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RPIM Meshless method for Numerical Solution of Natural Convection in Porous Square Cavity

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Abstract. This paper proposed meshless Radial Point Interpolation Meshless Methods (RPIM) method for numerical solution of natural convection in Darcy porous square cavity. It is assumed that Boussinesq approximation is valid to characteristic the buoyancy effect as the driving force of the fluid flow. The Galerkin global weak form is used to discretize the system equations. The multiquadratic radial basis function (RBF) is chosen as the shape and test function. Comparing the numerical results obtained using the proposed method with those obtained using the conventional methods shows very good agreement.

Keywords: RPIM, Meshless, natural convection, porous cavity.

INTRODUCTION

Natural convection in porous media plays an important role in industrial engineering, such as: industrial insulation, solar power collector, chemical reactor, geothermal energy system, etc. The phenomenon of natural convection is characterized by fluid flow due to differences in fluid density caused by heating or cooling. The fluid density will be reduced if it gets warmed up so that it will float and the area left behind will be filled by a relatively cold fluid. The relatively hot fluid as it approaches the colder wall increases its density so that it will flow down due to the gravitational pull. Thus the difference in density is the driving force of fluid circulation.

With the reasons mentioned above, natural convection attracted many researchers to research. Natural convection research is growing rapidly, especially numerical research. This is driven by the rapid development of high-speed digital computers. The results of numerical research turned out to show results close to the experimental results [1]. Thus, numerical calculations can be used to design tools, which are then validated by experimental results.

A large amount of research on natural convection heat transfer modeling in porous media has been done. Most of the numerical methods used are mesh or grid based methods, such as: Finite Difference Method (FDM) [2] [3] [4], Finite Element Method (FEM) [5] [6] [7] and Finite Volume Method (FVM) [8] [9]. Methods of such methods have proven their superiority for numerical modeling, especially with regard to accuracy and flexibility. The first step of the method makes mesh by dividing the domain of space into a number of elements and in each element there are a number of nodes connected to each other so as to form a kind of topology map. Nodes. The process of dividing the element along with its topology is not an easy task, so this process is often regarded as a weakness of the method.

Recently, the meshless methods have been proposed to circumvent the problem of mesh generation in the FDM FEM and FEV methods. In meshless methods, numerical solution is constructed entirely in a set of nodes and nodes connectivity is not needed. Prax, Sadat, and Salagnac [10] pioneered the use of meshless method, called Diffuse

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Approximation Method (DAM), for solving the natural convection in porous medium problem. This method replaced the FEM interpolation within an element by Moving Least Square (MLS) local interpolation, which is defined on nodes [11]. Singh and Bhargava [12] studied natural convection within a wave enclosure using element free Galerkin method. They demonstrated that the wavy surface enhanced the heat transfer rate and the obtained results have a good agreement with available benchmark data. Samimi and Pak [13] claimed that EFG method is rarely used in the field of fluid flow in porous media. They developed 3-dimensional EFG code to study the efficiency and the applicability of EFG method. The numerical results indicate that EFG method have a good performance. As the DAM, the EFG method uses MLS local interpolation as the shape function. The disadvantage of this approach is Dirichlet boundary condition cannot imposed exactly, since the MLS function does not have Kronecker delta property.

Liu and Gu [14] developed Radial Point Interpolation Meshless Methods (RPIM). The RPIM method solved the problem in global weak form similar to EFG method, but the field variable expanded in nodes using Radial Basis Function (RBF). The use of RBF made the Dirichlet boundary condition can be imposed exactly. It is observed that the development of RPIM for numerical solution of natural convection in Darcy porous square cavity problem has not been undertaken yet. Therefore, the purpose of this paper is to develop RPIM for solving the problem.

Governing Equations

The governing equations in terms of non-dimensional stream function (ψ) and temperature variables *(θ)* for the Darcian model may be written as follows [6]:

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=-\mathrm{Ra}\frac{\partial\theta}{\partial x} \quad (1a)$$

$$u=\frac{\partial\psi}{\partial y} \quad ; \quad v=-\frac{\partial\psi}{\partial x} \quad (1b)$$

$$\frac{\partial\theta}{\partial t}+u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2} \quad (1c)$$

where t is time, (x,y) are spatial coordinates, (u,v) are velocity components and Ra is Rayleigh number.

Numerical Solution Procedure

The RPIM solution procedure for unsteady the natural convection problem in the Darcy medium is described as follows:

Interpolation using RBF on a function $u(\mathbf{x})$ can be expressed as follows:

$$u(\mathbf{x})=\sum_{j=1}^{N}\phi_j u_j \quad (2a)$$

If it is written in matrix form, the above equation becomes:

$$u(\mathbf{x})=\boldsymbol{\varphi}(\mathbf{x})\mathbf{U_s}(\mathbf{x})=\mathbf{R}^{\mathbf{T}}\mathbf{U_s}(\mathbf{x})\mathbf{R_Q^{-1}}\mathbf{U_s} \quad (2b)$$

The vector $\mathbf{x}$ is spatial coordinates, N is the number of points present in the compact support domain (Fig. 1), $\mathbf{U_s}$ is the vector containing the value u at the node point and $\mathbf{R}$ is the Matrix containing the RBF interpolation, whereas $\boldsymbol{\varphi}$ is the shape function. The multiquadratic RBF function is selected as interpolation function:

$$R_i(x,y)=\left(r_i^2+C^2\right)^q=\left[(x-x_i)^2+(y-y_i)^2+C^2\right]^q \quad (3)$$

Constants C, q, η are the shape parameters. Variables in the model equations can be expressed in the same way:

$$v(\mathbf{x}) = \sum_{j=1}^{n} \phi_j v_j \;\; ; \;\; p(\mathbf{x}) = \sum_{j=1}^{n} \phi_j p_j \;\; ; \;\; \theta(\mathbf{x}) = \sum_{j=1}^{n} \phi_j \theta_j \tag{4}$$

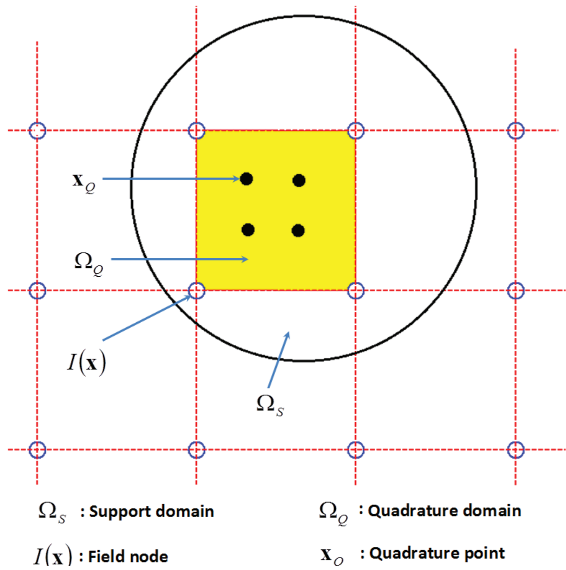

FIGURE 1. Support domain and quadrature domain

The Galerkin integration uses the same weight function as the shape function, equation (1c) is taken as Galerkin integration example:

$$\int_{\Omega_Q} \phi_i \left(\frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} - \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) \right) d\Omega = 0 \tag{5}$$

Substitute equation (4) into equation (5):

$$M_{ij} \frac{\partial \theta_j}{\partial t} + C_{ij} \theta_j + K_{ij} \theta_j = 0 \tag{6}$$

$$M_{ij} = \int_{\Omega_Q} \phi_i \phi_j d\Omega \tag{7a}$$

$$C_{ij} = \int_{\Omega_Q} \phi_i \left(u_j \frac{\partial \phi_j}{\partial x} + v_j \frac{\partial \phi_j}{\partial y} \right) d\Omega \tag{7b}$$

$$K_{ij} = \int_{\Omega_Q} \left(\frac{\partial \phi_i}{\partial x} \frac{\partial \phi_j}{\partial x} + \frac{\partial \phi_i}{\partial y} \frac{\partial \phi_j}{\partial y} \right) d\Omega \tag{7c}$$

Numerical integration is done using Gauss quadrature method in quadrature domain. In this paper, the quadrature domains are rectangular in shape so that numerical integration is performed using the 1-dimensional Gauss quadrature tensor product. Background mesh is still needed to determine the division and size of the quadrature domain. Temporal integration is done implicitly.

$$M_{ij}\theta_j^{n+1} + \Delta t K_{ij}\theta_j^{n+1} = M_{ij}\theta_j^{n} + \Delta t C_{ij}\theta_j^{n} \tag{8}$$

The stream function and velocity components are discretized in the same way:

$$K_{ij}\psi_j^{n+1} = D_{ij}\theta_j^{n+1} \tag{9}$$

$$M_{ij}u_j^{n+1} = D_{ij}^{y}\psi_j^{n+1} \quad ; \quad M_{ij}v_j^{n+1} = -D_{ij}^{x}\psi_j^{n+1} \tag{10}$$

$$D_{ij}^{x} = \int_{\Omega_Q} \phi_i \frac{\partial \phi_j}{\partial x} d\Omega \quad ; \quad D_{ij}^{y} = \int_{\Omega_Q} \phi_i \frac{\partial \phi_j}{\partial y} d\Omega \tag{11}$$

The RPIM algorithm is described as follows:

1. Determine the maximum time limit for the calculation and time step Δt, specify the initial condition ($t = 0$) for all variables (u, v, θ), coordinates of the nodes, basis function parameters, background mesh, Support domain and quadrature domain.
2. Find the derived matrix in Eq. (7a) - (7c) and (8).
3. The numerical calculation according to the time step begins.
4. Solve equation (8) to obtain the temperature θ for level (n+1).
5. Solve equation (9) and (10a) - (10b) to obtain the stream function and velocity components (*ψ, u, and v*) for level (n + 1).
6. Update time.
7. Check if it has reached the maximum time limit or not, if not back to step 2. If it is to step 7.
8. Write the data and finish

RESULTS AND DISCUSSION

The domain, distribution of the nodes and boundary condition of natural convection in a porous square cavity is depicted in Fig. 2. The left vertical wall are heated and the right wall is cooled. The top and bottom horizontal walls are adiabatic walls. Due to Darcy's law the slip boundary condition are applied to the walls, so the stream function is set to be zero at the walls.

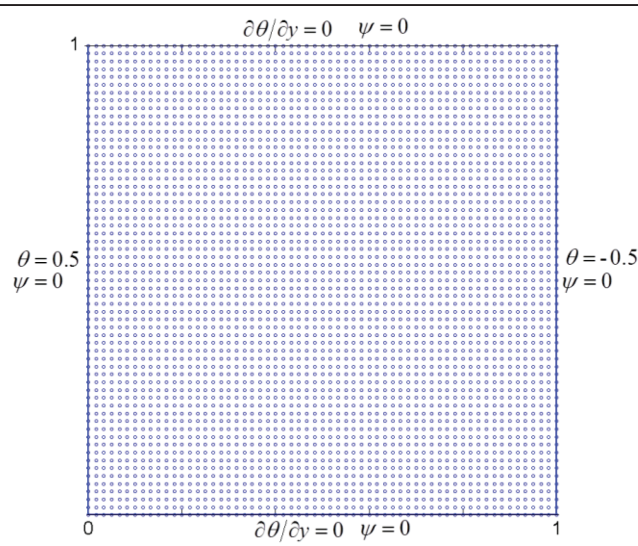

FIGURE 2. Nodes and boundary conditions

Numerical calculation is carried out by using 2 kinds of Rayleigh Number value, that is: Ra = 100 and Ra = 1000. All calculations use the same 61x61 nodes. At low Ra value heat transfer is dominated by conduction and if Ra value increases then conduction dominance weakens and convection becomes more dominant. This is evidenced by the increase of gradient temperature in the left and right wall as shown in Figure 3a and 3b. The increase in Ra value also causes the fluid flow velocity to increase so that the value of the stream function also increases, see Figure (4a) and (4b).

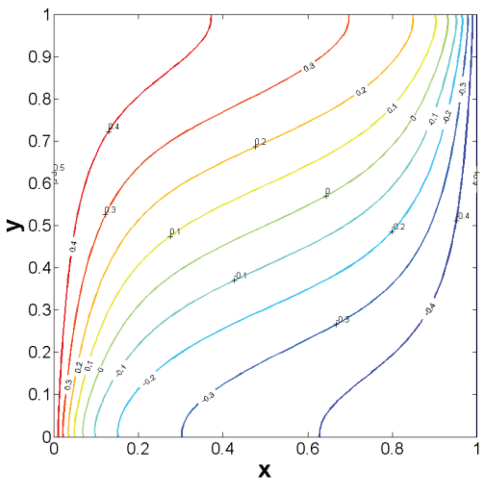

FIGURE 3a. Isotherm Ra = 100

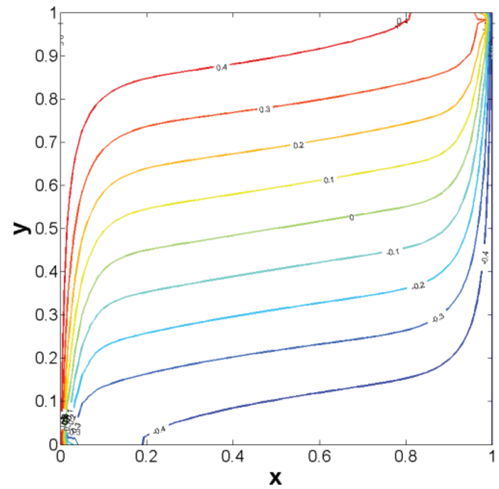

FIGURE 3b. Isotherm Ra = 1000

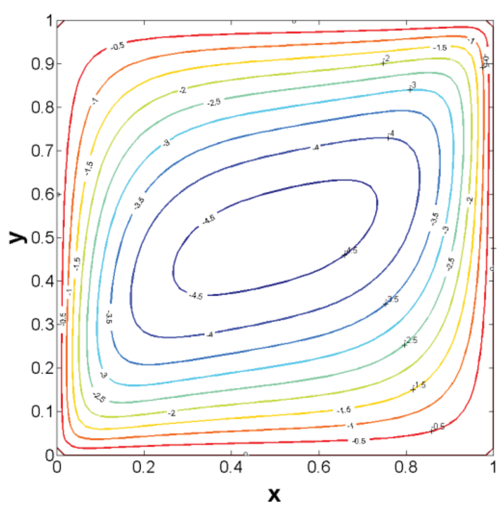

FIGURE 3b. Stream function at Ra = 100

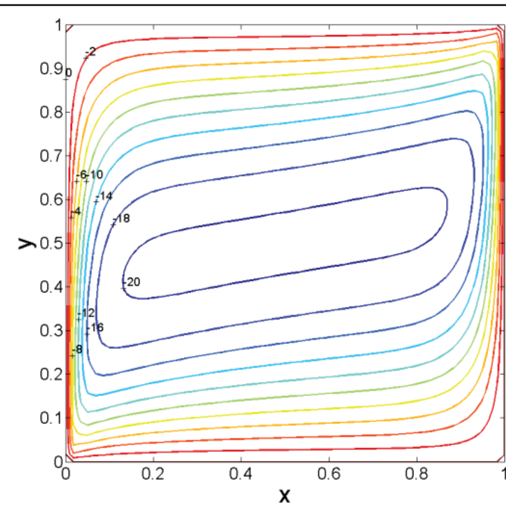

FIGURE 1. Stream function at Ra = 1000

Variations of Nusselt Number values are shown by Figure 4a and 4b, initially the Nu number values are high, and this is due to impulsive heating and cooling of the vertical wall. The Nu values then gradually decrease with time and finally reach the steady state. The increasing of the Ra number value causes the steady state to be achieved faster.

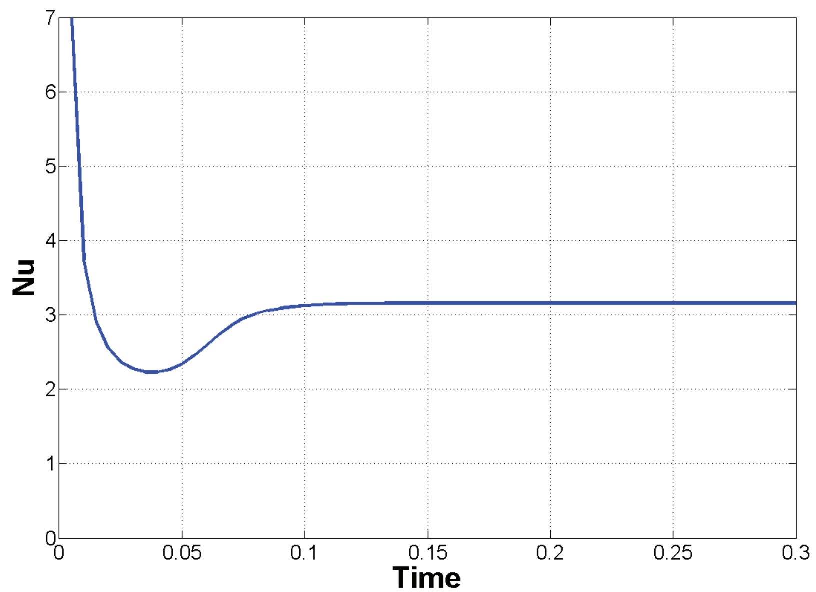

FIGURE 4a. Nu History at Ra = 100

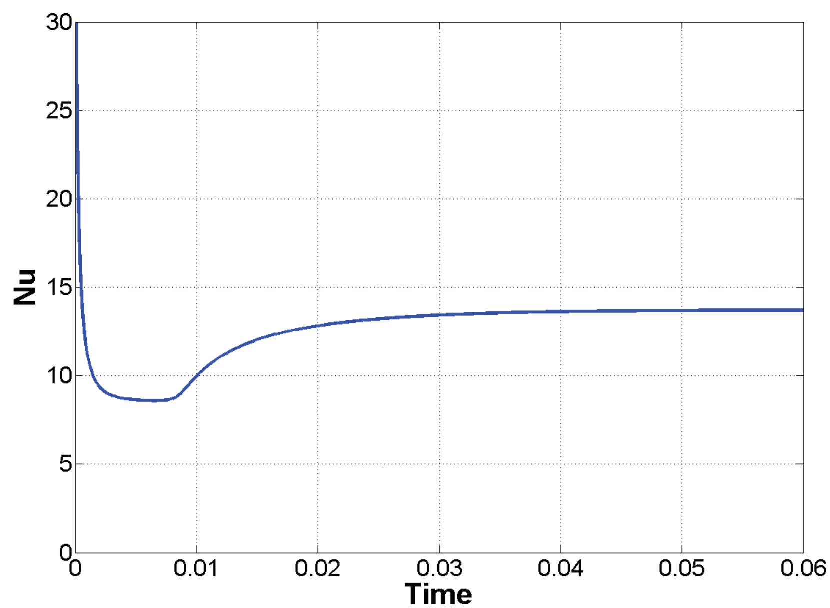

FIGURE 4b. Nu History at Ra = 1000

To test the accuracy of RPIM method, the results of numerical calculation are compared with the results of calculation of numerical method obtained in some journal literature. The results can be seen in Table 1, the comparison shows a very good agreement.

TABLE 1. Comparison of Nu

Author	Nu	
	Ra = 100	Ra = 1000
Walker and Homsy [15]	3.097	12.960
Manole and Lage [16]	3.118	13.637
Baytas [2]	3.160	14.060
Saeid and Pop [8]	3.002	13.726
Kumari and Nath [17]	3.114	13.675
Present results	3.181	13.703

CONCLUSION

A Radial Point Interpolation Meshless Methods (RPIM) is proposed to solve numerically natural convection in Darcy porous square cavity problem. Multiquadratic Radial base function is used as a shape and test function. The phenomenon of heat transfer and fluid flow due to the buoyant force can be simulated well. Comparison of RPIM numerical calculations with other methods shows that the suitability is very good. So it can be concluded that RIPM method has good accuracy for numerical solution of natural convection solution in Darcy porous square cavity.

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