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Prospect theory and asset allocation*

Ines Fortin[†] and Jaroslava Hlouskova[‡]

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Abstract

We study the asset allocation of an investor with prospect theory (PT) preferences. First, we solve analytically the two-asset problem of the PT investor for one risk-free and one risky asset and find that loss aversion and the reference return affect differently less ambitious investors and more ambitious investors. Second, we empirically investigate the performance of a PT portfolio when diversifying among a stock market index, a government bond and gold, in Europe and the US. We focus on investors with PT preferences under different scenarios regarding the reference return and the degree of loss aversion and compare their portfolio performance with the performance of investors under CVaR, risk neutral, linear loss averse and in particular mean-variance (MV) preferences. We find that, in the US, PT portfolios significantly outperform (in terms of returns) mean-variance portfolios in the majority of cases. Also with respect to risk-adjusted performance, PT investment outperforms MV investment in the US. Similar results, however, can not be observed in Europe. Finally, we analyze asymmetric effects along economic uncertainty and observe that PT investment leads to higher returns than MV investment in times of larger economic uncertainty, especially in the US.

Keywords: prospect theory, loss aversion, portfolio allocation, mean-variance portfolios, investment strategy

JEL classification: D81, G02, G11, G15

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1 Introduction

The mainstream expected utility model cannot explain many aspects of financial market characteristics. An alternative that has been proposed to describe investors' behavior under risk is prospect theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992). This theory can explain many of the anomalies observed in asset returns including the equity premium puzzle (Benartzi and Thaler, 1995; Barberis et al., 2001; An et al., 2020; Barberis et al., 2021). Experiments by Kahneman and Tversky found that the utility function does not depend on the absolute level of terminal wealth, as is hypothesized with the expected utility model, but depends on the change in the level of wealth. In addition, the probabilities assigned to the utility of the outcomes are expressed within a weighted function. These weight functions allow one to capture the tendency of people to underreact when faced with large probabilities and overreact to small probability events.[1] Kahneman and Tversky proposed a utility function that is defined over terminal wealth in relation to some reference level, such as the status quo, i.e., investors have reference-dependent preferences. In addition, they found that investors exhibit loss aversion, meaning that their disutility of a loss is greater than their utility of a gain of the same magnitude, i.e., investors are more sensitive when they experience a loss in financial wealth than when they experience a gain. Even when there is no commonly accepted measure of loss aversion in the literature and there are many different alternatives introduced (Abdellaoui et al., 2007), the main characteristic seems to be that the utility function is steeper in the domain of losses than in the domain of gains. The simplest form of such loss aversion is linear loss aversion, where the marginal utility of gains and losses is fixed. The optimal asset allocation decision under linear loss aversion has been studied widely, see, for example, Siegmann and Lucas (2005), Fortin and Hlouskova (2011), Best et al. (2014), and Best and Grauer (2016). Further, Kahnemann and Tversky found that investors prefer risk aversion options when they are confronted with gains while they are more willing to select risk-seeking options when confronted with losses. This behavior can be captured by an S-shaped utility function. We generalize linear loss aversion to S-shaped loss aversion and study optimal asset allocation in this setup.

In order to explain stock market anomalies Barberis et al. (2021) introduce a new model of asset prices in which investors evaluate risk according to prospect theory. This model incorporates all elements of prospect theory, accounts for investors' prior gains and losses, and makes quantitative predictions about an asset's average return based on empirical estimates of the asset's return volatility, return skewness, and past capital gain. With this

[1]This is referred to in the literature as cumulative prospect theory. However, in our study we do not use weight functions (subjective probabilities).

model, average assets' returns can thus be predicted under prospect theory preferences, as with the capital asset pricing model under mean-variance preferences. In our study, however, we consider the asset allocation problem of a given prospect theory investor, which is different from equilibrium asset pricing models.

Other studies examine the investor's asset allocation problem using the continuous-time framework and the martingale method (under the assumption that the market is complete) to solve the S-shaped utility maximization problem. For example, Berkelaar et al. (2004) derive the optimal investment strategies for two prospect theory utility functions. Chen et al. (2017) examine S-shaped preferences with a minimum performance constraint and inflation risk. He and Kou (2018) investigate the S-shaped utility maximization under a minimum guarantee. Dong and Zheng (2019) include short-selling and portfolio insurance constraints in the model, and Dong and Zheng (2020) impose trading and Value-at-Risk constraints; both apply the dual control method to solve the corresponding constrained optimization problem. In this framework, however, the solutions for the optimal wealth and trading strategy are not given directly, as in our discrete one-period setup.

There seem to be only very few studies which examine asset allocation under prospect theory preferences empirically.[2] For example, De Giorgi and Hens (2009) consider data of private clients and measure the clients' added value from holding prospect theory portfolio as compared to a mean-variance asset allocation; they find considerable monetary gains. In the empirical part of our study we also take the mean-variance portfolio as the main benchmark when assessing the performance of prospect theory portfolios and find that PT portfolios outperform mean-variance portfolios with respect to different performance measures. Grishina et al. (2017) compute prospect theory portfolios composed of up to 225 stocks using a differential evolution algorithm and a genetic algorithm. They take the stock index (in which the individual stocks are included) as the reference point, compare prospect theory models with index tracking models, and find that prospect theory portfolios perform better than index tracking models in bullish markets, and worse in bearish markets. We also compute prospect theory portfolios empirically, for three assets (stock, bond, gold) in the European and the US markets. We compare these portfolios with other benchmark portfolios, in particular with the traditional mean-variance portfolios, in terms of different performance measures. In addition, we look at potential differences between the prospect theory and mean-variance portfolios in periods of high and low economic uncertainty.

The remaining paper is organized as follows. In Section 2 we explore the two-asset

[2] As the S-shaped prospect theory utility function is not concave and the problem cannot be easily transformed to a sufficiently smooth higher dimensional concave problem, grid search algorithms or alternative special algorithms are necessary to solve the problem. These are computationally intensive, however, in particular for a large number of assets. In the empirical part of our paper we use the grid search method while Grishina et al. (2017), for instance, employ intelligent algorithms.

problem of an S-shaped prospect theory investor, with a risky and a risk-free asset, and derive properties of the optimal weight of the risky asset under the assumption of binomially and (generally) continuously distributed returns, both for the case when the reference point is equal to the risk-free rate and for the case when it is not. In Section 3 we implement different trading strategies of the prospect theory investor, who reallocates her portfolio on a monthly basis, and study the performance of the resulting optimal portfolio with respect to different performance measures that are based solely on portfolio returns (mean, median, realized returns), on risk-adjusted returns (Omega measure, Sharpe ratio, Sortino ratio, conditional value-at-risk) or on risk (volatility, downside volatility). We also compare the performance of the prospect theory portfolio with the performance of risk neutral, linear loss averse, conditional value-at-risk portfolios and, in particular, with the performance of traditional mean-variance portfolios. Finally, we analyze asymmetric effects along economic uncertainty by assessing whether prospect theory investors achieve higher returns in times of larger economic uncertainty, or in times of smaller economic uncertainty, than mean-variance investors. Section 4 provides a summary of the results and concludes.

2 Portfolio optimization under prospect theory preferences

We consider a loss averse investor characterized by the following S-shaped prospect theory value function of portfolio return r^p

$$v(r^p) = \left\{ \begin{array}{cc} \frac{(r^p-\hat{r})^{1-\gamma}}{1-\gamma}, & r^p > \hat{r} \\ \\ -\lambda\frac{(\hat{r}-r^p)^{1-\gamma}}{1-\gamma}, & r^p \leq \hat{r} \end{array} \right\} = \frac{1}{1-\gamma}\left[|\hat{r}-r^p|^{1-\gamma} - (1+\lambda)\left([\hat{r}-r^p]^+\right)^{1-\gamma}\right] \quad (2.1)$$

where $\hat{r} \in \mathbb{R}$ is the reference return with respect to which relative gains and losses are coded, $\gamma \in (0,1)$ is a parameter determining the curvature of the utility function for relative gains and losses (diminishing sensitivity parameter),[3] and $[t]^+$ denotes the maximum of 0 and t. Parameter $\lambda > 1$ is the penalty parameter that captures the degree of loss aversion making thus utility steeper in the loss domain ($r^p < \hat{r}$) than in the gain domain ($r^p > \hat{r}$). The investor's reduction in utility arising from a loss is greater (in absolute terms) than the marginal utility from a financial gain or, in other words, the investor is more sensitive when experiencing a loss than when experiencing a gain of the same size. Investors also display risk aversion in the domain of gains (the value function is concave for $r^p > \hat{r}$) but become risk lovers when they deal with losses (the value function is convex for $r^p < \hat{r}$). See Figure 1

[3]We assume that $\gamma \in (0,1)$ in order to be consistent with the experimental findings of Tversky and Kahneman (1992). Booij and van de Kuilen (2009) find the γ parameter to be (statistically) significantly less than unity.

for a graphical illustration of the value function, which is non-differentiable at the reference return.

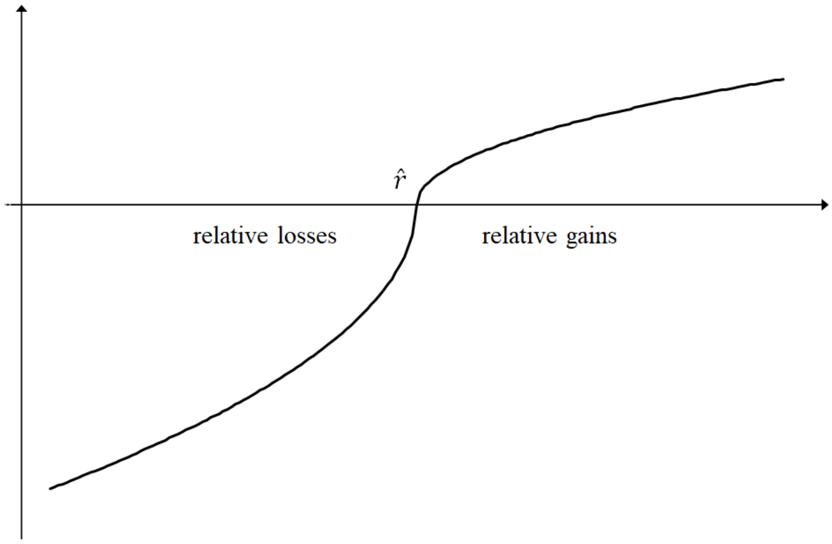

Figure 1: Value function of S-shaped prospect theory investor

We study the optimal asset allocation behavior of an investor with S-shaped prospect theory preferences. This behavior depends on the reference return $\hat{r}$ and, in particular, on whether this reference return is below, equal to, or above the risk-free interest rate. The position of the investor's reference return with respect to the risk-free rate is determined exogenously by the investor's incentive, e.g., investors with their reference return being below the risk-free rate ($\hat{r} < r^0$) can be viewed as less ambitious investors while investors with their reference return being above the risk-free rate ($\hat{r} > r^0$) can be viewed as more ambitious investors.

Investors maximize their expected utility of returns

$$\max_x \left\{ \mathbb{E}\left(v(\mathbf{r}'\mathbf{x})\right) \middle| \mathbf{A}\mathbf{x} \leq \mathbf{b} \right\} \tag{2.2}$$

where $\mathbf{x} = (x_1, \ldots, x_n)'$, with x_i denoting the proportion of wealth invested in asset i,[4] $i = 1, \ldots, n$, and $\mathbf{r}$ is the $n-$dimensional random vector of net returns, subject to the usual asset constraints $\mathbf{A}\mathbf{x} \leq \mathbf{b}$, where $\mathbf{A} \in \mathbb{R}^{m \times n}$, $\mathbf{b} \in \mathbb{R}^m$. Note that in general the proportion invested in a given asset may be negative or larger than one due to short-selling.

[4]Throughout this paper, prime ($'$) is used to denote matrix transposition and any unprimed vector is a column vector.

2.1 Analytical solution for one risk-free and one risky asset

To better understand the attitude with respect to risk of an investor with prospect theory preferences, we consider a simple two-asset world, where one asset is risk-free and the other is risky, and analyze what proportion of wealth is invested in the risky asset under prospect theory utility. Another motivation for looking at this problem is the following. When Tobin's separation theorem holds, the investor's investment decision problem can be simplified to deciding which proportion to invest in the safe asset and which to invest in some risky portfolio. As Levy et al. (2004) have shown, Tobin's separation principle does hold under the assumption of the Tversky and Kahneman's prospect theory utility.

Let r^0 be the certain (deterministic) return of the risk-free asset and let r be the (stochastic) return of the risky asset. Then the portfolio return is $R(x) = xr + (1 - x)r^0 = r^0 + (r - r^0)x$, where x is the proportion of wealth invested in the risky asset, and the maximization problem of the investor with prospect theory preferences is

$$\max_x \left\{ \mathbb{E}\left(v\left(\left(R(x)\right)\right)\right) = \mathbb{E}\left(v\left(r^0 + \left(r - r^0\right)x\right)\right) \mid x \in \mathbb{R} \right\} \tag{2.3}$$

with value function $v(\cdot)$ given by (2.1). As will be seen later, results will be sensitive to the position of the reference return with respect to the risk-free rate. A good interpretation of this position comes from writing down the portfolio return net of the reference return and seeing what happens if the investor stays out of the market ($x = 0$)

$$R(x) - \hat{r}|_{x=0} = r^0 - \hat{r}$$

Thus, if the residual of the relative portfolio return with respect to the reference point $\hat{r}$ with zero risky investment is positive, i.e., $\hat{r} < r^0$, and hence the investor is modest in setting her return goals, then even when she stays out of the market she will be in her *comfort zone*. On the other hand if the investor is more ambitious in setting her goals, i.e., $\hat{r} > r^0$, then the residual of the relative portfolio return with respect to the reference point with zero risky investment is negative and thus if she stays out of the market she will be not that well off and be in her *discomfort zone*.

The following two cases present characterizations of the optimal solution when the risky asset's return is binomially distributed (discrete distribution) and when it is (generally) continuously distributed.

The risky asset is binomially distributed

First we assume, for the sake of simplicity and because in this case we can show a number of results analytically, that the return of the risky asset follows a binomial distribution. We assume two states of nature: a good state of nature which yields return r_g such that $r_g > r^0$ and which occurs with probability p, and a bad state of nature which yields return r_b such that $r_b < r^0$ and which occurs with probability $1-p$, i.e., $r_b < r^0 < r_g$. In the good state of nature the portfolio thus yields return $R_g(x) = r^0 + (r_g - r^0)x$ with probability p and in the bad state of nature it yields return $R_b(x) = r^0 - (r^0 - r_b)x$ with probability $1-p$. Thus, based on (2.3), the expected prospect theory utility (value function) of the two-asset portfolio including the risk-free asset and the binomially distributed risky asset is the following continuous function

$$\mathbb{E}\left(v\left(R(x)\right)\right) = \left\{ \begin{array}{ll} \frac{1}{1-\gamma}\left[p\left(R_g(x)-\hat{r}\right)^{1-\gamma} + (1-p)(R_b(x)-\hat{r})^{1-\gamma}\right], & R_g(x) \geq \hat{r},\ R_b(x) \geq \hat{r} \\ \frac{1}{1-\gamma}\left[p(R_g(x)-\hat{r})^{1-\gamma} - \lambda(1-p)(\hat{r}-R_b(x))^{1-\gamma}\right], & R_g(x) \geq \hat{r},\ R_b(x) \leq \hat{r} \\ \frac{1}{1-\gamma}\left[-\lambda p(\hat{r}-R_g(x))^{1-\gamma} + (1-p)(R_b(x)-\hat{r})^{1-\gamma}\right], & R_g(x) \leq \hat{r},\ R_b(x) \geq \hat{r} \\ -\lambda\frac{1}{1-\gamma}\left[p(\hat{r}-R_g(x))^{1-\gamma} + (1-p)(\hat{r}-R_b(x))^{1-\gamma}\right], & R_g(x) \leq \hat{r},\ R_b(x) \leq \hat{r} \end{array} \right\} \tag{2.4}$$

To proceed with the analysis, we define the following threshold

$$K_\gamma = \frac{(1-p)(r^0-r_b)^{1-\gamma}}{p(r_g-r^0)^{1-\gamma}} \tag{2.5}$$

The next proposition presents the analytical solution of the prospect theory investor with preferences described by (2.1) and (2.3), who is less ambitious, i.e., her reference return is below the risk-free rate, and who is also sufficiently loss averse, i.e., λ is large enough.

Proposition 2.1 *Let $\mathbb{E}(r) > r^0$, $\hat{r} < r^0$, and $\lambda > \max\{K_\gamma, 1/K_\gamma\}$, where K_γ is defined by (2.5). Then there exists the only solution x^* of (2.3) such that*

$$x^* = \frac{\left(1-K_0^{1/\gamma}\right)(r^0-\hat{r})}{r^0-r_b+K_0^{1/\gamma}(r_g-r^0)} > 0 \tag{2.6}$$

Proof: See Appendix A. □

Note that solution (2.6) does not depend on the degree of loss aversion, λ, but a sufficiently large degree of loss aversion is needed to guarantee the monotonic properties of the prospect theory utility function in its certain domains. See the proof of proposition 2.1 in Appendix A for more detail.

Based on (2.6) we can formulate the following corollary, which formally states that when the reference return is below the risk-free rate then the investor is not sensitive to the degree of loss aversion (λ) and that she becomes more conservative with an increasing reference return, i.e., her investment in the risky asset decreases.

Corollary 2.1 *Let assumptions of proposition 2.1 be satisfied. Then the optimal solution of (2.3), x^*, has the following properties*

$$\frac{dx^*}{d\lambda} = 0$$

and

$$\frac{dx^*}{d\hat{r}} = -\frac{\left(1 - K_0^{1/\gamma}\right)}{r^0 - r_b + K_0^{1/\gamma}(r_g - r^0)} < 0 \tag{2.7}$$

Note that if $\tilde{v}(\cdot)$ is the power utility, namely $\tilde{v}(y) = \frac{y^{1-\gamma}}{1-\gamma}$, then the solution of the maximization of the expected power utility under the binomially distributed risky asset as specified above, namely $\max\{\mathbb{E}(\tilde{v}(R(x)) \,|\, x \in \mathbb{R}\}$, is $\tilde{x}^* = \frac{\left(1-K_0^{1/\gamma}\right)r^0}{r^0-r_b+K_0^{1/\gamma}(r_g-r^0)}$ and thus for $\hat{r} < r^0$ we have $x^* < \tilde{x}^*$ when $\hat{r} > 0$.[5] The proportion invested in the risky asset is thus smaller for the less ambitious prospect theory investor than for the investor characterized by power utility, as long as the reference return is positive.

The following proposition states that for a zero excess reference return ($\hat{r} = r^0$) the prospect theory investor stays out of the market ($x^* = 0$), i.e., everything is invested in the risk-free asset.[6]

Proposition 2.2 *Let $\hat{r} = r^0$ and $\lambda \geq \max\{K_\gamma, 1/K_\gamma\}$, where K_γ is defined by (2.5). Then the solution of (2.3) is $x^* = 0$.*

Proof: See Appendix A. □

Note that this is the only case when the portfolio of the prospect theory investor coincides with the portfolio of the mean-variance investor.[7]

[5] Note in addition that $x^* \geq \tilde{x}^*$ when $\hat{r} \leq 0$.

[6] This is also the case for the linear loss averse investor ($\gamma = 0$), see Fortin and Hlouskova (2011), but not for the loss averse investor with quadratic shortfall, where the optimal investment in the risky asset is strictly positive, see Fortin and Hlouskova (2015).

[7] As the variance of the portfolio return $r^p = xr + (1-x)r^0$ is $x^2\text{Var}(r)$, the minimum variance of the mean-variance portfolio is reached at $x = 0$.

Before proceeding further let us introduce the following notation

$$(x^*)^+ = \frac{\hat{r} - r^0}{r_g - r^0} \times \frac{\lambda^{1/\gamma} + \left(\frac{1}{K_0}\right)^{1/\gamma}}{\lambda^{1/\gamma} - \left(\frac{1}{K_\gamma}\right)^{1/\gamma}} \tag{2.8}$$

$$(x^*)^- = -\frac{\hat{r} - r^0}{r^0 - r_b} \times \frac{\lambda^{1/\gamma} + K_0^{1/\gamma}}{\lambda^{1/\gamma} - K_\gamma^{1/\gamma}} \tag{2.9}$$

$$\bar{p} = \frac{\left(r^0 - r_b\right)^{1-\gamma}}{\left(r^0 - r_b\right)^{1-\gamma} + \left(r_g - r^0\right)^{1-\gamma}} \tag{2.10}$$

Note that for the more ambitious investor, i.e., $\hat{r} > r^0$, is $(x^*)^+ > 0$ when $\lambda > 1/K_\gamma$ and $(x^*)^- > 0$ when $\lambda > K_\gamma$.

The following proposition presents the analytical solution of the investor with prospect theory preferences, see (2.1) and (2.3), who is more ambitious, i.e., her reference return exceeds the risk-free rate, and who is also sufficiently loss averse.

Proposition 2.3 *Let $\hat{r} > r^0$ and $\lambda > \max\{K_\gamma, 1/K_\gamma\}$, where K_γ is defined by (2.5). Then there exists the solution x^* of (2.3) such that*

$$x^* \left\{ \begin{array}{ll} = (x^*)^+ > 0, & \text{for } \; p > \bar{p} \\ = (x^*)^- < 0, & \text{for } \; p < \bar{p} \\ \in \{(x^*)^+, (x^*)^-\} & \text{for } \; p = \bar{p} \end{array} \right\} \tag{2.11}$$

which is unique for $p \neq \bar{p}$.

Proof: See Appendix A. □

Proposition (2.3) implies that the more ambitious sufficiently loss averse investor purchases the risky asset when the probability of the good state to occur is sufficiently large. In the other case, i.e., when the probability of the good state to occur is sufficiently small, she takes a short position in the stock market.

Based on (2.11) we can formulate the following corollary, which presents the sensitivity of the risky investment of the more ambitious investor ($\hat{r} > r^0$) with respect to her degree of loss aversion and her reference rate.

Corollary 2.2 *Let the assumptions of proposition 2.3 be satisfied. Then the optimal solu-*

tion has the following properties:

$$\frac{dx^*}{d\lambda} = \left\{ \begin{array}{r} -\frac{1}{\gamma}\frac{r_g - r_b}{r_g - r^0} \times \frac{\lambda^{1/\gamma - 1}}{K_0^{1/\gamma}\left[\lambda^{1/\gamma} - \left(\frac{1}{K_\gamma}\right)^{1/\gamma}\right]^2} < 0, \quad \text{for } p > \bar{p} \\ \\ \frac{1}{\gamma}\frac{r_g - r_b}{r^0 - r_b} \times \frac{K_0^{1/\gamma}\lambda^{1/\gamma - 1}}{\left[\lambda^{1/\gamma} - K_\gamma^{1/\gamma}\right]^2} > 0, \quad \text{for } p < \bar{p} \end{array} \right\} \tag{2.12}$$

and

$$\frac{dx^*}{d\hat{r}} = \left\{ \begin{array}{r} \frac{1}{r_g - r^0} \times \frac{\lambda^{1/\gamma} + \left(\frac{1}{K_0}\right)^{1/\gamma}}{\lambda^{1/\gamma} - \left(\frac{1}{K_\gamma}\right)^{1/\gamma}} > 0, \quad \text{for } p > \bar{p} \\ \\ -\frac{1}{r^0 - r_b} \times \frac{\lambda^{1/\gamma} + K_0^{1/\gamma}}{\lambda^{1/\gamma} - K_\gamma^{1/\gamma}} < 0, \quad \text{for } p < \bar{p} \end{array} \right\} \tag{2.13}$$

Corollary 2.2 implies the following findings regarding the comparative statics for the more ambitious investor. For a sufficiently large probability of the good state to occur the risk taking (i.e., the investment in the risky asset, x^*) decreases with an increasing degree of loss aversion while risk taking increases with an increasing level of ambition, $\hat{r}$. On the other hand, when the probability of the good state to occur is sufficiently small then the risk taking increases with an increasing loss aversion while it decreases with an increasing level of ambition.

Table 1 summarizes and contrasts the optimal investments into the risky asset of the linear loss averse (LLA) and the prospect theory (PT) investors (for more detail regarding the LLA investor see Fortin and Hlouskova, 2011). Note that the investment in the risky asset of the less ambitious LLA investor exceeds the investment in the risky asset of the less ambitious PT investor. The opposite holds for the more ambitious investor when the probability of the good state to occur is sufficiently large, i.e., when $p > \bar{p}$. If $p < \bar{p}$ then the PT investor takes a short position and thus the risk taking of the LLA investor again exceeds the risk taking of the PT investor. Note in addition that only when the reference return coincides with the risk-free rate is the investment in the risky asset the same for both investors, namely staying out of the market. Risk taking for the LLA investor is always positive when the reference return does not coincide with the risk-free rate.

The risky asset is continuously distributed

Now we assume that the risky asset's return r is continuously distributed with probability density function $f(\cdot)$ and cumulative distribution function $F(\cdot)$ such that $\mathbb{E}(|r|) = \int_{-\infty}^{+\infty} |r| f(r) dr < +\infty$ and $F(c) < 1$ for any $c \in \mathbb{R}$. These assumptions are satisfied when for

assumptions on $\hat{r}$	additional assumptions	solutions
$\hat{r} < r^0$	$\mathbb{E}(r) > r^0$	$0 < x^* < x^*_{\text{LLA}} = \frac{r^0-\hat{r}}{r^0-r_b}$
$\hat{r} = r^0$		$x^* = x^*_{\text{LLA}} = 0$
$\hat{r} > r^0$	$p > \bar{p}$	$(x^*)^+ = x^* > x^*_{\text{LLA}} = \frac{\hat{r}-r^0}{r_g-r^0} > 0$
$\hat{r} > r^0$	$p < \bar{p}$	$(x^*)^- = x^* < 0 < x^*_{\text{LLA}} = \frac{\hat{r}-r^0}{r_g-r^0}$

Table 1: Summary of optimal solutions under S-shaped prospect theory and linear loss aversion.
We assume that $\lambda > \max\{K_\gamma, 1/K_\gamma, 1/K_0\}$. Note that x^* is given by (2.6) when $\hat{r} < r^0$; $(x^*)^+$ is given by (2.8), $(x^*)^-$ by (2.9), $\bar{p}$ by (2.10) and x^*_{LLA} is given in Fortin and Hlouskova (2011).

instance the risky asset's return follows a normal distribution or a Gamma distribution.

Let r^p be a continuous random variable describing the stochastic portfolio return and $f_{r^p}(\cdot)$ be its probability density function. Then based on (2.1) we define the expected prospect theory utility of return r^p as

$$\mathbb{E}(v(r^p)) = \frac{1}{1-\gamma}\left(-\lambda\int_{-\infty}^{\hat{r}}(\hat{r}-z)^{1-\gamma}f_{r^p}(z)dz + \int_{\hat{r}}^{+\infty}(z-\hat{r})^{1-\gamma}f_{r^p}(z)dz\right) \tag{2.14}$$

and thus based on (2.14) the expected prospect theory utility function of portfolio return $R(x)$ is

$$\mathbb{E}(v(R(x))) = \begin{cases} \frac{(-x)^{1-\gamma}}{1-\gamma}\left[\int_{-\infty}^{z(x)}(z(x)-r)^{1-\gamma}f(r)dr - \lambda\int_{z(x)}^{+\infty}(r-z(x))^{1-\gamma}f(r)dr\right], & x<0 \\ \frac{1}{1-\gamma}\left(r^0-\hat{r}\right)^{1-\gamma}, & x=0 \text{ and } \hat{r}\le r^0 \\ -\frac{\lambda}{1-\gamma}\left(\hat{r}-r^0\right)^{1-\gamma}, & x=0 \text{ and } \hat{r}>r^0 \\ \frac{x^{1-\gamma}}{1-\gamma}\left[-\lambda\int_{-\infty}^{z(x)}(z(x)-r)^{1-\gamma}f(r)dr + \int_{z(x)}^{+\infty}(r-z(x))^{1-\gamma}f(r)dr\right], & x>0 \end{cases} \tag{2.15}$$

where $z(x) = \frac{\hat{r}-r^0}{x} + r^0$. It is easy to see that $\mathbb{E}(v(R(x)))$ is continuous in x, also for $x = 0$.

The problem we want to solve is

$$\max_x \{\, \mathbb{E}(v(R(x))) \,|\, x \in \mathbb{R}\,\} \tag{2.16}$$

The case when $\hat{r} = r^0$ is already solved in the literature (see Bernard and Ghossoub, 2010, or He and Zhou, 2011) and, as in the discrete case, the PT investor stays out of the market. This holds also for the mean-variance investor. We summarize it in the following proposition, where

$$K_\gamma = \frac{\int_{-\infty}^{r^0} (r^0 - r)^{1-\gamma} f(r) dr}{\int_{r^0}^{+\infty} (r - r^0)^{1-\gamma} f(r) dr} \tag{2.17}$$

Proposition 2.4 *Let $\hat{r} = r^0$ and $\lambda > \max\left\{K_\gamma, \frac{1}{K_\gamma}\right\}$ where K_γ is given by (2.17). Then $x^* = 0$ is the solution of problem (2.16).*

Proof: See Appendix A. □

For $\hat{r} \neq r^0$ we can find the solution only in its implicit form but we can still perform comparative statics and show how the solution depends on the degree of loss aversion λ, and the reference return $\hat{r}$.

The following propositions present sufficient conditions for the existence of a global maximum or at least local maxima of problem (2.16), namely strictly positive investment in the risky asset x^* that satisfies the following equation

$$\lambda \int_{-\infty}^{z(x^*)} \frac{r - r^0}{[z(x^*) - r]^\gamma} f(r) dr + \int_{z(x^*)}^{+\infty} \frac{r - r^0}{[r - z(x^*)]^\gamma} f(r) dr = 0 \tag{2.18}$$

where $z(x^*) = \frac{\hat{r} - r^0}{x^*} + r^0$. In more detail, the proposition below states that the optimal investment in the risky asset of a sufficiently loss averse less ambitious investor is strictly positive. To ease the exposition we introduce the following notation

$$\hat{K}_\gamma = \max_c \left\{ K_\gamma(c) = \frac{\int_{-\infty}^{c} (c - r)^{1-\gamma} f(r) dr - (c - r^0)^{1-\gamma}}{\int_{c}^{+\infty} (r - c)^{1-\gamma} f(r) dr} \,\Big|\, c \geq r^0 \right\} \tag{2.19}$$

Proposition 2.5 *Let $\mathbb{E}(r) > r^0$, $\hat{r} < r^0$, and $\lambda > \max\{\hat{K}_\gamma, 1/K_\gamma\}$, where K_γ is given by (2.17) and $\hat{K}_\gamma$ is given by (2.19). Then there exists a finite positive global maximum of problem (2.16), i.e., $x^* > 0$, which satisfies equation (2.18).*

Proof: See Appendix A. □

Note that $K_\gamma(r^0) = K_\gamma$ and thus from the assumption of proposition 2.5 it follows that $\lambda > K_\gamma$. Lemma 4.1 (see Appendix A) shows that $\hat{K}_\gamma < +\infty$, i.e., $\hat{K}_\gamma$ is bounded from above and thus there exist such λs that the assumption $\lambda > \max\{\hat{K}_\gamma, 1/K_\gamma\}$ of proposition 2.5 is satisfied.

The following proposition presents results again in the implicit form, for both less and more ambitious investors.

Proposition 2.6 *Let* $\mathbb{E}(r) > r^0$, $\hat{r} \neq r^0$, *and* $\lambda > \max\{K_\gamma, 1/K_\gamma\}$, *where* K_γ *is given by (2.17). Then there exists at least one local maximum of problem (2.16) such that* $x^* > 0$ *and any positive local maximum* $x^* > 0$ *satisfies (2.18). If there exists also a local maximum such that* $(x^*)^- < 0$ *then it satisfies the following equation*

$$\int_{-\infty}^{z((x^*)^-)} \frac{r - r^0}{[z((x^*)^-) - r]^\gamma} f(r)dr + \lambda \int_{z((x^*)^-)}^{+\infty} \frac{r - r^0}{[r - z((x^*)^-)]^\gamma} f(r)dr = 0 \qquad (2.20)$$

where $z((x^*)^-) = \frac{\hat{r} - r^0}{(x^*)^-} + r^0$. *Finally, any global maximum is finite, i.e.* $\mathbb{E}(v(R(x^*))) < +\infty$.

Proof: See Appendix A. □

The last proposition presents the sensitivity analysis of risky investment of both less and more ambitious investors with respect to their degrees of loss aversion and reference rates.

Proposition 2.7 *Let* $x^* > 0$ *be the optimal solution of (2.16) such that (2.18) is satisfied,* $\hat{r} \neq r^0$, *and let*

$$\lim_{r \to \pm\infty} |r|^{2-\gamma} f(r) = 0 \qquad (2.21)$$

Then x^* *has the following properties*

$$\frac{dx^*}{d\lambda} < 0 \qquad (2.22)$$

and

$$\frac{dx^*}{d\hat{r}} = \left\{ \begin{array}{ll} < 0, & \text{if } \hat{r} < r^0 \\ > 0, & \text{if } \hat{r} > r^0 \end{array} \right\} \qquad (2.23)$$

Proof: See Appendix A. □

Proposition 2.7 implies that any positive solution (investment in the risky asset) of (2.16) satisfying (2.18) decreases with an increasing degree of loss aversion and it also decreases with an increasing reference return for the less ambitious investor while the investment in the risky asset increases with an increasing reference return for the more ambitious investor. The reference return thus plays an important role in asset allocation. In particular, the proportion invested in the risky asset has a V-shaped pattern with respect to the reference return level. For relatively low reference returns, the investor prefers to accept more risk to earn more, rather than switch away from the stock market. For relatively high reference returns, the investor puts more money into the risky asset due to her risk-seeking behavior in the domain of losses.

A complete summary of the results including comparative statics with respect to loss aversion (λ) and the level of ambition ($\hat{r}$) for both the prospect theory investor and the linear loss averse investor[8] can be found in Table 2.

conditions on λ	conditions on $\hat{r}$	binomial	continuous
	S-shaped prospect theory		
		x^*	x^*
$\lambda > \max\{K_\gamma, \hat{K}_\gamma, 1/K_\gamma\}$	$\hat{r} < r^0$	> 0	> 0
$\lambda > 1/K_\gamma$, $p > \bar{p}$	$\hat{r} > r^0$	> 0	$-$
$\lambda > K_\gamma$, $p < \bar{p}$	$\hat{r} > r^0$	< 0	$-$
$\lambda > \max\{K_\gamma, 1/K_\gamma\}$	$\hat{r} \neq r^0$	$-$	> 0
	$\hat{r} = r^0$	$= 0$	$= 0$
		$dx^*/d\lambda$	$dx^*/d\lambda$
$\lambda > \max\{K_\gamma, \hat{K}_\gamma, 1/K_\gamma\}$	$\hat{r} < r^0$	$= 0$	$< 0^*$
$\lambda > 1/K_\gamma$, $p > \bar{p}$	$\hat{r} > r^0$	< 0	$-$
$\lambda > K_\gamma$, $p < \bar{p}$	$\hat{r} > r^0$	> 0	$-$
$\lambda > \max\{K_\gamma, 1/K_\gamma\}$	$\hat{r} \neq r^0$	$-$	$< 0^*$
	$\hat{r} = r^0$	$= 0$	$= 0$
		$dx^*/d\hat{r}$	$dx^*/d\hat{r}$
$\lambda > \max\{K_\gamma, \hat{K}_\gamma, 1/K_\gamma\}$	$\hat{r} < r^0$	< 0	$< 0^*$
$\lambda > 1/K_\gamma$, $p > \bar{p}$	$\hat{r} > r^0$	> 0	$-$
$\lambda > K_\gamma$, $p < \bar{p}$	$\hat{r} > r^0$	< 0	$-$
$\lambda > \max\{K_\gamma, 1/K_\gamma\}$	$\hat{r} > r^0$	$-$	$> 0^*$
	$\hat{r} = r^0$	$= 0$	$= 0$
	linear loss aversion		
		x^*	x^*
$\lambda > 1/K_0$	$\hat{r} \neq r^0$	> 0	> 0
	$\hat{r} = r^0$	$= 0$	$= 0$
		$dx^*/d\lambda$	$dx^*/d\lambda$
$\lambda > 1/K_0$	$\hat{r} \neq r^0$	$= 0$	< 0
	$\hat{r} = r^0$	$= 0$	$= 0$
		$dx^*/d\hat{r}$	$dx^*/d\hat{r}$
$\lambda > 1/K_0$	$\hat{r} < r^0$	< 0	< 0
	$\hat{r} > r^0$	> 0	> 0
	$\hat{r} = r^0$	$= 0$	$= 0$

Table 2: Summary of optimal solutions and sensitivities under S-shaped prospect theory and linear loss aversion.
We assume that $\mathbb{E}(r) > r^0$ and $\lambda > 1$. Note that K_γ for the binomial case is defined by (2.5) while for continuous case it is defined by (2.17). In addition, $\hat{K}_\gamma$ is defined by (2.19) and $\bar{p}$ by (2.10). 0^* denotes the cases when an additional assumption is required, namely $\lim_{r\to\pm\infty} |r|^{2-\gamma} f(r) = 0$.

[8] For more detail regarding the LLA investor see again Fortin and Hlouskova (2011).

3 Empirical application

In this section we investigate the performance of an optimal asset portfolio constructed by an investor with S-shaped prospect theory preferences. We study the benchmark scenario, where the penalty parameter is constant and the reference return is equal to zero percent, and three modified versions of the benchmark scenario. The first modification uses the risk-free interest rate as the reference point (risk-free scenario), the remaining two modifications employ time-changing versions of the penalty parameter, which depend on previous gains and losses, while the reference return is either zero, the risk-free interest rate or the portfolio return of the previous period. So, we consider two constant scenarios and two dynamic scenarios with respect to the penalty parameter. The first dynamic scenario describes the usual conservative loss averse investor, who becomes even more loss averse after losses (conservative scenario), while the second dynamic scenario describes a more aggressive non-conventional risk-seeking investor, who becomes less loss averse after losses and accepts further risk and gambles which offer a chance to break even (aggressive scenario). Our conservative and aggressive (break-even) scenarios are modified versions of the scenarios suggested by Barberis and Huang (2001) and Zhang and Semmler (2009), respectively.[9]

We consider the following scenarios, where the degree of the investor's loss aversion is updated according to certain rules based on the portfolio performance (gains or losses) of the previous period while the reference return is either zero, $\hat{r}_t = 0$, or the risk-free interest rate, $\hat{r}_t = r_t^0$, or it is the portfolio return of the previous period, $\hat{r}_t = r_{t-1}^p$. Thus, the value function adjusted for the time-changing penalty parameter and a certain reference return is

$$v(r_t^p) = \begin{cases} \frac{(r_t^p - \hat{r}_t)^{1-\gamma}}{1-\gamma}, & r_t^p \geq \hat{r}_t \\ \\ -\lambda_t \frac{(\hat{r}_t - r_t^p)^{1-\gamma}}{1-\gamma}, & r_t^p < \hat{r}_t \end{cases}$$

The conservative scenario is modeled as follows. If the investor has experienced gains, then her penalty parameter is equal to the pre-specified λ while, on the other hand, if the investor has experienced losses, then her loss aversion and thus her penalty parameter increases, i.e.,

$$\lambda_t = \begin{cases} \lambda, & r_{t-1}^p \geq r_{t-2}^p \;\; \text{(gains)} \\ \lambda + (z_{t-1} - 1), & r_{t-1}^p < r_{t-2}^p \;\; \text{(losses, } \lambda \text{ increases)} \end{cases} \tag{3.24}$$

[9] Note that although this analysis covers many empirical aspects of the problem a more thorough analysis is required to shed light on all details and we will deal with this in our future research. For example, as we focus mainly on loss aversion, the dynamic scenarios in our study update only the investor's loss aversion, and only after prior losses. They do not update the reference return and do not apply dynamic updates of both the reference return and loss aversion after prior gains. So we can not study here the house money effect, for instance.

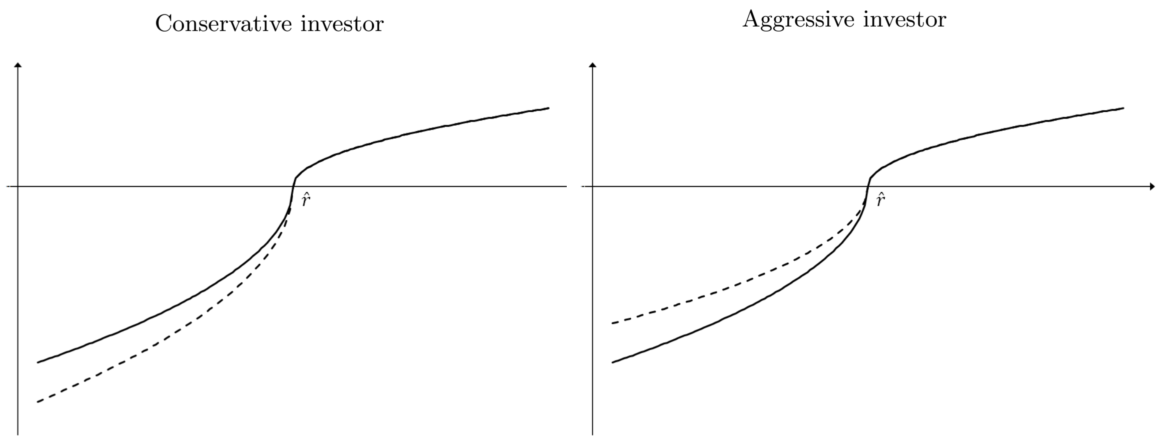

Figure 2: Value function of conservative and aggressive investors.
The value function after prior gains is plotted as a solid line, the value function after prior losses as a dashed line; $\hat{r}$ denotes the reference return.

where $z_t = \frac{1+r^p_{t-1}}{1+r^p_t} \geq 0$[10] and $\lambda_t \geq \lambda$. See the left plot in Figure 2 where the dashed line represents the higher loss aversion of a conservative investor after prior losses. At the same time the reference return of the conservative investor is either equal to zero, $\hat{r}_t = 0$, or to the risk-free interest rate, $\hat{r}_t = r^0$, or to the portfolio return of the previous period, $\hat{r}_t = r^p_{t-1}$.

The following scenario is partly based on the "break-even" effect as described in Zhang and Semmler (2009) and we refer to it as the aggressive scenario. The main idea is that sometimes both private and institutional investors may become more risk-seeking after losses in order to make up for previous losses. In other words, even if they have experienced a loss in the previous period, investors may be ready to incur further risks and accept gambles which offer them a chance to break even. In this case the loss implies a decreasing loss aversion due to the investor's increased risk-seeking. The gain, on the other hand, is treated as in the conservative scenario, i.e., the degree of loss aversion remains at the pre-specified level. The time-changing penalty parameter is then

$$\lambda_t = \begin{cases} \lambda, & r^p_{t-1} \geq r^p_{t-2} \ \text{(gains)} \\ \lambda + \left(\frac{1}{z_{t-1}} - 1\right), & r^p_{t-1} < r^p_{t-2} \ \text{(losses, } \lambda \text{ decreases)} \end{cases} \tag{3.25}$$

See the right plot in Figure 2 where the dashed line represents the lower loss aversion after prior losses of an aggressive investor. The reference return in the aggressive scenario is again either equal to zero or to the risk-free interest rate or to the portfolio return of the previous period. Note that in this case is $\lambda_t \leq \lambda$ and thus the investor decreases her degree of loss aversion. With the current lambda adjustment a sufficient condition for $\lambda_t \geq 1$ is $\lambda \geq 2$.[11]

[10] Note that in case of gains; i.e., $r^p_{t-1} \geq r^p_{t-2}$, is $z_{t-1} \leq 1$ and in case of losses; i.e., $r^p_{t-1} < r^p_{t-2}$, is $z_{t-1} > 1$.
[11] Note that this assumption might be violated in our empirical applications only for $\lambda = 1.5$, which we handle in our code such that if λ_t happens to be below one then we impose $\lambda_t = 1$.

In the empirical analysis we consider two geographical markets, the European and the US markets, where the euro area represents the European (EU) market. We consider three different types of assets among which the investor may select: a stock market index, a 10-year government bond[12] and gold.[13] We impose a budget constraint, i.e., $x_1 + x_2 + x_3 = 1$, and a no-short sale restriction, i.e., $x_i \geq 0$, $i = 1, 2, 3$.[14] Returns are computed as $r_t = P_t/P_{t-1} - 1$, where P_t is the monthly closing price at time t. All prices are extracted from Refinitiv Datastream from January 1983 to December 2020. The overall stock market indices for the EU and the US are as calculated by Datastream. Prices in the European markets are quoted in, or transformed to, Euro; prices in the US markets are quoted in US dollar, hence we consider European and US investors who completely hedge their respective currency risk. Figure 3 presents the price developments of the three assets for both the EU and the US, Table 9 in Appendix B provides the data description and the data sources, and Table 10 reports the summary statistics of asset returns including correlations. In general, the stock index exhibits a comparatively high risk and return, the government bond shows a much lower risk and return, and gold exhibits a relatively high risk but also a small return. The correlations between the stock index and the government bond as well as between the stock index and gold are slightly negative but close to zero, and the correlation between the government bond and gold is approximately 0.06 (in both the European and US markets, respectively). The relatively low correlations suggest that – at least in mean-variance portfolios – all three assets should be included in the optimal portfolio at non-negligible rates for diversification reasons.

The investor is assumed to re-optimize her portfolio once a month using monthly closing prices and considers an optimization sample of 36 months, i.e., three years. This yields an out-of-sample evaluation period from January 1986 until December 2020. We use different values of λ in all scenarios to allow for different degrees of loss aversion. Specifically, we let the penalty parameter be equal to 1.5, 2, 2.25, 2.5 and 3.[15] In addition we account for the following values of the risk aversion/risk loving – or diminishing sensitivity – parameter, namely $\gamma = 0$, 0.1, 0.5 and 0.9. The case $\gamma = 0$ represents a linear loss averse (LLA) investor and if, in addition, $\lambda = 1$ then the investor becomes risk neutral (linear).

For the European and US prospect theory investors we report optimization results for different scenarios, as described above. In particular, we present descriptive statistics in-

[12]We use the German 10-year government bond as a proxy, because euro area government bonds do not exist and we want to have an investable asset not some artificial aggregate.

[13]Note that gold has a low correlation with equity and bond prices, see Table 10, and hence including gold in the portfolio provides a natural hedge. Gold is also considered a safe haven in turbulent times.

[14]To solve problem (2.2) numerically, we apply the grid search method. The whole procedure is implemented in MATLAB R2021a and EViews 12.

[15]Note that the value $\lambda = 2.25$ is the one estimated by Kahneman and Tversky (see Tversky and Kahneman, 1992). Chapman et al. (2018) provide a median estimate of $\lambda = 1.99$ for the US in lab experiments.

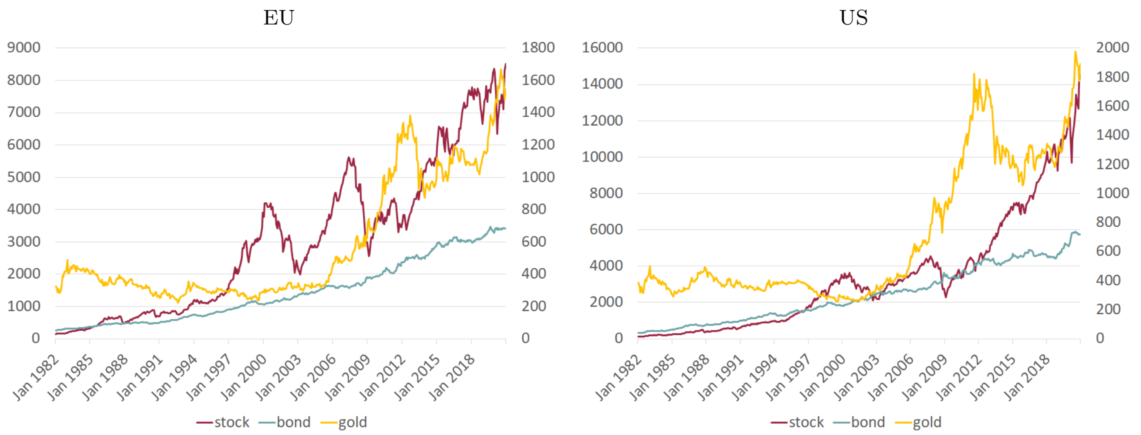

Figure 3: Asset prices in the EU (left) and the US (right). Stock prices are shown on the left axis, bond and gold prices are shown on the right axis.

cluding mean, standard deviation, downside volatility, conditional value at risk, and various risk-adjusted performance measures of the optimal portfolio returns as well as the average optimal portfolio weights. Risk-adjusted performance measures include the Sharpe and Sortino ratios and the Omega measure.[16]

3.1 Portfolio performance and asset allocation

As the main focus of our empirical analysis are S-shaped prospect theory (PT) investors, not linear loss aversion (LLA) investors, we present the results regarding the performance of PT portfolios and of benchmark portfolios, such as mean-variance (MV) and conditional value-at-risk (CVaR) models, in Tables 4 to 7. However, if the portfolio of the LLA investor (including the risk neutral investor who is a special case of the LLA investor) achieves the best performance then we explicitly mention this in the text.

Prospect theory versus mean-variance investment

Our empirical results suggest that PT investment leads to clearly higher means of portfolio returns, but also to much higher risk (measured by the volatility of portfolio returns) than traditional MV investment, for all types of PT investors[17] and for both the EU and US markets. If we consider risk-adjusted performance measures like Omega, the Sharpe ratio and the Sortino ratio, however, then the geographical market seems to matter. In this case MV investment outperforms PT investment in the EU, while PT investment outperforms

[16]The Sortino ratio is a modified version of the Sharpe ratio which uses downside volatility with respect to a target return (instead of standard deviation) as the denominator. The Omega measure is a ratio of upside potential of portfolio return relative to its downside potential with respect to a target return (see Shadwick and Keating, 2002). In our applications we take the risk-free interest rate as the target return.

[17]By all types of PT investors we mean investors with respect to all different values (under consideration) of $\hat{r}$, λ and γ, as well as both types of scenarios (constant and dynamic).

MV investment in the US in most cases. Considering the benchmark scenario in PT investment with $\gamma = 0.5$, for example, we observe an average mean of portfolio returns (over the different degrees of loss aversion) of 7.7% (9.8%) in the EU (US) market versus a mean of 6.2% (7.0%) for MV investment. The Sharpe ratio in this case is 49.9 (59.7) for PT investment versus 62.7 (57.4) for MV investment on average (again over the different degrees of loss aversion) in the EU (US) market. See Tables 11 and 12. We also observe that portfolio returns of PT investors significantly outperform portfolio returns of MV investors in the US market. For more details see Table 3 and the corresponding discussion in the subsection on means.

In addition, the PT investor shows a clearly different investment behavior with respect to implied asset weights from the MV investor. Figure 6 shows the optimal portfolio weights of stocks, bonds and gold for a PT investor (characterized by $\lambda = 2.25$, $\gamma = 0.5$ and $\hat{r} = 0$) and for an MV investor.[18] While the PT investor holds large proportions of stocks during certain time periods and her asset weights may sometimes vary strongly over short periods, the MV investor holds rather large proportions of bonds throughout the total time period and her asset weights do not change a lot over short periods.[19] In this particular case the means and standard deviations (in brackets) of the portfolio weights implied by the PT investor operating in the EU market are 32.9 (30.0), 56.9 (30.0), and 10.3 (11.9) for stocks, bonds, and gold, respectively. The corresponding numbers implied by MV investing are 10.6 (8.5), 80.7 (7.4), and 8.8 (7.1). If one invests in the US market the numbers implied by PT investing are 46.0 (31.6), 41.8 (32.2), and 12.3 (16.9) for stocks, bonds, and gold, respectively; the ones implied by MV investing are 20.7 (10.5), 64.6 (10.5), and 14.6 (14.2). See Tables 11 and 12. We can explain the "smooth" investment behavior of the MV investor, dominated by bonds, by her strong preferences for a low portfolio risk (measured by the variance of portfolio returns), which clearly favors bonds whose risk is constantly small and much lower than that of stocks and gold. The PT investor, however, seems to be driven more by the different levels of stock returns resulting, sometimes, in large changes in stock weights. This conjecture is verified by the high correlation between prospect theory stock weights and stock returns over some rolling window. For example, the correlation between the stock weight at a given time and the average of stock returns over the previous three

[18] Note that the particular example of the PT investor shows an investment behavior very similar to other types of PT investors, including conservative and aggressive types. So the following discussion holds overall for all PT investors.

[19] These different weight patterns across PT and MV investors are also suggested by the simple means and standard deviations of the portfolio weights presented in the bottom parts of Tables 11–18. In particular the high standard deviation of the weights implied by the PT investor vis-a-vis the much lower standard deviation of the weights implied by the MV investor reflects the non-smooth investment behavior of PT investors versus the smooth investment behavior of MV investors. For stocks and bonds, the standard deviation of the weight of the MV investor is often only a third, or even a forth, of the standard deviation of the weight of the PT investor.

years is 0.62 and 0.53 for the EU and the US, respectively.[20] We take a three-year return average as also the portfolio optimization process uses a rolling window of three years.

Different scenarios in prospect theory investment

Our empirical results suggest that for a given level of diminishing sensitivity and a given degree of loss aversion[21] the reference return seems to be a crucial factor in determining the performance of PT investment. This is in line with our theoretical finding that the optimal solution of a prospect theory investor in the two-asset case depends on her reference level. In particular the reference return seems to be more of a game changer than the investor's behavior regarding her update (or not) in her degree of loss aversion, i.e., whether the investor follows a constant or a conservative or an aggressive strategy. However, this is an observation based on the specific (limited) scenarios we consider, see footnote 9. For the reference return being equal to zero or the risk-free interest rate, the constant and dynamic scenarios yield very similar performance for both the European and the US markets. See Figure 4, which shows Omega measures for different PT investors with $\gamma = 0.5$.[22] Only when the reference return is equal to the portfolio return of the previous period is the situation different. In this case the difference between the performance of conservative and aggressive investors is more pronounced. In the US market the Omegas implied by aggressive investment are always (i.e., for all degrees of loss aversion) larger than the Omegas implied by conservative investment, while in the European market the Omegas implied by aggressive investment are larger for smaller degrees of loss aversion and smaller for higher degrees of loss aversion.[23] The similarities for zero and risk-free reference returns (across constant and dynamic scenarios) are mainly due to the small differences in the respective penalty parameters across the scenarios,[24] combined with portfolio returns being rather close to the reference return. In the case of the reference return being equal to the portfolio return of the previous period, the differences between the corresponding penalty parameters across

[20]These correlations are true for the specific examples shown in the graphs in Figure 6, for other types of PT investors the correlations are also large; and they are usually larger for smaller values of γ and for a lower degree of loss aversion. For example, for the reported type of PT investors with $\gamma = 0.1$ (instead of $\gamma = 0.5$) the correlations would be 0.78 and 0.65, for the EU and the US, respectively.

[21]By a "given degree of loss aversion" in dynamic scenarios we understand the degree of loss aversion after prior gains, which is constant. Also the degree of loss aversion after prior losses is usually very close to this value, see footnote 24.

[22]Similar observations apply for other performance measures.

[23]The investors' characteristics for which the aggressive scenarios lead to a better performance are, however, not similar across the three levels of diminishing sensitivity.

[24]Across all different types of investors (with respect to different γ, λ and $\hat{r}$) the loss aversion parameter is at most by 0.33 larger than the initially given value in the conservative scenarios, and it is at most by 0.25 smaller than the initially given value in the aggressive scenarios. Mostly, however, the deviations from the initial lambda are much smaller. In at least 90% of the deviations upwards (conservative scenario) the deviations are smaller than 0.08 (for a given investor), and in at least 90% of the deviations downwards (aggressive scenario) the absolute deviations are smaller than 0.07 (for a given investor).

conservative and aggressive scenarios are also small (maybe slightly larger than for $\hat{r} = 0$ and $\hat{r} = r^0$), but the portfolio returns are much further away from the reference returns, and thus the resulting differences in performance measures are more pronounced.

Prospect theory investment in the EU and US markets

When comparing prospect theory investment in the two markets, EU and US, we notice that the means of portfolio returns are clearly larger in the US than in the EU markets, and also risk-adjusted performance measures are mostly larger in the US than in Europe,[25] If one targets risk measures like volatility, however, then the European market seems to be the better choice, as volatilities are smaller in Europe than in the US. See Figure 5, which shows different performance measures (mean, Sharpe ratio, Omega measure, volatility) of portfolio returns implied by PT investment ($\gamma = 0.5$) for the zero and the risk-free reference returns, for different degrees of loss aversion, for the EU and the US markets.

With respect to the best performance we observe that PT investors show a different behavior in the EU and US markets. The best risk-adjusted performance[26] in the EU market is implied by the PT investor following the conservative scenario with the largest diminishing sensitivity ($\gamma = 0.9$), the largest degree of loss aversion ($\lambda = 3$) and a zero reference return, see Table 5. On the other hand, the best risk-adjusted performance in the US market is implied by the PT investor following the aggressive scenario with the largest diminishing sensitivity ($\gamma = 0.9$), $\lambda = 2$ and the risk-free reference return (see again Table 5). Though in both markets the largest risk-adjusted performance is implied by PT investors with the highest diminishing sensitivity, in the EU market the conservative strategy seems to be the way to go while in the US market the aggressive strategy provides the best (risk-adjusted) performance. Note also that the EU investor is more loss averse than the US investor and the reference return of the EU investor is always constant, namely zero, while it coincides with the risk-free investment (and thus is time changing) in the case of the US investor. Note finally that the EU investor allocates on average 60% of her investment in the bond market (followed by 29% in the stock market and 11% in gold) while the more aggressive US investor allocates on average 50% of her investment in the stock market (followed by 35% in the bond market and 15% in gold), see Tables 14 and 18.

[25] Only for the highest degree of loss aversion ($\lambda = 3$), risk-adjusted performance may be slightly better in Europe.

[26] What follows is equally true for the Omega measure, the Sharpe ratio and the Sortino ratio, for both the EU and US markets.

Mean

The best model for the EU market in terms of the largest mean of portfolio returns for any given level of γ, is the risk neutral (linear) model, which shows a mean of 12.9%. For the US market the largest mean, namely 13.6%, which is observed for an investor with $\gamma = 0.1$, is obtained by the PT model with $\lambda = 1.5$ under the conservative scenario with the reference level being the portfolio return from the previous period. For $\gamma = 0.5, 0.9$ the model with the largest mean in the US market is the linear model with a slightly smaller mean of 13.3%. The results (in terms of annual averages) are presented in Table 4. For the EU market the best models are found for the aggressive scenario with $\hat{r} = r^p$ for $\gamma = 0.1, 0.9$, and for the aggressive scenario with $\hat{r} = 0$ for $\gamma = 0.5$. Also for the US market the aggressive scenario is the one which implies the highest means (except for case $\gamma = 0.1$ when the best model is implied by the conservative scenario with $\hat{r} = r^p$). Note that for a given investor the means of PT portfolio returns in the US are by roughly two to three percentage points larger than the corresponding means of portfolio returns in the EU. Note in addition that the mean in general decreases with increasing λ, keeping other model parameters fixed (see Tables 11–18). This is in line with our theoretical results for the two-asset case and follows from Proposition 2.7, see (2.22).[27]

In the US market, PT portfolios perform significantly better than MV portfolios in the majority of cases, at the 10% significance level.[28] For a PT investor with $\gamma = 0.1$ this is true for all scenarios and for degrees of loss aversion of up to 2.25, for a PT investor with $\gamma = 0.5$ it is true nearly all the time, and for a PT investor with $\gamma = 0.9$ it is true for all portfolios except the ones generated by scenarios with a zero reference return, see Table 3. In the EU market, however, PT portfolios hardly ever significantly outperform MV portfolios, except for three cases, namely for an investor with $\gamma = 0.9$ and $\lambda = 3$, where the reference return is equal to zero (including the benchmark scenario as well as the conservative and aggressive scenarios), see Table 3. This difference between the portfolio performance implied by various types of PT investors in the EU and US is probably partly due to the higher returns of stock markets in the US than in the EU.

Median

The largest median of portfolio returns in the EU market, namely 14.9%, is achieved by the risk neutral investor for all degrees of diminishing sensitivity. This is also the case for

[27] Note that the expected portfolio return is $\mathbb{E}(r^p) = x\mathbb{E}(r - r^0) + r^0$ and thus for $\mathbb{E}(r) > r^0$ and $\frac{dx}{d\lambda} < 0$, see (2.22), it follows that $\frac{d\mathbb{E}(r^p)}{d\lambda} < 0$.

[28] Most of the time this is true for the 5% or 1% significance levels. We perform a Diebold-Mariano test (see Diebold and Mariano, 1995) to investigate whether returns of prospect theory investors exceed returns of mean-variance investors.

the US market for $\gamma = 0.5, 0.9$, with the median being 18.7%. However, for $\gamma = 0.1$ the largest median (18.9%) is achieved by the PT investor with $\lambda = 1.5$ and $\hat{r} = r^p$ under the aggressive scenario. We present a summary of the models (except LLA models) yielding largest medians in Table 4. The best models for the EU are implied by aggressive scenarios for $\gamma = 0.1, 0.5$, and by the risk-free scenario for $\gamma = 0.9$. The reference return of these models is the portfolio return of the previous period for $\gamma = 0.1$ and it is the risk-free rate for $\gamma = 0.5, 0.9$. All models implying the largest median in the EU have the lowest degree of loss aversion, namely $\lambda = 1.5$. For the US market the largest medians are found for the aggressive scenario with $\hat{r} = r^p$ and $\lambda = 1.5$ for $\gamma = 0.1, 0.5$, and for the aggressive scenario with $\hat{r} = r^p$ and $\lambda = 2.25$ for $\gamma = 0.9$. Medians of best models in the US are by roughly four percentage points larger than medians in the EU, for a given degree of diminishing sensitivity.

Largest means and medians of portfolio returns are implied in most cases by aggressive scenarios. Note that in both markets for $\gamma = 0.1$, the largest mean and median are implied by the same PT model with $\lambda = 1.5$ under the aggressive scenario with $\hat{r} = r^p$.[29] Note, in addition, that the largest median for any given degree of diminishing sensitivity is implied by the PT model with $\lambda = 1.5$ for the EU and by the PT model under the aggressive scenario with $\hat{r} = r^p$ for the US.

Risk-adjusted performance measures (Omega measure, Sharpe ratio, Sortino ratio, CVaR)

For the EU market the model that implies the largest Omega for $\gamma = 0.1$, namely 167.9, is the linear model. The risk neutral investor also achieves the largest Sharpe and Sortino ratios for $\gamma = 0.1, 0.5$, namely 63.2 (Sharpe ratio) and 98.6 (Sortino ratio). Note that these are the only cases with respect to all risk-adjusted measures, when the best models are not among the PT models. We report the results related to risk-adjusted performance in Table 5. For the EU the largest Omega, Sharpe and Sortino ratios are implied by the PT investor under the conservative scenario with $\hat{r} = 0$, $\lambda = 3$ and $\gamma = 0.9$.[30] For a smaller degree of diminishing sensitivity the best risk-adjusted performance is obtained by the mean-variance model. For the US the best performance with respect to Omega, the Sharpe and Sortino ratios for $\gamma = 0.1$ is obtained by the PT investor under the conservative scenario with $\hat{r} = r^p$ and $\lambda = 1.5$, while the best performance for $\gamma = 0.9$ is obtained by the PT model under the aggressive scenario with $\hat{r} = r^0$ and $\lambda = 2$. The largest CVaR

[29] Note that for the US market the largest mean (13.62%) is implied by the conservative scenario for $\gamma = 0.1$. However, the aggressive scenario with identical characteristics concerning diminishing sensitivity, loss aversion and the reference return implies nearly the same mean, namely 13.60%, which is only 0.02 percentage points lower.

[30] This is also the case for $\gamma = 0.5$ when the performance measure is Omega.

for both the EU and the US are achieved by the mean-variance investor for all degrees of diminishing sensitivity. Note that CVaR mostly increases with increasing λ (when all other model parameters are fixed), see Tables 11–18.

Risk measures (volatility and downside volatility)

The smallest risk measures for all degrees of diminishing sensitivity are implied by the mean-variance model in both the EU and the US markets. See Table 6 for a more detailed presentation. This does not come as a surprise as the mean-variance model explicitly targets the minimization of the variance, which is closely related to the reported risk measures. Note that volatility and downside volatility decrease with increasing λ (when all other model parameters are fixed).

Annual realized returns over the last 10, 5, 3 and 1 year(s)

We report the results with respect to the largest realized returns in Table 7. Regarding the EU market, in most cases the risk neutral investor achieves the largest realized returns (7.2% for the last 5 and 10 years, 7.8% for the last 3 years and 6.7% for the last year). The only exceptions are returns over the last year and the last 3 years for $\gamma = 0.9$, when the PT model with $\lambda = 1.5$ and $\hat{r} = r^p$ under the aggressive scenario achieves the largest realized returns (10.6% for last 3 years and 14.7% for the last year). Regarding the US market, the LLA model implies the largest returns only for $\gamma = 0.5$, when considering the last 10 years (11.6%); and for $\gamma = 0.1$, when looking at the last 5 years (11.5%). In both cases the LLA investor is characterized by the aggressive scenario with $\lambda = 1.5$ and $\hat{r} = r^p$. Note that the PT model with the same characteristics implies the largest realized returns over the last 10 years when $\gamma = 0.1$ and implies also the largest mean. Except for one case, the reference returns of investors with the largest realized returns are the portfolio returns from the previous period.

Note finally that in the vast majority of cases the means of PT portfolio returns exceed (sometimes quite substantially) the realized PT portfolio returns (on average by three percentage points), see Tables 11–18.[31]

[31] The rest of the tables presenting results of portfolio performance for a specific γ and scenario can be obtained from the authors upon request.

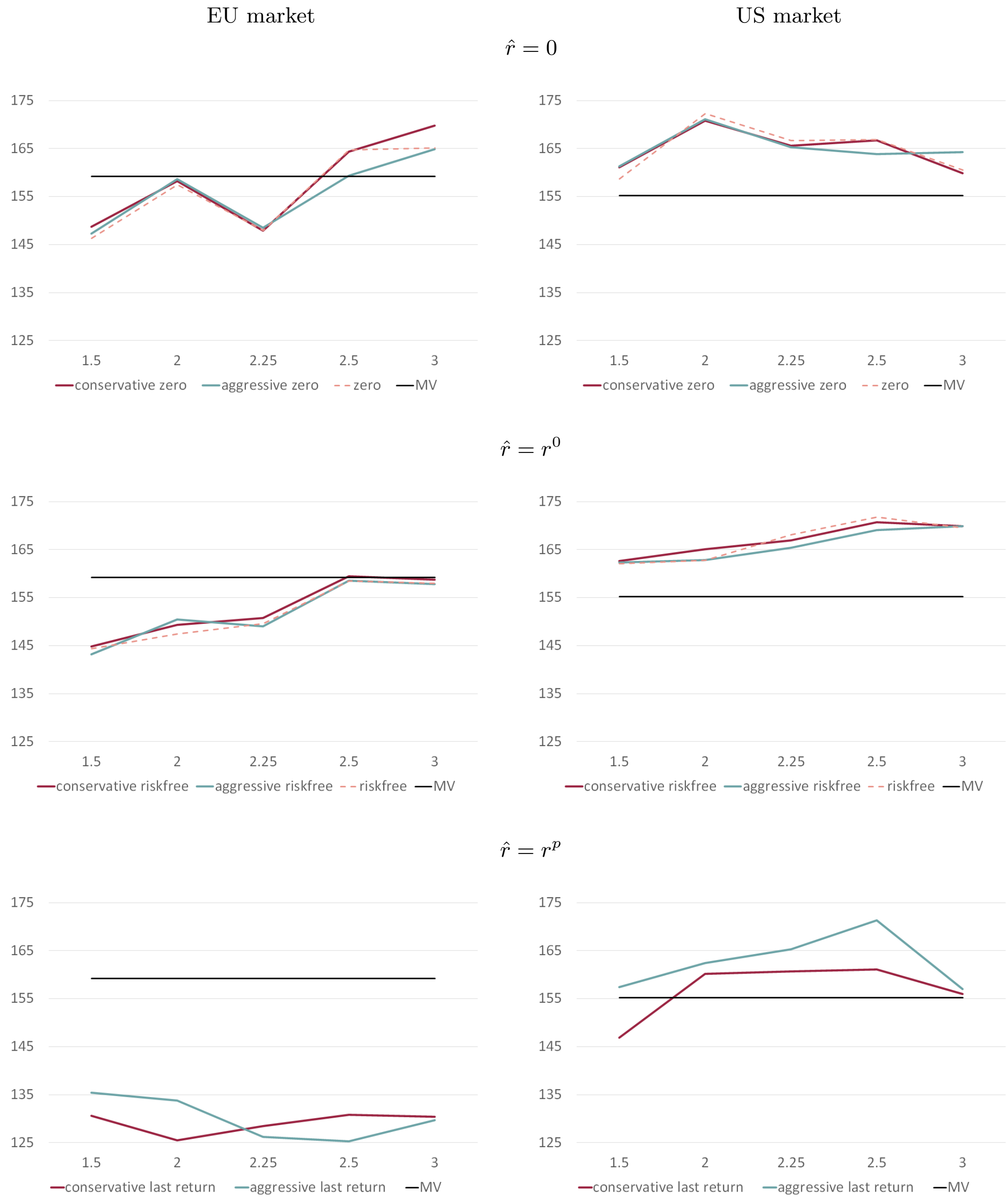

Figure 4: Conservative versus aggressive prospect theory investing: Omega measure. The graphs show the Omega measures of portfolio returns implied by conservative and aggressive prospect theory investing ($\gamma = 0.5$) for different degrees of loss aversion (shown on the x-axis) and for a given reference return (top row: $\hat{r} = 0$, middle row: $\hat{r} = r^0$, bottom row: $\hat{r} = r^p$) in the EU market (left) and the US market (right).

Figure 5: Prospect theory investing in the EU and the US.
The graphs show the means, Sharpe ratios, Omega measures, and volatilities of portfolio returns implied by a prospect theory investor ($\gamma = 0.5$) with reference returns of zero percent and the risk-free rate, for different degrees of loss aversion (shown on the x-axis) in the EU market (solid lines) and the US market (broken lines).

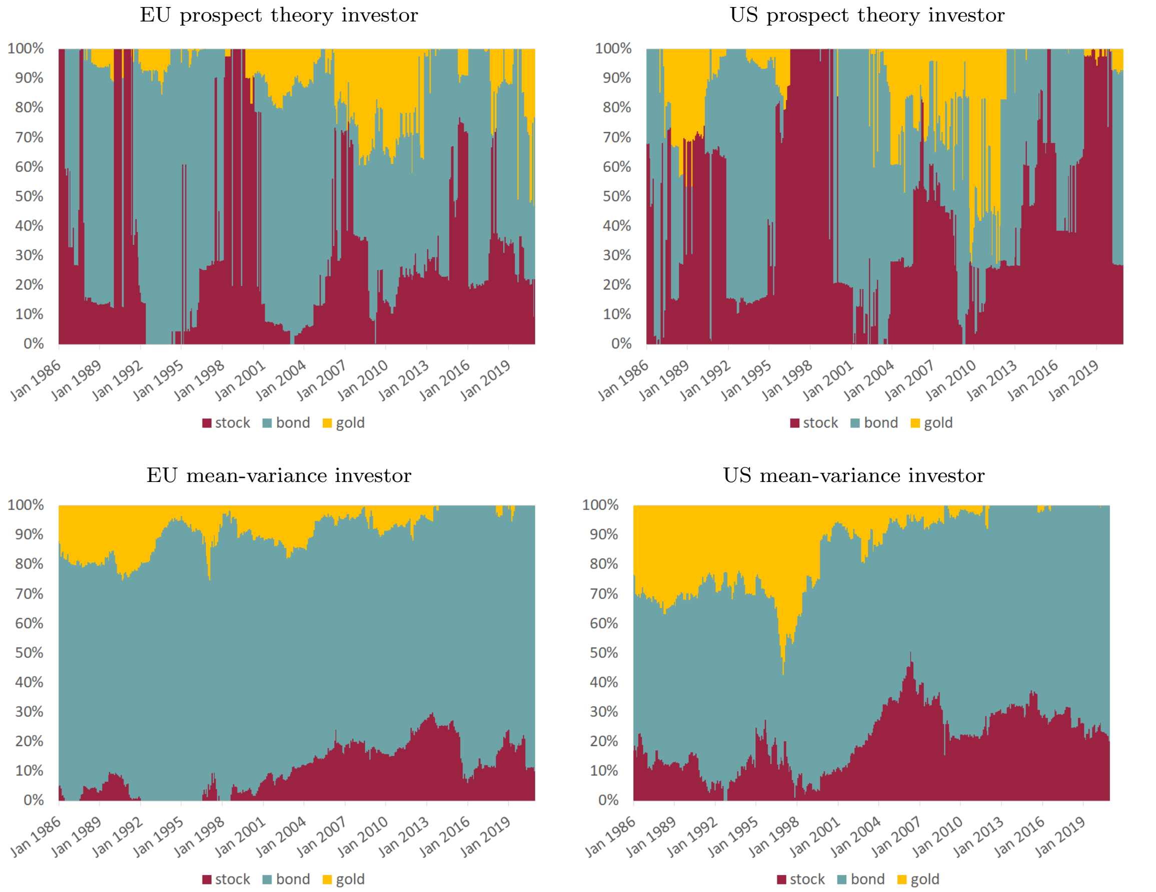

Figure 6: Optimal portfolio weights for prospect theory and mean-variance investors. The graph shows optimal asset weights for investors in the EU (left) and in the US (right) markets. The prospect theory investor is characterized by $\lambda = 2.25$, $\gamma = 0.5$ and a zero reference return.

scenario:	benchmark	risk-free		conservative			aggressive	
reference return:	$\hat{r} = 0$	$\hat{r} = r^0$	$\hat{r} = 0$	$\hat{r} = r^0$	$\hat{r} = r^p$	$\hat{r} = 0$	$\hat{r} = r^0$	$\hat{r} = r^p$
				EU				
$\boldsymbol{\gamma = 0.1}$								
$\lambda = 1.5$	2.25	2.54	2.01	2.38	3.70	2.31	2.49	3.91
$\lambda = 2$	2.02	1.89	1.80	1.85	1.93	1.82	1.78	1.92
$\lambda = 2.25$	1.35	1.51	1.50	1.59	1.57	1.33	1.57	1.35
$\lambda = 2.5$	1.20	1.35	1.28	1.37	0.95	1.23	1.33	1.08
$\lambda = 3$	0.79	0.74	0.76	0.70	1.06	0.79	0.71	1.07
$\boldsymbol{\gamma = 0.5}$								
$\lambda = 1.5$	1.55	1.78	1.69	1.72	1.18	1.68	1.69	1.75
$\lambda = 2$	1.77	1.45	1.82	1.59	0.26	1.83	1.70	1.43
$\lambda = 2.25$	0.97	1.43	0.95	1.49	0.46	1.01	1.38	0.23
$\lambda = 2.5$	1.57	1.72	1.56	1.79	0.60	1.33	1.73	0.02
$\lambda = 3$	1.32	1.38	1.53	1.43	0.38	1.31	1.40	0.33
$\boldsymbol{\gamma = 0.9}$								
$\lambda = 1.5$	1.77	2.16	2.12	1.91	1.94	2.00	2.16	0.79
$\lambda = 2$	1.28	1.90	1.37	2.15	0.16	1.47	1.99	1.09
$\lambda = 2.25$	1.90	1.72	1.82	1.75	-0.93	1.79	2.05	0.92
$\lambda = 2.5$	1.79	2.07	1.80	1.98	1.29	1.55	2.05	2.30
$\lambda = 3$	2.06*	1.63	2.29**	1.80	0.84	1.99*	1.53	1.13
				US				
$\boldsymbol{\gamma = 0.1}$								
$\lambda = 1.5$	5.01***	5.27***	4.70***	5.03***	6.27***	5.20***	5.31***	6.25***
$\lambda = 2$	2.94**	3.03**	2.88*	3.36**	4.20**	2.94**	3.10**	4.02**
$\lambda = 2.25$	2.50*	2.94**	2.45*	2.72*	3.44**	2.54*	2.91**	3.59**
$\lambda = 2.5$	1.82	2.18	1.73	2.08	2.71*	1.81	2.30*	2.59
$\lambda = 3$	1.41	1.56	1.31	1.52	2.48*	1.39	1.63	2.34
$\boldsymbol{\gamma = 0.5}$								
$\lambda = 1.5$	2.92*	3.84**	3.03*	3.88**	2.68	3.16*	3.91**	4.23**
$\lambda = 2$	3.30**	3.04*	3.21**	3.21**	4.03**	3.24**	3.07**	4.00**
$\lambda = 2.25$	2.77**	3.21**	2.66*	3.10**	4.10**	2.68*	3.04**	4.29**
$\lambda = 2.5$	2.53*	3.23**	2.51*	3.18**	4.01**	2.35*	3.08**	4.66***
$\lambda = 3$	1.82	2.82**	1.77	2.83**	2.90*	2.10*	2.83**	3.21*
$\boldsymbol{\gamma = 0.9}$								
$\lambda = 1.5$	2.46	4.28***	2.47	4.10***	3.62*	2.48	4.38***	3.39*
$\lambda = 2$	1.98	4.34***	1.97	4.27***	3.44*	1.93	4.36***	2.02
$\lambda = 2.25$	1.65	3.96***	1.77	4.12***	4.28**	1.58	4.09***	3.41*
$\lambda = 2.5$	1.80	4.08***	1.80	4.15***	4.05**	1.71	4.20***	3.26*
$\lambda = 3$	1.54	2.83*	1.39	2.89**	2.23	1.65	2.87*	3.89**

Table 3: Diebold-Mariano test for prospect theory returns being larger than mean-variance returns.
The table shows annualized estimated coefficients $\hat{c}$ from the regression $r_t^{PT} - r_t^{MV} = c + \varepsilon_t$. One (two, three) stars indicate significance at the 10% (5%, 1%) level, based on the HAC adjustment. Red numbers indicate the largest value for a given γ over all λs and scenarios.

	Mean	Scenario	Model	**Median**	Scenario	Model
EU						
$\gamma = 0.1$	10.30	aggressive	$\lambda = 1.5,\ \hat{r} = r^p$	10.62	aggressive	$\lambda = 1.5,\ \hat{r} = r^p$
$\gamma = 0.5$	8.11	aggressive	$\lambda = 2, ,\ \hat{r} = 0$	9.49	aggressive	$\lambda = 1.5,\ \hat{r} = r^0$
$\gamma = 0.9$	8.61	aggressive	$\lambda = 2.5,\ \hat{r} = r^p$	9.53	risk-free	$\lambda = 1.5,\ \hat{r} = r^0$
US						
$\gamma = 0.1$	13.62	conservative	$\lambda = 1.5,\ \hat{r} = r^p$	18.86	aggressive	$\lambda = 1.5,\ \hat{r} = r^p$
$\gamma = 0.5$	11.91	aggressive	$\lambda = 2.5,\ \hat{r} = r^p$	13.57	aggressive	$\lambda = 1.5,\ \hat{r} = r^p$
$\gamma = 0.9$	11.61	aggressive	$\lambda = 1.5,\ \hat{r} = r^0$	14.30	aggressive	$\lambda = 2.5,\ \hat{r} = r^p$

Table 4: Best performing scenarios and models with respect to the mean and median for the EU and the US markets. Presented values for means and medians are annual averages in percent.

	Omega	Scenario	Model	**CVaR**	Scenario	Model
EU						
$\gamma = 0.1$	159.23		MV	-27.23		MV
$\gamma = 0.5$	169.79	conservative	$\lambda = 3,\ \hat{r} = 0$	-27.23		MV
$\gamma = 0.9$	180.76	conservative	$\lambda = 3,\ \hat{r} = 0$	-27.23		MV
US						
$\gamma = 0.1$	173.28	conservative	$\lambda = 1.5,\ \hat{r} = r^p$	-31.16		MV
$\gamma = 0.5$	172.36	benchmark	$\lambda = 2,\ \hat{r} = 0$	-31.16		MV
$\gamma = 0.9$	181.59	aggressive	$\lambda = 2,\ \hat{r} = r^0$	-31.16		MV
	Sharpe	Scenario	Model	**Sortino**	Scenario	Model
EU						
$\gamma = 0.1$	62.74		MV	97.70		MV
$\gamma = 0.5$	62.74		MV	97.70		MV
$\gamma = 0.9$	69.82	conservative	$\lambda = 3,\ \hat{r} = 0$	116.87	conservative	$\lambda = 3,\ \hat{r} = r^0$
US						
$\gamma = 0.1$	68.70	conservative	$\lambda = 1.5,\ \hat{r} = r^p$	103.06	conservative	$\lambda = 1.5,\ \hat{r} = r^p$
$\gamma = 0.5$	67.57	aggressive	$\lambda = 2.5,\ \hat{r} = r^p$	106.28	aggressive	$\lambda = 2.5,\ \hat{r} = r^p$
$\gamma = 0.9$	71.71	aggressive	$\lambda = 2,\ \hat{r} = r^0$	109.87	aggressive	$\lambda = 2,\ \hat{r} = r^0$

Table 5: Best performing scenarios and models with respect to risk-adjusted performance measures (Omega, CVaR, Sharpe ratio, Sortino ratio) for the EU and the US markets.

	Volatility	Model	**Downside volatility**	Model
EU				
$\gamma = 0.1$	4.79	MV	2.69	MV
$\gamma = 0.5$	4.79	MV	2.69	MV
$\gamma = 0.9$	4.79	MV	2.69	MV
US				
$\gamma = 0.1$	5.77	MV	3.14	MV
$\gamma = 0.5$	5.77	MV	3.14	MV
$\gamma = 0.9$	5.77	MV	3.14	MV

Table 6: Best performing models with respect to risk measures (volatility and downside volatility) for the EU and the US markets.

	EU			US		
	Return	Scenario	Model	Return	Scenario	Model
	Last 10 years: 2011–2020					
$\gamma = 0.1$	5.67		MV	12.11	conservative	$\lambda = 1.5$, $\hat{r} = r^p$
$\gamma = 0.5$	6.01	aggressive	$\lambda = 3$, $\hat{r} = 0$	11.10	conservative	$\lambda = 2$, $\hat{r} = r^p$
$\gamma = 0.9$	6.72	aggressive	$\lambda = 2.5$, $\hat{r} = r^p$	14.07	conservative	$\lambda = 2.5$, $\hat{r} = r^p$
	Last 5 years: 2016–2020					
$\gamma = 0.1$	3.65	risk-free	$\lambda = 3$, $\hat{r} = r^0$	10.36	conservative	$\lambda = 1.5$, $\hat{r} = r^p$
		cons., aggr.	$\lambda = 3$, $\hat{r} = r^0$			
$\gamma = 0.5$	4.72	benchmark	$\lambda = 3$, $\hat{r} = 0$	11.73	aggressive	$\lambda = 2.5$, $\hat{r} = r^p$
		aggressive	$\lambda = 3$, $\hat{r} = 0$			
$\gamma = 0.9$	6.82	aggressive	$\lambda = 2.5$, $\hat{r} = r^p$	14.46	conservative	$\lambda = 2.5$, $\hat{r} = r^p$
	Last 3 years: 2018–2020					
$\gamma = 0.1$	3.86	aggressive	$\lambda = 3$, $\hat{r} = r^0$	10.19	conservative	$\lambda = 2.25$, $\hat{r} = r^p$
$\gamma = 0.5$	4.57	aggressive	$\lambda = 3$, $\hat{r} = 0$	11.95	aggressive	$\lambda = 2.5$, $\hat{r} = r^p$
$\gamma = 0.9$	10.63	aggressive	$\lambda = 1.5$, $\hat{r} = r^p$	14.74	conservative	$\lambda = 2$, $\hat{r} = r^p$
	Last year: 2020					
$\gamma = 0.1$	1.13		MV	16.67	risk-free	$\lambda = 3$, $\hat{r} = r^0$
					conservative	$\lambda = 3$, $\hat{r} = r^0$
$\gamma = 0.5$	5.55	aggressive	$\lambda = 1.5$, $\hat{r} = 0$	15.05		MV
$\gamma = 0.9$	14.65	aggressive	$\lambda = 1.5$, $\hat{r} = r^p$	23.32	conservative	$\lambda = 2.5$, $\hat{r} = r^p$

Table 7: Best performing scenarios and models with respect to realized annual returns over the last 10, 5, 3 and 1 year(s) for the EU and the US markets.

3.2 Asymmetric effects along economic uncertainty

We would like to analyze the extent to which the excess portfolio returns of prospect theory investing with respect to mean-variance investing depends on the state of the economy, namely economic uncertainty. We aim at assessing whether prospect theory investment provides larger returns in times of higher economic uncertainty – or in times of lower economic uncertainty – than returns implied by mean-variance investment. Interest in economic uncertainty has been spurred by a growing body of evidence showing that uncertainty rises sharply in recessions. Currently, popular measures of economic uncertainty are news-based economic policy uncertainty indices or the volatility of sovereign credit default swap spreads or indices measuring the degree of predictability of uncertainty, see Baker et al. (2016), Böck et al. (2021), and Jurado et al. (2015). We use the third type of indicator to measure economic (in)stability in the EU and the US.[32] We evaluate the statistical significance of optimal portfolio return differences between returns implied by the prospect theory preferences, $r_t^{PT}(\lambda, \hat{r}, \gamma, \text{scenario})$, and returns implied by mean-variance preferences, r_t^{MV}, (the benchmark model) in times of high and low economic uncertainty. To do so we estimate the following regression

$$r_t^{PT}(\lambda, \hat{r}, \gamma, \text{scenario}) - r_t^{MV} = c_0 + c_1 D_t + \varepsilon_t \tag{3.26}$$

where the explanatory variable D_t is the dummy variable representing periods of high uncertainty, if $D_t = 1$, and periods of low uncertainty, if $D_t = 0$. We consider $\lambda \in \{1.5, 2, 2.25, 2.5, 3\}$, $\gamma \in \{0.1, 0.5, 0.9\}$ and the scenario is either the benchmark, the risk-free, the conservative or the aggressive scenario. The dummy variable is derived using the estimator of a threshold level from the self-exiting threshold autoregression in levels[33] with p lags and with k lags in the threshold variable

$$u_t = \begin{cases} \phi_{01} + \sum_{i=1}^{p} \phi_{i1} u_{t-i} + \varepsilon_t, & \text{for } u_{t-k} \geq \gamma_\phi \\ \\ \phi_{02} + \sum_{i=1}^{p} \phi_{i2} u_{t-i} + \varepsilon_t, & \text{for } u_{t-k} < \gamma_\phi \end{cases} \tag{3.27}$$

where u_t is the uncertainty index, and the estimator of γ_ϕ, namely $\hat{\gamma}_\phi$, is the value of u_{t-k} that minimizes the sum of squared residuals in the non-linear regression (3.27).[34] Thus, $D_t = 1$ when $u_{t-k} \geq \hat{\gamma}_\phi$ and $D_t = 0$ when $u_{t-k} < \hat{\gamma}_\phi$. A positive and significant estimate

[32] The economic uncertainty indicator for the EU was calculated in Fortin et al. (2021). The corresponding indicator for the US (total macro uncertainty) was obtained from `https://www.sydneyludvigson.com/macro-and-financial-uncertainty-indexes`

[33] Uncertainty indicators for both markets (EU and US) are stationary variables.

[34] In the same way we determine lag lengths p and k.

of c_1 in equation (3.26) suggests that the prospect theory investor achieves higher portfolio returns than the MV investor in high uncertainty periods while a negative and significant estimate of c_1 suggests that the prospect theory investor achieves higher portfolio returns than the MV investor in low uncertainty periods.

We find the following results, see Table 8. For $\gamma = 0.1$ we do not find any evidence about significantly different PT and MV returns in either high uncertainty or low uncertainty periods. In all cases and in both markets are estimates of c_1, namely $\hat{c}_1$, insignificant, and in most cases is $\hat{c}_1 > 0$ in both markets. For $\gamma = 0.5$ are estimates of c_1 positive in all cases in the EU and in most cases in the US market, but $\hat{c}_1$ is significantly positive only in one case, namely for $\lambda = 1.5$ and $\hat{r} = r^p$ under the conservative scenario in the EU market. Hence, in this case the PT investor achieves significantly higher portfolio returns than the MV investor in periods of high economic uncertainty. Finally, for $\gamma = 0.9$ are estimates of c_1 positive in the majority of cases in the EU market and in all cases in the US market, and $\hat{c}_1$ is significantly positive in two cases for the EU market, namely when $\lambda = 3$ and the investor is conservative, and when $\lambda = 2.5$ and the investor is aggressive, where in both cases the reference return is the portfolio return from the previous period. Note that the latter aggressive scenario ($\gamma = 0.9$, $\lambda = 2.5$, $\hat{r} = r^p$) implies also the largest mean of portfolio returns and the largest realized annual returns over the last 10 and 5 years, see Tables 4, 7 and 15.[35] For the US market, the PT investor significantly outperforms the MV investor (in terms of returns) in periods of high economic uncertainty, when the reference return is the risk-free rate and $\lambda \in \{2, 2.25, 2.5\}$.[36] Note that the aggressive scenario with the risk-free reference return implies also the largest risk-adjusted performance (Omega, Sharpe ratio and Sortino ratio) when $\lambda = 2$, see Tables 5 and 18.

[35]The median of this aggressive scenario, i.e., when $\gamma = 0.9$, $\lambda = 2.5$ and $\hat{r} = r^p$, is 9.52% and thus is only by 0.01 percentage points smaller than the largest median for $\gamma = 0.9$, see Table 4.

[36]In addition, the PT investor significantly outperforms the MV investor (in terms of returns) in periods of high economic uncertainty when the investor is aggressive with $\hat{r} = r^p$ and $\lambda = 1.5$.

scenario:	benchmark	risk-free	conservative			aggressive		
reference return:	$\hat{r}=0$	$\hat{r}=r^0$	$\hat{r}=0$	$\hat{r}=r^0$	$\hat{r}=r^p$	$\hat{r}=0$	$\hat{r}=r^0$	$\hat{r}=r^p$
				EU				
$\boldsymbol{\gamma=0.1}$								
$\lambda=1.5$	7.36	7.64	4.42	6.47	12.30	10.82	11.18	13.66
$\lambda=2$	0.69	0.47	0.55	0.64	4.80	0.66	0.60	5.85
$\lambda=2.25$	1.43	1.65	1.20	1.77	2.73	1.27	1.81	2.21
$\lambda=2.5$	1.32	2.33	1.27	2.38	1.19	1.35	2.09	2.09
$\lambda=3$	1.07	1.14	1.02	1.24	-0.43	1.11	1.01	-0.33
$\boldsymbol{\gamma=0.5}$								
$\lambda=1.5$	3.54	3.03	2.97	1.30	10.50*	3.58	2.66	5.80
$\lambda=2$	1.31	0.58	1.21	0.87	5.96	1.89	0.66	10.32
$\lambda=2.25$	1.91	1.08	1.93	0.78	5.91	2.21	1.20	3.29
$\lambda=2.5$	3.50	3.04	3.53	3.35	5.46	3.44	3.10	4.98
$\lambda=3$	2.64	2.54	2.63	2.40	5.56	2.62	2.41	6.14
$\boldsymbol{\gamma=0.9}$								
$\lambda=1.5$	0.92	0.17	0.55	-1.10	8.47	0.78	0.25	6.97
$\lambda=2$	0.31	2.15	-0.33	1.96	-7.88	-0.56	2.05	2.97
$\lambda=2.25$	0.98	0.43	1.21	0.27	0.38	0.51	1.01	4.15
$\lambda=2.5$	1.17	3.11	1.16	2.70	5.32	1.12	3.54	7.96*
$\lambda=3$	1.42	-1.31	0.94	0.61	9.52*	1.55	-1.09	6.09
				US				
$\boldsymbol{\gamma=0.1}$								
$\lambda=1.5$	0.11	0.00	-0.94	-0.35	0.19	0.61	0.45	0.32
$\lambda=2$	0.55	0.11	0.61	0.25	-0.19	0.31	-0.04	0.16
$\lambda=2.25$	0.30	0.81	0.25	0.59	-0.87	0.71	1.25	-1.32
$\lambda=2.5$	0.49	0.64	0.33	0.48	-1.29	0.44	0.62	-0.73
$\lambda=3$	-0.21	-0.71	-0.54	-0.71	-0.45	-0.23	-0.55	-0.94
$\boldsymbol{\gamma=0.5}$								
$\lambda=1.5$	2.85	-0.26	2.79	-0.21	3.73	4.49	0.35	6.83
$\lambda=2$	2.52	3.05	2.60	2.76	3.18	2.52	2.99	4.93
$\lambda=2.25$	1.85	2.29	2.15	2.42	5.42	1.74	2.97	6.20
$\lambda=2.5$	1.64	2.22	1.62	2.39	5.32	1.67	2.34	7.07
$\lambda=3$	1.15	1.24	1.11	1.23	2.32	1.23	1.31	3.23
$\boldsymbol{\gamma=0.9}$								
$\lambda=1.5$	3.24	7.67	3.63	7.72	8.87	4.68	8.71	10.78*
$\lambda=2$	1.75	8.99*	1.56	9.03*	4.88	1.43	8.97*	2.33
$\lambda=2.25$	1.51	9.01*	1.47	9.13*	7.23	0.99	8.90*	1.51
$\lambda=2.5$	1.48	10.02*	1.03	8.97*	4.84	0.82	8.95*	7.25
$\lambda=3$	0.85	6.17	0.77	6.15	1.72	0.62	6.19	6.04

Table 8: Test for prospect theory returns being larger than mean-variance returns in times of high economic uncertainty.
The table shows annualized estimated coefficients $\hat{c}_1$ from the regression $r_t^{PT} - r_t^{MV} = c_0 + c_1 D_t + \varepsilon_t$. A star indicates significance at the 10% level, based on the HAC adjustment.

4 Conclusion

In this paper we investigate the behavior of an S-shaped prospect theory investor. In the theoretical part we derive the analytical closed form solution for a two-asset portfolio consisting of one risky asset and one risk-free asset. We examine the properties of the optimal weight of the risky asset under the assumptions of binomially and (generally) continuously distributed returns of the risky asset. For the assumption of binomial returns we find that the reference return plays a crucial role: we derive analytical solutions for the cases when the reference return is below the risk free-rate (less ambitious investor), when it coincides with the risk-free rate and when it exceeds the risk-free rate (more ambitious investor). We find that under certain conditions the risky investment of the less ambitious investor is strictly positive and does not depend on her degree of loss aversion and that her exposure to the risky asset decreases when her level of ambition (reference return) increases. When the reference return coincides with the risk-free rate then investor is out of the market and invests everything into the risk-free asset, as the mean-variance investor. Finally, risky investment of the more ambitious investor can be both strictly positive, when the probability of the good state to occur is sufficiently large, and strictly negative when the probability of the good state to occur is sufficiently small. In the former case the exposure to the risky asset decreases with an increasing degree of loss aversion and increases with an increasing level of ambition. In the latter case the behavior is opposite. The results for the S-shaped prospect theory (PT) investor are then compared with the results for the linear loss averse (LLA) investor. In addition to assuming binomial returns we examine the optimal portfolio choice under the assumption of a (general) continuous distribution for the return of the risky asset. In this case, however, the optimal exposure into the risky asset can only be expressed in an implicit form when the reference return differs from the risk-free interest. We can still derive comparative statics and find again that the reference return plays an important role.

In the empirical part we investigate the performance of optimal asset portfolios implied by PT preferences. We study two scenarios with a constant penalty parameter, where the reference return is either equal to zero or equal to the risk-free interest rate, and two dynamic scenarios, where the penalty parameter is time-changing conditional on previous gains and losses and the reference return is either zero, the risk-free rate or the portfolio return of the previous period. In one of the two dynamic scenarios the PT investor becomes more loss averse after losses (conservative scenario), in the other the PT investor becomes less loss averse after losses (aggressive scenario). The investor selects among three risky assets, a stock market index, a government bond index and gold, and she operates either in the EU or in the US market. We consider various performance measures, including risk-adjusted measures like the Omega measure and the Sharpe and Sortino ratios. In addition to PT portfolios, we examine optimal portfolios implied by LLA, risk neutral, conditional value-at-risk and, in particular, traditional mean-variance (MV) preferences.

There are many different findings. First, PT investment leads to clearly higher means of portfolio returns, but also to much higher risk, than MV investment, for all types of PT investors and for both the EU and US markets. We actually find that, in the US market, returns of PT portfolios are significantly larger than returns of MV portfolios for almost all types of PT investors while this is hardly the case in the EU. If we consider risk-adjusted performance, PT investment (mostly) outperforms MV investment in the US while MV investment outperforms PT investment in the EU. Then, rather surprisingly, the performance of optimal portfolios in the two dynamic scenarios is very similar, when the reference return is either zero or the risk-free rate, in both the EU and the US markets. So in these cases conservative and aggressive types of behavior lead to similar performance results. The situation is different when the reference return is equal to the portfolio return of the previous period. In this case the conservative and aggressive scenarios lead to (more) different performance measures, which is mainly due to the rather large distance of the portfolio return from the reference return. Our results also suggest that PT and MV investors show clearly different types of behavior with respect to portfolio re-balancing. While the PT investor holds large proportions of stocks during certain time periods and her asset weights may sometimes vary strongly over short periods, the MV investor holds rather large proportion of bonds all the time and her asset weights do not change substantially over short periods. Also unexpectedly, the risk neutral investor performs quite well empirically. In the EU market she performs best (also in terms of risk-adjusted performance) in most of the cases[37] while in the US the prospect theory investor usually performs best.[38]

In addition we examine whether PT and MV investing yield different returns in times of high/low economic uncertainty and we find that, especially in the US market, PT investment leads to higher portfolio returns than MV investment in times of larger economic uncertainty. So in times of higher uncertainty PT investment seems to be particularly beneficial.

Note that although our empirical analysis covers many aspects of the prospect theory asset allocation problem, a more thorough analysis is required to shed light on all details. For example, as we focus mainly on loss aversion, the dynamic scenarios in our study update only the investor's loss aversion parameter (and only after prior losses). They do not update the reference return and do not apply dynamic updates of both the reference return and loss

[37] In the others the PT investor performs best.

[38] The risk neutral investor does not much worse, however, in most cases.

aversion after both prior gains and losses. These effects will be explored in future research.

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Appendix A

Proof of proposition 2.1: To calculate the expected value of $v(R(x))$ with respect to the binomial distribution, we evaluate four options following from (2.4): (i) $R_b(x) > \hat{r}$ and $R_g(x) > \hat{r}$, (ii) $R_b(x) \leq \hat{r}$ and $R_g(x) > \hat{r}$, (iii) $R_b(x) > \hat{r}$ and $R_g(x) \leq \hat{r}$, and (iv) $R_b(x) \leq \hat{r}$ and $R_g(x) \leq \hat{r}$. Case (i) can occur only when $x \in \left[\frac{\hat{r}-r^0}{r_g-r^0}, \frac{r^0-\hat{r}}{r^0-r_b}\right]$, case (ii) occurs when $x \geq \max\left\{\frac{r^0-\hat{r}}{r^0-r_b}, \frac{\hat{r}-r^0}{r_g-r^0}\right\}$, case (iii) occurs when $x \leq \min\left\{\frac{r^0-\hat{r}}{r^0-r_b}, \frac{\hat{r}-r^0}{r_g-r^0}\right\}$, and case (iv) occurs when $x \in \left[\frac{r^0-\hat{r}}{r^0-r_b}, \frac{\hat{r}-r^0}{r_g-r^0}\right]$. Note that for $\hat{r} < r^0$, which is the assumption of this proposition, is case (iv) infeasible. Based on these we thus solve the following three maximization problems

$$\left.\begin{array}{rl} \max: & \mathbb{E}(v(R(x))) = \frac{1}{1-\gamma}\left[(1-p)(r^0-\hat{r}-(r^0-r_b)x)^{1-\gamma} + p(r^0-\hat{r}+(r_g-r^0)x)^{1-\gamma}\right] \\ \text{such that}: & -\frac{r^0-\hat{r}}{r_g-r^0} \leq x \leq \frac{r^0-\hat{r}}{r^0-r_b} \end{array}\right\} \text{(i)}$$

$$\left.\begin{array}{rl} \max: & \mathbb{E}(v(R(x))) = \frac{1}{1-\gamma}\left[-\lambda(1-p)(\hat{r}-r^0+(r^0-r_b)x)^{1-\gamma} + p(r^0-\hat{r}+(r_g-r^0)x)^{1-\gamma}\right] \\ \text{such that}: & x \geq \frac{r^0-\hat{r}}{r^0-r_b} \end{array}\right\} \text{(ii)}$$

$$\left.\begin{array}{rl} \max: & \mathbb{E}(v(R(x))) = \frac{1}{1-\gamma}\left[(1-p)(r^0-\hat{r}-(r^0-r_b)x)^{1-\gamma} - \lambda p(\hat{r}-r^0-(r_g-r^0)x)^{1-\gamma}\right] \\ \text{such that}: & x \leq -\frac{r^0-\hat{r}}{r_g-r^0} \end{array}\right\} \text{(iii)}$$

The idea of the proof is to show now that (i) is a concave problem with an unique maximum and as the objective function of (iii) is increasing at its domain and the objective function of (ii) is decreasing at its domain, and as $\mathbb{E}(v(R(x)))$ is continuous function, then the maximum of problem (i) coincides with the maximum of (2.3).

By differentiating the objective function of problem (i) we obtain

$$\frac{d}{dx}\mathbb{E}(v(R(x))) = -(1-p)[r^0-\hat{r}-(r^0-r_b)x]^{-\gamma}(r^0-r_b) + p[r^0-\hat{r}+(r_g-r^0)x]^{-\gamma}(r_g-r^0)$$

and

$$\begin{aligned} \frac{d^2}{dx^2}\mathbb{E}(v(R(x))) &= -\gamma(1-p)[r^0-\hat{r}-(r^0-r_b)x]^{-1-\gamma}(r^0-r_b)^2 \\ &\quad -\gamma p[r^0-\hat{r}+(r_g-r^0)x]^{-1-\gamma}(r_g-r^0)^2 < 0 \end{aligned}$$

which implies that (i) is a concave programming problem and thus the maximum satisfies the following first order conditions

$$\frac{d}{dx}\mathbb{E}(v(R(x))) = -(1-p)[r^0 - \hat{r} - (r^0 - r_b)x]^{-\gamma}(r^0 - r_b) + p[r^0 - \hat{r} + (r_g - r^0)x]^{-\gamma}(r_g - r^0) = 0$$

Thus,

$$(p(r_g - r^0))^{1/\gamma}\left[r^0 - \hat{r} - (r^0 - r_b)x\right] = ((1-p)(r^0 - r_b))^{1/\gamma}\left[r^0 - \hat{r} + (r_g - r^0)x\right]$$

which implies

$$\begin{aligned} x &= \frac{(p(r_g - r^0))^{1/\gamma} - ((1-p)(r^0 - r_b))^{1/\gamma}}{\left[(p(r_g - r^0))^{1/\gamma}(r^0 - r_b) + ((1-p)(r^0 - r_b))^{1/\gamma}(r_g - r^0)\right]}(r^0 - \hat{r}) \\ &= \frac{\left(1 - K_0^{1/\gamma}\right)(r^0 - \hat{r})}{r^0 - r_b + K_0^{1/\gamma}(r_g - r^0)} \end{aligned} \tag{4.28}$$

and this coincides with (2.6). Note that the conditions of the proposition imply that $0 < K_0 < 1$ (following from $r_b < r^0 < r_g$ and $\mathbb{E}(r) > r^0$) and thus $x^* > 0$.

Problem (iii) is increasing at its domain if

$$\frac{d}{dx}\mathbb{E}(v(R(x))) = -(1-p)[r^0 - \hat{r} - (r^0 - r_b)x]^{-\gamma}(r^0 - r_b) + \lambda p[\hat{r} - r^0 - (r_g - r^0)x]^{-\gamma}(r_g - r^0) > 0$$

which is guaranteed if

$$\lambda > \frac{(1-p)\left[\hat{r} - r^0 - (r_g - r^0)x\right]^{\gamma}(r^0 - r_b)}{p\left[r^0 - \hat{r} - (r^0 - r_b)x\right]^{\gamma}(r_g - r^0)} = \left(\frac{\frac{\hat{r} - r^0}{r_g - r^0} - x}{\frac{r^0 - \hat{r}}{r^0 - r_b} - x}\right)^{\gamma} K_\gamma \tag{4.29}$$

It follows from the assumptions of the theorem that $\lambda > K_\gamma$ and as $\frac{\frac{\hat{r} - r^0}{r_g - r^0} - x}{\frac{r^0 - \hat{r}}{r^0 - r_b} - x} < 1$ then (4.29) is satisfied and thus the objective function of (iii) is increasing.

Problem (ii) is decreasing at its domain if

$$\frac{d}{dx}\mathbb{E}(v(R(x))) = -\lambda(1-p)[\hat{r} - r^0 + (r^0 - r_b)x]^{-\gamma}(r^0 - r_b) + p[r^0 - \hat{r} + (r_g - r^0)x]^{-\gamma}(r_g - r^0) < 0$$

which is guaranteed if

$$\lambda \;>\; \frac{p(\hat{r} - r^0 + (r^0 - r_b)x)^{\gamma}(r_g - r^0)}{(1-p)(r^0 - \hat{r} + (r_g - r^0)x)^{\gamma}(r^0 - r_b)} = \left(\frac{\frac{\hat{r} - r^0}{r^0 - r_b} + x}{\frac{r^0 - \hat{r}}{r_g - r^0} + x}\right)^{\gamma} \frac{1}{K_\gamma} \tag{4.30}$$

It follows from assumptions of the theorem that $\lambda > \frac{1}{K_\gamma}$ and as $\frac{\frac{\hat{r} - r^0}{r^0 - r_b} + x}{\frac{r^0 - \hat{r}}{r_g - r^0} + x} < 1$ then (4.30) is satisfied and thus the objective function of (ii) is decreasing. This finishes the proof. □

Proof of proposition 2.2: Note that solving (2.3) boils down to solving problems (ii) and (iii) for $r^0 = \hat{r}$ as in cases (i) and (iv) is $x = 0$ the optimal solution. As the objective function of (iii) is increasing for $x \leq 0$ and the objective function of (ii) is decreasing for $x \geq 0$ then this implies that zero is the solution of (2.3). This finishes the proof. □

Proof of proposition 2.3: Based on (2.4) we consider the following four cases: (i) $R_b(x) > \hat{r}$ and $R_g(x) > \hat{r}$, (ii) $R_b(x) \leq \hat{r}$ and $R_g(x) > \hat{r}$, (iii) $R_b(x) > \hat{r}$ and $R_g(x) \leq \hat{r}$, and (iv) $R_b(x) \leq \hat{r}$ and $R_g(x) \leq \hat{r}$. Case (i) can occur only when $x \in \left[\frac{\hat{r}-r^0}{r_g-r^0}, -\frac{\hat{r}-r^0}{r^0-r_b}\right]$, case (ii) occurs when $x \geq \frac{\hat{r}-r^0}{r_g-r^0}$, case (iii) occurs when $x \leq -\frac{\hat{r}-r^0}{r^0-r_b}$, and case (iv) occurs when $x \in \left[-\frac{\hat{r}-r^0}{r^0-r_b}, \frac{\hat{r}-r^0}{r_g-r^0}\right]$. Note that for $\hat{r} > r^0$, which is the assumption of this proposition, is case (i) infeasible. Based on these we thus solve the following three maximization problems

$$\left.\begin{array}{rl} \max: & \mathbb{E}(v(R(x))) = \frac{1}{1-\gamma}\left[-\lambda(1-p)(\hat{r}-r^0+(r^0-r_b)x)^{1-\gamma}+p(r^0-\hat{r}+(r_g-r^0)x)^{1-\gamma}\right] \\ \text{such that}: & x \geq \frac{\hat{r}-r^0}{r_g-r^0} \end{array}\right\}\text{(ii)}$$

$$\left.\begin{array}{rl} \max: & \mathbb{E}(v(R(x))) = \frac{1}{1-\gamma}\left[(1-p)(r^0-\hat{r}-(r^0-r_b)x)^{1-\gamma}-\lambda p(\hat{r}-r^0-(r_g-r^0)x)^{1-\gamma}\right] \\ \text{such that}: & x \leq \frac{r^0-\hat{r}}{r^0-r_b} \end{array}\right\}\text{(iii)}$$

$$\left.\begin{array}{rl} \max: & \mathbb{E}(v(R(x))) = \frac{-\lambda}{1-\gamma}\left[(1-p)(\hat{r}-r^0+(r^0-r_b)x)^{1-\gamma}+p(\hat{r}-r^0-(r_g-r^0)x)^{1-\gamma}\right] \\ \text{such that}: & -\frac{\hat{r}-r^0}{r^0-r_b} \leq x \leq \frac{\hat{r}-r^0}{r_g-r^0} \end{array}\right\}\text{(iv)}$$

By differentiating (iv) we obtain

$$\frac{d}{dx}\mathbb{E}(v(R(x))) = -\lambda\left[(1-p)[\hat{r}-r^0+(r^0-r_b)x]^{-\gamma}(r^0-r_b) - p[\hat{r}-r^0-(r_g-r^0)x]^{-\gamma}(r_g-r^0)\right]$$

and

$$\begin{aligned}\frac{1}{\gamma}\frac{d^2}{dx^2}\mathbb{E}(v(R(x))) &= \lambda(1-p)[\hat{r}-r^0+(r^0-r_b)x]^{-1-\gamma}(r^0-r_b)^2 \\ &\quad +\lambda p[\hat{r}-r^0-(r_g-r^0)x]^{-1-\gamma}(r_g-r^0)^2 > 0 \qquad (4.31)\end{aligned}$$

which implies that (iv) is a convex programming problem and thus the maximum is reached in one of the corner points; i.e., $-\frac{\hat{r}-r^0}{r^0-r_b}$ or $\frac{\hat{r}-r^0}{r_g-r^0}$.

The first order conditions (FOC) for (iii) are

$$\frac{d}{dx}\mathbb{E}(v(R(x))) = -(1-p)[r^0-\hat{r}-(r^0-r_b)x]^{-\gamma}(r^0-r_b)+\lambda p[\hat{r}-r^0-(r_g-r^0)x]^{-\gamma}(r_g-r^0) = 0$$

It can be shown that $(x^*)^-$ satisfies the FOC and that (iii) is concave; i.e.,

$$\begin{aligned}\frac{1}{\gamma}\frac{d^2}{dx^2}\mathbb{E}(v(R(x))) &= \\ &-(1-p)[r^0-\hat{r}-(r^0-r_b)x]^{-1-\gamma}(r^0-r_b)^2+\lambda p[\hat{r}-r^0-(r_g-r^0)x]^{-1-\gamma}(r_g-r^0)^2 \\ &< 0\end{aligned}$$

for $\lambda > K_\gamma$ and $x > x_L \equiv \frac{\left[1+\left(\frac{\lambda}{K_{-1}}\right)^{1/(1+\gamma)}\right](\hat{r}-r^0)}{r_g-r^0-\left(\frac{\lambda}{K_{-1}}\right)^{1/(1+\gamma)}(r^0-r_b)} = -\frac{\hat{r}-r^0}{r^0-r_b}\times\frac{\left(\lambda^{\frac{1}{1+\gamma}}+K_{-1}^{\frac{1}{1+\gamma}}\right)}{\left(\lambda^{\frac{1}{1+\gamma}}-K_\gamma^{\frac{1}{1+\gamma}}\right)}$ and that $x_L < (x^*)^- < \frac{r^0-\hat{r}}{r^0-r_b}$ for $\lambda > K_\gamma$. Note in addition that (iii) is a convex problem for $x < x_L$ and that $\lim_{x\to-\infty}\mathbb{E}(v(R(x))) = -\infty$. This shows that $(x^*)^-$ is the only maximum of (iii).

The first order conditions (FOC) for (ii) are

$$\frac{d}{dx}\mathbb{E}(v(R(x))) = -\lambda(1-p)[\hat{r}-r^0+(r^0-r_b)x]^{-\gamma}(r^0-r_b)+p[r^0-\hat{r}+(r_g-r^0)x]^{-\gamma}(r_g-r^0) = 0$$

It can be shown that $(x^*)^+$ satisfies the FOC and that (ii) is concave; i.e.,

$$\begin{aligned}\frac{1}{\gamma}\frac{d^2}{dx^2}\mathbb{E}(v(R(x))) &= \\ &\lambda(1-p)[\hat{r}-r^0+(r^0-r_b)x]^{-1-\gamma}(r^0-r_b)^2-p[r^0-\hat{r}+(r_g-r^0)x]^{-1-\gamma}(r_g-r^0)^2 \\ &< 0\end{aligned}$$

for $\lambda > 1/K_\gamma$ and $x < x_U \equiv \frac{\left[1+\left(\frac{1}{\lambda K_{-1}}\right)^{1/(1+\gamma)}\right](\hat{r}-r^0)}{r_g-r^0-\left(\frac{1}{\lambda K_{-1}}\right)^{1/(1+\gamma)}(r^0-r_b)} = \frac{\hat{r}-r^0}{r_g-r^0}\times\frac{\left(\lambda^{\frac{1}{1+\gamma}}+\left(\frac{1}{K_{-1}}\right)^{\frac{1}{1+\gamma}}\right)}{\left(\lambda^{\frac{1}{1+\gamma}}-\left(\frac{1}{K_\gamma}\right)^{\frac{1}{1+\gamma}}\right)}$ and that $\frac{\hat{r}-r^0}{r_g-r^0} < (x^*)^+ < x_U$ for $\lambda > 1/K_\gamma$. Note in addition that (ii) is a convex problem for $x > x_U$ and that $\lim_{x\to-\infty}\mathbb{E}(v(R(x))) = -\infty$. This shows that $(x^*)^+$ is the only maximum of (ii) and thus the global maximum of (2.3) is achieved at $x^* = argmax\{\mathbb{E}(v(R((x^*)^+))), \mathbb{E}(v(R((x^*)^-)))$.

Next we analyze when $\mathbb{E}(v(R((x^*)^+))) > \mathbb{E}(v(R((x^*)^-)))$. It can be shown that

$$\mathbb{E}(v(R((x^*)^+))) = -\frac{1-p}{1-\gamma}\left[\frac{(r_g-r_b)(\hat{r}-r^0)}{r_g-r^0}\right]^{1-\gamma}\left[\lambda^{1/\gamma}-\left(\frac{1}{K_\gamma}\right)^{1/\gamma}\right]^\gamma < 0 \quad \text{as} \quad \lambda > \frac{1}{K_\gamma}$$

and

$$\mathbb{E}(v(R((x^*)^-))) = -\frac{1-p}{1-\gamma}\left[\frac{(r_g-r_b)(\hat{r}-r^0)}{r_g-r^0}\right]^{1-\gamma}\left[\left(\frac{\lambda}{K_\gamma}\right)^{1/\gamma}-1\right]^\gamma < 0 \quad \text{as} \quad \lambda > K_\gamma$$

and thus showing $\mathbb{E}(v(R((x^*)^+))) > \mathbb{E}(v(R((x^*)^-)))$ boils down to proving that

$$\lambda^{1/\gamma} - \left(\frac{1}{K_\gamma}\right)^{1/\gamma} < \left(\frac{\lambda}{K_\gamma}\right)^{1/\gamma} - 1$$

or

$$\lambda^{1/\gamma}\left(1 - K_\gamma^{1/\gamma}\right) > K_\gamma^{1/\gamma} - 1$$

where the last inequality is implied by $K_\gamma < 1$ which follows from $p > \bar{p}$.[39] The other cases follow directly. This concludes the proof. □

Proof of proposition 2.4: Note that the derivative of $\mathbb{E}(v(R(x)))$ with respect to x when $\hat{r} = r^0$ is

$$\frac{d}{dx}\mathbb{E}(v(R(x))) = \left\{ \begin{array}{lr} (-x)^{-\gamma}\left[\lambda\int_{r^0}^{+\infty}(r-r^0)^{1-\gamma}f(r)dr - \int_{-\infty}^{r^0}(r^0-r)^{1-\gamma}f(r)dr\right], & x<0 \\ \\ x^{-\gamma}\left[\int_{r^0}^{+\infty}(r-r^0)^{1-\gamma}f(r)dr - \lambda\int_{-\infty}^{r^0}(r^0-r)^{1-\gamma}f(r)dr\right], & x>0 \end{array}\right\} \quad (4.32)$$

and thus $\frac{d}{dx}\mathbb{E}(v(R(x))) > 0$ for $x < 0$ and $\lambda > K_\gamma$, and $\frac{d}{dx}\mathbb{E}(v(R(x))) < 0$ for $x > 0$ and $\lambda > 1/K_\gamma$. $\mathbb{E}(v(R(x)))$ is continuous for $x = 0$ as

$$\lim_{x\to 0^+}\mathbb{E}(v(R(x))) = \lim_{x\to 0^-}\mathbb{E}(v(R(x))) = 0 = \mathbb{E}(v(R(0)))$$

which follows from (2.15) and the assumption $\hat{r} = r^0$. Then, based on this and (4.32) is the maximum of (2.16) reached at zero when $\lambda > \max\left\{K_\gamma, \frac{1}{K_\gamma}\right\}$. This finishes the proof. □

Proof of proposition 2.5: The expected prospect theory utility function (2.15) is continuous as

$$\lim_{x\to 0^+}\mathbb{E}(v(R(x))) = \lim_{x\to 0^-}\mathbb{E}(v(R(x))) = \frac{(r^0-\hat{r})^{1-\gamma}}{1-\gamma} = \mathbb{E}(v(R(0))) \quad (4.33)$$

which follows from (2.15) and the assumption $\hat{r} < r^0$. Note that $\mathbb{E}(v(R(x))) < \mathbb{E}(v(R(0)))$ for $x < 0$ if

$$\begin{aligned}(1-\gamma)\mathbb{E}(v(R(x))) &= (-x)^{1-\gamma}\int_{-\infty}^{z(x)}[z(x)-r]^{1-\gamma}f(r)dr - (-x)^{1-\gamma}\lambda\int_{z(x)}^{+\infty}[r-z(x)]^{1-\gamma}f(r)dr \\ &< (r^0-\hat{r})^{1-\gamma}\end{aligned}$$

[39]Note that $K_\gamma < 1$ if and only if $p > \bar{p}$, $K_\gamma > 1$ if and only if $p < \bar{p}$, and $K_\gamma = 1$ if and only if $p = \bar{p}$.

where $z(x) = \frac{r^0 - \hat{r}}{-x} + r^0$. Thus

$$\lambda > \frac{\int_{-\infty}^{z(x)} \left[z(x) - r\right]^{1-\gamma} f(r)dr - \left[z(x) - r^0\right]^{1-\gamma}}{\int_{z(x)}^{+\infty} \left[r - z(x)\right]^{1-\gamma} f(r)dr} = K_\gamma\left(z(x)\right) \tag{4.34}$$

as $\int_{z(x)}^{+\infty} \left[r - z(x)\right]^{1-\gamma} f(r)dr > 0$.[40] The assumption of the theorem, namely $\lambda > \hat{K}_\gamma$ and the definition of $\hat{K}_\gamma$, see (2.19), imply that (4.34) holds and thus $\mathbb{E}(v(R(x))) < \mathbb{E}(v(R(0)))$ for $x < 0$.

Note that based on Leibniz integral rule the derivative of $\mathbb{E}(v(R(x)))$ with respect to x is

$$\frac{d}{dx}\mathbb{E}(v(R(x))) = \left\{ \begin{array}{l} \frac{1}{(-x)^\gamma} \int_{-\infty}^{z(x)} \frac{r - r^0}{[z(x)-r]^\gamma} f(r)dr + \frac{\lambda}{x^\gamma} \int_{z(x)}^{+\infty} \frac{r-r^0}{[r-z(x)]^\gamma} f(r)dr, \quad x < 0 \\ \\ \frac{\mathbb{E}(r) - r^0}{(r^0-\hat{r})^\gamma} > 0, \quad x = 0, \quad \hat{r} < r^0 \\ \\ \lambda\, \frac{\mathbb{E}(r)-r^0}{(\hat{r}-r^0)^\gamma} > 0, \quad x = 0, \quad \hat{r} > r^0 \\ \\ \frac{\lambda}{x^\gamma} \int_{-\infty}^{z(x)} \frac{r-r^0}{[z(x)-r]^\gamma} f(r)dr + \frac{1}{x^\gamma} \int_{z(x)}^{+\infty} \frac{r-r^0}{[r-z(x)]^\gamma} f(r)dr, \quad x > 0 \end{array} \right\} \tag{4.35}$$

where $z(x) = \frac{r^0 - \hat{r}}{-x} + r^0$. This follows from $\mathbb{E}(r) > r^0$ and

$$\lim_{x \to 0^+} \frac{d}{dx}\mathbb{E}(v(R(x))) = \lim_{x \to 0^-} \frac{d}{dx}\mathbb{E}(v(R(x))) = \int_{-\infty}^{+\infty} \frac{r - r^0}{(r^0 - \hat{r})^\gamma} f(r)dr = \frac{\mathbb{E}(r) - r^0}{(r^0 - \hat{r})^\gamma} > 0 \tag{4.36}$$

for $\hat{r} < r^0$ and from

$$\lim_{x \to 0^+} \frac{d}{dx}\mathbb{E}(v(R(x))) = \lim_{x \to 0^-} \frac{d}{dx}\mathbb{E}(v(R(x))) = \lambda \int_{-\infty}^{+\infty} \frac{r - r^0}{(\hat{r} - r^0)^\gamma} f(r)dr = \lambda \frac{\mathbb{E}(r) - r^0}{(\hat{r} - r^0)^\gamma} > 0 \tag{4.37}$$

for $\hat{r} > r^0$. Thus, for any $\hat{r} \neq r^0$ is $\mathbb{E}(v(R(x)))$ increasing in zero. Note in addition that based on (2.15) we obtain the following

$$\lim_{x \to +\infty} \mathbb{E}(v(R(x))) = +\infty \times \left[-\lambda \int_{-\infty}^{r^0} \left(r^0 - r\right)^{1-\gamma} f(r)dr + \int_{r^0}^{+\infty} \left(r - r^0\right)^{1-\gamma} f(r)dr \right] = -\infty \tag{4.38}$$

[40] Let $\phi(c) = \int_c^{+\infty} (r - c)^{1-\gamma} f(r)dr$ and let us assume that there exists $c_0 \in \mathbb{R}$ such that $\phi(c_0) = 0$. Then $f(r) = 0$ for any $r \geq c_0$ and $F(c_0) = \int_{-\infty}^{c_0} f(r)dr = \int_{-\infty}^{+\infty} f(r)dr = 1$, which is a contradiction to the assumption of the distribution of the risky asset's return. Thus, $\phi(c) > 0$ for any $c \in \mathbb{R}$.

as $\lambda > 1/K_\gamma$; i.e., the expression in the brackets is negative.[41]

Thus based on the fact that $\mathbb{E}(v(R(x))) < \mathbb{E}(v(R(0)))$ for $x < 0$, $\mathbb{E}(v(R(x)))$ being continuous (also at zero, see (4.33)), increasing at zero, see (4.35), for $x = 0$, and achieving $-\infty$ in infinity, see (4.38), it follows then that the solution x^* of (2.16) is positive and such that the first order conditions are satisfied; i.e., $\frac{d}{dx}\mathbb{E}(v(R(x^*))) = 0$ for $x^* > 0$, which coincides with (2.18). This finishes the proof. $\square$

Lemma 4.1 *Let $\mathbb{E}(r) > r^0$. Then the function $K_\gamma : [r^0, +\infty) \to \mathbb{R}$*

$$K_\gamma(c) = \frac{\int_{-\infty}^{c}(c-r)^{1-\gamma}f(r)dr - (c-r^0)^{1-\gamma}}{\int_{c}^{\infty}(r-c)^{1-\gamma}f(r)dr}$$

is bounded from above; i.e., there exists a constant $M \geq 0$ such that $K_\gamma(c) \leq M$ for any $c \in [r^0, +\infty)$.

Proof: Note that $c \in \mathbb{R}$ and that the denominator of $K_\gamma(c)$, namely $\int_c^{+\infty}(r-c)^{1-\gamma}f(r)dr$, is strictly positive and that both integrals $\int_{-\infty}^{c}(c-r)^{1-\gamma}f(r)dr$ and $\int_c^{+\infty}(r-c)^{1-\gamma}f(r)dr$ are nonnegative and finite for any $c \in \mathbb{R}$. The latter is a consequence of the assumption $\mathbb{E}(|r|) = \int_{-\infty}^{+\infty}|r|f(r)dr < +\infty$ and the inequality $|r-c|^{1-\gamma} \leq C_\gamma(1+|c|+|r|)$ for any $r, c \in \mathbb{R}$ where $C_\gamma > 0$ is a constant.

Let $c \geq r^0$ be fixed. The function $H(r) \equiv (c-r)^{1-\gamma}$ is concave on the set $(-\infty, c)$. We remind ourselves Jensen's inequality

$$\int_{-\infty}^{c} H(r)g(r)dr \leq H\left(\int_{-\infty}^{c} rg(r)dr\right)$$

where $g(r) = f(r)/F(c) \geq 0$ is such that $\int_{-\infty}^{c} g(r)dr = (1/F(c))\int_{-\infty}^{c} f(r)dr = 1$. Therefore

$$\begin{aligned}\int_{-\infty}^{c}(c-r)^{1-\gamma}f(r)dr &\leq F(c)\left(c - \int_{-\infty}^{c} rg(r)dr\right)^{1-\gamma} = (F(c))^{\gamma}\left(\int_{-\infty}^{c}(c-r)f(r)dr\right)^{1-\gamma} \\ &\leq \left(\int_{-\infty}^{c}(c-r)f(r)dr\right)^{1-\gamma} \qquad (4.39)\end{aligned}$$

For $c \geq 0$ we have

$$\int_{-\infty}^{c}(c-r)f(r)dr - (c-r^0) = c(F(c)-1) + r^0 - \int_{-\infty}^{c} rf(r)dr \leq r^0 - \int_{-\infty}^{c} rf(r)dr \quad (4.40)$$

Since $\mathbb{E}(r) > r^0$, there exists $c_* \geq \max\{0, r^0\}$ such that $\int_{-\infty}^{c} rf(r)dr > r^0$ for any $c \geq c_*$.

[41] Note that this property holds also for $\hat{r} > r^0$.

This and (4.40) imply that for any $c \geq c_*$

$$\int_{-\infty}^{c} (c-r)f(r)dr < c - r^0$$

which gives, together with (4.39), the following

$$\int_{-\infty}^{c} (c-r)^{1-\gamma} f(r)dr - (c-r^0)^{1-\gamma} < 0 \quad \text{for any } c \geq c_*.$$

Since the denominator of $K_\gamma(c)$ is strictly positive the function $K_\gamma(c)$ is continuous on a compact interval $[r^0, c_*]$. Hence it attains its maximum $M \geq 0$ on $[r^0, c_*]$. As $K_\gamma(c) < 0$ for $c \geq c_*$ the lemma follows. □

Proof of proposition 2.6: Based on (2.15) the following holds

$$\lim_{x \to -\infty} \mathbb{E}(v(R(x))) = +\infty \times \left[\int_{-\infty}^{r^0} \left(r^0 - r\right)^{1-\gamma} f(r)dr - \lambda \int_{r^0}^{+\infty} \left(r - r^0\right)^{1-\gamma} f(r)dr \right] = -\infty \tag{4.41}$$

The same holds also when x reaches $+\infty$, see (4.38). Then, it follows based on this, the continuity of $\mathbb{E}(v(R(x)))$ and the fact that $\mathbb{E}(v(R(x)))$ is increasing at zero, see (4.36) and (4.37), that there is at least one local maximum x^* of problem (2.16) such that $x^* > 0$ and (2.18) is satisfied. In addition, continuity of $\mathbb{E}(v(R(x)))$ and (4.41) imply that if there is any local maxima of problem (2.16), $(x^*)^-$, such that $(x^*)^- < 0$, then the first order conditions hold and (2.20) is satisfied. Finally, continuity of $\mathbb{E}(v(R(x)))$, (4.38) and (4.41) imply that any global maxima is finite. This concludes the proof. □

Proof of proposition 2.7: The proof is based on implicit function differentiation and equation (2.18). Let $x^* > 0$ be a solution of

$$\frac{d}{dx}\mathbb{E}(v(R(x))) = 0$$

and let it be fixed for all the following analysis. Thus, the first order conditions are satisfied and

$$G(\lambda, \hat{r}, x) \equiv \lambda \int_{-\infty}^{z(x)} \frac{r - r^0}{[z(x) - r]^\gamma} f(r)dr + \int_{z(x)}^{+\infty} \frac{r - r^0}{[r - z(x)]^\gamma} f(r)dr = 0 \tag{4.42}$$

where $z(x) = r^0 + \frac{\hat{r} - r^0}{x}$. Then

$$\frac{dx}{d\lambda} = -\frac{\frac{dG}{d\lambda}}{\frac{dG}{dx}} \quad \text{and} \quad \frac{dx}{d\hat{r}} = -\frac{\frac{dG}{d\hat{r}}}{\frac{dG}{dx}} \tag{4.43}$$

where $\hat{r}$ is fixed in the first case and λ is fixed in the second case. Note that $\frac{G(\lambda,\hat{r},x)}{x^\gamma} = \frac{d}{dx}\mathbb{E}(v(R(x)))$ and as x^* is the point of local maximum then $\frac{d^2\mathbb{E}(v(R(x))}{dx^2} < 0$ for $x = x^*$ which implies then that $\frac{dG(\lambda,\hat{r},x)}{dx} < 0$. Namely

$$\begin{aligned}\frac{d^2\mathbb{E}(v(R(x))}{dx^2} &= \frac{d}{dx}\left(x^{-\gamma}G(\lambda,\hat{r},x)\right) = -x^{-1-\gamma}\gamma G(\lambda,\hat{r},x) + x^{-\gamma}\frac{dG(\lambda,\hat{r},x)}{dx} \\ &= -\frac{\gamma}{x}\frac{d}{dx}\mathbb{E}(v(R(x))) + x^{-\gamma}\frac{dG(\lambda,\hat{r},x)}{dx} = x^{-\gamma}\frac{dG(\lambda,\hat{r},x)}{dx} < 0\end{aligned}$$

(4.42) implies that

$$\frac{dG(\lambda,\hat{r},x)}{d\lambda} = \int_{-\infty}^{z(x)} \frac{r-r^0}{[z(x)-r]^\gamma} f(r)dr \qquad (4.44)$$

For $\hat{r} < r^0$ and $x > 0$ is $z(x) < r^0$ and thus based on (4.44) is $\frac{dG(\lambda,\hat{r},x)}{d\lambda} < 0$. On the other hand, for $\hat{r} > r^0$ and $x > 0$ is $z(x) > r^0$ and thus $\int_{z(x)}^{+\infty} \frac{r-r^0}{[r-z(x)]^\gamma} f(r)dr > 0$. This and equation (4.42) imply that $\lambda\int_{-\infty}^{z(x)} \frac{r-r^0}{[z(x)-r]^\gamma} f(r)dr < 0$ and thus $\frac{dG(\lambda,\hat{r},x)}{d\lambda} < 0$. This and (4.43) imply that $\frac{dx}{d\lambda} < 0$.

Note that

$$\begin{aligned}\int \frac{r-r^0}{[r-z(x)]^\gamma}dr &= \frac{[r-z(x)]^{2-\gamma}}{2-\gamma} + \frac{[r-z(x)]^{1-\gamma}[z(x)-r^0]}{1-\gamma} \\ \int \frac{r^0-r}{([z(x)-r]^\gamma}dr &= -\frac{[z(x)-r]^{2-\gamma}}{2-\gamma} + \frac{[z(x)-r]^{1-\gamma}[z(x)-r^0]}{1-\gamma}\end{aligned}$$

Using this and condition (2.21) when implying per partes on (4.42) gives

$$\begin{aligned}G(\lambda,\hat{r},x) &= -\int_{z(x)}^{+\infty}\left(\frac{[r-z(x)]^{2-\gamma}}{2-\gamma} + \frac{[r-z(x)]^{1-\gamma}[z(x)-r^0]}{1-\gamma}\right)\frac{df(r)}{dr}dr \\ &\quad +\lambda\int_{-\infty}^{z(x)}\left(-\frac{[z(x)-r]^{2-\gamma}}{2-\gamma} + \frac{[z(x)-r]^{1-\gamma}[z(x)-r^0]}{1-\gamma}\right)\frac{df(r)}{dr}dr = 0\end{aligned} \qquad (4.45)$$

As $\frac{dz(x)}{dx} = \frac{r^0 - \hat{r}}{(x^*)^2}$ and $\frac{dz(x)}{d\hat{r}} = \frac{1}{x^*}$, equation $G(\lambda, \hat{r}, x) = 0$ given by (4.45) implies

$$\left.\begin{aligned} \frac{dG}{dx}\Big|_{x=x^*} &= \frac{dz(x)}{dx}\Bigg[\lambda \int_{-\infty}^{z(x)} \left(\frac{\gamma}{1-\gamma}[z(x)-r]^{1-\gamma} + \frac{z(x)-r^0}{[z(x)-r]^\gamma}\right)\frac{df(r)}{dr}dr \\ &\quad - \int_{z(x)}^{+\infty}\left(\frac{\gamma}{1-\gamma}[r-z(x)]^{1-\gamma} - \frac{z(x)-r^0}{[r-z(x)]^\gamma}\right)\frac{df(r)}{dr}dr\Bigg] \\ \frac{dG}{d\hat{r}}\Big|_{x=x^*} &= \frac{dz(x)}{d\hat{r}}\Bigg[\lambda \int_{-\infty}^{z(x)} \left(\frac{\gamma}{1-\gamma}[z(x)-r]^{1-\gamma} + \frac{z(x)-r^0}{[z(x)-r]^\gamma}\right)\frac{df(r)}{dr}dr \\ &\quad - \int_{z(x)}^{+\infty}\left(\frac{\gamma}{1-\gamma}[r-z(x)]^{1-\gamma} - \frac{z(x)-r^0}{[r-z(x)]^\gamma}\right)\frac{df(r)}{dr}dr\Bigg] \end{aligned}\right\} \quad (4.46)$$

Based on (4.43) and (4.46) the following holds

$$\frac{dx^*}{d\hat{r}} = \frac{x^*}{\hat{r} - r^0}\begin{cases} < 0, & \text{if } \hat{r} < r^0 \\ > 0, & \text{if } \hat{r} > r^0 \end{cases}$$

This concludes the proof. □

Appendix B: Data description and summary statistics

Abbr	Variable	Unit	Note	Source	Code	Start	Freq
EU Markets							
stock	EU stock market ind	Index		Ref DS	TOTMKEM(RI)˜E	1983:1	m
bond	German gov bond ind	Index	Total ret ind, 10 yrs	Ref DS	BMBD10Y(RI)	1983:1	m
gold	Gold bullion LBM	EUR	A.M. official fixing	Ref DS: ICE	GOLDBLN(OF)˜E	1983:1	m
risk-free	one-month LIBOR	Percent		Ref DS: ICE	BBDEM1M	1986:1	m
US Markets							
stock	US stock market ind	Index		Ref DS	TOTMKUS(RI)	1983:1	m
bond	US gov bond ind	Index	Total ret ind, 10 yrs	Ref DS	BMUS10Y(RI)	1983:1	m
gold	Gold bullion LBM	USD	A.M. official fixing	Ref DS: ICE	GOLDBLN(OF)	1983:1	m
risk-free	one-month LIBOR	Percent		Ref DS: ICE	BBUSD1M	1986:1	m

Table 9: Data description and sources.
Abbr = Abbreviation, freq = frequency, gov = government, ind = index, ret = return, yrs = years, Ref DS = Refinitiv Datastream, EUR = Euro, USD = US dollar, LBM = London Bullion Market, ICE = ICE Benchmark Administration Ltd., m = monthly. Monthly values are end-of-month values for a given month. Returns are calculated as $100\,(P_t/P_{t-1} - 1)$, where P_t is the price of the index observed in month t, and are quoted in percent. Note that the risk-free rate is used in the evaluation process, not in the optimization process. It is only available from January 1986, which is why we start with the asset data in January 1983, granting us an optimization period of three years. Assets in Europe are quoted in Euro (Deutsche Mark), assets in the US are quoted in US dollar. Prices for gold are quoted in Euro or US dollar per troy ounce.

	EU markets				US markets			
	stock	bond	gold	risk-free	stock	bond	gold	risk-free
Performance of one-month returns (in percent p.a.)								
Mean	12.28	6.62	4.77	3.04	13.61	7.36	5.13	3.50
Std.dev.	16.80	5.44	15.85	0.79	14.87	7.50	15.62	0.77
Skewness	-0.61	-0.18	0.22	0.57	-0.68	0.24	0.13	0.27
Kurtosis	1.89	0.11	1.34	-0.39	2.31	0.95	1.61	-1.13
VaR	-60.92	-22.29	-56.98	-0.42	-58.08	-28.95	-53.76	0.17
CVaR	-76.81	-28.13	-69.38	-0.50	-71.08	-38.44	-68.32	0.16
Minimum	-93.99	-49.47	-89.62	-0.59	-93.40	-58.63	-91.77	0.14
Maximum	546.04	90.58	653.20	9.88	376.67	209.02	731.56	10.06
Percentiles (in percent p.a.)								
5	-60.92	-22.29	-56.98	-0.42	-58.08	-28.95	-53.76	0.17
10	-43.88	-17.59	-44.99	-0.40	-41.24	-21.81	-43.82	0.20
25	-17.40	-6.51	-23.90	0.35	-15.64	-8.46	-24.90	0.66
50	18.58	8.56	1.92	3.18	18.54	6.07	-0.39	3.19
75	57.11	21.18	39.20	4.57	52.22	24.19	42.22	5.68
90	104.72	32.64	101.53	7.93	102.38	46.48	104.64	7.13
95	149.39	40.55	157.25	9.06	134.23	65.99	157.16	8.31
Correlations across EU and US assets								
stock, EU								
bond, EU	-0.07							
gold, EU	-0.08	0.06						
risk-free, EU	-0.04	0.07	-0.06					
stock, US	0.74	-0.09	-0.13	-0.03				
bond, US	-0.17	0.67	-0.02	0.10	-0.04			
gold, US	-0.13	0.05	0.79	-0.05	-0.06	0.06		
risk-free, US	0.02	-0.01	-0.07	0.72	0.02	0.07	-0.07	

Table 10: Summary statistics for EU and US markets.
Statistics are calculated on the basis of monthly returns and then annualized using discrete compounding, for the period January 1983 to December 2020 (for the period January 1986 to December 2020 in case of the risk-free rate). The annualized standard deviation is calculated by multiplying the monthly standard deviation with $\sqrt{12}$. Skewness and kurtosis are not adjusted. The risk-free rate, originally given in percent p.a., is first converted to percent per month using discrete compounding and then the statistics are computed similarly to the other data.

Appendix C: Empirical results

	MV	CVaR	Risk neutral	Linear loss aversion λ					S-shaped prospect theory, $\gamma = 0.5$ λ				
				1.5	2	2.25	2.5	3	1.5	2	2.25	2.5	3
Performance of one-month returns (in percent p.a.)													
Mean	6.17	5.75	12.92	9.42	8.36	7.69	7.36	7.03	7.81	8.04	7.19	7.84	7.57
Omega	159.23	143.34	167.86	150.45	151.48	149.66	148.38	149.69	146.31	157.46	147.98	164.78	165.16
Sharpe ratio	62.74	46.93	63.16	45.99	46.14	44.15	42.63	44.06	41.74	50.63	41.82	57.01	58.13
Sortino ratio	97.70	72.35	98.64	70.31	68.96	65.16	62.46	61.82	59.84	78.42	60.14	90.83	91.60
Additional descriptive statistics (in percent p.a.)													
Median	7.32	6.10	14.93	9.51	8.99	8.44	8.74	9.02	9.11	8.34	7.81	7.74	7.86
Volatility	4.79	5.54	15.19	13.47	11.21	10.23	9.83	8.79	11.09	9.58	9.63	8.16	7.54
Down. vol.	2.69	3.16	9.35	8.48	7.17	6.61	6.40	5.97	7.38	5.82	6.38	4.77	4.45
CVaR	-27.23	-31.10	-69.37	-66.91	-59.46	-56.03	-54.62	-51.37	-61.23	-52.28	-53.48	-44.46	-42.27
Skewness	-0.34	-0.10	-0.31	-0.27	-0.56	-0.71	-0.78	-1.53	-0.83	-0.07	-1.05	0.00	-0.11
Kurtosis	3.59	4.10	5.19	6.86	10.15	12.95	14.71	19.12	11.00	9.17	15.82	10.71	10.96
Realized returns (in percent p.a.)													
Last 10 years	5.67	5.18	7.19	4.52	4.15	4.50	4.51	5.03	4.44	5.19	4.94	5.73	5.99
Last 5 years	3.49	3.47	7.18	1.12	2.24	2.57	2.37	3.52	3.04	2.04	2.39	4.06	4.72
Last 3 years	3.23	3.50	7.83	1.26	2.62	2.84	2.21	3.56	3.77	2.50	2.90	4.35	4.57
Last year	1.13	-3.83	6.70	1.54	-2.41	-2.96	-3.95	-0.44	5.52	-0.79	-0.69	4.54	4.72
Mean portfolio weights (in percent)													
Stock	10.55	14.66	66.19	48.11	38.54	34.13	31.41	27.84	41.47	35.20	32.85	28.73	25.07
Bond	80.65	70.40	11.43	31.28	47.28	53.33	56.53	60.86	45.82	54.30	56.88	61.17	65.10
Gold	8.80	14.95	22.38	20.61	14.18	12.54	12.06	11.29	12.70	10.50	10.27	10.10	9.83
Standard deviation of portfolio weights													
Stock	8.51	8.57	47.31	42.57	36.20	32.03	29.62	26.03	35.64	31.51	29.95	25.98	21.28
Bond	7.42	16.49	31.82	38.66	35.41	32.54	30.91	27.89	35.98	31.51	29.99	26.93	22.73
Gold	7.10	12.71	41.68	35.66	22.75	18.67	16.54	13.65	18.27	12.80	11.92	11.03	10.05

Table 11: Portfolio performance of PT portfolios in the EU: benchmark scenario, $\hat{r} = 0$, $\gamma = 0.5$.
The table reports statistics of a monthly reallocated optimal portfolio based on an optimization period of 36 months as well as the average and standard deviation of the optimal asset weights. The table also reports results for portfolios implied by mean-variance (MV), conditional value-at-risk (CVaR), risk neutral, and linear loss averse investors. The evaluation period covers January 1986 to December 2020. Statistics are calculated on the basis of monthly returns and then annualized assuming discrete compounding. The annual standard deviation is computed as $\sigma_{pa} = \sqrt{12}\sigma_{pm}$.

	MV	CVaR	Risk neutral	Linear loss aversion λ					S-shaped prospect theory, $\gamma = 0.5$ λ				
				1.5	2	2.25	2.5	3	1.5	2	2.25	2.5	3
Performance Measures													
Mean	6.95	7.35	13.34	12.34	10.08	9.35	8.62	8.46	10.06	10.46	9.90	9.64	8.89
Omega	155.26	159.99	163.86	167.92	158.80	155.92	152.46	155.95	158.69	172.36	166.71	166.93	160.51
Sharpe ratio	57.37	58.99	62.23	63.43	55.15	53.36	50.51	53.19	54.56	65.46	61.21	61.06	56.38
Sortino ratio	91.75	93.98	92.15	94.49	78.09	75.44	71.24	77.06	78.40	99.15	91.38	91.46	84.18
Additional Descriptive Statistics													
Median	6.17	6.91	18.72	14.24	11.46	10.57	10.60	10.04	10.69	9.91	9.69	9.34	8.45
Volatility	5.77	6.28	15.33	13.50	11.59	10.66	9.83	9.04	11.66	10.31	10.14	9.76	9.27
Down. vol.	3.14	3.49	9.90	8.65	7.78	7.13	6.55	5.82	7.72	6.40	6.38	6.10	5.79
CVaR	-31.16	-33.82	-71.48	-66.52	-62.15	-59.59	-56.00	-51.24	-63.19	-55.62	-55.97	-53.93	-51.32
Skewness	-0.02	-0.07	-0.68	-0.78	-1.30	-1.16	-1.12	-0.93	-1.03	-0.69	-0.73	-0.75	-0.70
Kurtosis	4.54	6.10	5.20	6.92	9.93	8.40	8.76	8.79	7.97	6.83	6.98	7.52	7.74
Realized Returns													
Last 10 Years	7.72	7.64	9.21	10.24	6.84	6.01	6.06	6.26	6.79	6.81	6.78	6.91	6.95
Last 5 Years	8.00	7.41	8.90	9.79	6.75	6.70	6.40	6.82	5.14	7.43	7.72	8.04	7.86
Last 3 Years	9.43	9.14	4.50	8.29	6.83	7.34	6.31	7.19	3.74	7.77	7.79	8.88	8.86
Last Year	15.05	14.66	-7.97	4.01	6.44	7.70	11.76	14.08	-9.99	4.27	5.03	8.04	11.14
Mean Portfolio Weights													
Stock	20.74	25.78	65.48	58.99	49.04	45.89	42.21	38.63	50.92	46.34	45.95	43.47	41.47
Bond	64.64	55.68	6.43	17.51	34.52	38.67	43.72	48.52	34.06	40.48	41.79	45.35	48.38
Gold	14.62	18.54	28.10	23.51	16.44	15.44	14.08	12.84	15.02	13.18	12.26	11.18	10.15
Standard Deviation of Portfolio Weights													
Stock	10.49	13.46	47.54	41.39	35.08	32.53	29.84	27.02	33.99	32.18	31.57	30.32	28.81
Bond	10.46	16.18	24.53	30.12	31.74	31.62	30.70	28.92	33.51	32.64	32.23	31.42	29.72
Gold	14.20	15.77	44.95	35.11	22.64	20.09	17.29	14.89	22.62	18.65	16.88	15.45	13.61

Table 12: Portfolio performance of PT portfolios in the US: benchmark scenario, $\hat{r} = 0$, $\gamma = 0.5$.
The table reports statistics of a monthly reallocated optimal portfolio based on an optimization period of 36 months as well as the average and standard deviation of the optimal asset weights. The table also reports results for portfolios implied by mean-variance (MV), conditional value-at-risk (CVaR), risk neutral, and linear loss averse investors. The evaluation period covers January 1986 to December 2020. Statistics are calculated on the basis of monthly returns and then annualized assuming discrete compounding. The annual standard deviation is computed as $\sigma_{pa} = \sqrt{12}\sigma_{pm}$.

	MV	CVaR	Risk neutral	Linear loss aversion λ					S-shaped prospect theory, $\gamma = 0.5$ λ				
				1.5	2	2.25	2.5	3	1.5	2	2.25	2.5	3
Performance Measures													
Mean	6.17	5.75	12.92	9.60	8.21	7.63	7.26	7.03	7.95	8.11	7.24	7.58	7.55
Omega	159.23	143.34	167.86	151.30	148.71	148.22	146.86	149.66	147.31	158.64	148.55	159.35	164.86
Sharpe ratio	62.74	46.93	63.16	46.46	43.84	43.05	41.55	44.04	42.59	51.45	42.25	52.91	57.90
Sortino ratio	97.70	72.35	98.64	71.85	64.67	63.28	60.73	61.77	61.31	79.82	60.74	82.43	91.23
Additional Descriptive Statistics													
Median	7.32	6.10	14.93	9.51	8.99	8.44	8.62	9.02	8.96	8.64	7.85	7.76	7.86
Volatility	4.79	5.54	15.19	13.71	11.45	10.34	9.87	8.79	11.19	9.55	9.64	8.32	7.54
Down. vol.	2.69	3.16	9.35	8.53	7.44	6.71	6.43	5.97	7.42	5.80	6.38	5.00	4.45
CVaR	-27.23	-31.10	-69.37	-66.91	-61.36	-57.11	-54.86	-51.37	-61.30	-52.12	-53.64	-46.30	-42.17
Skewness	-0.34	-0.10	-0.31	-0.16	-0.62	-0.71	-0.78	-1.53	-0.81	-0.07	-1.05	-0.11	-0.11
Kurtosis	3.59	4.10	5.19	6.83	9.81	12.53	14.54	19.14	10.67	9.26	15.78	10.46	11.01
Realized Returns													
Last 10 Years	5.67	5.18	7.19	4.47	4.40	4.46	4.25	5.03	4.42	5.06	4.84	5.85	6.01
Last 5 Years	3.49	3.47	7.18	1.28	2.56	2.53	2.29	3.52	2.83	2.18	2.47	4.06	4.72
Last 3 Years	3.23	3.50	7.83	1.54	3.16	2.84	2.21	3.56	3.43	2.73	3.05	4.35	4.57
Last Year	1.13	-3.83	6.70	1.63	-0.92	-2.95	-3.95	-0.44	5.55	-0.79	-0.68	4.54	4.72
Mean Portfolio Weights													
Stock	10.55	14.66	66.19	48.54	39.01	34.45	31.50	27.84	41.94	35.13	33.18	28.98	25.10
Bond	80.65	70.40	11.43	30.54	46.54	52.96	56.43	60.88	45.19	54.39	56.53	60.93	65.06
Gold	8.80	14.95	22.38	20.93	14.45	12.59	12.07	11.28	12.87	10.49	10.29	10.09	9.84
Standard Deviation of Portfolio Weights													
Stock	8.51	8.57	47.31	42.69	36.65	32.23	29.77	26.03	36.21	31.67	30.22	26.22	21.30
Bond	7.42	16.49	31.82	38.54	35.70	32.62	31.03	27.90	36.54	31.69	30.19	27.08	22.75
Gold	7.10	12.71	41.68	36.14	23.41	18.83	16.60	13.66	18.67	12.84	11.92	11.04	10.04

Table 13: Portfolio performance of PT portfolios in the EU: aggressive scenario, $\hat{r} = 0$, $\gamma = 0.5$.
The table reports statistics of a monthly reallocated optimal portfolio based on an optimization period of 36 months as well as the average and standard deviation of the optimal asset weights. The table also reports results for portfolios implied by mean-variance (MV), conditional value-at-risk (CVaR), risk neutral, and linear loss averse investors. The evaluation period covers January 1986 to December 2020. Statistics are calculated on the basis of monthly returns and then annualized assuming discrete compounding. The annual standard deviation is computed as $\sigma_{pa} = \sqrt{12}\sigma_{pm}$.

	MV	CVaR	Risk neutral	Linear loss aversion λ					S-shaped prospect theory, $\gamma = 0.9$ λ				
				1.5	2	2.25	2.5	3	1.5	2	2.25	2.5	3
Performance Measures													
Mean	6.17	5.75	12.92	9.20	8.29	7.73	7.30	7.00	8.41	7.62	8.10	8.08	8.59
Omega	159.23	143.34	167.86	148.83	151.48	150.25	148.25	149.33	167.98	159.76	169.07	170.64	180.76
Sharpe ratio	62.74	46.93	63.16	44.76	46.08	44.57	42.69	43.78	60.04	54.74	61.86	63.01	69.82
Sortino ratio	97.70	72.35	98.64	67.84	68.92	65.82	61.93	61.24	97.31	85.46	100.05	103.25	116.87
Additional Descriptive Statistics													
Median	7.32	6.10	14.93	9.43	8.99	8.57	8.74	9.02	7.54	7.47	8.19	8.01	8.10
Volatility	4.79	5.54	15.19	13.36	11.07	10.22	9.70	8.76	8.66	8.11	7.92	7.74	7.70
Down. vol.	2.69	3.16	9.35	8.48	7.08	6.60	6.37	5.96	4.99	4.85	4.55	4.38	4.26
CVaR	-27.23	-31.10	-69.37	-66.97	-58.74	-56.03	-54.58	-51.37	-46.04	-45.33	-42.81	-41.48	-40.60
Skewness	-0.34	-0.10	-0.31	-0.31	-0.56	-0.71	-0.91	-1.56	-0.08	-0.29	-0.13	-0.07	-0.03
Kurtosis	3.59	4.10	5.19	6.96	10.44	12.99	15.14	19.33	8.15	8.79	8.86	9.07	9.20
Realized Returns													
Last 10 Years	5.67	5.18	7.19	4.08	4.24	4.56	4.55	5.06	6.01	4.73	5.20	5.11	5.80
Last 5 Years	3.49	3.47	7.18	0.59	2.41	2.57	2.37	3.58	3.55	3.00	2.42	2.38	4.35
Last 3 Years	3.23	3.50	7.83	0.46	2.74	2.84	2.21	3.66	3.21	1.30	2.73	2.20	4.30
Last Year	1.13	-3.83	6.70	-0.03	-2.41	-2.96	-3.95	-0.08	0.92	-1.55	-0.82	1.01	1.40
Mean Portfolio Weights													
Stock	10.55	14.66	66.19	48.01	37.96	34.05	31.08	27.79	33.24	31.56	31.21	29.57	28.57
Bond	80.65	70.40	11.43	32.05	47.93	53.47	56.83	60.89	55.53	57.97	58.24	59.49	60.45
Gold	8.80	14.95	22.38	19.94	14.11	12.47	12.09	11.32	11.23	10.47	10.55	10.94	10.98
Standard Deviation of Portfolio Weights													
Stock	8.51	8.57	47.31	42.50	35.64	32.01	29.24	26.00	29.19	27.34	26.62	25.53	24.56
Bond	7.42	16.49	31.82	38.72	35.13	32.54	30.65	27.89	29.80	27.84	27.51	27.12	26.55
Gold	7.10	12.71	41.68	34.71	22.61	18.59	16.49	13.64	12.10	10.26	9.88	10.13	10.01

Table 14: Portfolio performance of PT portfolios in the EU: conservative scenario, $\hat{r} = 0$, $\gamma = 0.9$.
The table reports statistics of a monthly reallocated optimal portfolio based on an optimization period of 36 months as well as the average and standard deviation of the optimal asset weights. The table also reports results for portfolios implied by mean-variance (MV), conditional value-at-risk (CVaR), risk neutral, and linear loss averse investors. The evaluation period covers January 1986 to December 2020. Statistics are calculated on the basis of monthly returns and then annualized assuming discrete compounding. The annual standard deviation is computed as $\sigma_{pa} = \sqrt{12}\sigma_{pm}$.

	MV	CVaR	Risk neutral	Linear loss aversion λ					S-shaped prospect theory, $\gamma = 0.9$ λ				
				1.5	2	2.25	2.5	3	1.5	2	2.25	2.5	3
Performance Measures													
Mean	6.17	5.75	12.92	10.64	8.80	7.98	7.69	7.67	7.01	7.32	7.15	8.61	7.37
Omega	159.23	143.34	167.86	154.49	145.96	142.39	142.32	145.74	133.27	137.20	132.84	149.42	134.43
Sharpe ratio	62.74	46.93	63.16	50.99	43.32	40.26	39.63	41.73	33.94	37.56	33.62	48.27	36.66
Sortino ratio	97.70	72.35	98.64	78.31	65.27	59.44	58.59	61.50	47.94	54.91	46.61	70.65	54.89
Additional Descriptive Statistics													
Median	7.32	6.10	14.93	11.38	9.30	9.39	8.48	7.73	7.52	6.94	9.45	9.52	7.56
Volatility	4.79	5.54	15.19	14.47	12.91	11.91	11.38	10.76	11.31	11.05	11.84	11.17	11.47
Down. vol.	2.69	3.16	9.35	9.06	8.23	7.74	7.37	6.98	7.60	7.14	8.15	7.24	7.25
CVaR	-27.23	-31.10	-69.37	-68.48	-65.40	-62.51	-60.50	-58.07	-62.14	-59.64	-66.05	-60.57	-58.45
Skewness	-0.34	-0.10	-0.31	-0.26	-0.35	-0.53	-0.56	-0.67	-0.73	-0.37	-0.79	-0.59	-0.16
Kurtosis	3.59	4.10	5.19	5.92	7.18	8.56	9.64	11.29	6.67	6.18	6.56	6.46	4.12
Realized Returns													
Last 10 Years	5.67	5.18	7.19	5.24	4.34	3.70	4.10	4.31	6.09	4.15	2.82	6.72	1.45
Last 5 Years	3.49	3.47	7.18	3.30	1.42	1.09	1.29	1.24	4.88	5.95	2.41	6.82	1.92
Last 3 Years	3.23	3.50	7.83	3.46	1.49	1.67	1.86	1.13	10.63	6.68	0.27	7.72	1.11
Last Year	1.13	-3.83	6.70	1.64	-2.24	-2.67	-1.70	-3.18	14.65	9.66	-6.03	8.64	-5.77
Mean Portfolio Weights													
Stock	10.55	14.66	66.19	58.53	49.56	45.64	42.82	39.17	46.83	49.49	54.97	50.24	43.97
Bond	80.65	70.40	11.43	19.91	33.51	40.12	43.65	47.76	32.24	34.39	27.51	33.83	30.97
Gold	8.80	14.95	22.38	21.56	16.93	14.24	13.53	13.07	20.93	16.12	17.52	15.94	25.05
Standard Deviation of Portfolio Weights													
Stock	8.51	8.57	47.31	44.54	40.25	38.21	36.81	34.76	34.32	30.43	30.12	34.01	33.56
Bond	7.42	16.49	31.82	35.28	37.49	36.90	35.73	34.32	28.93	28.79	31.25	31.41	32.02
Gold	7.10	12.71	41.68	38.36	29.91	23.31	20.51	18.26	26.66	23.66	23.79	23.52	32.61

Table 15: Portfolio performance of PT portfolios in the EU: aggressive scenario, $\hat{r} = r^p$, $\gamma = 0.9$.
The table reports statistics of a monthly reallocated optimal portfolio based on an optimization period of 36 months as well as the average and standard deviation of the optimal asset weights. The table also reports results for portfolios implied by mean-variance (MV), conditional value-at-risk (CVaR), risk neutral, and linear loss averse investors. The evaluation period covers January 1986 to December 2020. Statistics are calculated on the basis of monthly returns and then annualized assuming discrete compounding. The annual standard deviation is computed as $\sigma_{pa} = \sqrt{12}\sigma_{pm}$.

	MV	CVaR	Risk neutral	Linear loss aversion λ					S-shaped prospect theory, $\gamma = 0.1$ λ				
				1.5	2	2.25	2.5	3	1.5	2	2.25	2.5	3
Performance Measures													
Mean	6.95	7.35	13.34	12.80	10.83	9.99	9.67	9.21	13.62	11.42	10.62	9.84	9.59
Omega	155.26	159.99	163.86	168.67	158.29	153.98	152.85	152.83	173.28	162.62	158.52	153.11	155.29
Sharpe ratio	57.37	58.99	62.23	64.53	55.06	51.99	51.27	52.14	68.70	58.49	55.32	51.39	54.33
Sortino ratio	91.75	93.98	92.15	96.39	80.40	75.33	73.80	76.80	103.06	85.94	79.56	73.87	79.66
Additional Descriptive Statistics													
Median	6.17	6.91	18.72	15.84	12.83	11.98	11.80	10.62	18.32	13.61	13.00	11.54	11.37
Volatility	5.77	6.28	15.33	13.96	12.89	12.09	11.66	10.62	14.26	13.12	12.46	11.95	10.87
Down. vol.	3.14	3.49	9.90	8.93	8.42	7.92	7.68	6.76	9.08	8.52	8.25	7.89	6.97
CVaR	-31.16	-33.82	-71.48	-67.74	-65.21	-63.20	-62.07	-56.69	-68.40	-65.81	-64.42	-62.98	-58.76
Skewness	-0.02	-0.07	-0.68	-0.74	-0.88	-0.87	-0.90	-0.67	-0.73	-0.85	-1.06	-0.95	-0.69
Kurtosis	4.54	6.10	5.20	6.33	7.58	6.90	7.00	6.19	6.03	7.35	8.09	7.26	5.75
Realized Returns													
Last 10 Years	7.72	7.64	9.21	11.45	9.74	8.38	7.77	6.55	12.11	10.26	9.26	8.50	7.25
Last 5 Years	8.00	7.41	8.90	11.10	10.06	7.98	7.65	7.35	10.36	9.40	9.89	7.95	7.44
Last 3 Years	9.43	9.14	4.50	9.01	9.93	7.08	6.98	6.98	7.83	9.15	10.19	6.78	6.82
Last Year	15.05	14.66	-7.97	6.43	9.55	7.79	10.61	10.82	1.80	7.74	12.09	7.10	10.59
Mean Portfolio Weights													
Stock	20.74	25.78	65.48	62.20	56.26	53.50	51.30	48.13	63.17	57.86	54.70	53.21	49.58
Bond	64.64	55.68	6.43	13.27	24.26	28.84	32.03	37.04	12.62	22.78	27.16	30.21	35.58
Gold	14.62	18.54	28.10	24.53	19.49	17.66	16.67	14.83	24.21	19.36	18.14	16.59	14.84
Standard Deviation of Portfolio Weights													
Stock	10.49	13.46	47.54	43.36	39.25	38.20	36.98	34.86	43.78	39.64	38.95	37.79	36.50
Bond	10.46	16.18	24.53	29.23	30.23	30.65	30.87	30.82	28.43	30.85	31.88	31.47	31.87
Gold	14.20	15.77	44.95	37.77	28.15	24.45	21.95	18.59	39.02	28.95	25.75	22.53	18.63

Table 16: Portfolio performance of PT portfolios in the US: conservative scenario, $\hat{r} = r^p$, $\gamma = 0.1$.
The table reports statistics of a monthly reallocated optimal portfolio based on an optimization period of 36 months as well as the average and standard deviation of the optimal asset weights. The table also reports results for portfolios implied by mean-variance (MV), conditional value-at-risk (CVaR), risk neutral, and linear loss averse investors. The evaluation period covers January 1986 to December 2020. Statistics are calculated on the basis of monthly returns and then annualized assuming discrete compounding. The annual standard deviation is computed as $\sigma_{pa} = \sqrt{12}\sigma_{pm}$.

	MV	CVaR	Risk neutral	Linear loss aversion λ					S-shaped prospect theory, $\gamma = 0.9$ λ				
				1.5	2	2.25	2.5	3	1.5	2	2.25	2.5	3
Performance Measures													
Mean	6.95	7.35	13.34	12.80	10.83	9.99	9.67	9.21	10.80	10.62	11.50	11.26	9.33
Omega	155.26	159.99	163.86	168.67	158.29	153.98	152.85	152.83	155.45	156.70	166.15	160.22	149.15
Sharpe ratio	57.37	58.99	62.23	64.53	55.06	51.99	51.27	52.14	54.34	53.61	61.81	56.63	48.26
Sortino ratio	91.75	93.98	92.15	96.39	80.40	75.33	73.80	76.80	79.55	78.69	92.38	84.10	68.63
Additional Descriptive Statistics													
Median	6.17	6.91	18.72	15.84	12.83	11.98	11.80	10.62	12.38	12.39	14.18	14.30	11.64
Volatility	5.77	6.28	15.33	13.96	12.89	12.09	11.66	10.62	13.04	12.86	12.53	13.22	11.72
Down. vol.	3.14	3.49	9.90	8.93	8.42	7.92	7.68	6.76	8.47	8.35	7.98	8.52	7.80
CVaR	-31.16	-33.82	-71.48	-67.74	-65.21	-63.20	-62.07	-56.69	-66.08	-66.22	-63.73	-66.51	-63.81
Skewness	-0.02	-0.07	-0.68	-0.74	-0.88	-0.87	-0.90	-0.67	-0.74	-0.66	-0.65	-0.67	-0.82
Kurtosis	4.54	6.10	5.20	6.33	7.58	6.90	7.00	6.19	5.58	6.47	6.24	6.49	6.05
Realized Returns													
Last 10 Years	7.72	7.64	9.21	11.45	9.74	8.38	7.77	6.55	6.28	10.15	10.79	14.07	7.52
Last 5 Years	8.00	7.41	8.90	11.10	10.06	7.98	7.65	7.35	8.68	14.24	13.75	14.46	10.14
Last 3 Years	9.43	9.14	4.50	9.01	9.93	7.08	6.98	6.98	7.64	14.74	14.23	14.64	8.43
Last Year	15.05	14.66	-7.97	6.43	9.55	7.79	10.61	10.82	2.42	23.21	21.50	23.32	5.02
Mean Portfolio Weights													
Stock	20.74	25.78	65.48	62.20	56.26	53.50	51.30	48.13	60.10	62.74	62.13	57.80	55.61
Bond	64.64	55.68	6.43	13.27	24.26	28.84	32.03	37.04	17.03	21.72	19.35	19.64	33.60
Gold	14.62	18.54	28.10	24.53	19.49	17.66	16.67	14.83	22.87	15.54	18.51	22.56	10.80
Standard Deviation of Portfolio Weights													
Stock	10.49	13.46	47.54	43.36	39.25	38.20	36.98	34.86	39.11	36.66	36.68	40.37	39.95
Bond	10.46	16.18	24.53	29.23	30.23	30.65	30.87	30.82	26.05	29.42	25.40	30.34	38.58
Gold	14.20	15.77	44.95	37.77	28.15	24.45	21.95	18.59	28.86	25.41	27.18	32.78	18.09

Table 17: Portfolio performance of PT portfolios in the US: conservative scenario, $\hat{r} = r^p$, $\gamma = 0.9$.

The table reports statistics of a monthly reallocated optimal portfolio based on an optimization period of 36 months as well as the average and standard deviation of the optimal asset weights. The table also reports results for portfolios implied by mean-variance (MV), conditional value-at-risk (CVaR), risk neutral, and linear loss averse investors. The evaluation period covers January 1986 to December 2020. Statistics are calculated on the basis of monthly returns and then annualized assuming discrete compounding. The annual standard deviation is computed as $\sigma_{pa} = \sqrt{12}\sigma_{pm}$.

	MV	CVaR	Risk neutral	Linear loss aversion λ					S-shaped prospect theory, $\gamma = 0.9$ λ				
				1.5	2	2.25	2.5	3	1.5	2	2.25	2.5	3
Performance Measures													
Mean	6.95	7.35	13.34	12.59	10.07	9.69	8.90	8.40	11.61	11.59	11.31	11.42	10.00
Omega	155.26	159.99	163.86	169.14	158.63	159.29	155.41	154.70	177.74	181.59	178.08	181.25	166.21
Sharpe ratio	57.37	58.99	62.23	64.42	54.93	56.23	52.99	52.16	68.64	71.71	69.19	70.90	60.85
Sortino ratio	91.75	93.98	92.15	95.90	77.63	80.79	75.45	74.72	102.56	109.87	104.77	108.20	89.07
Additional Descriptive Statistics													
Median	6.17	6.91	18.72	15.24	11.87	10.84	10.18	9.92	11.64	11.16	11.15	10.80	9.91
Volatility	5.77	6.28	15.33	13.65	11.60	10.69	9.88	9.11	11.46	10.95	10.95	10.84	10.36
Down. vol.	3.14	3.49	9.90	8.76	7.81	7.02	6.53	5.94	7.25	6.73	6.81	6.68	6.65
CVaR	-31.16	-33.82	-71.48	-67.17	-62.55	-59.44	-56.53	-52.42	-60.75	-58.00	-58.98	-58.35	-58.12
Skewness	-0.02	-0.07	-0.68	-0.78	-1.31	-1.02	-1.04	-1.01	-0.82	-0.64	-0.66	-0.64	-0.88
Kurtosis	4.54	6.10	5.20	6.72	9.91	7.56	8.18	8.94	7.42	6.95	6.84	6.96	7.08
Realized Returns													
Last 10 Years	7.72	7.64	9.21	10.73	6.96	6.18	6.34	6.41	7.95	8.07	7.59	7.78	6.04
Last 5 Years	8.00	7.41	8.90	10.22	6.92	6.80	6.87	7.11	8.94	9.33	8.06	8.42	4.96
Last 3 Years	9.43	9.14	4.50	8.99	7.15	7.44	7.08	7.66	9.26	9.70	9.08	9.13	3.80
Last Year	15.05	14.66	-7.97	5.74	6.11	7.35	12.35	15.97	8.26	9.22	9.23	9.24	-5.39
Mean Portfolio Weights													
Stock	20.74	25.78	65.48	59.55	49.35	46.49	42.62	38.55	51.55	49.99	49.92	49.93	48.08
Bond	64.64	55.68	6.43	17.00	34.55	38.61	43.84	49.43	31.86	35.23	35.58	36.15	38.15
Gold	14.62	18.54	28.10	23.45	16.11	14.90	13.54	12.02	16.59	14.78	14.50	13.92	13.77
Standard Deviation of Portfolio Weights													
Stock	10.49	13.46	47.54	41.64	35.13	33.23	30.54	27.69	30.48	30.47	30.33	30.32	29.50
Bond	10.46	16.18	24.53	30.67	32.38	32.24	31.26	29.39	27.29	27.58	27.47	27.67	27.71
Gold	14.20	15.77	44.95	35.56	22.46	19.80	17.16	14.55	21.21	19.01	18.44	17.66	16.92

Table 18: Portfolio performance of PT portfolios in the US: aggressive scenario, $\hat{r} = r^0$, $\gamma = 0.9$.

The table reports statistics of a monthly reallocated optimal portfolio based on an optimization period of 36 months as well as the average and standard deviation of the optimal asset weights. The table also reports results for portfolios implied by mean-variance (MV), conditional value-at-risk (CVaR), risk neutral, and linear loss averse investors. The evaluation period covers January 1986 to December 2020. Statistics are calculated on the basis of monthly returns and then annualized assuming discrete compounding. The annual standard deviation is computed as $\sigma_{pa} = \sqrt{12}\sigma_{pm}$.