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Sergey Ivashchenko [*] ***and Rangan Gupta*** [**]

Forecasting using a Nonlinear DSGE Model

* Saint Petersburg Institute for Economics and Mathematics (Russian Academy of Sciences); Faculty of Economics of Saint-Petersburg State University; National Research University Higher School of Economics, Saint Petersburg, Russia.

Email: sergey.ivashchenko.ru@gmail.com

** Corresponding author. Department of Economics, University of Pretoria, Pretoria, South Africa

Email: rangan.gupta@up.ac.za

Abstract: A medium-scale nonlinear dynamic stochastic general equilibrium (DSGE) model was estimated (54 variables, 29 state variables, 7 observed variables). The model includes an observed variable for stock market returns. The root-mean square error (RMSE) of the in-sample and out-of-sample forecasts was calculated. The nonlinear DSGE model with measurement errors outperforms AR (1), VAR (1) and the linearised DSGE in terms of the quality of the out-of-sample forecasts. The nonlinear DSGE model without measurement errors is of a quality equal to that of the linearised DSGE model.

Keywords: Nonlinear DSGE; Quadratic Kalman Filter; Out-of-sample forecasts.

JEL-codes: E32; E37; E44; E47.

1. Introduction

One of the most popular approaches for analysis of the macroeconomic environment is the use of dynamic stochastic general equilibrium (DSGE) models (Herbst and Schorfheide, 2016). This type of model is the basis of modern macroeconomic theory and is widely used by central banks and other policy-making institutions (Tovar, 2009). DSGE models have strong microeconomic foundations. The advantage of such an approach is that these models are based on 'deep structural' parameters that are not influenced by economic policy (Wickens, 2008). Different econometric techniques are employed for model estimation, but the empirical literature has focused on the estimation of first-order linearised DSGE models (Tovar, 2009, Diebold et al., forthcoming).

Computation with linear approximation is much faster than higher-order approximation, but its behaviour can differ from that of more accurate approximations (see Collard and Juillard, 2001, Fernández-Villaverde and Rubio-Ramírez (2007)). Second-order approximation can make the difference between the behaviour of models and that of approximation much smaller (Pichler, 2008). Nonlinear approximations of DSGE models have several other advantages: in particular, they allow uncertainty to influence economic choices (Ruge-Murcia, 2012). The likelihood function is sharper for nonlinear approximations, which means a more accurate estimation of the parameters (An and Schorfheide, 2007; Fernández-Villaverde et al., 2010).

Because of these advantages of nonlinear estimation, forecasting is expected to be of higher quality relative to linear versions of DSGE models. Many studies demonstrate the high quality forecasting of the linear approximations of DSGE models relative to standard atheoretical models like the VAR (Adolfson et al., 2007; Smets and Wouters, 2004). But majority of these models forecast a small number of variables (Rubaszek and Skrzypczynski, 2008; Del Negro and Schorfheide, 2012). However, in some studies, medium-scale linearised DSGE model is indeed shown to outperform VAR and AR models in terms of out-of-sample forecasting, involving many observable variables (Ivashchenko, 2013).

In a few studies, small-scale nonlinear DSGE models are estimated (Pichler, 2008; Gust et al., 2012; Fernandez-Villaverde et al., 2010). Most of them use only three observed variables: output, the nominal interest rate and inflation (Amisano and Tristani, 2010; Pichler, 2008; Balcilar et al., 2014; Gust et al., 2012). A few studies use other observed variables: Doh (2011) uses additional data about the yield curve, while, Hall (2012) uses consumption instead of output. But in general, forecasting using nonlinear DSGE models have been restricted to small number of variables and only few studies (Pichler, 2008; Diebold et al., forthcoming), even though lot of research has been done in terms of estimating nonlinear DSGE models based on the particle filter (Herbst and Schorfheide, 2016). In terms of nonlinear DSGE models, regime-switching models have also gained lot of popularity recently (see for example, Farmer et al. (2009, 2011), Liu et al. (2009, 2011), Liu and Mumtaz (2011), and Alstadheim et al. (2013), Balcilar et al., (forthcoming)). More recently, Diebold et al., (forthcoming) emphasized that time-varying volatility is a key nonlinearity not only in financial data but also in macroeconomic time series. Hence, the importance of nonlinearities in exogenous driving processes of DSGE models needs to be considered as well. Diebold et al., (forthcoming) find that incorporating stochastic volatility in DSGE models of macroeconomic fundamentals markedly improves their density forecasts. However, again these models are based on smaller number of observables. Reverting back to nonlinear DSGE models with higher order approximation, new results demonstrate greater advantages of alternative approaches over the use of particle filters when estimating the nonlinear DSGE models (Andreasen, 2013; Ivashchenko, 2014; Kollmann, 2014).

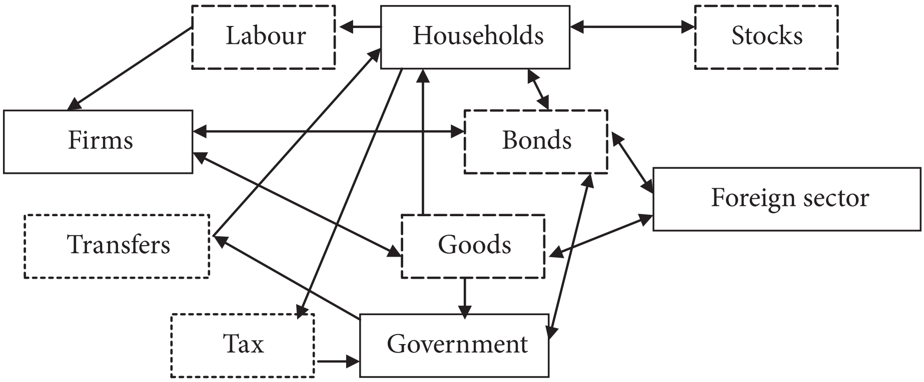
As indicated above, most estimated small-scale nonlinear DSGE models do not provide information about out-of-sample forecasts quality, barring a few. The forecasting quality of a nonlinear DSGE is nearly the same (or slightly worse) than that of a linearised DSGE model according to Pichler (2008). However, using the same model for South Africa, Balcilar et al., (2014) show that nonlinear DSGE model performs better than linear versions of the same. But these models, being small-scale, do not include observed variables that are sensitive to nonlinearities.

Given the existence of only few studies analyzing the forecasting ability of nonlinear DSGE models compared to its linear versions and also atheoretical models like the AR and VAR, this study presents an estimated medium-scale nonlinear DSGE model with seven observed variables, including stock market returns. Hence, we add to this literature of forecasting with nonlinear DSGE models, by building a bigger model that is more realistic, with higher number of observable variables, and also by including variables like the stock returns, which is known to be sensitive to nonlinearities. The model is then used for forecasting seven observables and compared with linear DSGE models and AR and VAR models. In addition, instead of using the particle filter, which is computationally slow for such medium-scale models, we use recent advances in the estimation of nonlinear DSGE models, and use the Quadratic Kalman Filter instead (Ivashchenko, 2014). To the best of our knowledge, this is the first attempt to build such a medium-scale nonlinear DSGE model, and also use it for forecasting key variables of the US economy. The rest of the paper is organized as follows: The DSGE model is described in section 2. Section 3 presents information on estimation techniques and the data utilized. Section 4 describes the estimation results and the quality of forecasts (in-sample and out-of-sample). Section 5 presents some conclusions.

2. Model

The DSGE model includes four types of agents: householders, firms, the government, and the foreign sector. The structure of the model is presented in Figure 1. The list of models variables presented at appendix (see table A1). The DSGE model includes central New-Keynesian features (for example, sticky price and adjustment costs in investment).

Figure 1. Structure of DSGE model



Note: Boxes with continuous border describe agents. The arrows describe flows, while dashed boxes describe type of flows that can be associated with markets and boxes with dotted lines describe policy related flows.

2.1 Householders

Households maximize the expected sum of their discounted utility functions (1) with budget restriction (2). Householders do not own capital, but they can invest in domestic stocks and bonds as a means of saving money. The utility function consists of the propensity to consume with a habit effect, the disutility of labour, money at the utility function, and the disutility of bond position deviation from preferred level.

$$E\left(\sum_{t=0}^{\infty} Z_{\beta,t}\left(\begin{array}{c}\dfrac{\left(C_t - h_C H_{t-1}\right)^{1-\omega_C}}{1-\omega_C} - Z_{L,t}\dfrac{L_t^{1+\omega_L}}{1+\omega_L} \\ + Z_{M,t}\dfrac{M_t}{P_t} - Z_{BH,t}\left(\dfrac{B_{H,t}}{P_t Z_t} - \mu_B\right)^2\end{array}\right)\right) \to \max_{B,C,L,M,X} \qquad (1)$$

$$P_t C_t + M_t + B_{H,t} + X_t S_t = (1-\tau_t) W_t L_t + M_{t-1} + R_{t-1} B_{H,t-1} + X_{t-1}\left(S_t + D_t\right) + T_{TR,t} \quad (2),$$

where C_t is consumption in period t, L_t is labour supply in period t, M_t is money stock in period t, P_t is the price of goods in period t, $B_{H,t}$ is the value of bonds bought by householders in period t, S_t is the price of stocks in period t, X_t is the amount of stocks bought by householders in period t, τ_t is the tax rate in period t, $T_{TR,t}$ is the transfer from government in period t, R_t is the interest rate on bonds in period t, and D_t is the dividends of stocks in period t. The first order conditions in terms of stable variables presented at appendix (A1)-(A6).

The model includes money for preventing problems with the description of the government, and also from the perspective of model determination. The dropping of money from the model would imply leaving out a single variable (M_t) and two equations (i.e. households' first order condition with respect to money, the exogenous rule for government). It would mean that number of variables would be larger than the number of equations, and the model becomes indeterminate (except for certain very specific situations). In the absence of money, the central bank cannot follow a Taylor-type rule, and control interest rate, because it cannot buy (or sell) bonds due to absence of another variable in its budget restriction. Cashless economy is mainly a simplification of the general equilibrium model under very specific conditions. We believe that such oversimplifications are very restrictive.

2.2 Finished goods-producing firms

Perfectly competitive firms produce the final good Y_t using the intermediate goods $Y_{j,t}$ and the CES production technology:

$$Y_t = \left(\int_0^1 Y_{j,t}^{(\theta-1)/\theta} dj \right)^{\theta/(\theta-1)} \tag{3}$$

Profit maximization and zero profit condition for the finished goods producers imply the following price level P_t and demand function for the intermediate good, j:

$$Y_{j,t} = \left(\frac{P_{j.t}}{P_t} \right)^{-\theta} Y_t \tag{4}$$

$$P_t = \left(\int_0^1 P_{j,t}^{1-\theta} dj \right)^{1/(1-\theta)} \tag{5}$$

2.3 Intermediate goods-producing firms

Firms maximize their expected discounted utility function (6) with restrictions. The utility function consists of dividends flow and two rigidities (stickiness of bond position and price stickiness in the Rotemberg form – Lombardo and Vestin, 2008). Firms are working in a market with monopolistic competition; therefore, they have a demand restriction (7). The budget restriction (8) and production function (9) is common. The restriction of capital evolution (10) contains investment rigidity.

$$E\left(\sum_{t=0}^{\infty}\left(\mathop{\Pi}_{k=0}^{t-1} R_k\right)^{-1}\left(\begin{array}{l} D_t - P_{F,t}Y_{F,t}\mu_{FB}\left(\dfrac{B_{F,t}}{P_t Z_t} - Z_{BF,t}\right) \\ - P_{F,t}Y_{F,t}Z_{P,t}\left(\dfrac{P_{F,t}}{P_{F,t-1}} - \overline{p}\right)^2 \end{array}\right)\right) \to \max_{\substack{D,B,P,Y,\\K,I,L}} \quad (6)$$

$$Y_{F,t} = \left(\frac{P_{F,t}}{P_t}\right)^{-\theta}\left(Y_{D,t}\right) \quad (7)$$

$$D_t + P_t I_t + W_t L_t + B_{F,t} = P_{F,t} Y_{F,t} + R_{t-1} B_{F,t-1} \quad (8)$$

$$Y_{F,t} = \left(Z_t L_t\right)^{Z_{\alpha,t}}\left(K_{t-1}\right)^{1-Z_{\alpha,t}} \quad (9)$$

$$K_t = (1-\delta)K_{t-1} + I_t\left(1 - Z_{I,t}\left(\frac{I_t}{I_{t-1}} - \overline{y}\right)^2\right) \quad (10)$$

where D_t is the dividends of the firm in period t, $Y_{F,t}$ is the output of firm F in period t, $P_{F,t}$ is the price of goods for firm F in period t, I_t is the demand for investments goods in period t, $Y_{D,t}$ is the aggregate demand in period t, P_t is the price level for domestic goods in period t, $B_{F,t}$ is the value of bonds bought by the firm in period t, K_t is the amount of capital used by the firm in period t, and L_t is the amount of labour used by the firm in period t. The first order conditions in terms of stable variables presented at appendix (A7)-(A17).

2.4 Government, foreign sector and balance equations

The government makes its decisions according to policy rules and budgetary restrictions. It has the following budgetary restriction:

$$P_t G_t + T_{TR,t} + B_{G,t} + M_{t-1} = \tau_t W_t L_t + R_{t-1} B_{G,t-1} + M_t \quad (11)$$

The monetary policy rule is as follows:

$$\ln(R_t) = \gamma_R \ln(R_{t-1}) + (1-\gamma_R)\left(\gamma_{RP}\left(\ln\left(\frac{P_t}{P_{t-1}}\right) - \overline{p}\right) + \gamma_{RY}\left(\ln\left(\frac{Y_{D,t}}{Y_{D,t-1}}\right) - \overline{y}\right) + Z_{R,t}\right) \quad (12)$$

The fiscal policy rules are as follows:

$$\ln\left(\frac{G_t}{Y_{D,t}}\right)=\gamma_G \ln\left(\frac{G_{t-1}}{Y_{D,t-1}}\right)+\left(1-\gamma_G\right)\begin{pmatrix} Z_{G,t}+\gamma_{GB}\left(\frac{B_{G,t}}{P_t Z_t}-\overline{b_G}\right)+ \\ +\gamma_{GY}\left(\ln\left(\frac{Y_{D,t}}{Y_{D,t-1}}\right)-\overline{y}\right)\end{pmatrix} \tag{13}$$

$$\ln\left(\frac{T_{TR,t}}{Y_{D,t}}\right)=\gamma_{TR} \ln\left(\frac{T_{TR,t-1}}{Y_{D,t-1}}\right)+\left(1-\gamma_{TR}\right)\begin{pmatrix} Z_{TR,t}+\gamma_{TRB}\left(\frac{B_{G,t}}{P_t Z_t}-\overline{b_G}\right)+ \\ +\gamma_{TRY}\left(\ln\left(\frac{Y_{D,t}}{Y_{D,t-1}}\right)-\overline{y}\right)\end{pmatrix} \tag{14}$$

$$\tau_t=\gamma_T\tau_{t-1}+\left(1-\gamma_T\right)\left(\gamma_{TB}\left(\frac{B_{G,t}}{P_t Z_t}-\overline{b_G}\right)+\gamma_{TY}\left(\ln\left(\frac{Y_{D,t}}{Y_{D,t-1}}\right)-\overline{y}\right)+Z_{T,t}\right) \tag{15}$$

The budget restriction of government implies that its expenditures involve government consumption, transfers to households, buying of bonds (this value is almost always negative), while sources of income are from labour tax, bonds from previous period and money creation. The monetary policy rule is a standard Taylor type rule. The fiscal policy rules suggest some smoothing, with dependence on the growth rate and position of the government debt.

The foreign sector is exogenous. It has a budgetary restriction (16) and is subject to an exogenous rule (17). Budget restriction of foreign sector is the balance of payment, which balances net export and financial flows (that consist of bonds flows). The rule for net export involves smoothing and dependence on foreign bond position, which prevents it from having an explosive trajectory.

$$NX_t P_t + B_{W,t} = R_{t-1}B_{W,t-1} \tag{16}$$

$$\left(\frac{NX_t}{Z_t}\right)=\gamma_{NX}\left(\frac{NX_{t-1}}{Z_{t-1}}\right)+\left(1-\gamma_{NX}\right)\left(\gamma_{NXB}\left(\frac{B_{W,t}}{P_t Z_t}-\overline{b_W}\right)+Z_{NX,t}\right) \tag{17}$$

In addition we must have the following restrictions: Each bond should be bought by someone (18), the amount of stocks is equal to one (19), and aggregate demand consists of consumption, investment, government consumption and net exports (20). (21) denotes how habit is formed, highlighting habit persistence.

$$B_{H,t} + B_{F,t} + B_{G,t} + B_{W,t} = 0 \tag{18}$$

$$X_t = 1 \tag{19}$$

$$Y_{D,t} = C_t + I_t + G_t + NX_t \tag{20}$$

$$H_t = h_h H_{t-1} + C_t \tag{21}$$

All the exogenous processes are AR (1) with the following parameterization:

$$z_{*,t} = \eta_{0,*,t}(1-\eta_{1,*,t}) + \eta_{1,*,t} z_{*,t-1} + \varepsilon_{*,t} \tag{22}$$

3. Estimation

Of the methods used for non-linear approximations of DSGE models, the perturbation method is the most widely used (Schmitt-Grohe and Uribe, 2004), and hence, it is used in this study. The maximum likelihood method is used for parameters estimation.

A few nonlinear filters can be used to calculate the likelihood function. One is the particle filter, which is used in most studies estimating nonlinear DSGE models (Pichler, 2008; Hall, 2012; Doh, 2011). However, it is too slow for implementation with medium-scale models. Another is the central difference Kalman filter (CDKF), which outperforms the particle filter (Andreasen, 2013). However, the quadratic Kalman filter (QKF) was used for the likelihood calculation because it produces a better quality of parameters estimations than the CDKF (Ivashchenko, 2014). The QKF was slightly slower than the CDKF (Ivashchenko, 2014), but after the program code was improved, it became six times faster and outperformed the CDKF in terms of speed.

The QKF is based on a normal approximation of density. Approximation with the perturbation method produces equation (23), which describes the data generating process for state variables (X_t). Equation (24) describes the dependence between observed variables (Y_t) and state variables. Exogenous shocks (ε_t) and measurement errors (u_t) have a normal distribution with zero mean and covariance matrices, Ω_ε and Ω_u.

$$X_t = \begin{bmatrix} B_X & B_\varepsilon \end{bmatrix} \begin{bmatrix} X_{t-1} \\ \varepsilon_t \end{bmatrix} + C + \begin{bmatrix} A_{xx} & A_{x\varepsilon} & 0 & A_{\varepsilon\varepsilon} \end{bmatrix} \begin{bmatrix} X_{t-1} \otimes X_{t-1} \\ X_{t-1} \otimes \varepsilon_t \\ \varepsilon_t \otimes X_{t-1} \\ \varepsilon_t \otimes \varepsilon_t \end{bmatrix} \tag{23}$$

$$Y_t = S + DX_t + u_t \qquad (24)$$

The updating step is similar with the Kalman filter, owing to the linearity of equation (24). The prediction step is based on an assumption of normal distribution of the state variables vector (X_{t-1}). The expected value of vector X_t is a function of the mean and covariance of vectors X_{t-1} and ε_t. The covariance of vector X_t is a function of the first, second, third, and fourth moments of vectors X_{t-1} and ε_t. However, the third and fourth moments of a vector with a normal distribution are a function of the mean and covariance. Thus, the QKF computes the first and second moments of the state variables vector and assumes that it has a normal distribution.

An alternative approach for nonlinear approximation is the pruning method (Kim et al., 2008), for which there is a nonlinear filter (Kollmann, 2014). It is faster than the QKF (before optimization of the program code) for small-scale models, but is much slower (by about five times) for medium-scale (with 20 state variables) models (Kollmann, 2014). The DSGE model described above has 54 variables (29 state variables); this was an additional reason for the usage of the QKF. The system of equations with rational expectation in terms of stable variables presented at appendix (A1)-(A29). The additional 25 equation are the following: 14 equations for exogenous process in form (22), 7 equations for observed variables and 4 equations for indicative variables (that are used for some analysis of model).

The model was estimated with quarterly data from the USA since 1985Q1 until 2013Q2. The following observed variables are used: logarithm of consumption as a fraction of GDP (obs_C); logarithm of government expenditure as a fraction of GDP (obs_G); logarithm of compensation of employees as a fraction of GDP (obs_{WL}); three-month euro-dollar deposit rate (obs_R); GDP growth rate (obs_Y); growth rate of the GDP deflator (obs_P); and MSCI USA gross return (obs_{STR}). The DSGE model was estimated four times (linearised model with the Kalman filter and second-order approximation with the QKF; with and without measurement errors for obs_{STR}).

The number of parameters is equal to 64 or 65 (depending on the existence of measurement errors). It means we have 12.28 to 12.47 observations per parameter (for the full sample estimation) and 9.91 to 10.06 observations per parameter (for the shortest sample involved in the forecasting exercise). Such ratios are acceptable from a practical view in the DSGE literature.

A relevant question to ask at this stage is: Why is the observed stock market return variable important? The answer to this is in the fact that the economy is highly influenced by expectations, and stock market-related variables are most strongly influenced by expectations. Thus, the usage of stock market returns, allow us to incorporate such influences in the model. This variable is more sensitive to nonlinear effects, and hence, should be of importance in a nonlinear structure, as it could be associated with nonlinear misspecifications. The stock market variable is assets of firms, so it is an additional variable that describe the situation related to capital owned by firms, and is especially important in case of investment rigidities.

4. Results

The estimation results are presented in Table 1 and Table A2 (see appendix). Some interesting details regarding the results are as follows. The monetary policy parameter γ_{RP} is less than 1. Many studies have obtained a value this parameter that is greater than 1 (1.045 – Fernandez-Villaverde et al., 2010; 1.66 – Smets and Wouters, 2004; 5.0 – Gust et al., 2012), but in others, it is less than 1 (0.63 – nonlinear estimation, Hall, 2012). Low values of γ_{RP} require additional comments: the log-likelihood value of the DSGE model with restriction (γ_{RP}>1) is less than 2900, which is much worse than with the other estimations (the QKF without measurement errors is 2947.5; the QKF with measurement errors is 2986.1; the linear estimation without measurement errors is 2920.8; and the linear estimation with measurement errors is 2986.9). Clearly, the linear and the nonlinear models with measurement errors tend to have the best fit, and hence, can be relied upon to provide more accurate estimate of this parameter. However, small estimates of these parameters do imply that the response of the interest rate to inflation shocks are actually quite marginal, and much less (though positive) than the estimate of 0.5 suggested in the original Taylor-rule, when we look in to a general equilibrium model. The OLS estimation of the monetary policy rule produces γ_{RP}=0.39. Hence, evidence of the existence of the Taylor principle is weak in our context.

Table 1. The DSGE model estimation results (part)

Param.	QKF				line			
	without measur.er.		with measur. error		without measur. error		with measur. error	
	Value	std	value	std	value	std	value	std
std ε_α	$2.60x10^{-01}$	$2.16x10^{-02}$	$2.95x10^{-01}$	$3.27x10^{-02}$	$8.02x10^{-02}$	$1.84x10^{-02}$	$6.83x10^{-02}$	$1.39x10^{-02}$
std ε_{BF}	$7.15x10^{-02}$	$1.01x10^{-01}$	$1.63x10^{-01}$	$5.74x10^{-02}$	$4.57x10^{-01}$	$2.45x10^{-01}$	$2.50x10^{+00}$	$3.04x10^{+00}$
std ε_{BH}	$5.52x10^{-01}$	$1.16x10^{-01}$	$4.65x10^{-01}$	$1.60x10^{-01}$	$2.86x10^{-04}$	$3.87x10^{+07}$	$2.46x10^{+00}$	$1.71x10^{+00}$
std ε_I	$3.55x10^{-08}$	$2.26x10^{-05}$	$1.92x10^{-09}$	$2.12x10^{-05}$	$4.84x10^{-04}$	$3.87x10^{+07}$	$4.61x10^{-04}$	$2.74x10^{+03}$
std ε_L	$2.25x10^{-08}$	$2.26x10^{-05}$	$1.13x10^{-08}$	$2.12x10^{-05}$	$1.00x10^{+02}$	$4.40x10^{-05}$	$7.85x10^{+01}$	$1.20x10^{+01}$
std ε_Y	$3.42x10^{-03}$	$2.65x10^{-04}$	$3.38x10^{-03}$	$1.82x10^{-04}$	$3.51x10^{-03}$	$4.19x10^{-04}$	$2.45x10^{-03}$	$3.07x10^{-04}$
std obs_{STR}	-	-	$6.70x10^{-02}$	$4.17x10^{-03}$	-	-	$6.60x10^{-02}$	$5.01x10^{-03}$
γ_G	$7.95x10^{-01}$	$2.18x10^{-02}$	$8.26x10^{-01}$	$1.77x10^{-02}$	$8.68x10^{-01}$	$2.02x10^{-02}$	$8.47x10^{-01}$	$2.86x10^{-02}$
γ_{GB}	$1.62x10^{+00}$	$2.05x10^{-01}$	$1.62x10^{+00}$	$3.58x10^{-01}$	$4.24x10^{-01}$	$6.32x10^{-02}$	$3.53x10^{-01}$	$2.34x10^{-01}$
γ_{GY}	$-2.41x10^{+00}$	$3.97x10^{-01}$	$-4.49x10^{+00}$	$3.70x10^{-05}$	$-5.00x10^{+00}$	$3.29x10^{-05}$	$-5.00x10^{+00}$	$2.33x10^{-05}$
γ_τ	$8.49x10^{-01}$	$1.26x10^{-02}$	$8.72x10^{-01}$	$1.57x10^{-02}$	$8.78x10^{-01}$	$1.88x10^{-02}$	$8.67x10^{-01}$	$1.88x10^{-02}$
$\gamma_{\tau B}$	$3.73x10^{-01}$	$8.31x10^{-02}$	$2.40x10^{-01}$	$8.37x10^{-02}$	$2.00x10^{-01}$	$4.45x10^{-02}$	$7.29x10^{-02}$	$4.57x10^{-02}$
$\gamma_{\tau Y}$	$-2.60x10^{+00}$	$2.74x10^{-01}$	$-2.88x10^{+00}$	$3.65x10^{-01}$	$-3.29x10^{+00}$	$2.34x10^{-01}$	$-4.54x10^{+00}$	$5.16x10^{-01}$
γ_R	$8.80x10^{-01}$	$2.11x10^{-02}$	$9.05x10^{-01}$	$1.52x10^{-02}$	$9.29x10^{-01}$	$1.64x10^{-02}$	$9.06x10^{-01}$	$2.04x10^{-02}$
γ_{RP}	$1.15x10^{-01}$	$5.64x10^{-02}$	$2.91x10^{-02}$	$1.55x10^{-01}$	$3.90x10^{-03}$	$3.07x10^{-02}$	$6.84x10^{-10}$	$2.09x10^{-05}$
γ_{RY}	$7.54x10^{-01}$	$1.24x10^{-01}$	$1.17x10^{+00}$	$3.08x10^{-01}$	$9.06x10^{-01}$	$2.12x10^{-01}$	$8.08x10^{-01}$	$1.50x10^{-01}$
$\eta_{0,\alpha}$	$5.99x10^{-01}$	$4.00x10^{-05}$	$5.99x10^{-01}$	$5.30x10^{-05}$	$6.00x10^{-01}$	$4.83x10^{-05}$	$5.97x10^{-01}$	$2.23x10^{-05}$
$\eta_{0,BF}$	$-8.80x10^{+00}$	$7.85x10^{-02}$	$-8.74x10^{+00}$	$2.96x10^{-01}$	$-1.26x10^{+01}$	$4.86x10^{-03}$	$-3.78x10^{+00}$	$8.70x10^{-01}$
$\eta_{0,BH}$	$1.94x10^{+01}$	$2.12x10^{+00}$	$1.92x10^{+01}$	$2.04x10^{+01}$	$1.95x10^{+01}$	$4.93x10^{-02}$	$1.30x10^{+00}$	$1.03x10^{+00}$
$\eta_{0,I}$	$1.84x10^{+01}$	$2.22x10^{+00}$	$1.19x10^{+01}$	$2.31x10^{+00}$	$1.21x10^{+01}$	$2.62x10^{+00}$	$7.79x10^{+00}$	$2.46x10^{+00}$
$\eta_{0,L}$	$2.78x10^{+00}$	$1.10x10^{-01}$	$2.84x10^{+00}$	$9.11x10^{-01}$	$3.94x10^{+00}$	$2.81x10^{-05}$	$4.14x10^{+00}$	$2.31x10^{-05}$
$\eta_{0,Y}$	$6.24x10^{-03}$	$5.76x10^{-05}$	$6.79x10^{-03}$	$1.37x10^{-04}$	$7.03x10^{-03}$	$2.72x10^{-05}$	$2.65x10^{-03}$	$2.38x10^{-05}$
$\eta_{1,BF}$	$9.06x10^{-01}$	$9.72x10^{-02}$	$9.67x10^{-01}$	$3.73x10^{-02}$	$8.05x10^{-01}$	$1.25x10^{-01}$	$9.72x10^{-01}$	$2.84x10^{-02}$
$\eta_{1,BH}$	$4.25x10^{-01}$	$1.56x10^{-01}$	$-2.22x10^{-01}$	$2.25x10^{-01}$	$-4.09x10^{-02}$	$3.73x10^{-02}$	$-1.13x10^{-01}$	$1.26x10^{-01}$
$\eta_{1,G}$	$-2.03x10^{-01}$	$5.43x10^{-02}$	$-2.65x10^{-01}$	$1.70x10^{-01}$	$-1.60x10^{-01}$	$1.30x10^{-01}$	$-1.61x10^{-02}$	$2.88x10^{+00}$
$\eta_{1,I}$	$-2.88x10^{-01}$	$2.21x10^{-01}$	$-1.84x10^{-01}$	$6.03x10^{-02}$	$1.98x10^{-03}$	$3.25x10^{-02}$	$5.54x10^{-03}$	$2.11x10^{+00}$
$\eta_{1,L}$	$1.08x10^{-01}$	$9.81x10^{-02}$	$-5.12x10^{-02}$	$2.03x10^{-01}$	$9.98x10^{-01}$	$4.72x10^{-05}$	$9.98x10^{-01}$	$1.82x10^{-05}$
$\eta_{1,M}$	$9.81x10^{-01}$	$1.19x10^{-02}$	$9.41x10^{-01}$	$1.18x10^{-02}$	$9.88x10^{-01}$	$3.20x10^{-03}$	$-2.01x10^{-01}$	$5.52x10^{-01}$
$\eta_{1,NX}$	$9.86x10^{-01}$	$3.42x10^{-03}$	$9.90x10^{-01}$	$2.17x10^{-03}$	$9.24x10^{-01}$	$2.24x10^{-02}$	$9.93x10^{-01}$	$9.42x10^{-04}$
$\eta_{1,P}$	$-9.32x10^{-03}$	$6.81x10^{-03}$	$-1.21x10^{-02}$	$7.11x10^{-03}$	$-4.51x10^{-05}$	$2.53x10^{-02}$	$5.55x10^{-04}$	$2.94x10^{+00}$
$\eta_{1,TB}$	$9.83x10^{-01}$	$9.17x10^{-03}$	$9.93x10^{-01}$	$5.18x10^{-03}$	$-1.22x10^{-02}$	$4.53x10^{-02}$	$-6.07x10^{-02}$	$2.44x10^{+00}$
$\eta_{1,Y}$	$2.17x10^{-01}$	$2.97x10^{-02}$	$1.66x10^{-01}$	$2.50x10^{-02}$	$1.86x10^{-01}$	$8.40x10^{-02}$	$3.08x10^{-02}$	$1.03x10^{-01}$
θ	$7.66x10^{+00}$	$7.65x10^{-04}$	$7.56x10^{+00}$	$5.04x10^{-02}$	$6.35x10^{+00}$	$2.49x10^{-05}$	$1.13x10^{+01}$	$2.32x10^{-05}$

Another important detail of the estimation results is high values of the standard deviation of the measurement errors (6.6% - QKF, 6.7% - line estimation, 7.2% - standard deviation of obs_{STR}). This could be a result of MSCI USA properties: it includes international companies (such as APPLE and JOHNSON & JOHNSON), which have a large portion of their production and sales in foreign countries. The identification of a few standard deviations (ε_{BH} and ε_I) is weak with linear approximation. However, this problem does not exist for the QKF. The standard deviation of ε_L is very sensitive to estimation technique (it is high for the linear estimation and almost zero for the QKF). Some autocorrelation coefficients ($\eta_{1,L}$, $\eta_{1,M}$ and $\eta_{1,TR}$) are sensitive to estimation technique as well (they are close to 1 with one estimation technique and close to 0 with another).

Table 2. RMSE of in-sample forecasts

	VAR(1)	AR(1)	DSGE QKF no meas.er.	DSGE QKF meas.er.	DSGE linear no meas.er.	DSGE linear meas.er.
obsC(+1)	**4.60x10^{-03}**	4.70x10^{-03}	5.29x10^{-03}	4.91x10^{-03}	5.09x10^{-03}	4.66x10^{-03}
obsG(+1)	7.90x10^{-03}	9.33x10^{-03}	**7.88x10^{-03}**	8.03x10^{-03}	9.21x10^{-03}	8.68x10^{-03}
obsY(+1)	**5.00x10^{-03}**	5.49x10^{-03}	5.68x10^{-03}	5.34x10^{-03}	5.72x10^{-03}	5.62x10^{-03}
obsP(+1)	**1.66x10^{-03}**	1.84x10^{-03}	2.22x10^{-03}	1.74x10^{-03}	2.22x10^{-03}	1.93x10^{-03}
obsWL(+1)	**6.49x10^{-03}**	7.17x10^{-03}	9.21x10^{-03}	6.98x10^{-03}	7.54x10^{-03}	6.72x10^{-03}
obsR(+1)	**1.07x10^{-03}**	1.29x10^{-03}	1.15x10^{-03}	1.15x10^{-03}	1.16x10^{-03}	1.13x10^{-03}
obsSTR(+1)	**6.59x10^{-02}**	7.04x10^{-02}	7.51x10^{-02}	7.44x10^{-02}	7.19x10^{-02}	7.22x10^{-02}
obsC(+2)	**5.36x10^{-03}**	5.41x10^{-03}	6.79x10^{-03}	6.57x10^{-03}	6.82x10^{-03}	5.84x10^{-03}
obsG(+2)	**1.26x10^{-02}**	1.58x10^{-02}	1.29x10^{-02}	1.29x10^{-02}	1.63x10^{-02}	1.45x10^{-02}
obsY(+2)	**5.22x10^{-03}**	5.76x10^{-03}	6.22x10^{-03}	5.69x10^{-03}	6.04x10^{-03}	6.13x10^{-03}
obsP(+2)	**1.88x10^{-03}**	2.01x10^{-03}	2.38x10^{-03}	1.96x10^{-03}	2.85x10^{-03}	2.03x10^{-03}
obsWL(+2)	**7.11x10^{-03}**	8.25x10^{-03}	1.16x10^{-02}	8.14x10^{-03}	9.93x10^{-03}	7.89x10^{-03}
obsR(+2)	**1.78x10^{-03}**	2.16x10^{-03}	1.90x10^{-03}	1.90x10^{-03}	1.97x10^{-03}	1.86x10^{-03}
obsSTR(+2)	**6.72x10^{-02}**	7.23x10^{-02}	7.32x10^{-02}	7.28x10^{-02}	7.23x10^{-02}	7.24x10^{-02}
obsC(+3)	**5.87x10^{-03}**	6.01x10^{-03}	8.78x10^{-03}	8.42x10^{-03}	8.51x10^{-03}	7.01x10^{-03}
obsG(+3)	**1.64x10^{-02}**	2.14x10^{-02}	1.72x10^{-02}	1.71x10^{-02}	2.28x10^{-02}	1.97x10^{-02}
obsY(+3)	**5.26x10^{-03}**	6.00x10^{-03}	6.67x10^{-03}	6.09x10^{-03}	6.35x10^{-03}	6.62x10^{-03}
obsP(+3)	**2.01x10^{-03}**	2.15x10^{-03}	2.68x10^{-03}	2.20x10^{-03}	3.57x10^{-03}	2.09x10^{-03}
obsWL(+3)	**7.48x10^{-03}**	9.72x10^{-03}	1.27x10^{-02}	9.31x10^{-03}	1.27x10^{-02}	9.36x10^{-03}
obsR(+3)	**2.40x10^{-03}**	2.90x10^{-03}	2.58x10^{-03}	2.60x10^{-03}	2.70x10^{-03}	2.50x10^{-03}
obsSTR(+3)	**6.76x10^{-02}**	7.26x10^{-02}	7.40x10^{-02}	7.31x10^{-02}	7.26x10^{-02}	7.27x10^{-02}
obsC(+4)	**6.64x10^{-03}**	6.68x10^{-03}	1.08x10^{-02}	1.03x10^{-02}	1.04x10^{-02}	8.39x10^{-03}
obsG(+4)	**1.99x10^{-02}**	2.68x10^{-02}	2.17x10^{-02}	2.17x10^{-02}	2.99x10^{-02}	2.53x10^{-02}
obsY(+4)	**5.32x10^{-03}**	6.05x10^{-03}	6.88x10^{-03}	6.09x10^{-03}	6.59x10^{-03}	6.86x10^{-03}
obsP(+4)	**2.11x10^{-03}**	2.26x10^{-03}	2.99x10^{-03}	2.46x10^{-03}	4.42x10^{-03}	2.27x10^{-03}
obsWL(+4)	**7.72x10^{-03}**	1.11x10^{-02}	1.41x10^{-02}	1.03x10^{-02}	1.62x10^{-02}	1.08x10^{-02}
obsR(+4)	**2.89x10^{-03}**	3.54x10^{-03}	3.15x10^{-03}	3.19x10^{-03}	3.31x10^{-03}	3.00x10^{-03}
obsSTR(+4)	**6.87x10^{-02}**	7.27x10^{-02}	7.43x10^{-02}	7.30x10^{-02}	7.27x10^{-02}	7.29x10^{-02}
average RMSE	**1.48x10^{-02}**	1.65x10^{-02}	1.71x10^{-02}	1.64x10^{-02}	1.76x10^{-02}	1.65x10^{-02}
root mean square RMSE	**2.64x10^{-02}**	2.86x10^{-02}	2.93x10^{-02}	2.88x10^{-02}	2.91x10^{-02}	2.87x10^{-02}
forecasts not worse than VAR	28	0	1	0	0	0
forecasts not worse than AR	28	28	8	16	7	14

The RMSE of the forecasts (for horizons one quarter-ahead till four- quarter-ahead) are presented in Table 2 (in-sample) and Table 3 (out-of-sample). Out-of-sample forecasts were computed for the last 22 quarters (this meant the re-estimation of parameters with dataset without the last quarter (from 1985Q1 until 2013Q1) and the computation of forecasts; the re-estimation without 2 quarters (from 1985Q1 until 2012Q4), and so on; the last re-estimation used dataset without 22 quarters – from 1985Q1 until 2007Q4).

Table 3. RMSE of out-of-sample forecasts

	VAR(1)	AR(1)	DSGE QKF no meas.er.	DSGE QKF meas.er.	DSGE linear no meas.er.	DSGE linear meas.er.
obsC(+1)	$5.23x10^{-03}$	$\mathbf{4.72x10^{-03}}$	$6.48x10^{-03}$	$5.03x10^{-03}$	$5.93x10^{-03}$	$5.76x10^{-03}$
obsG(+1)	$1.07x10^{-02}$	$1.29x10^{-02}$	$\mathbf{8.64x10^{-03}}$	$9.20x10^{-03}$	$9.97x10^{-03}$	$1.05x10^{-02}$
obsY(+1)	$7.33x10^{-03}$	$7.97x10^{-03}$	$8.05x10^{-03}$	$\mathbf{7.27x10^{-03}}$	$7.91x10^{-03}$	$8.49x10^{-03}$
obsP(+1)	$2.19x10^{-03}$	$2.21x10^{-03}$	$2.05x10^{-03}$	$\mathbf{1.86x10^{-03}}$	$2.42x10^{-03}$	$2.24x10^{-03}$
obsWL(+1)	$1.14x10^{-02}$	$1.13x10^{-02}$	$\mathbf{9.40x10^{-03}}$	$1.02x10^{-02}$	$1.12x10^{-02}$	$1.09x10^{-02}$
obsR(+1)	$\mathbf{1.22x10^{-03}}$	$1.52x10^{-03}$	$1.64x10^{-03}$	$1.55x10^{-03}$	$1.56x10^{-03}$	$1.55x10^{-03}$
obsSTR(+1)	$1.01x10^{-01}$	$\mathbf{9.79x10^{-02}}$	$1.07x10^{-01}$	$1.02x10^{-01}$	$9.83x10^{-02}$	$1.01x10^{-01}$
obsC(+2)	$6.91x10^{-03}$	$\mathbf{6.52x10^{-03}}$	$9.07x10^{-03}$	$7.40x10^{-03}$	$9.38x10^{-03}$	$7.78x10^{-03}$
obsG(+2)	$1.88x10^{-02}$	$2.39x10^{-02}$	$\mathbf{1.59x10^{-02}}$	$1.61x10^{-02}$	$1.93x10^{-02}$	$1.95x10^{-02}$
obsY(+2)	$8.31x10^{-03}$	$8.86x10^{-03}$	$9.09x10^{-03}$	$\mathbf{7.93x10^{-03}}$	$8.38x10^{-03}$	$9.65x10^{-03}$
obsP(+2)	$2.74x10^{-03}$	$2.51x10^{-03}$	$\mathbf{2.24x10^{-03}}$	$2.27x10^{-03}$	$2.78x10^{-03}$	$2.42x10^{-03}$
obsWL(+2)	$1.04x10^{-02}$	$1.18x10^{-02}$	$1.13x10^{-02}$	$\mathbf{9.23x10^{-03}}$	$1.17x10^{-02}$	$1.24x10^{-02}$
obsR(+2)	$\mathbf{1.72x10^{-03}}$	$2.14x10^{-03}$	$2.42x10^{-03}$	$2.34x10^{-03}$	$2.36x10^{-03}$	$2.33x10^{-03}$
obsSTR(+2)	$1.07x10^{-01}$	$1.02x10^{-01}$	$1.03x10^{-01}$	$9.96x10^{-02}$	$\mathbf{9.76x10^{-02}}$	$9.96x10^{-02}$
obsC(+3)	$8.52x10^{-03}$	$\mathbf{7.81x10^{-03}}$	$1.31x10^{-02}$	$1.00x10^{-02}$	$1.29x10^{-02}$	$1.01x10^{-02}$
obsG(+3)	$2.78x10^{-02}$	$3.42x10^{-02}$	$\mathbf{2.30x10^{-02}}$	$2.34x10^{-02}$	$2.93x10^{-02}$	$2.93x10^{-02}$
obsY(+3)	$9.04x10^{-03}$	$9.47x10^{-03}$	$9.98x10^{-03}$	$\mathbf{8.58x10^{-03}}$	$9.31x10^{-03}$	$1.08x10^{-02}$
obsP(+3)	$3.40x10^{-03}$	$2.87x10^{-03}$	$2.83x10^{-03}$	$\mathbf{2.80x10^{-03}}$	$3.62x10^{-03}$	$2.95x10^{-03}$
obsWL(+3)	$1.01x10^{-02}$	$1.37x10^{-02}$	$1.45x10^{-02}$	$\mathbf{1.00x10^{-02}}$	$1.29x10^{-02}$	$1.57x10^{-02}$
obsR(+3)	$\mathbf{2.00x10^{-03}}$	$2.56x10^{-03}$	$2.88x10^{-03}$	$2.90x10^{-03}$	$2.92x10^{-03}$	$2.87x10^{-03}$
obsSTR(+3)	$1.08x10^{-01}$	$1.03x10^{-01}$	$1.08x10^{-01}$	$1.02x10^{-01}$	$\mathbf{1.00x10^{-01}}$	$1.02x10^{-01}$
obsC(+4)	$8.96x10^{-03}$	$\mathbf{7.60x10^{-03}}$	$1.59x10^{-02}$	$1.26x10^{-02}$	$1.62x10^{-02}$	$1.27x10^{-02}$
obsG(+4)	$3.78x10^{-02}$	$4.34x10^{-02}$	$\mathbf{2.86x10^{-02}}$	$2.96x10^{-02}$	$3.80x10^{-02}$	$3.84x10^{-02}$
obsY(+4)	$9.04x10^{-03}$	$9.32x10^{-03}$	$1.04x10^{-02}$	$\mathbf{8.58x10^{-03}}$	$9.73x10^{-03}$	$1.13x10^{-02}$
obsP(+4)	$3.77x10^{-03}$	$\mathbf{3.02x10^{-03}}$	$3.15x10^{-03}$	$3.45x10^{-03}$	$4.20x10^{-03}$	$3.38x10^{-03}$
obsWL(+4)	$\mathbf{1.12x10^{-02}}$	$1.78x10^{-02}$	$2.03x10^{-02}$	$1.25x10^{-02}$	$1.75x10^{-02}$	$1.91x10^{-02}$
obsR(+4)	$\mathbf{2.11x10^{-03}}$	$3.07x10^{-03}$	$3.22x10^{-03}$	$3.38x10^{-03}$	$3.45x10^{-03}$	$3.35x10^{-03}$
obsSTR(+4)	$1.07x10^{-01}$	$1.02x10^{-01}$	$1.09x10^{-01}$	$1.01x10^{-01}$	$\mathbf{1.01x10^{-01}}$	$1.01x10^{-01}$
average RMSE	$2.30x10^{-02}$	$2.34x10^{-02}$	$2.36x10^{-02}$	$\mathbf{2.19x10^{-02}}$	$2.32x10^{-02}$	$2.35x10^{-02}$
root mean square RMSE	$4.16x10^{-02}$	$4.06x10^{-02}$	$4.17x10^{-02}$	$\mathbf{3.96x10^{-02}}$	$3.96x10^{-02}$	$4.02x10^{-02}$
forecasts not worse than VAR	28	12	11	19	6	9
forecasts not worse than AR	16	28	9	18	14	9

The VAR model produces the best in-sample forecasts; this may be explained by the larger number of parameters (VAR – 84 parameters, AR – 21 parameters, DSGE – 64 or 65 parameters, depending on the existence of measurement errors). The RMSEs of the out-of-sample forecasts are drastically higher than those of the in-sample forecasts because of the financial crisis of 2008-2009. The quality of the in-sample forecast with measurement errors is better for linear and quadratic estimations. However, the situation with out-of-sample forecasts is different: linear forecasts with measurement errors are worse than without measurement errors.

The in-sample quality of linear and quadratic forecasts with measurement errors is nearly the same. Quadratic forecasts with measurement errors outperform all other models in terms of out-of-sample RMSE. It outperforms each of the other models for more than two-thirds of the variables. A comparison of linear and quadratic estimation without measurement errors shows a small advantage for linear estimation (14 variables forecasts are better than with quadratic estimation of the same model), which is in linear with the results of Pichler (2008). It should be noted that forecasts (for 2, 3, and 4 quarters) of stock market returns by the DSGE model outperform the AR and VAR models, despite problems related to international companies.

The tables A3-A5 (see appendix) presents *p*-values of MSE-F test of McCracken (2007). It should be noted that it is a one-sided side test with alternative hypothesis that the unrestricted model (i.e., the model with larger number of parameters) performs better (produces a lower MSE) than the restricted model. However, our dataset is relatively short that could produce significant deviation of the density of the statistic from the corresponding asymptotic density. Thus, we compute finite sample density as follows. We use AR(1) process with full sample estimated parameters as DGP for drawing of 100 000 trajectories of 7 observed variables with length 114 observations. Then each draw is used for construction of out-of-sample forecast of AR(1), VAR(1) and models with 64-65 parameters. The DSGE model estimation is computationally expensive. So, we use an AR(1) model plus additional *i.i.d.* regressors, with the number of regressors chosen for achieving required number of parameters. The MSE-F statistic is computed for each draw and possible model pairs. In the process, we compute finite sample densities of the MSE-F statistic.

Tables A3-A5 shows that out of sample forecasting quality of models are insignificant in majority of the comparisons. However, the maximum cases of significant gains relative to the AR(1) model is achieved by nonlinear DSGE model without measurement errors (the second best is the linearized version of the same). However, the comparison with VAR(1) model shows that the linearized DSGE with measurement errors is not significantly outperformed by VAR(1) model for the majority of the cases. From Table A5, it is evident that the nonlinear DSGE model with measurement error in general does well, especially relative to the linear model without measurement errors.

5. Conclusion

The medium-scale nonlinear DSGE model was estimated in this study. The DSGE model includes stock market returns, but observed data (MSCI USA gross return) describes international companies. Thus, measurement errors (for the stock returns variable) increase the quality of the model with nonlinear estimation (however, it does not change the quality of the linear estimated model). Measurement errors have a high standard deviation.

The quality of the out-of-sample forecasts of the DSGE models without measurement errors is almost equal (slightly worse) to those of AR (1) and VAR (1) models. The quality of the DSGE model with linear and nonlinear estimations is actually equal. In the case of the existence of measurement errors, the situation is different: the nonlinear DSGE model outperforms all other models (including linearised DSGE). Thus, this study finds that nonlinear DSGE models are more sensitive to misspecification (a negative effect of sharper likelihood), and that achieving an advantage from nonlinear approximation requires a more realistic model than when compared to a linearised model.

Compliance with Ethical Standards:

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References

1. Adolfson M., Lindé J., and Villani M. (2007). Forecasting performance of an open economy DSGE model. *Econometric Reviews,* 26(2-4), 289-328.
2. Alstadheim, R., Bjornland, H.C. and Maih, J. (2013). Do central banks respond to exchange rate movements? A Markov-switching structural investigation. Working Paper 2013/24, Norges Bank.
3. Amisano G. and Tristani O. (2010). Euro area inflation persistence in an estimated nonlinear DSGE model. *Journal of Economic Dynamics and Control,* 34(10), 1837-1858.
4. Andreasen, M. M. (2013). Non-Linear DSGE Models and the Central Difference Kalman Filter. Journal of Applied Econometrics, 28(6), 929-955.
5. An S. and Schorfheide F. (2007). Bayesian analysis of DSGE models // *Econometric Reviews,* 26(2-4), 113-172.
6. Balcilar M., Gupta R., and Kotze K. (2014). Forecasting macroeconomic data for an emerging market with a nonlinear DSGE model. Economic Modelling, 44(1), 215-228.
7. Balcilar, M., Gupta, R., and Kotze, K. (forthcoming). Forecasting South African Macroeconomic Variables with a Markov-Switching Small Open-Economy Dynamic Stochastic General Equilibrium Model. Empirical Economics.
8. Collard F. and Juillard M. (2001). Accuracy of stochastic perturbation methods: The case of asset pricing models. *Journal of Economic Dynamics and Control,* 25(6-7), 979-999.
9. Del Negro M., Schorfheide F. (2012). DSGE model-based forecasting. Staff Reports from Federal Reserve Bank of New York, No 554.
10. Doh T., 2011. Yield curve in an estimated nonlinear macro model. *Journal of Economic Dynamics and Control,* 35(8), 1229-1244.
11. Farmer, R.E., Waggoner, D.F. and Zha, T. (2009). Understanding Markov-switching rational expectations models. Journal of Economic Theory, 144(5), 1849-1867.
12. Farmer, R.E., Waggoner, D.F. and Zha, T. (2011). Minimal state variable solutions to Markov-switching rational expectations models. Journal of Economic Dynamics and Control, 35(12), 2150-2166.
13. Fernandez-Villaverde J., Guerron P.A., Rubio-Ramirez J.F. (2010). Reading the recent monetary history of the United States, 1959-2007. *Federal Reserve Bank of St. Louis Review,* 92(4), 311-338.
14. Fernandez-Villaverde, J., and Rubio-Ramirez, J.F. (2007). Estimating Macroeconomic Models: A Likelihood Approach. Review of Economic Studies, 74(4), 1059-1087.

15. Gust C., Lopez-Salido D., Smith M. E. (2012). The empirical implications of the interest-rate lower bound. No 2012-83, Finance and Economics Discussion Series from Board of Governors of the Federal Reserve System (U.S.).
16. Hall J. (2012). Consumption dynamics in general equilibrium. MPRA Paper from University Library of Munich, Germany.
17. Herbst, E., and Schorfheide, F. (2016). Tempered Particle Filtering. PIER Working Paper 16-017.
18. Ivashchenko S. (2013). Dynamic stochastic general equilibrium model with banks and endogenous defaults of firms. *Journal of the New Economic Association,* 19(3), 27-50.
19. Ivashchenko S. (2014). DSGE model estimation on the basis of second-order approximation. *Computational Economics,* 43(1), 71-82.
20. Kim J., Kim S., Schaumburg E., Sims C. (2008). Calculating and using second-order accurate solutions of discrete-time dynamic equilibrium models. *Journal of Economic Dynamics and Control,* 32, 3397-3414.
21. Kollmann R. (2014). Tractable latent state filtering for non-linear DSGE models using a second-order approximation and pruning. *Computational Economics,* DOI 10.1007/s10614-013-9418-3
22. Liu, P. and Mumtaz, H. (2011). Evolving Macroeconomic Dynamics in a Small Open Economy: An Estimated Markov Switching DSGE Model for the UK. Journal of Money, Credit and Banking, 43(7), 1443-1474.
23. Liu, Z., Waggoner, D. and Zha, T. (2009). Asymmetric Expectation Effects of Regime Shifts in Monetary Policy. Review of Economic Dynamics, 12(2), 284-303.
24. Liu, Z., Waggoner, D.F. and Zha, T. (2011). Sources of macroeconomic fluctuations: A regime- switching DSGE approach. Quantitative Economics, 2(2), 251-301.
25. Lombardo G., Vestin D. (2008). Welfare implications of Calvo vs. Rotemberg-pricing assumptions. *Economics Letters,* 100(2), 275-279.
26. McCracken M. (2007). Asymptotics for out of sample tests of Granger causality. *Journal of Econometrics*, 2007, vol. 140, issue 2, pages 719-752
27. Pichler P. (2008). Forecasting with DSGE models: The Role of nonlinearities. *The B.E. Journal of Macroeconomics,* 8(1), 1-35.
28. Rubaszek M., Skrzypczyński P. (2008). On the forecasting performance of a small-scale DSGE model. *International Journal of Forecasting,* 24(3), 498-512.
29. Ruge-Murcia F.J. (2012). Estimating nonlinear DSGE models by the simulated method of moments: With an application to business cycles. *Journal of Economic Dynamics and Control,* 36(6), 914-938.

30. Schmitt-Grohe S., Uribe M. (2004). Solving dynamic general equilibrium models using a second-order approximation to the policy function. *Journal of Economic Dynamics and Control,* 28(4), 755-775.
31. Smets F.R., Wouters R. (2004). Forecasting with a Bayesian DSGE model: An Application to the Euro area. *Journal of Common Market Studies,* 42(4), 841-867.
32. Tovar C. E. (2009). DSGE models and central banks. Economics-The Open-Access, Open-Assessment E-Journal, 3(16), 1-31.
33. Wickens M.R. (2008). *Macroeconomic Theory–A Dynamic General Equilibrium Approach.* Princeton University Press: Princeton, NJ.

Appendix

Table A1. The DSGE model variables

Variable	Description	Stationary variable
$B_{F,t}$	Value of bonds bought by firms in period t	$b_{F,t} = B_{F,t} / P_t Z_t$
$B_{G,t}$	Value of bonds bought by government in period t	$b_{G,t} = B_{G,t} / P_t Z_t$
$B_{H,t}$	Value of bonds bought by households in period t	$b_{H,t} = B_{H,t} / P_t Z_t$
$B_{W,t}$	Value of bonds bought by foreign sector in period t	$b_{W,t} = B_{W,t} / P_t Z_t$
C_t	Consumption at time t	$c_t = \ln(C_t / Z_t)$
D_t	Dividends at time t	$d_t = (C_t / Z_t)$
G_t	Government expenditure at time t	$g_t = \ln(G_t / Z_t)$
H_t	Habit at time t	$h_t = \ln(H_t / Z_t)$
I_t	Investments at time t	$i_t = \ln(I_t / Z_t)$
K_t	Capital at time t	$k_t = \ln(K_t / Z_t)$
L_t	Labour at time t	$l_t = \ln(L_t)$
M_t	Money stock in period t	$m_t = \ln\left(M_t / P_t Z_t \right)$
NX_t	Net export in period t	$nx_t = (NX_t / Z_t)$
P_t	Price of goods in period t	$p_t = \ln(P_t / P_{t-1})$
$P_{F,t}$	Price for goods of firm F in period t	$p_{F,t} = \ln(P_{F,t} / P_t)$
R_t	Interest rate in period t	$r_t = \ln(R_t)$
S_t	Price of stocks in period t	$s_t = \ln\left(S_t / P_t Z_t \right)$
τ_t	Tax rate in period t	$\tau_t = \tau_t$
$T_{TR,t}$	Transfer from government in period t	$\tau_{TR,t} = \ln\left(T_{TR,t} / P_t Z_t \right)$
W_t	Wage in period t	$w_t = \ln\left(W_t / P_t Z_t \right)$
X_t	Amount of stocks bought by householders in period t	$x_t = X_t$
$Y_{D,t}$	Aggregate demand in period t	$y_{D,t} = \ln(Y_{D,t} / Z_t)$
$Y_{F,t}$	Output of firm F in period t	$y_{F,t} = \ln(Y_{F,t} / Z_t)$
$Z_{\alpha,t}$	Exogenous process corresponding to elasticity of production function	$z_{\alpha,t} = Z_{\alpha,t}$
$Z_{\beta,t}$	Exogenous process corresponding to intertemporal preferences of households	$z_{\beta,t} = \ln(Z_{\beta,t} / Z_{\beta,t-1})$
$Z_{BF,t}$	Exogenous process corresponding to conventional level of debt pressure	$z_{BF,t} = Z_{BF,t}$
$Z_{BH,t}$	Exogenous process corresponding to stickiness of households' bond position	$z_{BH,t} = \ln(Z_{BH,t} / Z_t^{1-\omega_C})$
$Z_{G,t}$	Exogenous process corresponding to government expenditure	$z_{G,t} = \ln(Z_{G,t})$
$Z_{I,t}$	Exogenous process corresponding to decreasing efficiency of investments	$z_{I,t} = \ln(Z_{I,t})$

$Z_{L,t}$	Exogenous process corresponding to households' amount of labour	$z_{L,t} = \ln\left(Z_{L,t} / Z_t^{1-\omega_C}\right)$
$Z_{M,t}$	Exogenous process corresponding to liquidity preferences of households	$z_{M,t} = \ln\left(Z_{M,t} / Z_t^{-\omega_C}\right)$
$Z_{NX,t}$	Exogenous process corresponding to net export	$z_{NX,t} = Z_{NX,t}$
$Z_{P,t}$	Exogenous process corresponding to level of price stickiness	$z_{P,t} = \ln\left(Z_{P,t}\right)$
$Z_{R,t}$	Exogenous process corresponding to monetary policy	$z_{R,t} = Z_{R,t}$
$Z_{\tau,t}$	Exogenous process corresponding to taxation policy	$z_{\tau,t} = Z_{\tau,t}$
$Z_{TR,t}$	Exogenous process corresponding to transfers policy	$z_{TR,t} = Z_{TR,t}$
Z_t	Exogenous process corresponding to technological development	$z_t = \ln\left(Z_t / Z_{t-1}\right)$

Table A2. The DSGE model estimation results

	QKF				Linear			
Param.	**without measur.er.**		**with measur. error**		**without measur.er.**		**with measur. error**	
	value	**std**	**value**	**std**	**value**	**std**	**value**	**std**
std ε_α	$2.60\text{x}10^{-01}$	$2.16\text{x}10^{-02}$	$2.95\text{x}10^{-01}$	$3.27\text{x}10^{-02}$	$8.02\text{x}10^{-02}$	$1.84\text{x}10^{-02}$	$6.83\text{x}10^{-02}$	$1.39\text{x}10^{-02}$
std ε_β	$5.91\text{x}10^{-10}$	$2.26\text{x}10^{-05}$	$3.77\text{x}10^{-08}$	$2.12\text{x}10^{-05}$	$7.10\text{x}10^{-07}$	$3.55\text{x}10^{-05}$	$3.11\text{x}10^{-05}$	$2.09\text{x}10^{-05}$
std ε_{BF}	$7.15\text{x}10^{-02}$	$1.01\text{x}10^{-01}$	$1.63\text{x}10^{-01}$	$5.74\text{x}10^{-02}$	$4.57\text{x}10^{-01}$	$2.45\text{x}10^{-01}$	$2.50\text{x}10^{+00}$	$3.04\text{x}10^{+00}$
std ε_{BH}	$5.52\text{x}10^{-01}$	$1.16\text{x}10^{-01}$	$4.65\text{x}10^{-01}$	$1.60\text{x}10^{-01}$	$2.86\text{x}10^{-04}$	$3.87\text{x}10^{+07}$	$2.46\text{x}10^{+00}$	$1.71\text{x}10^{+00}$
std ε_G	$2.32\text{x}10^{-02}$	$3.42\text{x}10^{-03}$	$2.65\text{x}10^{-02}$	$6.84\text{x}10^{-03}$	$3.55\text{x}10^{-02}$	$1.08\text{x}10^{-02}$	$1.36\text{x}10^{-07}$	$2.09\text{x}10^{-05}$
std ε_I	$3.55\text{x}10^{-08}$	$2.26\text{x}10^{-05}$	$1.92\text{x}10^{-09}$	$2.12\text{x}10^{-05}$	$4.84\text{x}10^{-04}$	$3.87\text{x}10^{+07}$	$4.61\text{x}10^{-04}$	$2.74\text{x}10^{+03}$
std ε_L	$2.25\text{x}10^{-08}$	$2.26\text{x}10^{-05}$	$1.13\text{x}10^{-08}$	$2.12\text{x}10^{-05}$	$1.00\text{x}10^{+02}$	$4.40\text{x}10^{-05}$	$7.85\text{x}10^{+01}$	$1.20\text{x}10^{+01}$
std ε_M	$1.77\text{x}10^{+00}$	$5.06\text{x}10^{-01}$	$8.70\text{x}10^{-01}$	$1.57\text{x}10^{-01}$	$1.00\text{x}10^{+02}$	$4.40\text{x}10^{-05}$	$5.60\text{x}10^{-01}$	$4.58\text{x}10^{-01}$
std ε_{NX}	$3.70\text{x}10^{+00}$	$9.97\text{x}10^{-01}$	$4.07\text{x}10^{+00}$	$1.08\text{x}10^{+00}$	$2.55\text{x}10^{-01}$	$8.46\text{x}10^{-02}$	$8.41\text{x}10^{-01}$	$7.94\text{x}10^{-02}$
std ε_P	$1.04\text{x}10^{-01}$	$4.24\text{x}10^{-02}$	$8.54\text{x}10^{-02}$	$1.75\text{x}10^{-02}$	$6.22\text{x}10^{-04}$	$5.60\text{x}10^{+01}$	$4.61\text{x}10^{-04}$	$1.59\text{x}10^{+03}$
std ε_R	$1.93\text{x}10^{-02}$	$3.34\text{x}10^{-03}$	$2.02\text{x}10^{-02}$	$2.22\text{x}10^{-03}$	$2.69\text{x}10^{-02}$	$5.06\text{x}10^{-03}$	$2.14\text{x}10^{-02}$	$3.90\text{x}10^{-03}$
std ε_τ	$2.76\text{x}10^{-02}$	$7.52\text{x}10^{-03}$	$2.05\text{x}10^{-08}$	$2.12\text{x}10^{-05}$	$5.96\text{x}10^{-02}$	$9.97\text{x}10^{-03}$	$3.83\text{x}10^{-02}$	$7.79\text{x}10^{-03}$
std ε_{TR}	$2.28\text{x}10^{-08}$	$2.26\text{x}10^{-05}$	$4.75\text{x}10^{-09}$	$2.12\text{x}10^{-05}$	$2.66\text{x}10^{-06}$	$3.55\text{x}10^{-05}$	$5.33\text{x}10^{-06}$	$2.09\text{x}10^{-05}$
std ε_Y	$3.42\text{x}10^{-03}$	$2.65\text{x}10^{-04}$	$3.38\text{x}10^{-03}$	$1.82\text{x}10^{-04}$	$3.51\text{x}10^{-03}$	$4.19\text{x}10^{-04}$	$2.45\text{x}10^{-03}$	$3.07\text{x}10^{-04}$
std obs_{STR}	-	-	$6.70\text{x}10^{-02}$	$4.17\text{x}10^{-03}$	-	-	$6.60\text{x}10^{-02}$	$5.01\text{x}10^{-03}$
γ_{NX}	$2.09\text{x}10^{-01}$	$2.44\text{x}10^{-01}$	$3.05\text{x}10^{-01}$	$2.70\text{x}10^{-01}$	$-2.21\text{x}10^{-01}$	$6.76\text{x}10^{-02}$	$-2.48\text{x}10^{-01}$	$1.22\text{x}10^{-01}$
γ_{NXB}	$5.00\text{x}10^{+00}$	$7.56\text{x}10^{-05}$	$3.68\text{x}10^{+00}$	$1.18\text{x}10^{+00}$	$3.03\text{x}10^{-01}$	$5.32\text{x}10^{-02}$	$2.25\text{x}10^{-01}$	$1.16\text{x}10^{-01}$
γ_G	$7.95\text{x}10^{-01}$	$2.18\text{x}10^{-02}$	$8.26\text{x}10^{-01}$	$1.77\text{x}10^{-02}$	$8.68\text{x}10^{-01}$	$2.02\text{x}10^{-02}$	$8.47\text{x}10^{-01}$	$2.86\text{x}10^{-02}$
γ_{GB}	$1.62\text{x}10^{+00}$	$2.05\text{x}10^{-01}$	$1.62\text{x}10^{+00}$	$3.58\text{x}10^{-01}$	$4.24\text{x}10^{-01}$	$6.32\text{x}10^{-02}$	$3.53\text{x}10^{-01}$	$2.34\text{x}10^{-01}$
γ_{GY}	$-2.41\text{x}10^{+00}$	$3.97\text{x}10^{-01}$	$-4.49\text{x}10^{+00}$	$3.70\text{x}10^{-05}$	$-5.00\text{x}10^{+00}$	$3.29\text{x}10^{-05}$	$-5.00\text{x}10^{+00}$	$2.33\text{x}10^{-05}$
γ_{TR}	$7.81\text{x}10^{-01}$	$6.61\text{x}10^{-02}$	$8.44\text{x}10^{-01}$	$4.10\text{x}10^{-02}$	$9.78\text{x}10^{-01}$	$9.71\text{x}10^{-03}$	$9.92\text{x}10^{-01}$	$9.80\text{x}10^{-03}$
γ_{TRB}	$-3.65\text{x}10^{-02}$	$1.20\text{x}10^{-01}$	$-4.28\text{x}10^{-01}$	$3.12\text{x}10^{-01}$	$6.51\text{x}10^{-01}$	$2.07\text{x}10^{-01}$	$-1.35\text{x}10^{-01}$	$2.96\text{x}10^{-01}$
γ_{TRY}	$-4.63\text{x}10^{+00}$	$9.94\text{x}10^{-01}$	$2.47\text{x}10^{+00}$	$9.30\text{x}10^{-01}$	$-5.00\text{x}10^{+00}$	$3.29\text{x}10^{-05}$	$5.00\text{x}10^{+00}$	$1.75\text{x}10^{-05}$
γ_τ	$8.49\text{x}10^{-01}$	$1.26\text{x}10^{-02}$	$8.72\text{x}10^{-01}$	$1.57\text{x}10^{-02}$	$8.78\text{x}10^{-01}$	$1.88\text{x}10^{-02}$	$8.67\text{x}10^{-01}$	$1.88\text{x}10^{-02}$
$\gamma_{\tau B}$	$3.73\text{x}10^{-01}$	$8.31\text{x}10^{-02}$	$2.40\text{x}10^{-01}$	$8.37\text{x}10^{-02}$	$2.00\text{x}10^{-01}$	$4.45\text{x}10^{-02}$	$7.29\text{x}10^{-02}$	$4.57\text{x}10^{-02}$
$\gamma_{\tau Y}$	$-2.60\text{x}10^{+00}$	$2.74\text{x}10^{-01}$	$-2.88\text{x}10^{+00}$	$3.65\text{x}10^{-01}$	$-3.29\text{x}10^{+00}$	$2.34\text{x}10^{-01}$	$-4.54\text{x}10^{+00}$	$5.16\text{x}10^{-01}$

γ_R	8.80x10^{-01}	2.11x10^{-02}	9.05x10^{-01}	1.52x10^{-02}	9.29x10^{-01}	1.64x10^{-02}	9.06x10^{-01}	2.04x10^{-02}
γ_{RP}	1.15x10^{-01}	5.64x10^{-02}	2.91x10^{-02}	1.55x10^{-01}	3.90x10^{-03}	3.07x10^{-02}	6.84x10^{-10}	2.09x10^{-05}
γ_{RY}	7.54x10^{-01}	1.24x10^{-01}	1.17x10^{+00}	3.08x10^{-01}	9.06x10^{-01}	2.12x10^{-01}	8.08x10^{-01}	1.50x10^{-01}
h_C	6.33x10^{-01}	1.80x10^{-05}	6.34x10^{-01}	3.59x10^{-05}	6.29x10^{-01}	2.80x10^{-05}	6.31x10^{-01}	2.23x10^{-05}
h_h	3.50x10^{-01}	1.68x10^{-04}	3.50x10^{-01}	1.24x10^{-04}	3.52x10^{-01}	2.78x10^{-05}	3.45x10^{-01}	2.19x10^{-05}
μ_F	-1.13x10^{-03}	8.15x10^{-03}	-2.17x10^{-03}	7.66x10^{-01}	-5.47x10^{-08}	5.11x10^{+00}	6.36x10^{-08}	6.35x10^{+01}
μ_H	5.79x10^{+00}	1.01x10^{-01}	6.01x10^{+00}	4.32x10^{-01}	5.28x10^{+00}	2.53x10^{-03}	1.08x10^{+00}	8.89x10^{-01}
$\eta_{0,\alpha}$	5.99x10^{-01}	4.00x10^{-05}	5.99x10^{-01}	5.30x10^{-05}	6.00x10^{-01}	4.83x10^{-05}	5.97x10^{-01}	2.23x10^{-05}
$\eta_{0,\beta}$	-2.00x10^{-02}	2.44x10^{-05}	-2.00x10^{-02}	2.15x10^{-05}	-9.33x10^{-03}	2.93x10^{-05}	-1.98x10^{-02}	2.38x10^{-05}
$\eta_{0,BF}$	-8.80x10^{+00}	7.85x10^{-02}	-8.74x10^{+00}	2.96x10^{-01}	-1.26x10^{+01}	4.86x10^{-03}	-3.78x10^{+00}	8.70x10^{-01}
$\eta_{0,BH}$	1.94x10^{+01}	2.12x10^{+00}	1.92x10^{+01}	2.04x10^{+01}	1.95x10^{+01}	4.93x10^{-02}	1.30x10^{+00}	1.03x10^{+00}
$\eta_{0,G}$	-2.00x10^{+00}	5.07x10^{-05}	-2.00x10^{+00}	2.55x10^{-05}	-1.98x10^{+00}	1.02x10^{-03}	-1.95x10^{+00}	1.06x10^{-02}
$\eta_{0,I}$	1.84x10^{+01}	2.22x10^{+00}	1.19x10^{+01}	2.31x10^{+00}	1.21x10^{+01}	2.62x10^{+00}	7.79x10^{+00}	2.46x10^{+00}
$\eta_{0,L}$	2.78x10^{+00}	1.10x10^{-01}	2.84x10^{+00}	9.11x10^{-01}	3.94x10^{+00}	2.81x10^{-05}	4.14x10^{+00}	2.31x10^{-05}
$\eta_{0,M}$	-1.84x10^{+00}	2.15x10^{-01}	-1.71x10^{+00}	9.92x10^{-01}	-2.75x10^{+00}	2.91x10^{-05}	-1.43x10^{+00}	2.28x10^{-05}
$\eta_{0,NX}$	1.18x10^{-01}	3.05x10^{-03}	1.10x10^{-01}	1.57x10^{-02}	1.78x10^{-01}	2.52x10^{-05}	7.42x10^{-02}	2.38x10^{-05}
$\eta_{0,P}$	6.13x10^{+00}	8.96x10^{-02}	6.41x10^{+00}	9.05x10^{-02}	5.33x10^{+00}	1.25x10^{-01}	4.62x10^{+00}	2.11x10^{-01}
$\eta_{0,R}$	8.83x10^{-03}	2.33x10^{-03}	1.14x10^{-02}	1.25x10^{-03}	1.35x10^{-02}	2.71x10^{-05}	8.34x10^{-03}	2.38x10^{-05}
$\eta_{0,\tau}$	6.00x10^{-01}	4.12x10^{-05}	5.99x10^{-01}	5.39x10^{-05}	5.49x10^{-01}	2.80x10^{-05}	4.84x10^{-01}	2.21x10^{-05}
$\eta_{0,TR}$	-1.77x10^{+00}	2.14x10^{-04}	-1.77x10^{+00}	4.02x10^{-05}	-1.98x10^{+00}	1.03x10^{-03}	-2.27x10^{+00}	1.51x10^{-02}
$\eta_{0,Y}$	6.24x10^{-03}	5.76x10^{-05}	6.79x10^{-03}	1.37x10^{-04}	7.03x10^{-03}	2.72x10^{-05}	2.65x10^{-03}	2.38x10^{-05}
$\eta_{1,\alpha}$	9.75x10^{-01}	3.04x10^{-03}	9.74x10^{-01}	2.76x10^{-03}	9.47x10^{-01}	6.77x10^{-03}	9.38x10^{-01}	8.19x10^{-03}
$\eta_{1,\beta}$	2.55x10^{-02}	4.50x10^{-02}	6.06x10^{-02}	7.31x10^{-02}	-1.95x10^{-01}	1.28x10^{-01}	-2.33x10^{-03}	3.00x10^{+00}
$\eta_{1,BF}$	9.06x10^{-01}	9.72x10^{-02}	9.67x10^{-01}	3.73x10^{-02}	8.05x10^{-01}	1.25x10^{-01}	9.72x10^{-01}	2.84x10^{-02}
$\eta_{1,BH}$	4.25x10^{-01}	1.56x10^{-01}	-2.22x10^{-01}	2.25x10^{-01}	-4.09x10^{-02}	3.73x10^{-02}	-1.13x10^{-01}	1.26x10^{-01}
$\eta_{1,G}$	-2.03x10^{-01}	5.43x10^{-02}	-2.65x10^{-01}	1.70x10^{-01}	-1.60x10^{-01}	1.30x10^{-01}	-1.61x10^{-02}	2.88x10^{+00}
$\eta_{1,I}$	-2.88x10^{-01}	2.21x10^{-01}	-1.84x10^{-01}	6.03x10^{-02}	1.98x10^{-03}	3.25x10^{-02}	5.54x10^{-03}	2.11x10^{+00}
$\eta_{1,L}$	1.08x10^{-01}	9.81x10^{-02}	-5.12x10^{-02}	2.03x10^{-01}	9.98x10^{-01}	4.72x10^{-05}	9.98x10^{-01}	1.82x10^{-05}
$\eta_{1,M}$	9.81x10^{-01}	1.19x10^{-02}	9.41x10^{-01}	1.18x10^{-02}	9.88x10^{-01}	3.20x10^{-03}	-2.01x10^{-01}	5.52x10^{-01}
$\eta_{1,NX}$	9.86x10^{-01}	3.42x10^{-03}	9.90x10^{-01}	2.17x10^{-03}	9.24x10^{-01}	2.24x10^{-02}	9.93x10^{-01}	9.42x10^{-04}
$\eta_{1,P}$	-9.32x10^{-03}	6.81x10^{-03}	-1.21x10^{-02}	7.11x10^{-03}	-4.51x10^{-05}	2.53x10^{-02}	5.55x10^{-04}	2.94x10^{+00}
$\eta_{1,R}$	5.10x10^{-01}	7.57x10^{-02}	4.06x10^{-01}	7.98x10^{-02}	4.22x10^{-01}	8.86x10^{-02}	4.50x10^{-01}	8.65x10^{-02}
$\eta_{1,\tau}$	5.61x10^{-01}	5.98x10^{-02}	5.64x10^{-01}	2.08x10^{-01}	6.29x10^{-02}	6.47x10^{-02}	1.42x10^{-01}	1.31x10^{-01}
$\eta_{1,TR}$	9.83x10^{-01}	9.17x10^{-03}	9.93x10^{-01}	5.18x10^{-03}	-1.22x10^{-02}	4.53x10^{-02}	-6.07x10^{-02}	2.44x10^{+00}
$\eta_{1,Y}$	2.17x10^{-01}	2.97x10^{-02}	1.66x10^{-01}	2.50x10^{-02}	1.86x10^{-01}	8.40x10^{-02}	3.08x10^{-02}	1.03x10^{-01}
ω_C	1.17x10^{+00}	4.96x10^{-02}	1.15x10^{+00}	2.16x10^{-01}	1.21x10^{+00}	2.88x10^{-05}	1.20x10^{+00}	2.27x10^{-05}
ω_L	1.58x10^{-01}	5.57x10^{-02}	1.85x10^{-02}	8.34x10^{-02}	2.85x10^{-03}	2.58x10^{-05}	3.28x10^{-07}	2.09x10^{-05}
δ	1.00x10^{-02}	2.28x10^{-05}	1.00x10^{-02}	2.11x10^{-05}	1.00x10^{-02}	2.98x10^{-05}	2.70x10^{-02}	2.38x10^{-05}
θ	7.66x10^{+00}	7.65x10^{-04}	7.56x10^{+00}	5.04x10^{-02}	6.35x10^{+00}	2.49x10^{-05}	1.13x10^{+01}	2.32x10^{-05}

Table A3. *p*-value of out-of-sample forecast quality test vs. AR(1)

	VAR(1)	DSGE QKF no meas.er.	DSGE QKF meas.er.	DSGE linear no meas.er.	DSGE linear meas.er.
obsC(+1)	2.3%	0.1%	18.2%	0.5%	1.0%
obsG(+1)	96.8%	100.0%	99.9%	99.3%	98.1%
obsY(+1)	75.3%	34.7%	82.4%	43.6%	13.9%
obsP(+1)	25.0%	75.2%	96.1%	8.1%	32.1%
obsWL(+1)	20.1%	96.2%	84.8%	45.1%	59.4%
obsR(+1)	11.7%	100.0%	25.0%	100.0%	28.4%
obsSTR(+1)	99.0%	0.0%	28.9%	0.0%	27.7%
obsC(+2)	7.2%	0.6%	23.4%	2.5%	3.9%
obsG(+2)	89.2%	99.5%	98.8%	96.1%	92.8%
obsY(+2)	66.7%	36.3%	76.6%	43.5%	18.0%
obsP(+2)	26.4%	66.5%	90.5%	12.6%	32.9%
obsWL(+2)	24.9%	88.9%	73.9%	42.6%	52.8%
obsR(+2)	18.2%	100.0%	28.5%	100.0%	31.1%
obsSTR(+2)	98.6%	0.0%	33.4%	0.0%	32.2%
obsC(+3)	11.3%	1.9%	26.7%	5.3%	7.3%
obsG(+3)	82.9%	98.2%	96.6%	92.1%	87.6%
obsY(+3)	65.3%	37.6%	74.6%	44.3%	20.2%
obsP(+3)	27.9%	62.3%	85.9%	15.5%	33.7%
obsWL(+3)	27.8%	82.6%	67.9%	42.2%	50.3%
obsR(+3)	22.4%	100.0%	31.2%	100.0%	33.2%
obsSTR(+3)	98.4%	0.0%	36.1%	0.0%	35.0%
obsC(+4)	14.7%	3.6%	29.1%	8.0%	10.1%
obsG(+4)	78.5%	96.6%	94.1%	88.5%	83.5%
obsY(+4)	66.3%	39.4%	74.5%	45.8%	22.1%
obsP(+4)	29.3%	60.8%	83.2%	17.3%	34.7%
obsWL(+4)	30.0%	78.2%	64.4%	42.3%	49.1%
obsR(+4)	25.4%	100.0%	32.9%	100.0%	34.8%
obsSTR(+4)	98.2%	0.0%	38.5%	0.0%	37.3%
Forecasts significantly better than AR(1)	1	8	0	6	2

Note: The alternative hypothesis is that the unrestricted models perform better than the AR(1) model.

Table A4. ***p*****-value of out-of-sample forecast quality test vs. VAR(1)**

	DSGE QKF no meas.er.	DSGE QKF meas.er.	DSGE linear no meas.er.	DSGE linear meas.er.
obsC(+1)	99.7%	9.9%	95.0%	89.3%
obsG(+1)	0.0%	0.1%	2.9%	16.9%
obsY(+1)	89.3%	24.6%	83.3%	97.8%
obsP(+1)	3.3%	0.1%	91.4%	49.8%
obsWL(+1)	0.0%	0.4%	18.3%	6.7%
obsR(+1)	0.0%	32.8%	0.0%	26.1%
obsSTR(+1)	100.0%	99.9%	100.0%	99.9%
obsC(+2)	99.0%	13.8%	91.7%	85.0%
obsG(+2)	0.1%	0.6%	5.7%	20.2%
obsY(+2)	85.6%	24.8%	79.0%	96.2%
obsP(+2)	5.7%	0.3%	87.4%	46.6%
obsWL(+2)	0.2%	1.5%	21.5%	10.3%
obsR(+2)	0.0%	33.3%	0.0%	27.9%
obsSTR(+2)	100.0%	99.8%	100.0%	99.8%
obsC(+3)	97.8%	17.7%	88.1%	81.0%
obsG(+3)	0.3%	1.7%	9.1%	24.1%
obsY(+3)	83.9%	26.3%	77.3%	95.2%
obsP(+3)	7.8%	0.7%	84.6%	46.6%
obsWL(+3)	0.6%	3.1%	25.2%	14.0%
obsR(+3)	0.0%	35.5%	0.0%	30.8%
obsSTR(+3)	100.0%	99.7%	100.0%	99.7%
obsC(+4)	96.3%	21.0%	85.2%	78.0%
obsG(+4)	0.8%	3.0%	11.9%	27.1%
obsY(+4)	82.9%	27.2%	76.3%	94.5%
obsP(+4)	9.4%	1.0%	82.7%	47.0%
obsWL(+4)	1.2%	5.0%	28.0%	17.3%
obsR(+4)	0.0%	37.4%	0.0%	33.2%
obsSTR(+4)	100.0%	99.6%	100.0%	99.6%
Forecasts insignificantly worse than VAR(1)	15	17	23	28

Note: The alternative hypothesis is that the unrestricted models perform better than the VAR(1) model.

Table A5. ***p*****-value of out-of-sample forecast quality test for different DSGE models**

H0:	DSGE QKF no meas.er. equal to model;	DSGE QKF no meas.er. equal to model;	DSGE QKF no meas.er. equal to model;	DSGE QKF meas.er. equal to model;	DSGE line no meas.er. equal to model;	DSGE line no meas.er. equal to model;
H1:	**DSGE QKF meas.er. is better**	**DSGE line no meas.er. is better**	**DSGE line meas.er. is better**	**DSGE line meas.er. is better**	**DSGE QKF meas.er. is better**	**DSGE line meas.er. is better**
obsC(+1)	29.3%	95.0%	90.6%	99.3%	0.1%	6.0%
obsG(+1)	2.0%	15.0%	35.4%	0.5%	79.7%	99.1%
obsY(+1)	45.5%	86.4%	97.5%	81.7%	7.8%	78.3%
obsP(+1)	1.5%	92.0%	63.3%	57.8%	8.0%	89.5%
obsWL(+1)	5.1%	38.4%	23.6%	27.6%	87.0%	97.5%
obsR(+1)	50.5%	0.0%	45.4%	65.9%	100.0%	100.0%
obsSTR(+1)	99.8%	100.0%	99.9%	63.7%	0.0%	0.0%
obsC(+2)	32.8%	91.4%	86.1%	98.0%	0.6%	10.0%
obsG(+2)	4.4%	19.6%	37.5%	1.6%	75.2%	97.6%
obsY(+2)	46.3%	82.8%	95.7%	78.2%	10.8%	74.8%
obsP(+2)	3.2%	88.2%	61.3%	56.6%	11.8%	85.3%
obsWL(+2)	8.8%	40.1%	27.6%	31.3%	82.3%	94.9%
obsR(+2)	50.4%	0.0%	46.2%	63.5%	100.0%	100.0%
obsSTR(+2)	99.6%	100.0%	99.7%	62.3%	0.0%	0.0%
obsC(+3)	35.5%	87.5%	81.8%	95.8%	1.7%	14.0%
obsG(+3)	7.4%	23.8%	39.5%	3.5%	71.4%	95.4%
obsY(+3)	46.6%	80.9%	94.4%	76.4%	12.6%	73.3%
obsP(+3)	5.1%	85.1%	59.9%	55.8%	14.8%	82.3%
obsWL(+3)	12.6%	41.9%	30.9%	34.2%	78.2%	91.7%
obsR(+3)	50.3%	0.0%	46.8%	61.4%	100.0%	100.0%
obsSTR(+3)	99.5%	100.0%	99.6%	62.0%	0.0%	0.0%
obsC(+4)	37.3%	84.3%	78.6%	93.4%	3.2%	17.4%
obsG(+4)	10.3%	26.7%	41.0%	5.6%	68.9%	93.0%
obsY(+4)	46.6%	79.8%	93.7%	75.5%	13.6%	72.5%
obsP(+4)	6.5%	83.3%	59.3%	55.4%	16.7%	80.2%
obsWL(+4)	15.9%	42.7%	33.3%	36.3%	75.0%	88.8%
obsR(+4)	50.4%	0.0%	47.3%	60.0%	100.0%	100.0%
obsSTR(+4)	99.4%	100.0%	99.4%	61.5%	0.0%	0.0%
Number times H0 rejected	4	4	0	3	8	4

The model equations are as follows:

$$e^{z_{\beta,t+1}+(1-\omega_C)*z_{Y,t+1}}\lambda_{HB,t+1}e^{r_t-z_{Y,t+1}-p_{t+1}}+2(-b_{H,t}+\mu_B)e^{z_{BH,t}}-\lambda_{HB,t}=0 \quad \text{(A1)}$$

$$(e^{c_t}-h_C e^{h_{t-1}+z_{Y,t}})^{-\omega_C}e^{c_t}-\lambda_{HB,t}e^{c_t}=0 \quad \text{(A2)}$$

$$-e^{z_{L,t}+l_t+l_t\omega_L}+\lambda_{HB,t}(1-\tau_t)e^{w_t+l_t}=0 \quad \text{(A3)}$$

$$e^{z_{\beta,t+1}+(1-\omega_C)z_{Y,t+1}}\lambda_{HB,t+1}e^{m_t-p_{t+1}-z_{Y,t+1}}+e^{z_{M,t}+m_t}-\lambda_{HB,t}e^{m_t}=0 \quad \text{(A4)}$$

$$-e^{z_{\beta,t+1}+(1-\omega_C)z_{Y,t+1}}\lambda_{HB,t+1}(-d_{F,t+1}-e^{s_{t+1}}))-\lambda_{HB,t}e^{s_t}=0 \quad \text{(A5)}$$

$$\begin{aligned}&e^{c_t}+e^{s_t}+e^{m_t}+b_{H,t}-(1-\tau_t)e^{w_t+l_t}-e^{m_{t-1}-p_t-z_{Y,t}}\\&-b_{H,t-1}e^{r_{t-1}-z_{Y,t}-p_t}-(d_{F,t}+e^{s_t})-e^{\tau_{TR,t}}=0\end{aligned} \quad \text{(A6)}$$

$$(-b_{F,t}+z_{BF,t})=0 \quad \text{(A7)}$$

$$1-\lambda_{FB,t}=0 \quad \text{(A8)}$$

$$\begin{aligned}&-2e^{-r_t+z_{Y,t+1}+p_{t+1}}(-e^{p_{t+1}}+e^{\bar{p}})e^{y_{F,t+1}+z_{P,t+1}+p_{t+1}}\\&-e^{y_{F,t}+z_{P,t}}(e^{p_t}-e^{\bar{p}})^2+(-2e^{p_t}+2e^{\bar{p}})e^{y_{F,t}+z_{P,t}+p_t}\\&-e^{y_{F,t}+z_{BF,t}}(b_{F,t}-\overline{b_F})^2+e^{y_{F,t}}-\theta\lambda_{FD,t}=0\end{aligned} \quad \text{(A9)}$$

$$-e^{y_{F,t}+z_{P,t}}(e^{p_t}-e^{\bar{p}})^2-e^{y_{F,t}+z_{BF,t}}(b_{F,t}-\overline{b_F})^2+e^{y_{F,t}}-\lambda_{FD,t}-\lambda_{FP,t}=0 \quad \text{(A10)}$$

$$-e^{w_t+l_t}+z_{\alpha,t}\lambda_{FP,t}=0 \quad \text{(A11)}$$

$$e^{-r_t+z_{Y,t+1}+p_{t+1}}((1-\sigma)e^{k_t-z_{Y,t+1}}\lambda_{FK,t+1}-(-1+z_{\alpha,t})\lambda_{FP,t+1})=e^{k_t}\lambda_{FK,t} \quad \text{(A12)}$$

$$\begin{aligned}&2e^{-r_t+z_{Y,t+1}+p_{t+1}}e^{i_{t+1}}z_{I,t+1}(e^{i_{t+1}-i_t+z_{Y,t+1}}-e^{\eta_{0,Y}})e^{i_{t+1}-i_t+z_{Y,t+1}}\lambda_{FK,t+1}-e^{i_t}=\\&=(-e^{i_t}(1-z_{I,t}(e^{i_t-i_{t-1}+z_{Y,t}}-e^{\eta_{0,Y}})^2)+2e^{i_t}z_{I,t}(e^{i_t-i_{t-1}+z_{Y,t}}-e^{\eta_{0,Y}})e^{i_t-i_{t-1}+z_{Y,t}})\lambda_{FK,t}\end{aligned} \quad \text{(A13)}$$

$$d_{F,t}+e^{i_t}+e^{w_t+l_t}+b_{F,t}-e^{y_{F,t}}-b_{F,t-1}e^{r_{t-1}-z_{Y,t}-p_t}=0 \quad \text{(A14)}$$

$$e^{k_t}-(1-\sigma)e^{k_{t-1}-z_{Y,t}}-e^{i_t}(1-z_{I,t}(e^{i_t-i_{t-1}+z_{Y,t}}-e^{\eta_{0.Y}})^2)=0 \quad \text{(A15)}$$

$$y_{F,t}-y_{D,t}=0 \quad \text{(A16)}$$

$$y_{F,t}-z_{\alpha,t}l_t-(1-z_{\alpha,t})(k_{t-1}-z_{Y,t})=0 \quad \text{(A17)}$$

$$r_t=\gamma_R*r_{t-1}+(1-\gamma_R)(\gamma_{RP}(p_t-\bar{p})+\gamma_{RY}(y_t-\bar{y})+z_{R,t}) \quad \text{(A18)}$$

$$\tau_t=\gamma_T\tau_{t-1}+(1-\gamma_T)(\gamma_{TY}(y_t-\bar{y})+\gamma_{TB}(b_{G,t}-\overline{b_G})+z_{\tau,t}) \quad \text{(A19)}$$

$$(g_t-y_{D,t})=\gamma_G(g_{t-1}-y_{D,t-1})+(1-\gamma_G)(\gamma_{GY}(y_t-\bar{y})+\gamma_{GB}(b_{G,t}-\overline{b_G})+z_{G,t}) \quad \text{(A20)}$$

$$(\tau_{TR,t} - y_{D,t}) = \gamma_{TR}(\tau_{TR,t-1} - y_{D,t-1}) + (1-\gamma_{TR})(\gamma_{TRY}(y_t - \overline{y}) + \gamma_{TRB}(b_{G,t} - \overline{b_G}) + z_{TR,t}) \quad \text{(A21)}$$

$$nx_t = \gamma_{NX} nx_{t-1} + (1-\gamma_{NX})(\gamma_{NXB}(b_{W,t} - \overline{b_W}) + z_{NX,t}) \quad \text{(A22)}$$

$$nx_t + b_{W,t} = e^{r_{t-1} - p_t - z_{Y,t}} b_{W,t-1} \quad \text{(A23)}$$

$$b_{G,t} + b_{H,t} + b_{F,t} + b_{W,t} = 0 \quad \text{(A24)}$$

$$e^{y_{F,t}} = e^{c_t} + e^{i_t} + e^{g_t} + nx_t \quad \text{(A25)}$$

$$0 = (1-\theta) p_{F,t} \quad \text{(A26)}$$

$$y = y_{D,t} - y_{D,t-1} + z_{Y,t} \quad \text{(A27)}$$

$$e^{h_t - z_{Y,t}} = h_h e^{h_{t-1}} + e^{c_{t-1}} \quad \text{(A28)}$$

$$x_t = 1 \quad \text{(A29)}$$