Predicting Stock Market Movements with a Time-Varying Consumption-Aggregate Wealth Ratio[#]

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ABSTRACT

We develop a time-varying measure of *cay* (cay^{TVP}) using time-varying cointegration, and then compare the predictive ability of cay^{TVP} with *cay* and a Markov-switching *cay* (cay^{MS}) for excess stock returns and volatility in the US over the period 1952:Q2-2015:Q3, using a *k*-th order nonparametric causality-in-quantiles test. We find that time-varying cointegration exists between consumption, asset wealth, and labor income. In addition, while there is no evidence of predictability of volatility of excess returns from *cay*, cay^{MS}, or cay^{TVP}, they tend to act as strong predictors of stock returns, with cay^{TVP} being important during the bearish phases of the equity market.

Keywords: Consumption-aggregate wealth ratio; Time-varying cointegration; Stock returns; Volatility; Nonparametric causality-in-quantiles test.

JEL Codes: C22, G10.

1. Introduction

Lettau and Ludvigson (2001), using a wide class of optimal models of consumer behaviour showed in an influenctial paper that the consumption-aggregate wealth (human capital plus asset holdings) ratio (*cay*) summarizes expected returns on aggregate wealth or the market portfolio and, hence, is a strong predictor of US quarterly stock returns. Ever since, a large number of studies have confirmed this finding for other developed and emerging countries (see for example, Sousa, 2010, 2015; Afonso and Sousa, 2011; Rapach and Zhou, 2013; Rocha Armada *et al.* 2015; Caporale and Sousa, 2016). Moreover, Ludvigson and Ng (2007), show that *cay* can also predict volatility of US excess returns. More recently, Bianchi *et al.* (2016) provide evidence of infrequent shifts, or breaks, in the mean of *cay*. As a result, the authors introduce a Markov-switching version of the consumption-wealth ratio, i.e., cay^{MS}, and show that, relative to the classic *cay*, the regime-switching version has superior predictive power for quarterly excess stock market returns.

At this stage it must be pointed out that, as is standard practice in the literature on stock returns predictability, the above-mentioned studies rely on linear predictive regression frameworks when predicting stock returns and volatility based on the consumption-aggregate wealth ratios. But, as has recently been shown by Bekiros and Gupta (2015) and Balcilar *et al.* (2017a), the relationship between stock-market movements and *cay* or cay^{MS} is in fact nonlinear, and hence, the linear models used in the literature are misspecified and results derived from them cannot be relied upon. Interestingly, Bekiros and Gupta (2015) investigate the predictability of US stock returns and its volatility emanating from *cay* and cay^{MS} using the *k*-th order conditional mean-based nonparametric causality test of Nishiyama *et al.* (2011), and find no evidence of predictability. However, more recently, Balcilar *et al.* (2017a) using a quantiles-based extension of the above test as developed by Balcilar *et al.* (2016a), show that while stock-market volatility is still unpredictable, *cay* and cay^{MS} can predict the equity premium. However, cay^{MS} performs better than *cay* only at certain quantiles (in general the moderately-lower ones below the median), and not over the entire conditional distribution.

We first contribute to the literature on the instability of the consumption function, as indicated by Bianchi *et al.* (2016), by estimating a time-varying cointegration equation capturing the relationship between consumption, wealth, and income. In this regard, we use the (single-equation) smooth time-varying cointegration (STVC) model of Park and Hahn (1999), as an extension of the single-equation cointegration approaches that Lettau and

Ludvigson (2001) and Bianchi *et al.* (2016) use to obtain estimates of the consumption-aggregate wealth ratio. The STVC framework renders it possible to test whether cointegration exists and, if so, whether the long-run relationship is in fact time varying. Using this framework, we are able to recover the evolution of the wealth and income elasticities, through time-varying parameters. Hence, we can capture any instability in the consumption function (if it exists) appropriately. Once we have estimated the STVC model, we recover the residuals of the model and derive a new measure of the consumption-aggregate wealth ratio, which we call cay^{TVP}. Then, as part of our second objective, as is standard practice in the literature, we use cay^{TVP} to predict the US equity premium over the quarterly period from 1952:Q2 to 2015:Q3, and compare its performance with the classical *cay* and its Markov-switching version, cay^{MS}. Given the evidence of nonlinearity in the relationship between consumption-aggregate wealth ratios and excess stock returns as shown by Bekiros and Gupta (2015), and following Balcilar *et al.* (2017a), we conduct the predictability analysis based on the *k*-th order nonparametric causality-in-quantiles test that has been recently developed by Balcilar *et al.* (2016a).

The test of Balcilar *et al.* (2016a) studies higher order causality over the entire conditional distribution of the dependent variable and is inherently based on a nonlinear dependence structure between the variables, as captured by data-driven nonparametric functions. It essentially combines the causality-in-quantiles test of Jeong *et al.* (2012) and the higher-moment nonparametric causality test of Nishiyama *et al.* (2011).[1] To the best of our knowledge, this is the first paper that develops a measure of consumption-aggregate wealth ratio from a time-varying cointegration model, and then uses this measure to evaluate its predictive power of the equity premium and its volatility, relative to *cay* and cay^{MS}, based on a nonparametric causality-in-quantiles framework.

The rest of this research is organized as follows: Section 2 describes the econometric frameworks of time-varying cointegration and higher-moment nonparametric causality-in-quantiles test. Section 3 presents the data and discusses the empirical results. Finally, Section 4 concludes.

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[1] As indicated by Balcilar *et al.* (2016a), the causality-in-quantile approach has the following novelties: Firstly, it is robust to misspecification errors as it detects the underlying dependence structure between the examined time series. Secondly, via this methodology, it makes it possible to test for not only causality-in-mean (1st moment), but also causality that may exist in the tails of the distribution of the variables. Finally, it renders it possible also to investigate causality-in-variance and, thus, study higher-order dependency.

2. Econometric Methodologies

2.1. Time Varying Cointegration Test

According to Park and Hahn (1999), the STVC model can be specified as follows:

$$c_{qj} = \beta_0 + \alpha_q + \beta'_{qj}X_{qj} + \varepsilon_{qj} = \beta_0 + \alpha_q + \beta_{1(qj)}a_{qj} + \beta_{2(qj)}y_{qj} + \varepsilon_{qj}, \tag{1}$$

where, subscripts q and j indicate the quarter $q(=1, 2, 3)$ of year $j(=1, 2, 3,...,T)$; c_{qj} is the natural logarithm of real consumption; $X_{qj} = (a_{qj}, y_{qj})'$ represents a vector of explanatory variables, i.e., natural logarithms of real asset wealth (a) and real labor income (y); α_q denotes the fixed quarterly effect to capture seasonality present in the consumption function; β_0 denotes the constant term, and ε_{qj} denotes the error term. $\beta_{qj} = (\beta_{1(qj)}, \beta_{2(qj)})'$ is a vector of time-varying parameters corresponding to $X_{qj} = (a_{qj}, y_{qj})'$. Specifically, $\beta_{qj} = \beta(t/n)$ is a smooth function defined on the inteval [0,1], where n is the number of observations, and t is the order of observation in the total sample given by $t=4(j-1)+q$. Using the flexible Fourier function to approximate the time-varying parameter, β_{qj}, Eq. (1) can be rewritten as:

$$c_{qj} = \beta_0 + \alpha_q + \gamma'_k X_{kqj} + \varepsilon_{kqj}, \tag{2}$$

where, $\varepsilon_{kmj} = \varepsilon_{mj} + [\beta(t/n) - \beta_k(t/n)]X_{kqj}$, $X_{kqj} = f_k(t/n) \otimes X_{qj}$, and $f_k = (1, \lambda, \varphi'_1(\lambda),..., \varphi'_k(\lambda))'$, with $\lambda \in [0,1]$, $\varphi_i(\lambda) = (\cos 2\pi i\lambda, \sin 2\pi i\lambda)'$, $i=1,2,....k$, and $\gamma_k = (\gamma'_{k,1}, \gamma'_{k,2},..., \gamma'_{k,2(k+1)})'$; $\otimes$ denotes the Kronecker product.

In order to obtain an efficient estimator that can generate a valid inference for the time-varying parameters, the canonical cointegrating regression (CCR, hereafter) proposed by Park (1992) is used to estimate Eq. (2). In this way, Park and Hahn (1999) show that the CCR estimator of β_k is a consistent estimator of β.

There are two model-specification tests proposed by Park and Hahn (1999) for the STVC model. The first test examines the null hypothesis of STVC against the alternative hypothesis of a spurious regression with non-stationary errors. The second test investigates the null hypothesis of the fixed-coefficient cointegration model against the alternative hypothesis of the STVC model. The first test statistic is given by:

$$\tau^* = \frac{RSS_{TVC} - RSS^s_{TVC}}{\omega^2_*}, \tag{3}$$

where, RSS_{TVC} and RSS_{TVC}^s represent the sum of squared residuals from the CCR estimation for Eq. (2), and the same equation augmented with s additional superfluous regressors (such as time polynomial terms, t, t^2, t^3,…,t^s), respectively, and ω_*^2 is the conditional long-run variance of the residuals from the transformed regression. The limit distribution of τ^* is a chi-square distribution with s degrees of freedom.

The second tests statistic is given by:

$$\tau_1^* = \frac{RSS_{FC} - RSS_{FC}^s}{\omega_*^2}, \tag{4}$$

where, RSS_{FC} and RSS_{FC}^s represent the sum of squared residuals from CCR estimation for Eq. (2) obtained by restricting the parameters to be constant over time, and the same equation augmented with s additional superfluous regressors, respectively. The limit distribution of τ_1^* is also a chi-square distribution with s degrees of freedom.

2.2. Nonparametric Causality-in-Quantiles Test

Next, we briefly present the methodology for the detection of nonlinear causality via a hybrid approach developed by Balcilar *et al.* (2016a), which in turn is based on the frameworks of Nishiyama *et al.* (2011) and Jeong *et al.* (2012). Our description of the technical details of the test is relatively compact and heavily draws on Balcilar *et al.* (2016b, d; 2017b, 2017c), who have introduced this test into the empirical finance literature.

We start by denoting excess returns by y_t and the predictor variable (in our case, *cay*, *cay^{MS}*, or *cay^{TVP}*) as x_t. We further let $Y_{t-1} \equiv (y_{t-1},...,y_{t-p})$, $X_{t-1} \equiv (x_{t-1},...,x_{t-p})$, $Z_t = (X_t, Y_t)$ and $F_{y_t|Z_{t-1}}(y_t, Z_{t-1})$ and $F_{y_t|Y_{t-1}}(y_t, Y_{t-1})$ denote the conditional distribution functions of y_t given Z_{t-1} and Y_{t-1}, respectively. If we let denote $Q_\theta(Z_{t-1}) \equiv Q_\theta(y_t \mid Z_{t-1})$ and $Q_\theta(Y_{t-1}) \equiv Q_\theta(y_t \mid Y_{t-1})$, we have $F_{y_t|Z_{t-1}}\{Q_\theta(Z_{t-1}) \mid Z_{t-1}\} = \theta$ with probability one. As a result, the (non)causality in the q-th quantile hypotheses to be tested are:

$$H_0: \quad P\{F_{y_t|Z_{t-1}}\{Q_q(Y_{t-1}) \mid Z_{t-1}\} = q\} = 1, \tag{5}$$

$$H_1: \quad P\{F_{y_t|Z_{t-1}}\{Q_q(Y_{t-1}) \mid Z_{t-1}\} = q\} < 1. \tag{6}$$

Jeong *et al.* (2012) use the distance measure $J = \{\varepsilon_t E(\varepsilon_t \mid Z_{t-1}) f_z(Z_{t-1})\}$, where ε_t is the regression error term and $f_z(Z_{t-1})$ is the marginal density function of Z_{t-1}. The regression

error ε_t emerges based on the null hypothesis in (1), which can only be true if and only if $E[1\{y_t \le Q_\theta(Y_{t-1}) \mid Z_{t-1}\}] = \theta$ or, expressed in a different way, $1\{y_t \le Q_\theta(Y_{t-1})\} = \theta + \varepsilon_t$, where $\mathbf{1}\{\times\}$ is the indicator function. Jeong *et al.* (2012) show that the feasible kernel-based sample analogue of J has the following format:

$$\hat{J}_T = \frac{1}{T(T-1)h^{2p}} \sum_{t=p+1}^{T} \sum_{s=p+1, s \ne t}^{T} K\left(\frac{Z_{t-1} - Z_{s-1}}{h}\right) \hat{e}_t \hat{e}_s, \qquad (7)$$

where $K(\cdot)$ is the kernel function with bandwidth h, T is the sample size, p is the lag order, and $\hat{e}_t$ is the estimate of the unknown regression error, which is given by

$$\hat{e}_t = 1\{y_t \pounds Q_q(Y_{t-1})\} - q. \qquad (8)$$

$\hat{Q}_\theta(Y_{t-1})$ is an estimate of the θth conditional quantile of y_t given Y_{t-1}, and we estimate $\hat{Q}_\theta(Y_{t-1})$ using the nonparametric kernel method as

$$\hat{Q}_\theta(Y_{t-1}) = \hat{F}_{y_t \mid Y_{t-1}}^{-1}(\theta \mid Y_{t-1}), \qquad (9)$$

where $\hat{F}_{y_t \mid Y_{t-1}}(y_t \mid Y_{t-1})$ is the *Nadarya-Watson* kernel estimator given by

$$\hat{F}_{y_t \mid Y_{t-1}}(y_t \mid Y_{t-1}) = \frac{\sum_{s=p+1, s^1 t}^{T} L\left((Y_{t-1} - Y_{s-1})/h\right) 1(y_s \pounds y_t)}{\sum_{s=p+1, s^1 t}^{T} L\left((Y_{t-1} - Y_{s-1})/h\right)}, \qquad (10)$$

with $L(\cdot)$ denoting the kernel function and h the bandwidth.

As an extension of Jeong *et al.* (2012)'s framework, Balcilar *et al.* (2016a) develop a test for the *second* moment. In particular, we can test the causality between *cay*, *cay^MS*, or *cay^TVP* and (excess) stock returns volatility. Adapting the approach in Nishiyama *et al.* (2011), higher-order quantile causality can be specified in terms of the following hypotheses as

$$H_0: \quad P\{F_{y_t^k \mid Z_{t-1}}\{Q_q(Y_{t-1}) \mid Z_{t-1}\} = q\} = 1 \qquad \text{for } k = 1, 2, \ldots, K, \qquad (11)$$

$$H_1: \quad P\{F_{y_t^k \mid Z_{t-1}}\{Q_q(Y_{t-1}) \mid Z_{t-1}\} = q\} < 1 \qquad \text{for } k = 1, 2, \ldots, K. \qquad (12)$$

We can integrate the entire framework and test whether x_t *Granger causes* y_t in quantile θ *up to the kth moment* using Eq. (11) to construct the test statistic in Eq. (10) for each k. The causality-in-variance test can be calculated by replacing y_t in Eqs. (7) and (8) with y_t^2 - measuring the volatility of excess stock returns. However, one can show that it is difficult to combine the different statistics for each $k = 1, 2, \ldots, K$ into a single statistic for the joint null in Eq. (11), because the statistics are mutually correlated (Nishiyama *et al.*, 2011). Balcilar *et al.*

(2016a), thus, propose a sequential-testing method as described in Nishiyama *et al.* (2011). First, as in Balcilar *et al.* (2016a), we test for nonparametric Granger causality in the *first* moment (*i.e.* $k = 1$). Nevertheless, failure to reject the null for $k = 1$ does not automatically lead to no-causality in the *second* moment. Thus, we construct the test for $k = 2$, as discussed in detail in Balcilar *et al.* (2016a).

The empirical implementation of causality testing via quantiles entails specifying three key parameters: the bandwidth h, the lag order p, and the kernel type for $K(\cdot)$ and $L(\cdot)$. We use a lag order (of one) based on the Schwarz information criterion (SIC), which is known to choose a parsimonious model as compared with other lag-length selection criteria. The SIC criterion helps to overcome the issue of over-parameterization that typically arises in studies using nonparametric frameworks. For each quantile, we determine the bandwidth parameter (h) by using the leave-one-out least-squares cross validation method. Finally, for $K(\cdot)$ and $L(\cdot)$, we use Gaussian kernels.

3. Data and Empirical Results

Our quarterly dataset comprises excess stock returns, *cay*, *cay^{MS}*, and *cay^{TVP}*, with data on *cay* and *cay^{MS}* obtained from Professor Martin Lettau's website: http://faculty.haas.berkeley.edu/lettau/data_cay.html. In order to compute *cay^{TVP}*, data on real values of personal consumption expenditures (*c*), asset holdings, i.e., net hosehold worth (*a*), and labor income (*y*) are also obtained individually from the above data weblink of Professor Lettau. As our plan is to compare the predictive ability of the three alternative formulations of the consumption-wealth ratio, we standardize them by dividing the actual series by the corresponding standard deviations. Our data spans over the quarterly period of 1952:Q2-2015:Q3, with the start and end dates being driven by data availability (of the variables under consideration) at the time of writing of this paper.

Excess stock-market returns are computed as the excess returns of a market index (*exsr*) over the risk-free asset return, which is common practice in the relevant literature. Specifically, we calculate the continuously compounded log returns of the Center for Research in Security Prices (CRSP) index (including dividends) minus the 3-month Treasury bill rate. We also compute the volatility of excess stock-market returns (*exsv*) using the squared values of *exsr*.[2]

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[2] *exsr* and *exsv* are negatively and positively skewed respectively, with both having excess kurtosis. Both these variables have non-normal distributions, as statistically suggested by the rejection of the null hypothesis of normality under the Jarque-Bera test at the highest level of significance. This evidence of heavy tails for both

Data on the value-adjusted CSRP index and the risk free rate are obtained from Professor Amit Goyal's website: http://www.hec.unil.ch/agoyal/. Note that, as pointed out by Lettau and Ludvigson (2001), the CRSP Index (which includes the NYSE, AMEX, and NASDAQ) is believed to provide a better proxy for non-human components of total asset wealth because it is a much broader measure than the S&P 500 index.

We start off with our analysis of time-varying cointegration.[3] In Table 1, we observe that not only is the fixed-coefficient cointegration model rejected in favour of the STVC model (at the 1% level of significance), but we are also unable to reject the null hypothesis of time-varying cointegration (at the conventional 5% level of significance) against the alternative of no cointegration. In other words, the two tests of Park and Hahn (1999) not only suggest that cointegration exists, but it does so in a time-varying fashion, thus justifying the use of the STVC model to detect the long-run relationship between consumption, asset wealth, and labor income.

Table 1. Time Varying Cointegration Tests (1952Q1-2015Q3)

| Model Specification Test	H_0: Fixed Coefficient Cointegration Model H_A: STVC Model	H_0: STVC Model H_A: No Cointegration
	τ_1^*	τ^*
Test Statistic	402.9754***	4.7763
1% Critical Value	13.2767	13.2767
5% Critical Value	9.4877	9.48777

Note: *** represents significance at 1% level.

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exsr and _exsv,_ in turn, provides a preliminary motivation to use the causality-in-quantiles test. Complete details of these results are available upon request from the authors.

[3] As is standard practice, as a pre-test requirement, we also checked for whether _c_, _a_ and _y_ are I(1) processes using the linear Augmented Dickey-Fuller (ADF, Dickey and Fuller, 1981) and Phillips-Perron (PP, Phillips and Perron, 1988) tests, and the nonlinear unit root test of Kapetanios et al., (2003, KSS). The results reported in Table A1 confirm that the variables are indeed unit-root processes, and hence satisfy the data requirement for a test of cointegration.

Figure 1. Time-Varying Wealth and Income Elasticities

1(a): Wealth Elasticity:
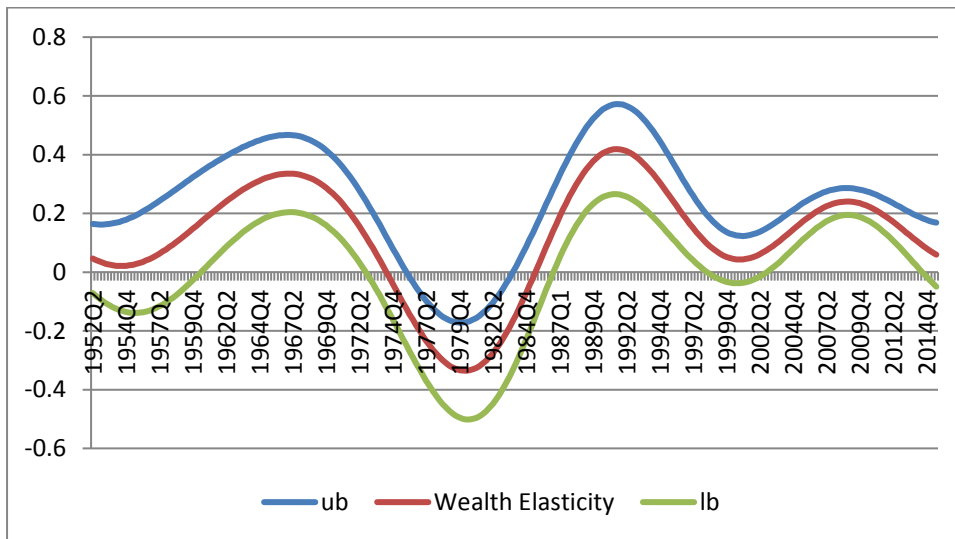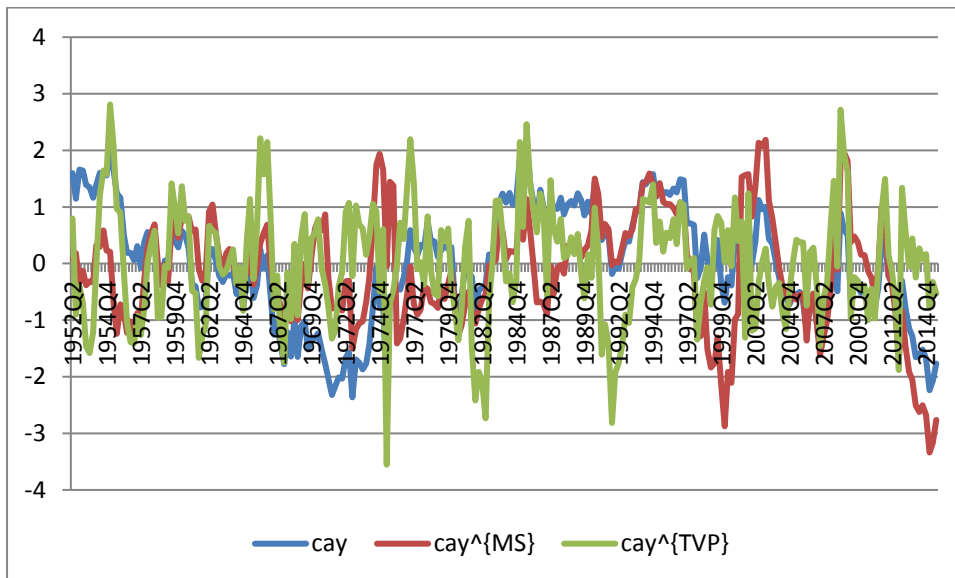

1(b): Income Elasticity:
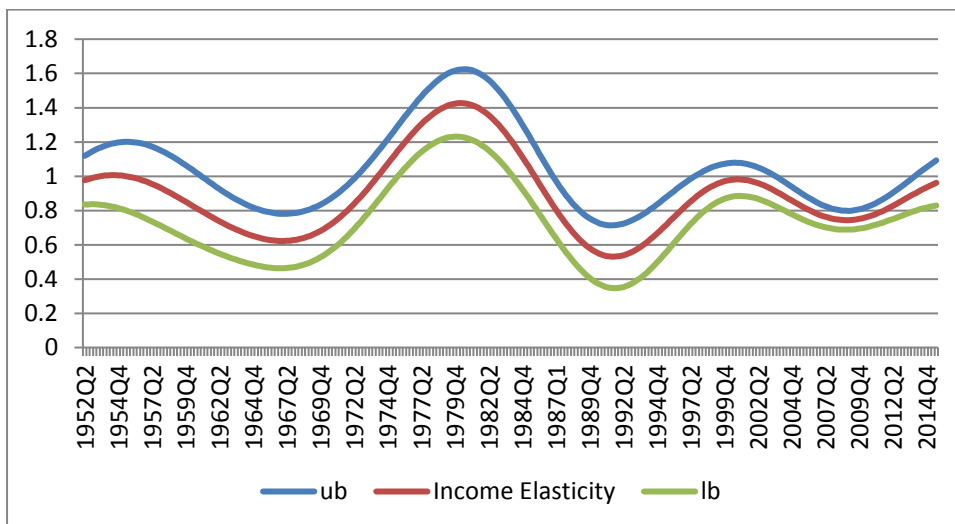
Note: ub and lb denotes the 95% upper and lower confidence bands respectively.

Given the existence of time-varying cointegration, we present in Figure 1 the paths of the time-varying elasticities of consumption with respect to wealth and income, with the elasticities being more or less mirror image of each other. The wealth and income effects evolve in a smooth fashion over time, with the parameter corresponding to labor income being consistently significant at the 5% level over the entire sample period. In general, barring some periods at the beginning and end of the sample, the wealth effect is also significant. While the income effect is always positive, the wealth effect is negative over the period from 1974:Q3 to 1985:Q2, with the negative effect being significant at the 5% level

starting in 1976:Q1. Note that, given the wealth-effect channel, the general underlying logic is that a rise in asset prices, especially if it is considered to be permanent, increases expected income of households and, hence, consumption, while also making it easier for firms to finance investment opportunities (Simo-Kengne et al., 2015). But, as pointed out by Ludwig and Sløk (2004), it is also possible that stakeholders draw on their wealth as positive asset returns increase their financial investment and, hence, reduce their consumption. In other words, the wealth effect can either increase or decrease consumption, with the final effect depending on the two forces discussed above.

Having estimated the STVC model, we present in Figure 2 the residuals from the time-varying cointegration equation, i.e., cay^{TVP}, along with cay and cay^{MS}.[4] As can be seen from the figure, the three standardized measures of the consumption-wealth ratio have a similar pattern of movement, though the cay^{TVP} in general tends to fluctuate more than the other two measures.[5]

Figure 2. Plots of standardized cay, cay^{MS} and cay^{TVP}

Note: cay^{MS} and cay^{TVP} stands for cay^{MS} and cay^{TVP} respectively.

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[4] The estimated wealth and income elasticities, as tabulated in Table 1 of Bianchi et al., (2017), used to derive cay and cay^{MS} respectively are: 0.1246 (0.0952, 0.1540) and 0.7815 (0.7486, 0.8144), and 0.2679 (0.2505, 0.2852) and 0.6071 (0.5873, 0.6270), with the 95 percent confidence bands in parentheses.

[5] While the mean of cay is zero, that of cay^{MS} and cay^{TVP} are negative and positive respectively, with all the measures showing strong evidence of autocorrelation and heteroskedasticity. Complete details of these results are available upon request from the authors.

Next, we turn to the causality-in-quantiles test that we use to evaluate the performance of the three measures of the consumption-wealth ratios with regard to predicting excess return and volatility.[6] Figures 3 and 4 display the results from the causality-in-quantiles test for excess stock returns (*exsr*) and volatility (*exsv*). We find that *cay*, cay^{MS}, and cay^{TVP} predict *exsr*, in general, over its entire conditional distribution, with the exception of the quantile 0.10 of *exsr*, which cannot be predicted by cay^{MS}, and 0.90, which cannot be explained by both *cay* and cay^{TVP}. It is important to note (and in contrast to the results reported by Bianchi *et al.* 2016 based on a misspecified linear predictive regression model, Bekiros and Gupta, 2015), it is not necessarily true that cay^{MS} is always the stronger predictor (in terms of significance) of *exsr* when compared to *cay*, which in turn, corroborates the findings of Balcilar *et al.* (2017a). If we look at the results closely, we observe that cay^{TVP} is the strongest predictor of *exsr* over the quantile range of 0.10 to 0.30, while *cay* does best at quantiles 0.50-0.60, 0.70-0.75 and 0.85, and cay^{MS} having the strongest causal influence (in terms of significance) over the remaining quantiles, i.e., 0.35-0.45, 0.65, 0.80 and 0.90.[7] However, we find that *cay*,

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[6] We also conducted standard linear Granger causality tests involving the three alternative measures of the consumption-aggregate wealth ratios. As in Bianchi *et al.* (2016) and Balcilar *et al.* (2017a), the linear model confirmed that cay^{MS} is a stronger predictor ($\chi^2(1)$=9.0214, *p*-value=0.0027) than *cay* ($\chi^2(1)$=8.4507, *p*-value=0.0036) for *exsr*, however, there was no evidence of predictability with regard to cay^{TVP}($\chi^2(1)$=0.5428, *p*-value=0.4613). But, as in Balcilar *et al.* (2017a), the Brock *et al.* (1996, BDS) test of nonlinearity and the Bai and Perron (2003) tests of multiple structural breaks indicated that the linear models with three measures of consumption-aggregate wealth ratios (considered in turn) are misspecified, and hence, results from the standard Granger tests for noncausality cannot be relied upon. In other words, we need to base our inference on a nonparametric approach, which is robust to such misspecifications, and in the process provides us with a strong motivation to use the *k*-th order causality-in-quantiles test. When we conducted the BDS test on the errors for the equity premium equation and the volatility equation, we could not reject the null hypothesis of *iid* residuals for the various quantiles (0.10, 0.20, …, 0.90) considered, suggesting that there is no remaining nonlinearity between the equity premium and its volatility with the three predictors considered in turn. In other words, the nonparametric framework of ours is not misspecified. Complete details of tests for linear Granger noncausality, nonlinearity, and multiple structural breaks are available upon request from the authors.

[7] As an anonymous referee pointed out, our analysis is silent on whether the gains in predictability across the three alternative measures of the consumption-wealth ratio over the various quantiles of the conditional distribution are statistically significant. This is indeed correct. However, we follow the extant literature of in-sample predictability (see Rapach and Zhou, 2013), as well as Bianchi et al., (2017), in drawing the conclusions based on the strength of the test statistic, after the predictors have been standardized. In order to assess statistical significance, we would need in-sample predicted values of *exsr*, and then we could apply standard forecast comparison tests (like Diebold and Mariano, 1995), which in turn, requires parameter estimates of the nonparametric causality-in-quantiles framework. However, in a nonparametric framework, this is not straightforward. To do so, we would need to employ the first-order partial derivative. Estimation of the partial derivatives for nonparametric models can experience complications because nonparametric methods exhibit slow convergence rates, which can depend on the dimensionality and smoothness of the underlying conditional-expectation function. One could look at a statistic that summarizes the overall effect or the global curvature (i.e., the global sign and magnitude), but not the entire derivative curve. In this regard, a natural measure of the global curvature is the average derivative (AD). One could use the conditional pivotal quantile, based on approximation or the coupling approach of Belloni et al., (2017), to estimate the partial ADs. The pivotal coupling approach additionally can approximate the distribution of AD using Monte Carlo simulation. We leave this for future research and follow, at this stage, the extant literature on in-sample predictability comparisons across alternate predictors based on the strength of the causality statistic. Although one should not expect to have a one-to-one correspondence between our nonparametric causality-in-quantiles model and a standard

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cay^{MS}, and cay^{TVP} fail to predict $exsv$ over its entire conditional distribution - a result in line with the work of Balcilar *et al.* (2017a), but unlike that of Ludvigson and Ng (2007), with the result from the latter study being unreliable due to the possibility of misspecification of the linear model (Bekiros and Gupta, 2015).

In sum, the predictive ability of the three measures of the consumption-wealth ratio for $exsr$ is contingent on the state of the stock market. In general, cay^{TVP} is the most important of the three measures during a bear market, and cay performs strongly around the median to the moderately higher quantiles, i.e., when the market is performing in normal to good phases. The third measure, cay^{MS}, in turn, tends to do well during a bull market, and for moderately poor phases. Hence, an investor who wants to predict stock returns to carry out portfolio allocation needs to rely on all three measures of the consumption-wealth ratio, as each of them individually caries valuable information about future stock returns at various phases of the stock market. In other words, an investor cannot rely all the time on a single measure of the consumption-wealth ratio to make his or her portfolio-allocation decision, but rather needs to condition his or her choice of the appropriate consumption-wealth ratio on and estimate of the state of the equity market.[8,9] Note that, individually though each of the measures tend to have their strongest predictability around the median (below or above) of the conditional distribution, which corresponds to when the market functions in its normal mode, resulting in the hump-shaped nature of the test statistic. This could be an indication of agents in the equity market herding when the market is either in its bear and bull phases (Balcilar et al., 2016d), while when the market is performing normally, investors look to

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quantile-regression approach, applying the Diebold and Mariano (1995) test to the in-sample predictions recovered across the various quantiles indicated that in the majority of cases predictability is statistically different between cay, cay^{MS} and cay^{TVP}. Complete details of these results are available upon request from the authors.

[8] Based on the suggestion of an anonymous referee we also estimated a GARCH model (specifically the GJR-GARCH, Glosten et al., 1993, model to capture leverage) and also conducted the the Nishiyama et al., (2011) test. Under the GARCH framework, we observed that cay^{TVP} failed to predict excess returns and volatility, while predictability was again restricted to returns only with cay and cay^{MS}, with the latter being a stronger predictor. These results are in line with our linear Granger tests (discussed in Footnote 6). When we used the Nishiyama et al., (2011) test, cay, cay^{MS} and cay^{TVP} all predicted excess returns, but not volatility, with cay^{TVP} being the strongest predictor followed by cay and then cay^{MS}. Given that our quantiles-based results are more informative than these conditional-mean based models (because we study the entire conditional distribution), and because our results does not suffer from misspecification involving linearity, we do not present these results formally, but make them available upon request. However, with the Nishiyama et al., (2011) test being the predecessor to our test, we report these results in Table A2 at the end of the paper.

[9] While, it is true that the alternative consumption-wealth ratios tend to carry more accurate information for predicting excess stock returns relative to its competitors conditional on the state of the market, i.e., at specific points of the conditional distribution, on average over the quantiles, the test statistic under cay tends to have the highes value of 3.3277.

obtain information from predictors (in our case the various consumption-wealth ratios) to try and make their portfolio holdings more profitable.

Figure 3. Causality-in-quantiles of Excess stock returns (*exsr*)
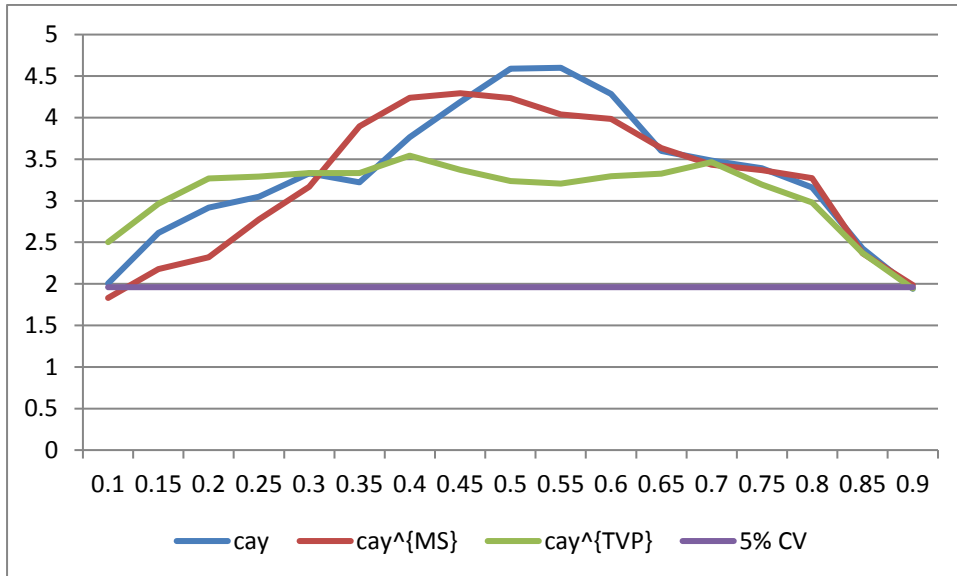
Note: See Notes to Figure 2; the horizontal axis depicts the various quantiles and the vertical axis measures the test statistic.

Figure 4. Causality-in-quantiles of excess stock returns volatility (*exsv*)
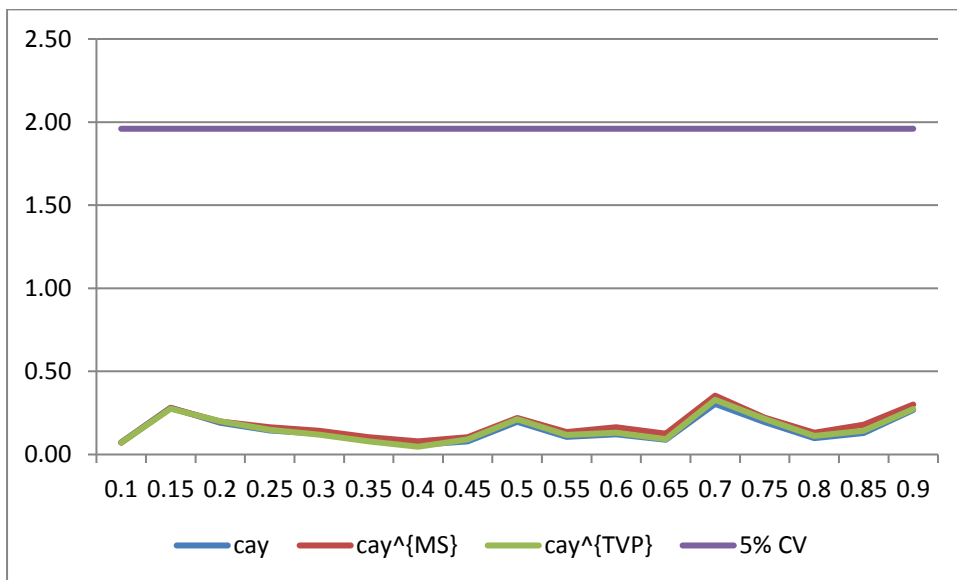
Note: See Notes to Figure 3.

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4. Conclusion

We have contributed to this literature on the consumption-aggregate wealth ratio (*cay*) and the predictability of equity-market fluctuations initiated by Lettau and Ludvigson (2001), in that (a) we have developed a time-varying measure of *cay* (cay^{TVP}) using time-varying cointegration techniques, (b) we have compared the predictive ability of the cay^{TVP} measure with the predictive ability of the *cay* and the Markov-switching *cay* (cay^{MS} introduced by Bianchi *et al.* 2016) measures with regard to excess stock returns and volatility in the US over the period 1952:Q2-2015:Q3, and, (c) we have analyzed the predictive ability of the three measures of the consumption-aggregate wealth ratio using a nonparametric causality-in-quantiles test developed by Balcilar *et al.* (2016a). This test renders it possible to analyze higher-order predictability over the entire conditional distributions of returns and volatility and, being a nonparametric test, it controls for the widely-reported misspecification associated with linear predictive models of stock-markets returns.

We find that cointegration exists, in a time varying fashion, between consumption, asset wealth, and labor income. In addition, while there is no evidence of predictability for excess returns volatility from any of the three measures of the consumption-aggregate wealth ratios, they tend to act as strong predictors (in terms of significance) of stock returns, with each of the ratios being important at certain phases of the equity market. In general, cay^{TVP} is important during a bear market, *cay* performs strongly when the market is performing in the normal to good phases, and cay^{MS}, in turn, tends to do well during a bull market and for moderately poor phases. Hence, investors can obtain important information from all three measures, contingent on the state the stock market is in.

As part of future research, it would be interesting to extend our study in order to examine whether our results regarding the predictive value of the three measures of the consumption-aggregate wealth ratio continue to hold in an out-of-sample forecasting exercise because there is not necessarily a one-to-one cooorespondence between in-sample and out-of-sample predictabilityy (Rapach and Zhou, 2013; Bonaccolto *et al.* forthcoming). One of the limitations of our in-sample predictability analysis, as is the case with any other Granger tests for noncausality, is that, while we can discuss whether a variable has predictive content for the dependent variable, we are not able to measure the extent of (statistical and economic) gains derived from using the predictor in question. An out-of-sample exercise would allow this issue to be addressed as we would be able to compare by how much the forecast error

and economic utility produced from the model are better or worse than those obtained from a benchmark model (i.e., the model without the predictor).[10]

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[10] Because the out-of-sample forecasting model of the causality-in-quantiles test is still under development, we could only obtain preliminary results, which in turn suggested that the model with *cay^{TVP}* tends to perform the best (especially at lower quantiles) in forecasting *exsr*, with it producing gains in terms of the mean squared forecast errors ranging from 8% to 1% relative to the benchmark. The model with *cay* (especially around the median) follows next with gains ranging from 5% to 1%. Complete details of these results are available upon request from the authors.

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APPENDIX:

Table A1. Unit Root Tests

Panel (a): Data in Levels

Variable	ADF		PP		KSS	
	Constant	Constant + Trend	Constant	Constant + Trend	Constant	Constant + Trend
c	-1.273	-1.789	-1.450	-1.291	-1.287	-1.948
a	-0.062	-2.846	-0.195	-3.099	-0.165	-3.452
y	-2.026	-1.110	-1.922	-1.225	-1.792	-1.482

Panel (b): Data in First-Differences

Variable	ADF		PP		KSS	
	Constant	Constant + Trend	Constant	Constant + Trend	Constant	Constant + Trend
c	-6.560***	-7.664***	-11.830***	-11.893***	-5.215***	-6.169***
a	-12.626***	-12.609***	-12.929***	-12.913***	-8.001***	-7.987***
y	-15.571***	-15.764***	-15.637***	-15.806***	-11.929***	-12.040***

Note: *** indicates rejection of the null hypothesis of a unit root at 1% level of significance.

Table A2: Nishiyama et al., (2011) Test Results

Dependent variable	Independent variable	Mean (Returns)	Volatility (Variance)
Exr	cay	16.282**	5.978
	cay^{MS}	15.932**	8.798
	cay^{TVP}	22.430**	5.861

Note: ** indicates rejection of the null hypothesis of non-Granger causality of returns (volatility) at the 5% critical value of 14.38.