MEKATRONIKA
e-ISSN: 2637-0883
VOL. 3, ISSUE 2, 57 – 64
DOI: https://doi.org/10.15282/mekatronika.v3i2.7354

ORIGINAL ARTICLE

Articulated Robot Arm

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ABSTRACT – In medical rehabilitation programs, trajectory tracking is used to increase the repeatability of joint movement and the patient's recovery in the early phases of rehabilitation. In order to achieve that, the robotic arm has been implemented since it can provide a precise and move in almost perfect motion. This manuscript aim to develop and simulate a 2DOF robotic arn that will able to tracking the trajectory successfully. Hence, in order to achieved that a modeling, simulation, and control of a Two Degree of Freedom (2-DOF) Robot Arm is being discussed in this manuscript. First, the robot specifications, as well as Robot Kinematics forward and inverse kinematics of a 2-DOF robot arm, are provided. The dynamics of the 2-DOF robot arm were then formulated in order to obtain motion equations by using the Eular-Lagrange Equation. For the controller of the robot, a control design was created utilising a PID controller. All the data is recorded from the margin of error as well as the overshoot and peak settling time is being record via matlab. The data is differentiate by with with controller, with PI and PID, in which the error is less than 12.5 and 1.63 consecutively. The data that being gathered show that a controller best suited in this rehabilitation robot.

ARTICLE HISTORY
Received: 12th June 2021
Revised: 16th July 2021
Accepted: 30th July 2021

KEYWORDS
Upper-limb
Robotic arm
Trajectory Tracking
Rehabilitation

INTRODUCTION

Rehabilitation robotics is one of the many sectors that gain lot of positive value. This can be seen through lot of robotic rehabilitation program and such program can bring a great value to humanity. The effectiveness in healing the upper extremity movements and locomotor abilities of a weak athlete or even in stroke population showed a better result in the clinical demonstration [1].

Rehabilitation robot arms can be seen made in two different type which is exoskeleton robot and end-effector robot. The exoskeleton robot is an external structural system with human-like joints and connections. Whereas an end effector is a device that connects to the wrist of a robot and allows it to communicate with its mission. Both robotics type required a controller in utilizing to enhance or enable the rehabilitation process by assisting the path mapping and trajectory tracking. Hence to design a suitable controller for motion tracking, ensuring the aided hand can follow the reference trajectory as desired, is one of the notable challenges in developing a virtual robot arm and control [2].

Upper limb exoskeletons are joined to the human arm at many points and are meant to work in tandem with the human upper limb [3]. In the sagittal plane, the two linkages depict motions such as flexion/extension for both the shoulder and elbow joints. This can be configure as 2DOF linked robotic arm. The two-link model is stated to be restricted along the sagittal plane by the concept of constant human contact.

There will be three objective to achieved which is first is to develop a mathematical modelling of 2-DOF robotic manipulator and monitor the robot arm's output in 2 joint angle positions and observe the responses before implementing the model in the virtual robot arm.The first modelling configuration will be the kinematic analysis. There are two methods for performing kinematic analysis. A robot's dynamic model is the second modelling configuration concerned with the movement and forces involved in the robot arm, and it develops a mathematical link between the position of the robot joint variables and the robot's dimensional parameters. Then, the Euler-Lagrange approach is employed in this study, which relies on computing the total Kinematic and Potential Energies of the robot arm to derive the Lagrangian ($\mathscr{L}$) of the entire system in order to compute the force or torque given to each joint [4].Another objective is to implement the PID controller with the robotic arm in order controlling the trajectory in reducing error and overshoot rate of motion. The PID controller may assess the torques that must be supplied to the robot's limbs so that the error between the measured joint angles and the intended joint angles (joint angles error) approaches a constant linearly[5].Lastly, is to conduct prototyping model of a two degrees of freedom robot arm (2-DOF) that are analytical with comprehensive analysis. This will be future researched.

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METHODOLOGY : 2DOF UPPER LIMB SYSTEM

Mathematical modelling of the upper limb system

To understand the significant challenges in dynamic modelling, the dynamics of a robot arm are explicitly determined using the Lagrange-Euler formulation. Figure 1 depicts a schematic picture of the robot arm's two degrees of freedom (DOF) with the robot arm links 1 and 2, joint displacement is θ_1 and θ_2, link lengths are l1 and l2, m1 and m2 represent the masses of each link, and τ_1 and τ_2 are torque for the links 1 and 2, respectively.

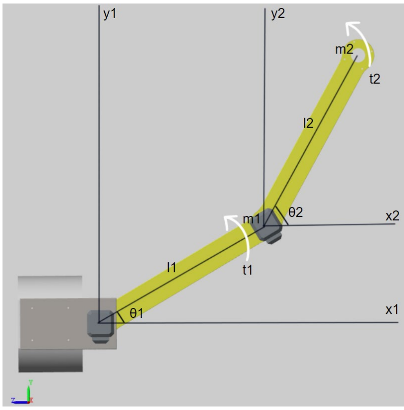

Figure 1. Diagram of the robot arm's two degree of freedom

Properties and assumptions for dynamic modeling in equation (1), are as follows

1. The actuators dynamics (motor and gear boxes) is not taken into account.
2. The effect of friction forces is assumed to be negligible.
3. The mass of each link is assumed to be concentrated at the end of each link.
4. Matrix M (θ) is symmetric and positive definite.

First, the system's kinetic and potential energies are computed; the kinetic energy of the manipulator as a function of joint position and velocity is stated as:

$$K(\theta,\dot{\theta}) = \frac{1}{2}\dot{\theta}\, M(\theta)\dot{\theta} = \frac{m_i v^2}{2} \quad (1)$$

Where, $M(\theta)$ is the NxN manipulator mass matrix and the subscript I denote 1 and 2.

As a result, the sum of the kinetic energies (K_1 and K_2) of the individual link is the total kinetic energy of the robot arm.

$$K(\theta,\dot{\theta}) = \sum_i^n K_i(\theta,\dot{\theta}) \quad (2)$$

$$K(\theta,\dot{\theta}) = \frac{m_1 {v_1}^2}{2} + \frac{m_2 {v_2}^2}{2} \quad (3)$$

To determine K_1 and K_2, are differentiated the position equations for m_1 at A as ill as m_2 at B are stated and is differentiate the two positions using inner product to find their corresponding velocity.

$$x_1 = L_1 S_1 \quad (4)$$

$$y_1 = L_1 C_1 \quad (5)$$

$$x_2 = L_1 S_1 + L_1 S_{12} \quad (6)$$

$$y_2 = -L_1 C_1 - L_2 C_{12} \quad (7)$$

Velocity is defined as,

$$v^2 = \|v^2\| = v^t v \tag{8}$$

$$v^2{}_1 = [l_1 C_1 \dot{\theta}_1 \quad -l_1 S_1 \dot{\theta}_1] \begin{bmatrix} l_1 C_1 \dot{\theta}_1 \\ -l_1 S_1 \dot{\theta}_1 \end{bmatrix} \tag{9}$$

$$v = \frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \tag{10}$$

$$v_1^2 = l_1^2({C_1}^2 + {S_1}^2)\dot{\theta}_1 = l_1^2 {\dot{\theta}_1}^2 \tag{11}$$

v_2^2 is obtain in the same way similarly.

$$v_2^2 = l_1^2 C1^2 {\dot{\theta}_1}^2 + 2l_1 l_2 C_1 C_{12}\left({\dot{\theta}_1}^2 + \dot{\theta}_1 \theta_2\right) + l_2^2 C_{12}^2 (\dot{\theta}_1 + \dot{\theta}_2)^2 + l_1^2 S_1^2 {\dot{\theta}_1}^2 + 2l_1 l_2 S_1 S_{12}\left({\dot{\theta}_1}^2 + \dot{\theta}_1 \theta_2\right) + l_2^2 S_{12}^2 (\dot{\theta}_1 + \dot{\theta}_2)^2 \tag{12}$$

To simplify the derivation, i will refer to trigonometry as:

$$v_2^2 = l_1^2 {\dot{\theta}_1}^2 + l_2^2\left(\dot{\theta}_1 + \dot{\theta}_2 + 2\dot{\theta}_1 \dot{\theta}_2\right) + 2l_1 l_2 C_2 ({\dot{\theta}_1}^2 + \dot{\theta}_1 \theta_2) \tag{13}$$

v_1^2 v_2^2 (2)

$$K_1(\theta, \dot{\theta}) = \frac{1}{2} m_1 l_1^2 {\dot{\theta}_1}^2 \tag{14}$$

$$K_2(\theta, \dot{\theta}) = \frac{1}{2} m_1 l_1^2 {\dot{\theta}_1}^2 + \frac{1}{2} m_2 l_2^2 ({\dot{\theta}_1}^2 + {\dot{\theta}_2}^2 + 2\dot{\theta}_1 \dot{\theta}_2) + m_2 l_1 l_2 C_2 ({\dot{\theta}_1}^2 + \dot{\theta}_1 \theta_2) \tag{15}$$

So that the total kinetic energy of the robot arm is obtained from equations (14) and (15) and presented as

$$K(\theta, \dot{\theta}) = \frac{1}{2}(m_1 + m_2) l_1^2 {\dot{\theta}_1}^2 + \frac{1}{2} m_2 l_2^2 ({\dot{\theta}_1}^2 + {\dot{\theta}_2}^2 + 2\dot{\theta}_1 \dot{\theta}_2) + m_2 l_1 l_2 C_2 ({\dot{\theta}_1}^2 + \dot{\theta}_1 \theta_2) \tag{16}$$

The potential energy of the upper limb system is the sum of the potential energies of links 1 and 2, calculated with reference to the datum (zero potential energy) at the axis of rotation.;

$$\mathrm{U}(\theta) = \sum_{i}^{n} U_i(\theta) \tag{17}$$

$$\mathrm{U}(\theta) = -(m_1 + m_2) g l_1 C_1 - (m_2) g l_2 C_{12} \tag{`8}$$

$$L(\theta, \dot{\theta}) = K - U \tag{19}$$

$$L(\theta, \dot{\theta}) = \frac{1}{2}(m_1 + m_2) l_1^2 {\dot{\theta}_1}^2 + \frac{1}{2} m_2 l_2^2 ({\dot{\theta}_1}^2 + {\dot{\theta}_2}^2 + 2\dot{\theta}_1 \dot{\theta}_2) + m_2 l_1 l_2 C_2 \left({\dot{\theta}_1}^2 + \dot{\theta}_1 \theta_2\right) + (m_1 + m_2) g l_1 C_1 + (m_2) g l_2 C_{12} \tag{20}$$

From this Lagrangian, the dynamic systems equations of the motion are given by:

$$\frac{\mathrm{d}}{dt}\frac{\partial \mathrm{L}}{\partial \dot{\theta}} - \frac{\partial \mathrm{L}}{\partial \theta} = \tau_i \tag{21}$$

τ_i

θ_i

θ_1

$$\frac{\partial \mathrm{L}}{\partial \dot{\theta}_1} = (m_1 + m_2) l_1^2 \dot{\theta}_1 + m_2 l_2^2 (\dot{\theta}_1 + \dot{\theta}_2) + 2 m_2 l_1 l_2 C_2 (\dot{\theta}_1) + m_2 l_1 l_2 C_2 (\dot{\theta}_2) \tag{22}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_1} = [(m_1+m_2)l_1^2 + m_2 l_2^2 + 2m_2 l_1 l_2 C_2]\ddot{\theta}_1 + [m_2 l_2^2 + m_2 l_1 l_2 C_2]\ddot{\theta}_2 - 2m_2 l_1 l_2 S_2 \dot{\theta}_1 \dot{\theta}_2 - m_2 l_1 l_2 S_2 \ddot{\theta}_2{}^2 \quad (23)$$

$$\frac{\partial L}{\partial \theta} = -(m_1+m_2) g l_1 S_1 - (m_2) g l_2 S_{12} \quad (24)$$

$$\tau_1 = [(m_1+m_2)l_1^2 + m_2 l_2^2 + 2m_2 l_1 l_2 C_2]\ddot{\theta}_1 + [m_2 l_2^2 + m_2 l_1 l_2 C_2]\ddot{\theta}_2 - 2m_2 l_1 l_2 S_2 \dot{\theta}_1 \dot{\theta}_2 - m_2 l_1 l_2 S_2 \ddot{\theta}_2{}^2 + (m_1+m_2) g l_1 S_1 + (m_2) g l_2 S_{12} \quad (25)$$

θ_2

$$\frac{\partial L}{\partial \dot{\theta}_2} = m_2 l_2^2(\dot{\theta}_1 + \dot{\theta}_2) + m_2 l_1 l_2 C_2(\dot{\theta}_1) \quad (26)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_2} = m_2 l_2^2(\ddot{\theta}_1 + \ddot{\theta}_2) + m_2 l_1 l_2 C_2(\ddot{\theta}_1) - m_2 l_1 l_2 S_2(\dot{\theta}_1 \dot{\theta}_2) \quad (27)$$

$$\frac{\partial L}{\partial \theta_2} = -m_2 l_1 l_2 S_2\left(\dot{\theta}_1{}^2 + \dot{\theta}_1 \dot{\theta}_2\right) - m_2 g l_2 S_{12} \quad (28)$$

$$\tau_2 = [m_2 l_2^2 + m_2 l_1 l_2 C_2]\ddot{\theta}_1 + [m_2 l_2^2]\ddot{\theta}_2 + m_2 l_1 l_2 S_2 \dot{\theta}_1{}^2 + (m_2) g l_2 S_{12} \quad (29)$$

The deduced dynamic equations of motion may be stated in terms of the inertial matrix components, centrifugal force, Coriolis force vector, and gravity force, and are stated as;

$$M(\Theta) = \begin{bmatrix} m_{11} m_{12} \\ m_{21} m_{22} \end{bmatrix}$$

$$m_{11} = I_1 + I_2 + m_1 L_1^2 + m_2(L_1^2 + L_2^2 + 2L_1 L_2 C_2) \quad (30)$$

$$m_{12} = I_2 + m_2(L_2^2 + L_1 L_2 C_2) \quad (31)$$

$$m_{21} = I_2 + m_2(L_2^2 + L_1 L_2 C_2) \quad (32)$$

$$m_{22} = I_2 + m_2 L_2^2 \quad (33)$$

$$C(\Theta, \dot{\theta}) = \begin{bmatrix} c_{11} c_{12} \\ c_{21} c_{22} \end{bmatrix}$$

$$c_{11} = -m_2 L_1 L_2 (2\dot{\theta}_2) S_2 \quad (34)$$

$$c_{12} = -m_2 L_1 L_2 (\dot{\theta}_2) S_2 \quad (35)$$

$$c_{21} = m_2 L_1 L_2 (\dot{\theta}_1) S_2 \quad (36)$$

$$c_{12} = 0 \quad (37)$$

$$G(\theta) = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}$$

$$g_1 = (m_1 + m_2) g L_1 S_1 + m_2 g L_2 (S_{12}) \quad (38)$$

$$g_2 = m_2 g L_2 (S_{12}) \quad (39)$$

The associated non-linear differential equations developed from the Lagrangian approach are commonly used to express the dynamic equations for the motion control;

$$M(\Theta)\ddot{\theta} + C(\Theta, \dot{\theta})\dot{\theta} + G(\theta) = \tau \quad (40)$$

Controller Design

For successful control of the robot arm, a proportional-integral-derivate controller (PID) is being used. This design used multiple PID controllers because arm1 and arm2 are interdependent; in fact, there is a substantial connection between both of the linked arm. However, the coupling effect must be separated in order to acquire enough flexibility to effectively control individually. The main goal is to improve or put the robotic arms in the desired location. To do this, the determined desired (set point) joint angle θ^d, and the goal of robot control is being design as the input torque in equation (39) . Hence, that the regulation error is

$$(\theta) = \frac{\tau - M(\Theta)\ddot{\theta} + C(\Theta, \dot{\theta})\dot{\theta}}{G} \quad (41)$$

$$\tilde{\theta} = \theta^d - \theta \quad (42)$$

θ^d : The desired joint [rad]
$\tilde{\theta}$: Angle error [rad]
Θ : Actual joint angle [rad]

Furthermore, the PID control law is stated in terms of error, $\tilde{\theta}$ as:

$$\tau_{pid} = K_p\tilde{\theta} + K_i\int_0^t \tilde{\theta}(t)dt + K_d\tilde{\theta} \tag{43}$$

Figure 3.2 illustrates a closed-loop system for a two-degree-of-freedom mechanical arm, which gives insight into the modelling and control aspects of the robot arm.

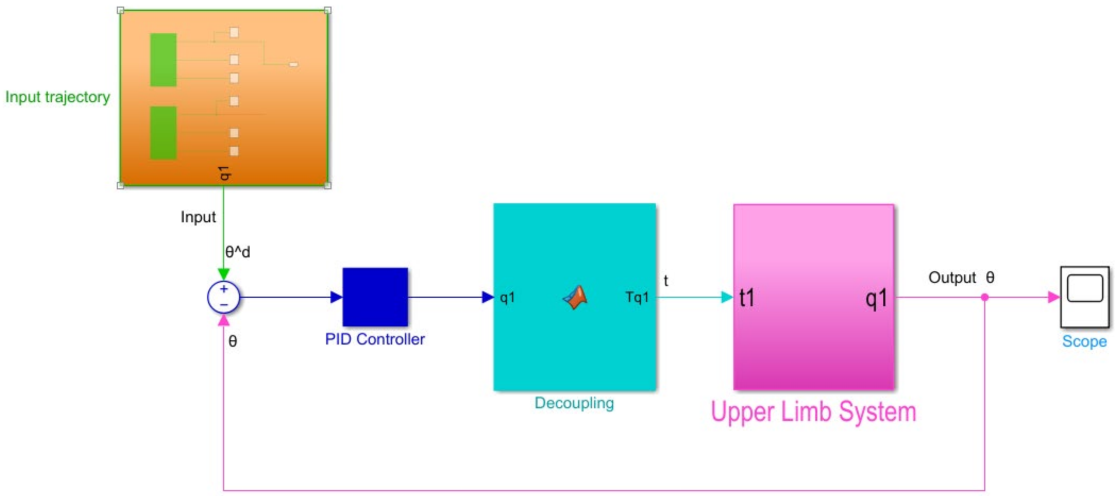

Figure 2 Close-loop system for two degree of freedom

The closed loop equation of the robot arm is obtained by substituting the control action τ_{pid} in equation (43) into the robot model (40).

$$M(\Theta)\ddot{\theta} + C(\Theta,\dot{\theta})\dot{\theta} + G(\theta) = K_p\tilde{\theta} + \xi + K_d\tilde{\theta}\tau = \tau_{pid} \tag{44}$$

Input Trajectory

To establish the movements that are comfortable and efficient for rehabilitation, the robot should travel along trajectories that are similar to those of normal human motion. Studies have identified that normal human movements follow a trajectory that minimises total jerk based on voluntary movement data [6].

When the sixth time derivative of the trajectory function equals zero, the minimal jerk trajectory occurs. This problem has a general solution of a fifth-order polynomial in time (45), where and are the six constants to be found.

$$x(t) = a_0 + a_1t + a_2t^2 + a_3t^3 + a_4t^4 + a_5t^5 \tag{45}$$

$$\dot{x}(t) = a_1 + 2a_2t + 3a_3t^2 + 4a_4t^3 + 5a_5t^4 \tag{46}$$

$$\ddot{x}(t) = 2a_2 + 6a_3t + 12a_4t^2 + 20a_5t^3 \tag{47}$$

This equation will be used to get a smooth trajectory in the matlab simulink as input block.

EXPERIMENTAL RESULTS

For the simulation, the experiment is carried out via simulation on Matlab Simulink. First, start by creating a block diagram for each matrix in the dynamic equation. These block diagram will be use in the upperlimb system or it was being labelled as plant. These are the parameter used as in Table 1.

Table 1. Parameter for Mass, Link1, Link2.

Parameter	Link1	Link2
Mass (kg)	1.47	1.47
Length (m)	0.25	0.25
Gravity (ms^{-2})	9.81	9.81

A virtual projection is being created and will be acting as the input signal or θ. It is virtually to give a motion from 0 then 90 then 0 again. This was being done to mimic human arm motion during rehabilitation. Figure 3 shows the overall layout for the simulation.

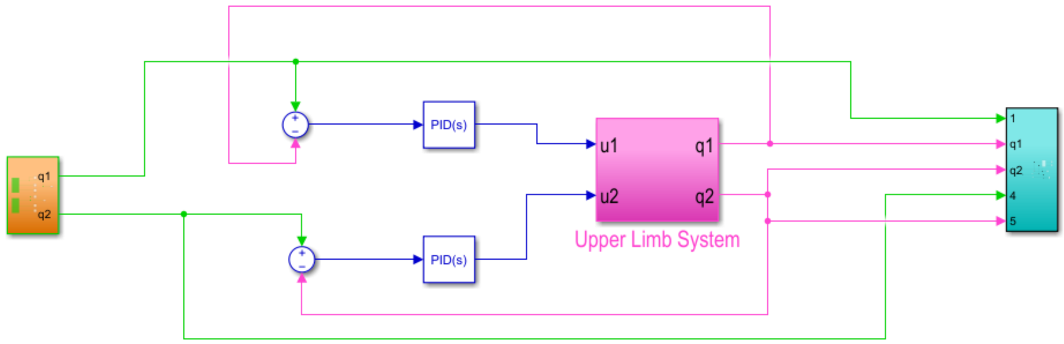

Figure 3. The simulation's overall design.

At the early stage, the PID was not yet implemented. Its originally start with an input block moving directly into the upper limb system. Figure 4 shows the result for the input and output of the system.

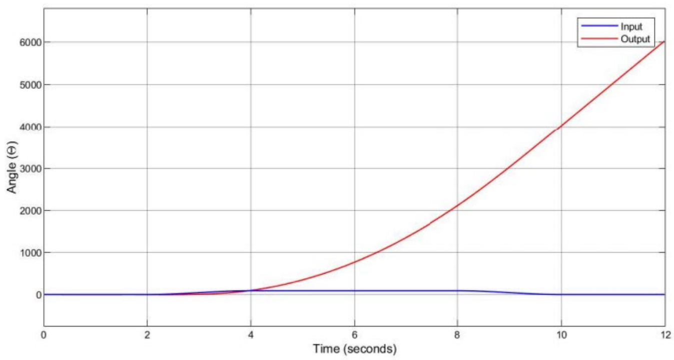

Figure 4. Displacement of the input and output without feedback.

The result of the simulation without any feedback is deviate really far from the virtual motion suggested. This give an insignificance result for the robot arm. Hence, to get a better output motion, the decision to add the feedback into the classical PI and PID controller with parameter as in Table 2 is being made. This feedback produced a good result hereafter. It started out as an oscillating motion with a PI controller. This demonstrates that there is no steady state loss, which is undesirable for a rehabilitation robot. Figure 5 show the result for the PI controller. It gave error of 12.5 degree compare to the virtual.

Table 2. The parameter for the Proportion, Integral and Derivative value.

Parameter	Link1	Parameter	Link2
Kp	12.3	Kp	12.3
Ki	34.50	Ki	34.50
Kd	25	Kd	25

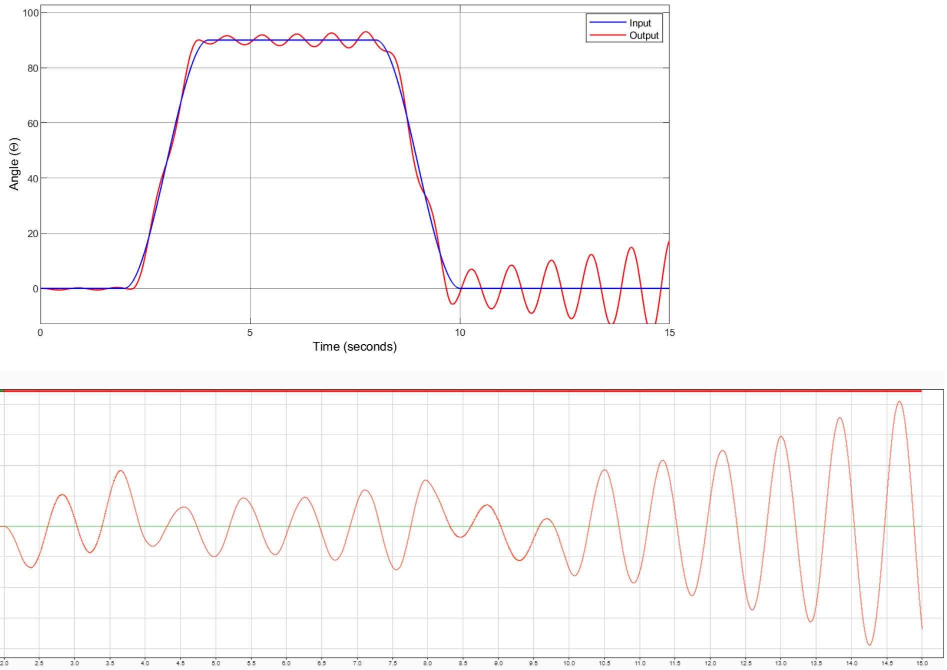

Figure 5. Perfomance of PI controller.

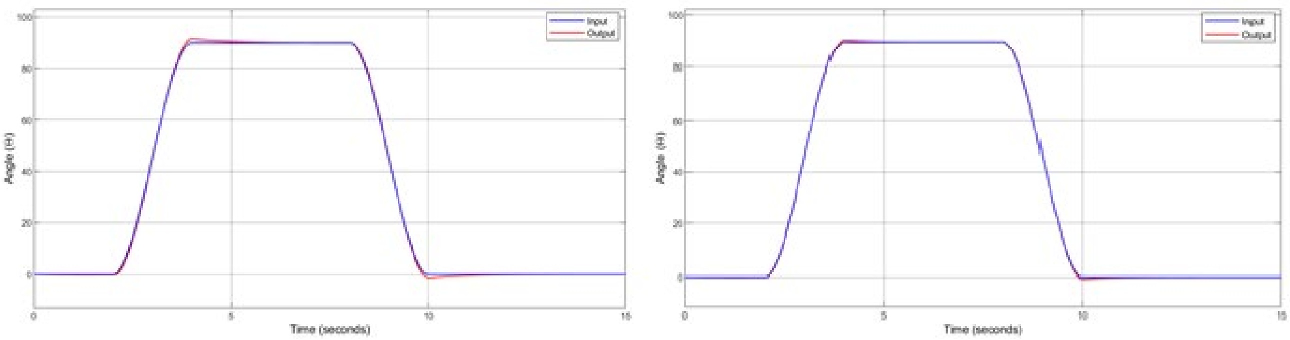

Figure 6. Angle displacement for the output and input in Link 1 and Link 2.

Later the PID is being introduced to the equation and the aforesaid parameter is being modified. In comparison to the virtual projection, the feedback was able to achieve a good motion with a slight overshoot. The findings achieved after PID deployment are shown in Figures 6 and 7.

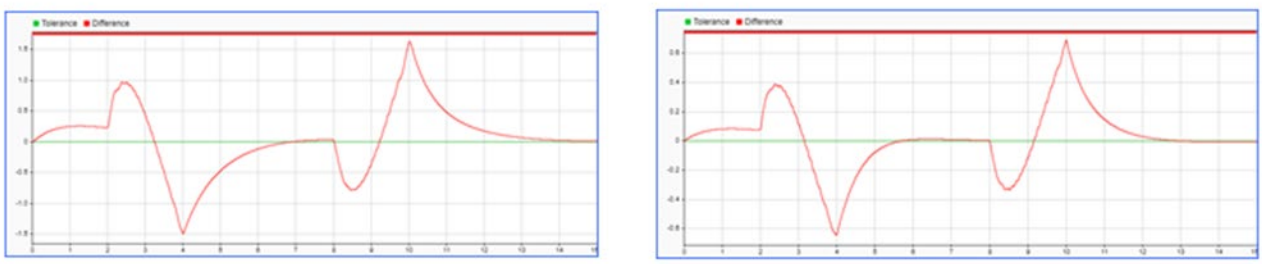

Figure 7. Error displacement for the Link 1 and Link 2

Based on Figure 7, the result proved that maximum trajectory error is less than 1.63 which quite considerably good for a robotic arm. This error means the virtual projection of the motion is almost the same as the output of the simulation

PID

▼ Transitions

High	8.994e+01
Low	-2.233e-02
Amplitude	8.996e+01
+ Edges	1
+ Rise Time	1.183 s
+ Slew Rate	60.820 (/s)
- Edges	1
- Fall Time	1.186 s
- Slew Rate	-60.661 (/s)

▼ Overshoots / Undershoots

+ Preshoot	-0.025 %
+ Overshoot	1.546 %
+ Undershoot	1.998 %
+ Settling Time	782.493 ms
- Preshoot	0.658 %
- Overshoot	1.999 %
- Undershoot	1.546 %
- Settling Time	785.770 ms

PI

▼ Transitions

High	8.765e+01
Low	-5.301e-01
Amplitude	8.819e+01
+ Edges	1
+ Rise Time	1.043 s
+ Slew Rate	67.661 (/s)
- Edges	1
- Fall Time	1.223 s
- Slew Rate	-57.678 (/s)

▼ Overshoots / Undershoots

+ Preshoot	-0.601 %
+ Overshoot	5.919 %
+ Undershoot	1.995 %
+ Settling Time	1.808 s
- Preshoot	7.831 %
- Overshoot	10.634 %
- Undershoot	10.044 %
- Settling Time	--

Figure 8. Performance of controller

Figure 8 compares the result between two simulation which is with PID and with PI. This show that the average amount of time required for each rising edge to cross from the lower-reference level to the upper-reference level in PID is higher than PI. For the overshoot percentage, average highest aberration in the region immediately following each rising transition in PID is really low compare to PI. The settling time is the time after the signal crosses into and remains in the tolerance region around the high-state level after passing through the mid-reference level instant. It shows here that PID have much better settling time. Based of above perfomance,the simulation can conclusively said that PID gave a better result.

CONCLUSION

This paper reports the first attempt to A 2-DOF robotic manipulator with a linearized mathematical model and it can be derived base on kinematic and dynamic equation. Once the error condition is met, the manipulation of the data will being introduced. The analysis confirmed that the proposed 2-DOF robotic manipulator is quite good paired with a PID feedback.

FUTURE WORKS

For future research, different type of feedback shall be considered to enhance further the 2-DOF robotic manipulator. The work can be enhance and be protoype with a more complexed design simulation. These virtual prototype can be done by creating a Solidwork design and be analyse by Matlab Simsmechanic for more comprehensive analysis. The design will be more detail with its own strength material and Finite element analysis can be done easily for good measure of use. These type of future works is really appropriate for student to implement their knowledge in variety of topic.

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