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Transit network design with meta-heuristic algorithms and agent based simulation

Obiora A. Nnene * Johan W. Joubert ** Mark H.P. Zuidgeest *

* Centre for Transport Studies, Faculty of Engineering and the Built Environment, University of Cape Town, Private Bag X3 Rondebosch 7701, South Africa (e-mail: nnnobi002@myuct.ac.za)
** Centre for Transport Development, Industrial Engineering, University of Pretoria, Pretoria, 0083, South Africa

Abstract:
In this work, a transit network design problem is presented. The problem is identified as a typical large-scale complex system. Subsequently, it is decomposed into its sub-components. The first two sub-components, which encompass the network design and frequency setting problems, are then tackled by means of an innovative solution framework that combines a genetic algorithm with agent-based travel demand modelling. An analysis of results obtained from applying the proposed method to different testing scenarios shows that it is capable of designing transit networks that address the individual and collective perspectives of different stakeholders. Hence it can be used as a viable decision support tool for policy makers in the transportation network sector.

Keywords: transit network design, genetic algorithm, agent based modelling, large scale complex system, optimisation

1. INTRODUCTION

Urban transportation networks provide spatial connection between points of travel demand production or attraction within a given geographical space. They also facilitate the efficient movement of people, goods and information. When considered as a large-scale complex system, it is easy to observe such features as the presence of a large number of human agents with their stochastic decision making and interactions on the network; non-linear phenomenon like congestion where a localised incident such as a vehicle crash can lead to network wide travel delays; and interconnectivity between the network and other external dynamic systems like the environment or economy [Rodrigue et al., 2013]. Owing to these features, designing a transportation network is a very arduous endeavour that will be implausible to consider as a single system. A more realistic approach to tackling them would be to decompose the system into more tractable sub-systems, thereby allowing the easier handling of the latter [Filip and Leiviska, 2009]. Ceder [2015] identifies four sub-systems of the urban transportation network design problem (UTNDP) as follows: route design, frequency setting, vehicle crew assignment and vehicle scheduling. This work focuses on the first two sub-problems, namely route design and frequency setting. The first deals with the provision of transportation network routes relative to their configuration and level of demand utilisation. The second sub-problem focuses on providing schedules that would enable operators satisfy the revealed demand for travel on the routes. Since there should be a feedback loop between an urban transportation route and its service frequency in order to meet the travel demand, the two problems should be solved simultaneously. However, the intractable nature of the problem and drawbacks of conventional travel demand modelling has limited the ability of researchers to achieve this goal. Specifically, modelling travel demand which by nature, is stochastic with static models like the four step travel demand model; and the difficulty of encoding both elements (transportation network and operational frequencies) as a single decision variable within a solution scheme such as meta-heuristic algorithms have impacted the possibility of solving this problem. Consequently, a sequential ideation of the problems is often done in the literature [Ceder, 2007].

Therefore, the goal of this work is to propose a network design approach that combines a genetic algorithm with an agent-driven travel demand model simulation as a way of addressing these limitations, in a transit setting. The agent-based model (ABM) procedure replaces the traditional conventional traffic assignment that has been used in nearly all transit network design efforts in the literature. The case for using of ABM is made owing to the fact that it espouses the modelling of agents' activity and by consequence the trips connecting these activities rather than looking at trips in isolation. The resulting network solutions will then be analysed; looking closely at how it impacts stakeholders such as the *users* and *operators* on the network. The remainder of this paper is structured as follows: section two discusses the theoretical background for the proposed solution method. Section three presents the mathematical model upon which the problem is set, while section four outlines the component algorithms of the solution technique. In section five, the

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proposed solution is applied to a large scale network in the city of Cape Town South Africa and its results discussed; especially as it affects passengers and service operators. In the final section, conclusions on the work are drawn.

2. LITERATURE

2.1 Introduction

The design of urban transportation networks is broadly classified as a transit network design problem (TNDP). In the literature, the problem deals with the optimised design of transit route networks and the scheduling of their operational frequencies. It is often presented as the minimisation of transit costs or the maximisation of network utilities subject to some feasibility constraints on available resources. This is done with the aim of achieving a compromise network solution for all stakeholder across the transportation area being analysed. Normally, the problem is represented by an objective function which is a mathematical expression of the stakeholder goals to be optimised. Newell [1979] first identified the TNDP as a *non-linear* programming problem. It is further classified as *np-hard* by Fan and Machemehl [2004]. Some features of the TNDP include: one or more decision variables, an objective or cost function, a user behaviour sub routine for evaluating passenger behaviour on the network, transit demand which is usually represented by an origin-destination (OD) matrix (replaced by passengers' daily plans in this paper) and feasibility constraints.

Two major ways of solving the TNDP in the literature are conventional and heuristic approaches. The former, is able to find a unique solution to the TNDP using analytical algorithms. The major criticism of this solution approach is that they are hardly applicable to real life large scale transit network problems, since, getting a closed form expression for the objective function is computationally too expensive [Chakroborty, 2003]. On the other hand, heuristics by nature, cannot search for an exact solution to the TNDP. They rather obtain suitable approximate solutions of a global optimum solution—assuming the latter exists. In any case, their solutions are normally considered acceptable give the relatively smaller amount of time they use to find the solution. For this reason heuristics, especially meta-heuristic algorithms, have been widely adopted with positive outcome in the solution of large scale TNDP(s).

Some works in the literature, which, demonstrate the application of meta-heuristics to the TNDP are Chen et al. [2017], Cipriani et al. [2012], Pattnaik et al. [1998].

In Pattnaik et al. [1998], the authors formulated a two-stage model involving the generation of feasible routes and determining the best options among them. Their objective function minimizes a total travel cost expression, which, is a summation of the total travel time representing the user perspective and total bus kilometres, which, denotes the operator perspective. In stage one they first generated feasible transit network routes with heuristic procedure. The authors then assign travel demand, with the aid of a heuristic that was used in Baaj and Mahmassani [1990]. The heuristic assigned travel demand with priority given to routes based on the number of transfers on the route. In the second stage of the model, the authors implemented a genetic algorithm (GA) to search for an optimized solution. Binary variables were used to denote each route and the GA was coded using two different techniques namely, fixed and variable string coding. Their model was tested on a small network in south India.

Cipriani et al. [2012], proposed a two stage model comprising a route generation phase and a genetic algorithm. Their model dealt with a simultaneous determination of a sub-optimal set of routes and their frequencies. The objective function was to minimize total costs, while, their decision variables were route network and their frequencies. Constraints used in the model were route length, load capacity and maximum line frequency. In the first phase of their model, three types of routes (types A, B and C) were initially generated from the study network using different criteria. A-type routes connected high demand nodes and addressed the user perspective of minimized transfers and increased trip directness. B-type routes on the other hand, represented the operator's perspective. These routes connected major transit centres like rail stations. Their travel demand assignment was done with a flow concentration procedure used earlier by Carrese and Gori [2002]. This is essentially an iterative All or Nothing traffic assignment, which aggregates demand volumes on links, and those with the highest volumes considered the best. C-type routes in addition were the existing bus network routes. In the second stage of the model, a GA implemented in parallel, was used to get the optimal network from the pool of generated routes.

Lastly, Chen et al. [2017] proposed a model to optimise suburban routes with sparse travel demand for airport access. They sought to minimise total access time to the system. They chose pick-up locations and the visiting sequence of the locations as their decision variables. The problem was modelled as a discrete optimisation. Their travel demand model was similar to the flow concentration technique first used by Cipriani et al. [2012]. They developed a small case study of the problem and solved it with a dynamic programming approach. However, to solve a more realistic life-size network problem, a meta-heuristic—artificial bee colony algorithm was used. The work was applied to two suburban bus routes in Melbourne Australia.

In this paper, the optimised design of a transit network and its operational frequencies is achieved by combining a typical meta-heuristic—GA, with agent-based travel demand modelling.

2.2 Genetic Algorithm

Genetic algorithms are meta-heuristic search procedures that can be used to find efficient solutions to optimisation problems. In the literature, they are classified as *bio-inspired* algorithms because their operations mimic the principle of natural genetics. GAs work by enabling the realisation of newer and presumably better generations of solutions from existing ones. A typical genetic algorithm framework consists of a population of solutions or chromosomes. Each chromosome is made up of genes that depend on the particular representation of the chromosome. Furthermore, the algorithm has operators, namely *selection*, *crossover* and *mutation*. The action of these operators on the current population gives rise to offspring solutions which are generally assumed to be fitter or perform better than their *progenitors*.

In the context of the transit network design problem, individual network solutions are the chromosomes that make up the population. The genes in a chromosome are the routes in the network. The best performing chromosome or network in the population represents a global optimum solution. However, in very difficult problems like the TNDP it is not feasible to determine that a solution is the global optimum. Therefore, efficient local optimum solutions obtained within a reasonable time frame are generally considered as acceptable. An initial population of chromosomes is usually set in the GA. This is followed by the evaluation of each individual in the population to determine their fitness score or objective function value. Individuals with the best scores then have a higher possibility of being selected as parents for the reproduction of offspring. These processes are achieved by the actions of the earlier mentioned genetic operators. The procedure continues iteratively, until a predefined termination criteria is attained.

To apply a GA to a problem, the chromosomes need to be encoded in a way that is applicable to the operators of the GA. Taking a historic look at the literature, string and binary representation are the most common representations used when solving the TNDP with GAs. Over the years, GA-based models have become one of the most efficient methods for solving the TNDP [Buba and Lee, 2018, Kepaptsoglou and Karlaftis, 2009, Nnene et al., 2017].

2.3 Agent-based modelling

At this stage it is important to briefly discuss how passenger behaviour which has been identified as a major component in the proposed network design solution framework will be modelled. An agent based travel demand simulation model is used; the technique is based on the microsimulation of people's activities in a geographical area. Travel demand is, hence, generated by the trips connecting those activities. It was conceived in an attempt to overcome the limitations of trip based models such as the four step model, which were increasingly unable to respond to dynamic and more complex transport planning scenarios [Castiglione et al., 2015]. The foundational assumption employed in the model, is that an individual's decision to engage in an activity is based on the following factors: the agent or decision maker; the set of activity options or choice sets; and certain heuristic rules, which, govern the choice sets and define the boundaries outside which they become unrealistic. The two major components of an agent-based travel demand model are 1) an activity based demand generation and 2) a dynamic traffic assignment. Activity based demand generation consists of creating a synthetic population and their daily activity schedule using the concept of a *random realisation* presented in Balmer et al. [2006], which basically means the creation of a virtual population that share the same demographic structure as that of the survey or census of a real population. After generating the demand, a dynamic traffic assignment is used to distribute the generated demand on the basis of users route choices. This method of traffic assignment was developed as an improvement over the static assignment, with the major enhancement being its ability to generate time dependent traffic or link volumes see [Friedrich et al., 2000, Kaufman et al., 1991].

2.4 Innovation

The work discussed in this paper, draws lessons from the earlier discussed [Cipriani et al., 2012, Pattnaik et al., 1998]. However, two major distinctions from those works are; 1) conventional travel demand models like the four step model used for travel behaviour modelling in [Cipriani et al., 2012], is replaced with an agent based model in this work. 2) our definition of a more robust encoding for the decision variable encoding, than that of [Pattnaik et al., 1998]. Firstly, by using an agent based simulation, for user behaviour modelling and evaluation of solutions within the GA solution framework, we will more correctly describe the stochastic behaviour of stakeholders and more accurately predict the decisions they take in response to the changes that occur on the network in real time. Therefore, since, the problem's objective functions are evaluated based on the outcome of modelling people travel behaviour, the effect is that the final network solutions, would be better suited to respond to the travel demand in the transportation area being studied. Secondly, In this paper an innovative encoding is used that is based on a java script object notation (JSON) data structure [Crockford, 2011]. This representation is markedly different from other works in the literature as it accommodates the encoding of each network with a detailed operational schedule. The advantage of this approach, is that it enables the simultaneous handling of the route network design and frequency setting sub-problems of the TNDP, which, have previously been handled consecutively (see section 1).

3. MATHEMATICAL MODEL

In this paper, the network is represented as a graph $\mathbf{G} = (\mathbf{N}, \mathbf{L})$ which is a multi-connection of a finite sets of $\mathbf{n} \in \mathbf{N}$ nodes and $\mathbf{l} \in \mathbf{L}$ links. The objective function in "(1)" is a linear summation of user and operator costs see "(2)" and "(3)" below. Users view cost in terms of their total travel time, while, operators are concerned with the total operational cost. Therefore, by minimising this objective function, the total cost incurred on the network will be optimised for both stakeholders: users and operators.

$$Z = z_1 + z_2 \tag{1}$$

$$z_1 = W_1(a * \sum_{r=1} t_r q_r) \tag{2}$$

$$z_2 = W_2(b * \sum_{r=1} l_r f_r + e * \sum_{r=1} t_r f_r) \tag{3}$$

subject to agent based stochastic user equilibrium on the network:

$$q_r^n = \tau(c(x\{q_r^n\})) \tag{4}$$

and some feasibility conditions on route length, frequency and vehicle fleet:

$$l_{min} \leq l_r \leq l_{max} \tag{5}$$

$$f_{min} \leq f_r \leq f_{max} \tag{6}$$

$$r_{tot} \leq R \tag{7}$$

Where:
Z = objective function;
z_1 = user cost function;
W_1 = weight factor for user cost;
a = monetary unit value for user travel time;
t_r = travel time on route r;
q_r = total travel demand on route r;
z_2 = operator cost function;
W_2 = weight factor for operator cost;
b = monetary unit value for vehicle mileage;
l_r = length of route r;
f_r = frequency on route r;
e = monetary unit value for vehicle operating time;
q_r^n = individual agent demand on the route r;
n = index of the agent;
τ = agent based probabilistic route choice model;
$c(x)$ = network costs;
$\{q_r^n\}$ = all individual agent route demands on the network;
l_{min} = minimum route length;
l_{max} = maximum route length;
f_{min} = minimum frequency value;
f_{max} = maximum frequency value;
r_{tot} = number of designed routes;
R = maximum specified number of routes;

The objectives are subject to agent-based stochastic user equilibrium, which, describes the simulation of the individual traveller's behaviour on the network represented by "(4)". This approach of modelling travel behaviour extends the traditional stochastic user equilibrium because rather than model flows on routes in the form of productions and attractions, each individual traveller's demand and behaviour is modelled. Furthermore, the route and mode choices used in the traditional user equilibrium is expanded to include other dimensions such as destination choice. Lastly, a stochastic network loading is utilised with time dependent trip departure times.

Other feasibility constraints for the model are those on, route length, frequency and vehicle fleet size which are seen in "(5)" to "(7)". These are put in place to define the allowable limiting conditions for the allocation of resources on the network. Equation (5), is a route length constraint introduced to specify the upper and lower bounds outside which it would be illogical to operate a bus service. Such constraint, prevents the algorithm from proposing to operate a bus service on a route where walking is preferred due to its short distance, or operating one on routes that are extremely long which prevents keeping adequate bus schedules Cipriani et al. [2012]. Equation (6), is the frequency feasibility constraint introduced to represent the maximum and minimum operable frequency on each route within the bus network. It is usually dependent on the available fleet size and transit demand for each route. Equation (7), is a constraint on the maximum number of routes, which is generally determined by transit authorities who stipulate the size of their network or number of routes. In practice this constraint is dictated by the available resources at the disposal of the authorities to operate the routes.

4. SOLUTION PROCEDURE

The proposed solution is a three step model that involves: a heuristic network generation technique, an agent based travel demand simulation network analysis procedure and a GA. The generation step, involves creating feasible network options from which an initial population is selected by the GA. In the analysis phase of the, an agent based simulation model known as MATSim [Horni et al., 2016] is used to evaluate each network solution. Lastly, a search of the solutions is done with a genetic algorithm to obtain an efficient solution.

4.1 Network Generation

The input for this stage includes: nodes of an existing transit network, minimum and maximum route length; and the number of routes per network. The network generation heuristic first reads in the nodes data, it then obtains the shortest paths between all origin destination pairs in the data using a k-shortest path algorithm [Yen, 1971]. In this way, all possible routes are enumerated between the origin destination pairs. Each path must then be checked for the route length feasibility condition. Lastly, networks are built by randomly choosing a user specified number of shortest paths from the created set. This process is repeated, until the user specified number or pool of *feasible* candidate network solutions are generated. Using this process, a pool of 1500 feasible networks were generated. Each network is allowed to contain a user-specified number of routes.

4.2 Network Analysis

This step of the model involves setting up a MATSim simulation. In the introductory part of this section, it was indicated that MATSim is used to analyse the network solutions. Its inputs include the network selected by the GA, a synthetic population of agents and their travel demand or (24-hour activity plan), which, is created from the fare collection data of an existing transit service, with the aid of a heuristic. Other inputs are, an initial schedule of transit operations on the routes of the network, comprising a timetable—detailed fleet schedule and vehicle departures. The final input is the fleet of transit vehicles that will operate the schedules.

MATSim works, by iteratively, simulating users initial demand and optimising them in three steps namely *execution*, *scoring* and *replanning*. Execution, involves simulating the agent's plans in an efficient queue-based simulation. Two factors that influence the agent's travel behaviour during execution, are the start time of their activities, and the space they occupy on the network on the way to their activity locations. The latter impacts how an agent executes their plan, as a likely build up of congestion on a route, could delay the travel time of agents that would have used the route. After execution, the performance of the agents' plans are scored using a utility function, that allocates values to different time components such as waiting time, travel time and time spent on the activity. The function describes the agent's experience on the network. It also measures, how well an agent's plan performed. After the plan scores are obtained, the agent database is updated. Lastly, about 10% of the agents are allowed to *replan* or modify their original plan. This makes it possible for an agent to improve its plan in subsequent iterations. During the simulation, the agents' actions are written to an events file. When the simulation ends, the events file are analysed. The objective function, of the transit network design problem, is then evaluated from the simulation results. The obtained score, is used in the GA in step three of the model to determine the survival of the feasible solutions.

4.3 Solution Search

The genetic algorithm (GA) starts by randomly choosing 100 networks from the pool of feasible networks created in section 4.1. These are then initialised as the first population. The solutions are encoded in a JSON format. This facilitates the easy manipulation of the transit schedule file that is originally in XML format. However, this has the implication that the GA operators have to be customised to enable them manipulate the JSON representation. After defining the appropriate encoding scheme, the solutions are evaluated with the simulation model in section 4.2 and assigned their respective fitness values which are obtained by analysing the result of the simulation. Subsequently, pairs of parent solutions are drawn from the population based on their fitness with the GA's selection operator. The other operators known as crossover and mutation are then used to manipulate the selected pairs leading to the reproduction of a new generation of offspring (networks). A custom single point crossover and a polynomial mutation was used in this work. The termination criteria is the number of generations. Consequently, the offspring obtained at the final generation represents the optimised solution to the transit network design problem at hand.

5. TESTING AND RESULTS

To determine the ability of the proposed model solution to design a large-scale public transportation network, it is applied to the improvement of a bus rapid transit network in the City of Cape Town, South Africa. The network consists of 472 nodes and, currently, about 46 operational routes. This test involves three network design scenarios in which the relative weight factors for the user and operator components of the objective function in "(2)" and "(3)" are altered. Results obtained of various performance indicators pertaining to the network stakeholders, are presented in the tables 1 and 2 below. Subsequently, the results are discussed in this section.

Table 1. Aggregate transit network performance indicators for the identified scenarios

Indicators	Users	Operators	Balanced
Total demand (pax)	38569	38569	38569
Satis. demand (pax)	34216	29590	31654
Unsat. demand (pax)	4353	8979	6915
Utilisation (%)	88.71	76.72	82.07
Veh. dist (km)	45215.15	48567.20	42452.99
Veh. time (hr)	1507.17	1618.91	1348.43
Op Cost ('000)	2137	2484	2178
Obj function/utility	3489543	3521928	3364508

Table 2. Average performance indicators at the route level for the identified scenarios

Indicators	Users	Operators	Balanced
Number of routes	46	46	46
Route density (pax/route)	744	643	634
Avg veh dist. (km/route)	982.94	1055.81	922.89
Avg veh time (hr/route)	32.76	35.19	29.31
Avg op cost ('000)	46.45	54.00	47.35

In the first scenario, the transit user is given priority by setting the weights in an 80:20 split between the user and operator. From the results presented in table 1, it can be observed, that there is a higher satisfied demand and network utilisation than the operators and balanced scenarios. Vehicle mileage and times are, however, less than only the operator's perspective. This can be attributed to the fact that users opt for direct routes, which, are generally shorter than more circuitous ones. This, therefore, implies that passenger demand on direct routes are served; leaving out the majority of travel demand on longer routes. Operationally, the result is realistic, as a transit network designed with a bias for the user, will contain direct routes that guarantees the *shortest travel time* for passengers. Hence, more people will be encouraged to use the service. This also implies that circuitous routes and those running through transfer points will be minimal or totally excluded in the solution.

In the second scenario the weightings are now changed to prioritise the service operator, using a 20:80 split in their favour. Again, in table 1, the results show that the operator has a greater vehicle mileage and operational hours than the user-centric solution. However, it satisfies less demand. This may be due to the fact, that while transit operators try to maximise network coverage by using circuitous routes, this may ultimately discourage some passenger who want to use only the direct routes. A network that is skewed in favour of the operator will largely contain routes that are longer than the preference of users.

Lastly, an optimal transit network solution would contain a mix of direct routes and other more circuitous ones. Hence, in the last scenario, a balance is struck between the perspectives of the user and operator, by assigning equal weightings to both stakeholders. Since direct routes reduce the ability of operators to cover demand occurring along more circuitous paths. An optimised solution is balanced between the user and operator's cost perspectives. This is revealed in the last column of table 1 where the indicators can be seen to have values between the user and operator perspectives. This indicates, that this scenario is a compromise between the earlier mentioned ones. The solution also has the least objective function score, showing that it is indeed an efficient solution as it minimises cost for all stakeholder.

The outcomes discussed above are reinforced in table 2 where the balanced network solution has indicator values that compromise between the users and operator perspective. However in terms of route density, it maintains a higher value than the others. The latter is a measure of how easily people can access the transit network. This result shows that the balanced solution is the most attractive for all stakeholders and it also offers better access to public transit services. Overall, these results show that suitable network design solutions can be obtained by setting appropriate weight factors in the model.

6. CONCLUSION

The connection between the quality of urban transit networks and the general well being of a society is well documented in the literature. Therefore it is important, that on a continuing basis efforts are taken to improve the design of urban transit networks. This paper presented an innovative design procedure that reduces the cost of utilisation for users and that of operations for the service provider. By combining meta-heuristics with agent-based models in network design framework, better networks can potentially be design because of the improved understanding and ability to model human behaviour on the network. The results show that the proposed design technique is capable of developing network solutions that respond to the stated objective of the network designer or policy maker. Having successfully applied the scheme to the context of a uni-modal public transit in the city of Cape Town, future efforts should be geared towards expanding its application to design multi-modal transit networks.

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