie, Bollerslev, and Mikkelsen (1996) establish a framework for reproducing long-term dependence in electricity price volatility. For the fractional-differentiation parameter $d \in (0, 1)$, the conditional variance in the FIGARCH(1, d, 1) model can be formalized as

$$\begin{aligned}\sigma_t^2 &= \omega + \left[1 - \beta\mathrm{L} - (1-\phi\mathrm{L})(1-\mathrm{L})^d\right]\varepsilon_t^2 + \beta\sigma_{t-1}^2 \\ &= \omega(1-\beta)^{-1} + \psi(\mathrm{L})\varepsilon_t^2,\end{aligned} \tag{10}$$

where $\psi(\mathrm{L}) = \psi_1\mathrm{L} + \psi_2\mathrm{L}^2 + \cdots = [1 - (1-\beta\mathrm{L})^{-1}(1-\phi\mathrm{L})(1-\mathrm{L})^d]$ with $\psi_i \geq 0$ for $i = 1, 2, \ldots, \infty$.[13]

To ensure the non-negativity of the ψ_i-coefficients, the following parameter conditions have to be satisfied:

10 Liu, di Matteo, and Lux (2007) find that other base distributions (e.g., adequately parameterized lognormal and gamma) produce very similar results in empirical applications.

11 δ offers the GARCH-type processes the flexibility to capture the Taylor-property (Taylor 1986). See He and Teräsvirta (1999) for a detailed discussion.

12 We note that the EGARCH process specifies the log variance, so that some positivity constraints on the parameters in Eq. (9) can be relaxed. However, EGARCH volatility forecasts may have optimality properties for log variances, but are biased with respect to (non-log) variances, due to Jensen's inequality.

13 The ψ_i-coefficients in the series expansion can be obtained via the following recursions: $\psi_1 = \phi - \beta + d$; $\psi_i = \beta\psi_{i-1} + \left(\frac{i-1-d}{i} - \phi\right)\chi_{d,i-1}$ for $i = 2, \ldots, \infty$, where $\chi_{d,i} = \chi_{d,i-1}(i-1-d)/i$ and $\chi_{d,1} = d$. In our estimation procedure, we impose a truncation at lag 1000.

$$\beta - d \leq \phi \leq \frac{(2-d)}{3} \tag{11}$$

and

$$d\left[\phi - \frac{(1-d)}{2}\right] \leq \beta(d - \beta + \phi). \tag{12}$$

Conrad and Haag (2006) establish generalized restrictions on the parameters in the FIGARCH model.

4. *Fractionally integrated APARCH (FIAPARCH) model:*

Tse (1998) merges the FIGARCH framework with the asymmetric power ARCH (APARCH) model of Ding, Granger, and Engle (1993). In the resulting FIAPARCH(1,d,1) specification, the volatility equation is given by

$$\sigma_t^\delta = \omega(1-\beta)^{-1} + \psi(\mathrm{L})(|\varepsilon_t| - \rho\varepsilon_t)^\delta, \tag{13}$$

where $\delta, \omega > 0, 0 < \beta < 1$, and with lag polynomial $\psi(\mathrm{L})$ as defined in the FIGARCH framework with $\psi_i \geq 0$ for $i = 1, 2, \ldots, \infty$, and fractional differentiation parameter $d \in (0,1)$. $\rho(|\rho| < 1)$ is an asymmetry parameter, ensuring that positive and negative shocks of identical absolute value can have asymmetric effects on conditional volatility. For $\delta = 0$, $\rho = 0$, the FIAPARCH(1,d,1) model reduces to FIGARCH(1,d,1).

5. *Smooth transition FIGARCH (STFIGARCH) model:*

In order to simultaneously account for long-memory and nonlinear volatility dynamics, Kiliç (2011) proposes the STFIGARCH(1, d, 1) process, by formalizing the conditional variance as

$$\begin{aligned}\sigma_t^2 = \omega + \left[1 - \left[\tilde{\beta} G(z_t, \tau, c)\mathrm{L} + \beta(1 - G(z_t, \tau, c))\mathrm{L}\right] - (1 - \phi\mathrm{L})(1 - \mathrm{L})^d\right]\varepsilon_t^2 \\ + \tilde{\beta} G(z_t, \tau, c)\sigma_{t-1}^2 + \beta(1 - G(z_t, \tau, c))\sigma_{t-1}^2.\end{aligned} \tag{14}$$

In Eq. (14), the transition function $G(\cdot)$ is defined as in Eq. (4), the FIGARCH elements are from Eq. (10), and β and $\tilde{\beta}$ are the volatility dynamics parameters.

6. *Smooth transition FIAPARCH (STFIAPARCH) model:*

The STFIAPARCH model is a natural generalization of Kiliç's (2011) STFIGARCH process, with the objective of accounting for the Taylor-property (Taylor 1986). The STFIAPARCH(1,d,1) volatility equation is

$$\begin{aligned}\sigma_t^\delta = \omega + \left[1 - \left[\tilde{\beta} G(z_t, \tau, c)\mathrm{L} + \beta(1 - G(z_t, \tau, c))\mathrm{L}\right] - (1 - \phi\mathrm{L})(1 - \mathrm{L})^d\right](|\varepsilon_t| - \rho\varepsilon_t)^\delta \\ + \tilde{\beta} G(z_t, \tau, c)\sigma_{t-1}^\delta + \beta(1 - G(z_t, \tau, c))\sigma_{t-1}^\delta,\end{aligned} \tag{15}$$

where the elements in Eq. (15) are adopted from the previous volatility Eqs. (4), (13) and (14). The parameter $\delta > 0$ enables the STFIAPARCH model to capture the Taylor-property.

7. *Markov-switching GARCH (MSGARCH) model*:

We consider the two-regime MSGARCH(1,1) approach suggested by Haas, Mittnik, and Paolella (2004), where the conditional variance equation in regime i is given by

$$\sigma_{t,i}^2 = \omega_i + \alpha_i \varepsilon_{t-1}^2 + \beta_i \sigma_{t-1}^2, \quad i = 1, 2, \tag{16}$$

with $\omega_i > 0$, $\alpha_i, \beta_i \geq 0$. In this framework, the regime dynamics are governed by the two-state Markov-chain regime-indicator Δ_t with irreducible, aperiodic (2×2) transition probability matrix $\mathbf{P} = (p_{ij}) = (\Pr\{\Delta_t = j | \Delta_{t-1} = i\})$ for i, $j = 1, 2$. It is assumed that ε_t and Δ_t are independent. Liu (2006) presents details on stationarity conditions and the existence of moments.

8. *Markov-switching FIGARCH (MSFIGARCH) model:*

Combining the FIGARCH process with the Markov-switching approach from Haas, Mittnik, and Paolella (2004), we obtain the MSFIGARCH(1,d,1) model with the regime-i volatility equation

$$\sigma^2_{t,i} = \omega_i + \left[1 - \beta_i \mathrm{L} - (1 - \phi_i \mathrm{L})(1 - \mathrm{L})^{d_i}\right]\varepsilon^2_t + \beta_i \sigma^2_{t-1,i}, \quad i = 1, 2. \tag{17}$$

In Eq. (17), β_i, ϕ_i are regime-specific autoregressive and moving average parameters. The (regime-specific) fractional differencing parameters satisfy $0 < d_i < 1$.

9. *Markov-switching FIAPARCH (MSFIAPARCH) model:*

The MSFIAPARCH(1,d,1) model generalizes the MSFIGARCH(1,d,1) model to capture the Taylor-property. The volatility equation in regime i is given by

$$\sigma^{\delta_i}_{t,i} = \omega_i + \left[1 - \beta_i \mathrm{L} - (1 - \phi_i \mathrm{L})(1 - \mathrm{L})^{d_i}\right](|\varepsilon_t| - \rho_i \varepsilon_t)^{\delta_i} + \beta_i \sigma^{\delta_i}_{t-1,i}, \quad i = 1, 2, \tag{18}$$

where all parameters correspond, apart from indexation, to their counterparts in the FIAPARCH and MSFIGARCH volatility specifications.

4.3 Volatility forecasting specifications

Prior to selecting our volatility forecasting models, we set up a vector autoregressive (VAR) framework with the objective of testing for potential volatility spillovers from adjacent markets and/or trading days (to assess, for example, whether volatility at time t in one state might be affected by volatility in another state at date t–1 and earlier). For this purpose, we used the state-specific daily squared returns as volatility measures in our VAR model and conducted two types of Granger causality tests: (i) multivariate tests of the null hypothesis that volatility in one specific state (e.g., NSW) does not Granger-cause ($\nrightarrow$) volatility in all remaining states (QLD, SA, TAS, VIC), via an F-type test statistic. (ii) Bivariate causality tests of the null hypothesis that volatility in one specific state (e.g., NSW) does not Granger-cause ($\nrightarrow$) volatility in a second specific state alone (e.g., QLD), again using an appropriate F-test.[14]

Table 3 displays the results of the multivariate and bivariate Granger causality tests for the daily squared returns. As a robustness check, we implemented all tests using (i) the full data set (3776 observations), (ii) the last half, and (iii) the last quarter of the data set (1888 and 944 observations). The multivariate tests (column 'Remaining states' in Table 3) reveal (i) no evidence of any volatility spillovers for the full data set, and (ii) minimal evidence that volatility in Victoria might Granger-cause volatility in the remaining states for the last half and the last quarter of the data set (at 5 and 1% levels).[15] By contrast, the bivariate tests indicate various causality relations, when applied to the full data set. In particular, volatility in South Australia (SA) and Victoria (VIC) appears to (bivariately) Granger-cause volatility in NSW, QLD, VIC and NSW, QLD, SA at the 1% level. However, many of these causality relations seem to vanish with the shortened data sets, as shown in the middle and the lower blocks of Table 3. In view of these results, the univariate volatility forecasting framework described in Sections 4.1 and 4.2 appears acceptable for our study. Nonetheless, establishing a fully-fledged multivariate volatility forecasting approach would be desirable, a challenging task that we leave for future research (see our concluding comments in Section 6).

We briefly report the complete (univariate) specifications (i.e. the explicit mean-volatility models), which we use in our forecasting analysis of Australian electricity-price volatility in Section 5. We experimented with

14 In both types of test, we selected the optimal lag lengths via the Bayesian information criterion (BIC). In the computation of the multivariate test statistics, we used a heteroscedasticity-robust covariance-matrix estimator.

15 Since we conducted a multitude of tests with the same data set, our results might be misleading due to data-snooping distortions. As a precautionary measure, we disclose the Granger-causality results ($\nrightarrow$,$\rightarrow$) in Table 3, as obtained from testing at the 1% level.

Table 3: Granger causality tests for daily squared returns.

State	**Remaining states**	**NSW**	**QLD**	**SA**	**TAS**	**VIC**
Full data set: 01/01/2006 – 03/05/2016 (3776 observations)						
NSW	↛ (0.2499)		↛ (0.1317)	→ (0.0000)	↛ (0.8063)	↛ (0.4749)
QLD	↛ (0.5778)	↛ (0.7510)		↛ (0.1104)	↛ (0.8625)	↛ (0.4749)
SA	↛ (0.1008)	→ (0.0000)	→ (0.0000)		↛ (0.1776)	→ (0.0000)
TAS	↛ (0.6732)	→ (0.0025)	↛ (0.8404)	↛ (0.5568)		↛ (0.0461)
VIC	↛ (0.5579)	→ (0.0000)	→ (0.0000)	→ (0.0000)	↛ (0.0110)	
Last half of the data set: 03/03/2011 – 03/05/2016 (1888 observations)						
NSW	↛ (0.2201)		↛ (0.4164)	↛ (0.0260)	↛ (0.2908)	↛ (0.3604)
QLD	↛ (0.5577)	↛ (0.2047)		↛ (0.5717)	↛ (0.9977)	↛ (0.7366)
SA	↛ (0.3937)	→ (0.0000)	↛ (0.7846)		↛ (0.9182)	→ (0.0041)
TAS	↛ (0.9844)	↛ (0.3136)	↛ (0.5987)	↛ (0.7611)		↛ (0.7406)
VIC	↛ (0.0147)	→ (0.0000)	↛ (0.7807)	→ (0.0000)	↛ (0.1242)	
Last quarter of the data set: 02/10/2013 – 03/05/2016 (944 observations)						
NSW	↛ (0.4179)		↛ (0.4732)	↛ (0.0259)	↛ (0.6191)	↛ (0.3045)
QLD	↛ (0.5812)	↛ (0.3077)		↛ (0.2909)	↛ (0.6526)	↛ (0.7272)
SA	↛ (0.2075)	→ (0.0000)	↛ (0.8005)		↛ (0.3785)	→ (0.0045)
TAS	↛ (0.0684)	↛ (0.5140)	↛ (0.5118)	↛ (0.6737)		↛ (0.3892)
VIC	→ (0.0000)	→ (0.0000)	↛ (0.8771)	→ (0.0000)	↛ (0.2405)	

The five Australian states are abbreviated as NSW, New South Wales; QLD, Queensland; SA, South Australia; TAS, Tasmania; VIC, Victoria. $y \rightarrow x$ ($y \nrightarrow x$) denotes 'y does (does not) Granger-cause x'. p-values are in parentheses. The displayed results ($\nrightarrow$ versus $\rightarrow$) are obtained from testing at the 1% level.

several combinations of the 2 mean and 12 volatility specifications, and ultimately decided to use the following mean-volatility models:

– Mean equation: STARFIMA [Eq. (2)]
  Volatility equations: GARCH [Eq. (9)], GJR [Eq. (9)], EGARCH [Eq. (9)], APARCH [Eq. (9)], FIGARCH [Eq. (10)], STFIGARCH [Eq. (14)], STFIAPARCH [Eq. (15)]

– Mean equation: MSARFIMA [Eq. (5)]
  Volatility equations: MSGARCH [Eq. (16)], MSFIGARCH [Eq. (17)], MSFIAPARCH [Eq. (18)]

We emphasize that any volatility specification is unequivocally assigned either to (i) the STARFIMA, or (ii) the MSARFIMA mean equation. Thus, to economize on space, we henceforth denote the complete specifications by their volatility nomenclature (e.g., GARCH, GJR, MSGARCH).

5 Volatility-forecasting performance

In this section, we evaluate the quality of volatility forecasts for the alternative models presented in Section 4. To this end, we divided each of our five regional data sets into appropriate in-sample and out-of-sample periods and then applied a daily rolling window, in order to fix the number of observations used to estimate the respective models over time. We estimated all econometric specifications with the maximum likelihood techniques suggested in the original articles.

Prior to estimating the STARFIMA and MSARFIMA specifications from Eqs. (2) and (5), we corrected all series for deterministic (weekly and annual) seasonalities via the trigonometric functions

$$s_t = a_1 \sin(2\pi t) + a_2 \cos(2\pi t) + a_3 \sin(4\pi t) + a_4 \cos(4\pi t) + a_5,$$

with parameters $a_1, \ldots, a_5$, and the annualized time factor t. For the STARFIMA specification, we selected the number of transitions and the lag lengths via the Bayesian information criterion (BIC). Specifically, we selected one transition, and the lag lengths (i) $p = 4,\ q = 0$ for New South Wales, and (ii) $p = 2,\ q = 0$ for Queensland, South Australia, Tasmania and Victoria. For the MSARFIMA and the MSGARCH-type models, we set $p = q = 1$ (see Bollerslev, Engle, and Nelson 1994). For the MSARFIMA estimation, we used the Durbin-Levinson-Viterbi algorithm, as described in Tsay and Härdle (2009).

To initialize the rolling window for each region, we separated the in-sample from the out-of-sample period at the first breakpoint detected in the five sub-data sets via the modified ICSS algorithm, as proposed by Sansó, Arragó, and Carrion (2004). Table 2 reports all breakpoints detected via this algorithm in the date format "dd/mm/yyyy". Thus, the initializing in-sample (out-of-sample) periods are given by (i) 01/01/2006 – 01/11/2009 (02/11/2009 – 04/05/2016) for New South Wales and Tasmania, (ii) 01/01/2006 – 15/02/2010 (16/02/2010 – 04/05/2016) for Queensland, (iii) 01/01/2006 – 0/02/2010 (11/02/2010 – 04/05/2016) for South Australia, and (iv) 01/01/2006 – 22/04/2010 (23/04/2010 – 04/05/2016) for Victoria.

5.1 Forecast evaluation criteria

For each state, we computed the model-specific volatility forecasts for the four alternative forecast horizons $h = 1,\ 5,\ 10,\ 20$ trading days. In order to evaluate forecasting performance, we need an adequate proxy for the latent conditional variance. Patton (2011) theoretically motivates the choice of the realized variance RV_t from Eq. (1) as a proxy, with the objective of avoiding distortions in ranking the competing forecasts. Nevertheless, besides the realized variance, we also consider the daily squared return, x_t^2, which has been used as a proxy in many studies.[16]

In a first step, we assess forecast accuracy on the basis of the two most frequently used measures, the root mean squared error (RMSE) and the mean absolute error (MAE), which are given respectively by

16 Awartani and Corradi (2005) recommend using the proxy 'squared return', when comparing the relative predictive forecasting performance of alternative models under a quadratic loss function. It is well-known that the proxy 'daily squared return' may be noisy and lead to problematic rankings of the competing forecasting models. However, as shown in Patton (2011), the concrete form of the loss function can help to consistently rank volatility forecasts in the presence of noise (see Footnote 17 below).

$$\mathrm{RMSE} = \sqrt{T^{-1}\sum_{t=1}^{T}\left(\widehat{\sigma}^2_{t,M} - \sigma^2_t\right)^2}, \tag{19}$$

$$\mathrm{MAE} = T^{-1}\sum_{t=1}^{T}\left|\widehat{\sigma}^2_{t,M} - \sigma^2_t\right|, \tag{20}$$

where $\widehat{\sigma}^2_{t,M}$ denotes the volatility forecast for date t (given the forecast horizon h), M the specific model from which the forecast is obtained (e.g., $\widehat{\sigma}^2_{t,\mathrm{GARCH}(1,1)}$), σ^2_t is either (i) the actually observed realized variance (RV_t), or (ii) the daily squared return x^2_t, and T denotes the number of out-of-sample observations.

In a second step, we make statistical inferences about the relative forecasting performance of our alternative volatility models by reporting the results of the Equal Predictive Ability (EPA) test suggested by Diebold and Mariano (1995), and the Superior Predictive Ability (SPA) test from Hansen (2005). The EPA test enables us to compare the forecasting accuracy of two competing models (say M_1 and M_2) by considering the loss differential

$$d_t = g(e_{t,M_1}) - g(e_{t,M_2}), \tag{21}$$

where $e_{t,M_1} = \widehat{\sigma}^2_{t,M_1} - \sigma^2_t$ and $e_{t,M_2} = \widehat{\sigma}^2_{t,M_2} - \sigma^2_t$ are the model-specific forecast errors at date t and the loss function $g(\cdot)$ either denotes the squared error loss $g(e_{t,M_i}) = e^2_{t,M_i}$ or the absolute error loss $g(e_{t,M_i}) = |e_{t,M_i}|$.[17] Then, the null hypothesis of the EPA test states that there is no difference in the forecast accuracy between two competing models:

$$H_0 : \mathbb{E}(d_t) = 0 \quad \text{for all} \quad t. \tag{22}$$

For large sample sizes, an appropriate test statistic of the EPA test is given by

$$\mathrm{EPA} = \frac{\overline{d}}{\sqrt{1/T\sum_{k=-N}^{N}\widehat{\gamma}(k)}}, \tag{23}$$

where $\overline{d} = 1/T\sum_{t=1}^{T} d_t$, N is the nearest integer larger than $T^{1/3}$ and

$$\widehat{\gamma}(k) = \frac{1}{T}\sum_{t=|k|+1}^{T}\left(d_t - \overline{d}\right)\left(d_{t-|k|} - \overline{d}\right).$$

Following Diebold and Mariano (1995), the test statistic EPA is approximately standard normally distributed under the null hypothesis in large samples.[18]

In contrast to the EPA test, the SPA test suggested by Hansen (2005) enables us to compare a benchmark forecast model M_0 with K competitive forecast models $M_1, \ldots, M_K$ under a given loss function. In line with Eq. (21), we define the loss differential between the benchmark model M_0 and the alternative model $M_k \in \{M_1, \ldots, M_K\}$ as

$$d_{t,M_k} = g(e_{t,M_0}) - g(e_{t,M_k}). \tag{24}$$

Based on these K loss differentials, we can state the null hypothesis that the benchmark model M_0 is not inferior to any of the other K competing models as

$$H_0 : \max\{\mathbb{E}(d_{t,M_1}), \ldots, \mathbb{E}(d_{t,M_K})\} \leq 0 \quad \text{for all} \quad t. \tag{25}$$

In order to express the test statistic of the SPA test, we define the sample mean of the kth loss differential as $\overline{d}_{M_k} = 1/T\sum_{t=1}^{T} d_{t,M_k}$ and consider the estimated variance $\widehat{\mathrm{Var}}(\sqrt{T}\cdot\overline{d}_{M_k})$ for $k = 1, \ldots, K$. We note that this latter variance is estimated by using a bootstrap method (Hansen 2005). One way to test the null hypothesis from Eq. (25) is now to consider the test statistic

17 According to Proposition 4 in Patton (2011), the (mean) squared error belongs to the family of robust homogeneous loss functions.

18 We refer the reader to Diebold (2015), who provides a detailed overview and an in-depth discussion of the EPA framework.

$$\mathrm{SPA} = \max\left\{\frac{\sqrt{T}\bar{d}_{M_1}}{\widehat{\mathrm{Var}}\left(\sqrt{T}\cdot\bar{d}_{M_1}\right)},\ldots,\frac{\sqrt{T}\bar{d}_{M_K}}{\widehat{\mathrm{Var}}\left(\sqrt{T}\cdot\bar{d}_{M_K}\right)}\right\}, \tag{26}$$

the p-values of which can be obtained via a stationary bootstrap procedure.

5.2 Out-of-sample forecasting results

We report our forecasting results separately under the respective volatility proxies 'realized variance' and 'daily squared return'.

5.2.1 The volatility proxy 'realized variance'

Table 4 reports the root mean squared (RMSE) and mean absolute forecast errors (MAE) for the 12 volatility specifications across the five Australian states, where the forecast horizons were chosen as 1, 5, 10, and 20 trading days. *Prima facie*, the forecast errors appear to be rather similar for all volatility models in all five Australian states. This rough forecast-error analysis may suggest that all volatility models exhibit quasi indistinguishable forecasting performance across all Australian states and over all forecast horizons.

In order to inferentially compare the volatility forecasting performance among the distinct specifications, we applied the EPA and SPA tests, as described in Section 5.1. Tables 5 and 6 report the results, for the computation of which we used 5000 bootstrap samples to establish p-values of the SPA tests. For the EPA tests in Table 5, we chose the MSM model as the benchmark specification (Model 2), against which we compare all other specifications. Only in very few cases does the MSM model significantly outperform any of the other specifications at the 5% level. In particular, this is the case for (i) the EGARCH model in New South Wales for the horizons $h = 5$ (squared error loss) and $h = 1$ (absolute error loss), and (ii) the STFIGARCH model in South Australia for the horizons $h = 5$, 10 (squared error loss) and $h = 1$, 5, 10 (absolute error loss) and in Tasmania for all horizons under both error losses. In total, this amounts to a significant EPA volatility-forecasting outperformance of the MSM model over any other specification in only 3.41 % (15 out of 440) of the cases analyzed.

We emphasize that the EPA analysis from Table 5 only provides pairwise performance comparisons of the MSM model with each of the other specifications. It does not provide an overall evaluation of the MSM volatility forecasting performance, when compared with all other volatility specifications simultaneously. Statistical evidence on this latter issue is revealed in the results of the SPA tests in Table 6, where the null hypothesis states that the benchmark model is not inferior to any of the other 11 competing models. When considering the MSM specification as the benchmark model, the SPA tests in Table 6 always reject the null hypothesis across all five states, and for all forecast horizons under both error losses (that is, in 40 out of 40 tests) at the 5% level. This yields the robust result that for each of the 40 volatility forecast settings (that is, across five states, four forecast horizons, two error loss functions) there is at least one competing specification that significantly outperforms the MSM model. Viewed from this angle, it is interesting to note that the MSFIAPARCH model performs best with 20 out of 40 (20/40) rejections of the null hypothesis at the 5% level, followed by MSFIGARCH (23/40), STFIAPARCH (24/40), APARCH (30/40), FIAPARCH (30/40), MSGARCH (30/40), GARCH (32/40), FIGARCH (32/40), GJR (35/40), EGARCH (38/40), STFIGARCH(39/40), and MSM (40/40).

Overall, we are not able to identify a unique model that systematically outperforms all alternative specifications across the five Australian states at every forecast horizon, when using the volatility proxy 'realized variance'. At least, it appears that some MSGARCH- and STGARCH-type specifications produce better volatility forecasts than other specifications, and may therefore serve as tools in power derivative pricing. However, our overall finding is compatible with the results of previous forecasting studies, using data from deregulated electricity markets. In a comparative study, comprising 47 published works on forecasting electricity prices, Aggarwal, Saini, and Kumar (2009) conclude that

Table 4: Root mean squared errors (RMSE) and mean absolute errors (MAE).

Model	Forecast horizon (trading days)											
	1	5	10	20	1	5	10	20	1	5	10	20
	Realized variance as proxy								Daily squared return as proxy			
	RMSE				MAE				RMSE			
	New South Wales											
MSM	27.479	27.497	27.515	27.438	4.671	4.695	4.729	4.770	0.658	0.673	0.673	0.624
GARCH	27.456	27.487	27.497	27.416	4.661	4.646	4.656	4.676	0.605	0.704	0.683	0.694
GJR	27.406	27.495	27.510	27.431	4.653	4.665	4.699	4.735	0.631	0.698	0.689	0.640
EGARCH	28.388	27.502	27.518	27.439	4.967	4.699	4.730	4.769	0.737	0.705	0.680	0.631
APARCH	27.485	27.499	27.516	27.435	4.672	4.693	4.718	4.735	0.696	0.689	0.682	0.632
FIGARCH	27.437	27.489	27.498	27.425	4.659	4.661	4.680	4.702	0.602	0.698	0.669	0.629
FIAPARCH	27.454	27.486	27.503	27.423	4.611	4.633	4.668	4.709	0.629	0.685	0.686	0.645
MSGARCH	27.447	27.480	27.482	27.400	4.660	4.653	4.655	4.634	0.618	0.753	0.712	0.747
MSFIGARCH	27.440	27.490	27.500	27.419	4.665	4.662	4.678	4.694	0.623	0.704	0.672	0.644
MSFIAPARCH	27.439	27.468	27.482	27.404	4.572	4.584	4.616	4.655	0.637	0.695	0.697	0.652
STFIGARCH	27.437	27.490	27.503	27.426	4.661	4.661	4.680	4.701	0.612	0.693	0.677	0.627
STFIAPARCH	27.387	27.479	27.496	27.418	4.617	4.612	4.637	4.672	0.682	0.685	0.673	0.640
	Queensland											
MSM	35.781	35.788	35.788	35.803	7.789	7.793	7.797	7.847	0.530	0.555	0.559	0.559
GARCH	35.766	35.765	35.741	35.712	7.765	7.730	7.698	7.703	0.517	0.622	0.661	0.776
GJR	35.766	35.768	35.746	35.720	7.765	7.739	7.712	7.723	0.522	0.633	0.667	0.775
EGARCH	35.769	35.780	35.776	35.790	7.771	7.760	7.753	7.801	0.610	0.566	0.570	0.574
APARCH	35.776	35.771	35.758	35.765	7.775	7.732	7.710	7.755	0.545	0.579	0.598	0.637
FIGARCH	35.767	35.775	35.766	35.770	7.770	7.754	7.746	7.770	0.503	0.574	0.577	0.582
FIAPARCH	35.762	35.780	35.778	35.796	7.738	7.755	7.765	7.819	0.495	0.565	0.566	0.568
MSGARCH	35.764	35.763	35.733	35.699	7.763	7.720	7.685	7.691	0.503	0.637	0.677	0.797
MSFIGARCH	35.774	35.780	35.775	35.782	7.781	7.768	7.765	7.800	0.500	0.570	0.573	0.578
MSFIAPARCH	35.763	35.780	35.779	35.797	7.740	7.757	7.768	7.821	0.493	0.564	0.565	0.568
STFIGARCH	35.767	35.776	35.767	35.772	7.769	7.756	7.749	7.776	0.517	0.589	0.592	0.595
STFIAPARCH	35.756	35.778	35.772	35.791	7.749	7.757	7.763	7.812	0.519	0.580	0.572	0.570
	South Australia											
MSM	24.974	24.983	24.986	28.882	6.221	6.221	6.226	6.690	0.564	0.594	0.593	0.593
GARCH	24.959	24.971	24.972	28.855	6.212	6.203	6.199	6.638	0.544	0.623	0.628	0.646
GJR	24.945	24.972	24.979	28.868	6.207	6.217	6.219	6.669	0.538	0.655	0.647	0.645
EGARCH	24.962	24.984	24.989	28.888	6.214	6.230	6.241	6.710	0.525	0.596	0.593	0.593
APARCH	24.965	27.473	25.849	29.497	6.223	6.772	6.953	6.903	0.692	0.793	0.801	0.889
FIGARCH	24.956	24.969	24.970	28.858	6.211	6.195	6.185	6.625	0.525	0.619	0.620	0.636
FIAPARCH	24.959	24.983	24.987	28.887	6.211	6.229	6.237	6.709	0.514	0.603	0.602	0.602
MSGARCH	24.960	24.971	24.973	28.856	6.207	6.200	6.196	6.634	0.547	0.618	0.621	0.631
MSFIGARCH	24.973	24.987	24.990	28.886	6.237	6.236	6.242	6.708	0.548	0.596	0.598	0.605
MSFIAPARCH	24.952	24.989	24.992	28.879	6.208	6.246	6.255	6.707	0.651	0.825	0.844	0.845
STFIGARCH	24.947	24.964	24.965	28.854	6.210	6.190	6.183	6.622	0.482	0.629	0.621	0.651
STFIAPARCH	24.929	24.964	24.969	28.866	6.195	6.181	6.185	6.646	0.450	0.623	0.618	0.619
	Tasmania											
MSM	21.332	21.356	21.372	21.422	8.672	8.698	8.709	8.761	0.387	0.401	0.401	0.402
GARCH	21.320	21.345	21.357	21.399	8.653	8.662	8.657	8.693	0.378	0.413	0.416	0.428
GJR	21.318	21.342	21.352	21.396	8.650	8.654	8.646	8.684	0.375	0.416	0.420	0.434
EGARCH	21.324	21.350	21.363	21.409	8.658	8.680	8.684	8.724	0.368	0.403	0.402	0.404
APARCH	21.329	21.347	21.356	21.392	8.663	8.673	8.664	8.683	0.391	0.404	0.406	0.415
FIGARCH	21.320	21.343	21.353	21.388	8.654	8.665	8.659	8.688	0.378	0.415	0.417	0.432
FIAPARCH	21.298	21.325	21.340	21.391	8.597	8.623	8.637	8.692	0.387	0.415	0.416	0.417

Table 4: (continued)

						Tasmania						
MSGARCH	21.317	21.342	21.352	21.394	8.650	8.659	8.652	8.685	0.380	0.416	0.415	0.421
MSFIGARCH	21.339	21.364	21.381	21.431	8.684	8.712	8.725	8.779	0.391	0.404	0.404	0.405
MSFIAPARCH	21.298	21.324	21.342	21.391	8.597	8.625	8.638	8.693	0.392	0.418	0.420	0.422
STFIGARCH	21.320	21.340	21.348	21.375	8.656	8.660	8.646	8.659	0.377	0.416	0.420	0.448
STFIAPARCH	21.302	21.344	21.360	21.406	8.648	8.668	8.679	8.732	0.354	0.405	0.404	0.405
						Victoria						
MSM	30.125	30.130	30.136	33.602	5.308	5.319	5.342	5.926	0.311	0.323	0.322	0.321
GARCH	30.113	30.118	30.117	33.570	5.292	5.267	5.248	5.774	0.294	0.368	0.386	0.459
GJR	30.108	30.118	30.129	33.591	5.290	5.293	5.300	5.861	0.340	0.426	0.421	0.435
EGARCH	31.077	30.130	30.136	33.603	5.530	5.314	5.332	5.917	0.351	0.328	0.322	0.322
APARCH	30.120	30.128	96.621	35.973	5.304	5.299	8.027	8.890	0.336	0.410	0.430	0.441
FIGARCH	30.113	30.118	30.120	33.573	5.293	5.275	5.268	5.801	0.294	0.378	0.392	0.443
FIAPARCH	30.114	30.126	30.132	33.600	5.267	5.284	5.307	5.892	0.285	0.332	0.332	0.334
MSGARCH	30.112	30.115	30.117	33.560	5.295	5.270	5.251	5.761	0.304	0.381	0.397	0.459
MSFIGARCH	30.119	30.127	30.132	33.591	5.308	5.313	5.331	5.909	0.294	0.329	0.324	0.329
MSFIAPARCH	30.115	30.126	30.133	33.600	5.267	5.284	5.307	5.892	0.285	0.331	0.331	0.332
STFIGARCH	30.112	30.119	30.123	33.572	5.294	5.278	5.268	5.799	0.287	0.402	0.425	0.500
STFIAPARCH	30.101	30.113	30.117	33.583	5.245	5.239	5.253	5.831	0.291	0.359	0.365	0.376

The volatility models are abbreviated as MSM, Markov-switching multifractal; GARCH, generalized autoregressive conditional heteroscedasticity; GJR, Glosten-Jagannathan-Runkle GARCH; EGARCH, exponential GARCH; APARCH, asymmetric power ARCH; FIGARCH, fractionally integrated GARCH; FIAPARCH, fractionally integrated APARCH; MSGARCH, Markov-switching GARCH; MSFIGARCH, Markov-switching FIGARCH, MSFIAPARCH, Markov-switching FIAPARCH; STFIGARCH, smooth transition FIGARCH; STFIAPARCH, smooth transition FIAPARCH.

> … there is no systematic evidence of out-performance of one model over other models on a consistent basis.

In similar vein, Weron (2014) reports mixed forecasting performance of the same model in different studies.

5.2.2 The volatility proxy 'daily squared return'

In order to check whether the choice of the volatility proxy affects the forecasting results, we now address the proxy 'daily squared return'. Following the recommendation of Awartani and Corradi (2005), we only consider the squared loss (see Footnote 16). Tables 4–6 reveal that the forecasting results differ substantially from those under the volatility proxy 'realized variance'. From Table 4, we find that the MSM model now has the unambiguously lowest RMSEs (among all 12 specifications) for the horizons $h = 5, 10, 20$ across all of the five Australian states. This picture is broadly confirmed by the EPA and SPA tests in Tables 5 and 6. For example, the SPA tests in Table 6—with the MSM specification as the benchmark model—reject the null hypothesis (that MSM is not inferior to any of the other 11 competing models) at the 5% level only in 1 out of 20 (1/20) cases. According to this ranking, the MSM model now performs best, followed by EGARCH (4/20), MSGARCH (8/20), APARCH (9/20), MSFIGARCH (10/20), STFIGARCH (10/20), FIGARCH (11/20), STFIGARCH (12/20), FIAPARCH (13/20), GJR (14/20), GARCH (16/20), and MSFIAPARCH (16/20). Obviously, the choice of the volatility proxy strongly impacts on this type of ranking among the forecast models.

6 Conclusion

In this paper, we analyze electricity price dynamics by means of a unique intraday data set covering 5-min electricity prices for five Australian states. In a multifractal detrended fluctuation analysis, we find that

Table 5: Equal predicitve ability (EPA) tests (p-values).

Model 1	Model 2	Forecast horizon (trading days)											
		Realized variance as proxy								Daily squared return as proxy			
		Squared error loss				Absolute error loss				Squared error loss			
		1	5	10	20	1	5	10	20	1	5	10	20
New South Wales													
GARCH	MSM	0.991	0.998	1.000	1.000	0.992	1.000	1.000	1.000	0.984	0.001	0.284	0.008
GJR		0.938	0.796	0.999	0.999	0.978	1.000	1.000	1.000	0.641	0.010	0.043	0.017
EGARCH		0.058	0.033	0.092	0.151	0.024	0.231	0.458	0.544	0.045	0.056	0.111	0.085
APARCH		0.092	0.148	0.283	0.965	0.279	0.677	1.000	1.000	0.000	0.012	0.067	0.056
FIGARCH		0.954	0.997	0.987	0.999	0.989	1.000	1.000	1.000	0.946	0.042	0.626	0.292
FIAPARCH		1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.000	0.000	0.023
MSGARCH		0.952	0.994	0.997	1.000	0.978	1.000	1.000	1.000	0.917	0.047	0.325	0.364
MSFIGARCH		0.922	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.425	0.079	0.510	0.020
MSFIAPARCH		0.971	0.998	0.998	1.000	0.984	1.000	1.000	1.000	0.908	0.000	0.107	0.002
STFIGARCH		0.929	0.975	0.999	0.999	0.899	1.000	1.000	1.000	0.863	0.009	0.515	0.105
STFIAPARCH		1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.982	0.000	0.000	0.000
Queensland													
GARCH	MSM	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.716	0.010	0.000	0.000
GJR		0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.606	0.014	0.001	0.000
EGARCH		0.988	0.993	0.991	1.000	0.958	1.000	1.000	1.000	0.086	0.030	0.000	0.000
APARCH		0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.052	0.000	0.000	0.000
FIGARCH		0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.877	0.068	0.018	0.008
FIAPARCH		1.000	1.000	0.994	0.998	1.000	1.000	1.000	1.000	0.931	0.041	0.009	0.013
MSGARCH		0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.761	0.100	0.061	0.027
MSFIGARCH		1.000	1.000	0.993	1.000	1.000	1.000	1.000	1.000	0.619	0.054	0.051	0.036
MSFIAPARCH		0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.812	0.009	0.000	0.000
STFIGARCH		0.986	0.997	0.998	0.999	0.982	1.000	1.000	1.000	0.888	0.002	0.000	0.009
STFIAPARCH		1.000	1.000	0.991	0.995	1.000	1.000	1.000	1.000	0.931	0.042	0.014	0.019
South Australia													
GARCH	MSM	0.999	0.994	1.000	1.000	0.999	1.000	1.000	1.000	0.765	0.031	0.028	0.008
GJR		0.998	0.955	0.996	0.995	0.994	0.743	0.736	0.999	0.654	0.096	0.100	0.081
EGARCH		0.981	0.307	0.037	0.003	0.960	0.007	0.001	0.000	0.831	0.234	0.371	0.400
APARCH		0.896	0.157	0.155	0.156	0.374	0.083	0.107	0.130	0.093	0.153	0.155	0.156

Table 5: (continued)

						South Australia							
FIGARCH		0.999	0.998	1.000	1.000	0.999	1.000	1.000	1.000	0.845	0.024	0.011	0.000
FIAPARCH		1.000	0.505	0.371	0.012	1.000	0.028	0.014	0.001	0.941	0.011	0.005	0.024
MSGARCH		0.963	0.984	1.000	1.000	0.972	1.000	1.000	1.000	0.827	0.057	0.034	0.000
MSFIGARCH		0.989	0.999	1.000	0.998	0.999	1.000	1.000	1.000	0.886	0.004	0.007	0.019
MSFIAPARCH		1.000	0.998	1.000	1.000	1.000	1.000	1.000	1.000	0.756	0.034	0.027	0.004
STFIGARCH		0.706	0.009	0.047	0.126	0.000	0.000	0.009	0.075	0.897	0.139	0.019	0.003
STFIAPARCH		1.000	0.151	0.198	0.662	1.000	0.066	0.064	0.009	0.240	0.149	0.150	0.153
						Tasmania							
GARCH	MSM	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.803	0.015	0.013	0.000
GJR		1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.842	0.005	0.002	0.000
EGARCH		0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.882	0.086	0.286	0.026
APARCH		0.992	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.172	0.081	0.007	0.000
FIGARCH		1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.786	0.039	0.060	0.002
FIAPARCH		1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.489	0.000	0.000	0.002
MSGARCH		1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.795	0.033	0.037	0.000
MSFIGARCH		0.986	1.000	1.000	0.998	1.000	1.000	1.000	1.000	0.877	0.024	0.084	0.119
MSFIAPARCH		1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.725	0.024	0.039	0.001
STFIGARCH		0.000	0.000	0.000	0.012	0.000	0.000	0.000	0.001	0.138	0.121	0.089	0.047
STFIAPARCH		1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.175	0.000	0.000	0.002
						Victoria							
GARCH	MSM	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000	0.809	0.002	0.000	0.000
GJR		0.997	0.918	1.000	1.000	1.000	1.000	1.000	1.000	0.256	0.017	0.010	0.001
EGARCH		0.081	0.574	0.612	0.301	0.059	0.985	1.000	0.990	0.087	0.072	0.500	0.214
APARCH		0.886	0.974	0.135	0.125	0.987	1.000	0.097	0.094	0.000	0.110	0.135	0.125
FIGARCH		0.999	0.989	1.000	1.000	1.000	1.000	1.000	1.000	0.771	0.003	0.000	0.000
FIAPARCH		1.000	1.000	1.000	0.932	1.000	1.000	1.000	1.000	0.937	0.000	0.000	0.000
MSGARCH		0.995	0.997	1.000	1.000	1.000	1.000	1.000	1.000	0.836	0.003	0.000	0.000
MSFIGARCH		1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.790	0.000	0.000	0.000
MSFIAPARCH		0.999	0.992	1.000	1.000	1.000	1.000	1.000	1.000	0.632	0.003	0.000	0.000
STFIGARCH		0.995	0.959	0.953	0.974	0.574	0.999	1.000	0.989	0.837	0.010	0.069	0.001
STFIAPARCH		1.000	1.000	1.000	0.936	1.000	1.000	1.000	1.000	0.938	0.001	0.000	0.000

p-values obtained for the null hypothesis that, for a given forecast horizon, there is no difference in forecast accuracy between Model 1 and Model 2, as opposed to the one-sided alternative that the forecasts from Model 1 are inferior to those from Model 2. The volatility models are abbreviated as MSM, Markov-switching multifractal; GARCH, generalized autoregressive conditional heteroscedasticity; GJR, Glosten-Jagannathan-Runkle GARCH; EGARCH, exponential GARCH; APARCH, asymmetric power ARCH; FIGARCH, fractionally integrated GARCH; FIAPARCH, fractionally integrated APARCH; MSGARCH, Markov-switching GARCH; MSFIGARCH, Markov-switching FIGARCH; MSFIAPARCH, Markov-switching FIAPARCH; STFIGARCH, smooth transition FIGARCH; STFIAPARCH (smooth transition FIAPARCH).

Table 6: Superior predicitve ability (SPA) tests (p-values).

Benchmark model	Forecast horizon (trading days)											
	Realized variance as proxy								Daily squared return as proxy			
	Squared error loss				Absolute error loss				Squared error loss			
	1	5	10	20	1	5	10	20	1	5	10	20
New South Wales												
MSM	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.015	1.000	0.669	0.629
GARCH	0.035	0.016	0.093	0.015	0.000	0.000	0.000	0.000	0.799	0.002	0.486	0.000
GJR	0.510	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.692	0.027	0.157	0.050
EGARCH	0.061	0.000	0.000	0.000	0.010	0.000	0.000	0.000	0.061	0.103	0.313	0.073
APARCH	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.011	0.203	0.046
FIGARCH	0.276	0.004	0.005	0.000	0.000	0.000	0.000	0.000	0.993	0.104	0.743	0.617
FIAPARCH	0.001	0.001	0.003	0.001	0.000	0.000	0.000	0.000	0.223	0.000	0.003	0.050
MSGARCH	0.258	0.004	0.005	0.000	0.000	0.000	0.000	0.000	0.304	0.122	0.797	0.871
MSFIGARCH	0.857	0.065	0.097	0.007	0.000	0.000	0.000	0.000	0.252	0.174	0.949	0.001
MSFIAPARCH	0.237	0.085	0.532	0.695	0.000	0.000	0.000	1.000	0.389	0.003	0.111	0.000
STFIGARCH	0.152	0.003	0.018	0.000	0.000	0.000	0.000	0.000	0.194	0.022	0.862	0.109
STFIAPARCH	0.285	1.000	0.468	0.305	1.000	1.000	1.000	0.024	0.064	0.000	0.000	0.000
Queensland												
MSM	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.146	0.918	1.000	1.000
GARCH	0.134	0.020	0.024	0.000	0.000	0.004	0.000	0.000	0.025	0.009	0.001	0.000
GJR	0.111	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.116	0.026	0.006	0.000
EGARCH	0.039	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.082	0.044	0.000	0.000
APARCH	0.002	0.001	0.001	0.000	0.000	0.036	0.000	0.000	0.084	0.000	0.000	0.000
FIGARCH	0.028	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.385	0.067	0.042	0.001
FIAPARCH	0.181	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.316	0.096	0.048	0.006
MSGARCH	0.007	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.147	0.123	0.082	0.041
MSFIGARCH	0.886	0.000	0.000	0.000	0.025	0.000	0.000	0.000	0.355	0.096	0.152	0.065
MSFIAPARCH	0.083	1.000	1.000	1.000	0.000	1.000	1.000	1.000	0.491	0.013	0.000	0.000
STFIGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.515	0.001	0.000	0.000
STFIAPARCH	0.226	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.997	0.084	0.033	0.009
South Australia												
MSM	0.020	0.007	0.000	0.000	0.000	0.000	0.000	0.000	0.126	0.946	0.924	0.929
GARCH	0.078	0.006	0.020	0.641	0.001	0.000	0.000	0.000	0.012	0.018	0.011	0.002
GJR	0.110	0.003	0.000	0.002	0.141	0.000	0.000	0.000	0.112	0.073	0.076	0.059
EGARCH	0.016	0.000	0.000	0.000	0.049	0.000	0.000	0.000	0.263	0.478	0.562	0.575
APARCH	0.022	0.116	0.106	0.115	0.000	0.050	0.060	0.081	0.103	0.106	0.106	0.115
FIGARCH	0.085	0.017	0.051	0.090	0.107	0.000	0.248	0.093	0.267	0.015	0.004	0.000
FIAPARCH	0.128	0.002	0.000	0.000	0.111	0.000	0.000	0.000	0.621	0.011	0.005	0.005
MSGARCH	0.006	0.745	0.894	0.672	0.002	0.000	0.889	1.000	0.060	0.078	0.049	0.000
MSFIGARCH	0.914	0.856	0.106	0.002	1.000	1.000	0.404	0.000	0.911	0.011	0.000	0.000
MSFIAPARCH	0.097	0.012	0.012	0.449	0.189	0.000	0.000	0.003	0.022	0.018	0.007	0.001
STFIGARCH	0.015	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.187	0.130	0.015	0.001
STFIAPARCH	0.148	0.013	0.002	0.001	0.163	0.000	0.000	0.000	0.148	0.262	0.241	0.102
Tasmania												
MSM	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.309	1.000	0.734	1.000
GARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.208	0.004	0.001	0.000
GJR	0.000	0.000	0.000	0.000	0.000	0.000	0.025	0.000	0.310	0.002	0.000	0.000
EGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.442	0.082	0.303	0.005
APARCH	0.000	0.000	0.000	0.006	0.000	0.000	0.000	0.000	0.167	0.069	0.001	0.000
FIGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.190	0.028	0.039	0.001
FIAPARCH	0.551	0.246	1.000	0.012	0.554	0.816	0.632	0.000	0.057	0.000	0.000	0.000

Table 6: (continued)

						Tasmania						
MSGARCH	0.000	0.000	0.003	1.000	0.000	0.000	0.020	1.000	0.223	0.020	0.018	0.000
MSFIGARCH	0.414	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.915	0.030	0.044	0.071
MSFIAPARCH	0.000	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.111	0.009	0.013	0.000
STFIGARCH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.198	0.134	0.087	0.039
STFIAPARCH	0.872	0.754	0.077	0.013	0.446	0.184	0.368	0.000	0.034	0.000	0.000	0.000
						Victoria						
MSM	0.005	0.000	0.000	0.003	0.000	0.000	0.000	0.000	0.106	1.000	0.755	0.979
GARCH	0.020	0.156	0.789	0.143	0.000	0.000	0.902	0.000	0.548	0.003	0.000	0.000
GJR	0.040	0.383	0.000	0.015	0.000	0.000	0.000	0.000	0.111	0.031	0.021	0.004
EGARCH	0.095	0.000	0.000	0.005	0.043	0.000	0.000	0.000	0.095	0.122	0.791	0.152
APARCH	0.025	0.000	0.114	0.122	0.000	0.000	0.098	0.112	0.002	0.133	0.114	0.122
FIGARCH	0.020	0.030	0.011	0.089	0.000	0.000	0.000	0.000	0.621	0.009	0.002	0.000
FIAPARCH	0.004	0.000	0.000	0.004	0.000	0.000	0.000	0.000	0.955	0.000	0.000	0.000
MSGARCH	0.004	0.138	0.008	0.083	0.000	0.000	0.000	0.000	0.913	0.012	0.003	0.000
MSFIGARCH	1.000	0.677	0.685	0.024	1.000	1.000	0.172	0.000	0.689	0.000	0.000	0.000
MSFIAPARCH	0.017	0.684	0.926	0.878	0.000	0.000	0.187	0.888	0.041	0.007	0.001	0.000
STFIGARCH	0.004	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.525	0.020	0.039	0.000
STFIAPARCH	0.006	0.000	0.000	0.004	0.000	0.000	0.000	0.000	0.943	0.000	0.000	0.000

p-values obtained for the null hypothesis that the benchmark model is not inferior to any of the other competing models. The volatility models are abbreviated as MSM, Markov-switching multifractal; GARCH, generalized autoregressive conditional heteroscedasticity; GJR, Glosten-Jagannathan-Runkle GARCH; EGARCH, exponential GARCH; APARCH, asymmetric power ARCH; FIGARCH, ractionally integrated GARCH; FIAPARCH, fractionally integrated APARCH; MSGARCH, Markov-switching GARCH; MSFIGARCH, Markov-switching FIGARCH; MSFIAPARCH, Markov-switching FIAPARCH; STFIGARCH, smooth transition FIGARCH; STFIAPARCH, smooth transition FIAPARCH.

electricity price returns exhibit multifractal behavior, encouraging us to model the returns as a smooth-transition autoregressive fractionally integrated moving process with a Markov-switching multifractal volatility component (STARFIMA-MSM model). In an out-of-sample volatility-forecasting analysis, we compare the MSM model with several alternative GARCH- and MSGARCH-type specifications. Our major findings are twofold. On the one hand, the (multifractal) MSM process can compete (to some degree) with the alternative volatility specifications, when using the latent volatility proxy 'realized variance'. On the other hand, the MSM model outperforms all competing models under the volatility proxy 'daily squared return' in terms of superior predictive ability. The choice of the volatility proxy substantially affects the ranking of the forecast models, a result which conforms to the theoretical considerations in Patton (2011).

A desirable and useful line of future research could consist of extending our univariate volatility-forecasting framework to a genuine multivariate volatility model, in order to capture volatility co-movements and spillovers among the five Australian states. In Section 4.3 we attempted to approach these issues via Granger-causality testing and found empirical evidence of spillover effects, encouraging the derivation of an econometrically feasible multivariate multifractal framework.

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