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Political Economy Models of Conflict

Francisco Moraiz
University of Surrey

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A mis padres Jesús y Aurora.

Abstract

We present a study of conflict from an economic perspective. We start by reviewing the approach to conflict in the economic sciences. We model conflict as a process of allocation of resources into two main technologies, production and appropriation. Then we complement this framework by allowing participants to negotiate. We introduce models of bargaining with complete and incomplete information.

We incorporate the cost of conflict and this ensures that negotiated settlements always produce a more efficient outcome. The possibility of conflict arises as a result of incomplete information, which takes the form of informational asymmetry about the cost of conflict. We find endogenous war equilibrium outcomes and compare the outcome of optimal resource equilibria with arbitrary non-equilibria allocations.

We also present some empirical evidence in the literature supporting the choice of utility models of conflict and present new results showing support for our propositions.

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Finally, I would like to say that the responsibility of this work lies only with the author. As sooner or later, all theories are proven to be wrong, I have no illusions that this will be no exception. In the meantime I will enjoy a pleasant mild sensation of achievement and relief.

Preface

This PhD thesis forms part of a series of studies carried out by a group of researchers, denominated the Arms Trade Group, whose objective is to provide a set of linked political economy models of the arms trade. These models can be used to organize and interpret the available evidence on the effects of military expenditure on security and welfare, and to improve policy formulations.

The economic repercussions of military expenditure have been the object of increasing attention by researchers in the last few years. Many of these studies highlight the benefits derived from lower defence spending and the conversion of the 'military industrial base' into civilian production. These effects have been grouped under a common denomination: the 'peace dividend'- a term that has gone beyond economic analysis and is widely use by media, politicians and activist, containing some general ideas about the beneficial effects of reducing the military budget on growth, security and welfare. At the same time there are increasing pressures to expand military expenditure and the arms trade, both at the supply and the demand side of the economy.

The supply side pressures presage that the trade could expand because arms markets and firms are experiencing a deep transformation. On the one hand, they need to sell abroad to compensate for declining budgets at home. On the other hand, firms are undertaking a process of rationalization, merging with other international firms and diversifying their production which makes it more difficult for national governments to control the arms trade.

On the demand side, we are facing a climate of uncertainty which makes it difficult to forecast the potential use of arms sales/purchases. The subject of arms transfers is at the hard-core of the international security agenda. However, there is a lack of conceptual structure which could provide some guidelines for policy makers. Since unilateral reductions of military expenditure could have serious security repercussions, it is necessary to provide some theoretical explanations of the driving force underlying the need for arms; that it to say, conflict.

This work tries to extend the understanding of the demand for arms beyond the explanations provided by the models of arms races. In these models, security is understood as a function of the proportion of arms that a country holds compared to its potential enemies.

We are going to try to understand the demand for arms by producing a scenario where military expenditure is directly related to conflict. For this, we will understand conflict as a special distributional mechanism that has many parallels with market exchange, but also fundamental differences.

In the first chapter we study the different definitions of conflict and produce an economic definition which will be consistent with our modelling choices. Since conflict is a very heterogeneous phenomenon, we try to restrict its definition to the activities where – without prejudice to other discipline – economic science can provided better explanations. These are mainly the choices amongst rational decision makers of the allocation of resources to productive and appropriative activities.

In the second chapter we review some of the models in the economic literature which are directly relevant to our study. These are classified into two main groups. The first category covers the models of rent-seeking activities and optimal allocation of resources. The second focuses on models of bargaining and negotiation. In the third chapter we present a model that merges both traditions. In this model, countries must allocate their initial resources between income production and military capability production. Once they have decided their respective optimal allocations, a process of bargaining takes place. Parties decide the share of income that they can claim according to their bargaining power. Bargaining power in this model is generated not only by military capability, but also by the cost of a hypothetical war and other features of the negotiating process.

In the fourth chapter we construct a model of conflict with incomplete information and perfect allocation of resources between productive and 'rent-seeking' activities. The process of distribution of resources can also take two forms: a conflict whose outcome is decided by the resources allocated to that activity, or a negotiated settlement. We incorporate the cost of conflict and this ensures that negotiated settlements always produce a more efficient outcome. The possibility of conflict arises because of incomplete information which takes the form of informational asymmetry about the cost of conflict. We find endogenous war equilibrium outcomes and compare the outcome of optimal resource equilibria with arbitrary non-equilibria allocations.

In the last chapter we support the previous ideas with empirical information. We review two well known paradigms in the empirical analysis of conflict. We find increasing support for the so called *utility models of war* but we do not find support for the *democratic liberal peace*. We follow the models and use the data sets of the reviewed papers. However, we introduce some changes to the dependent variable in these models. In particular, we only consider conflict if physical violence is used. Our model is based on, and interpreted in the light of the game theoretic analysis presented in earlier chapters.

Chapter 1

Defining Conflict

1.1 Introduction

In a broad sense conflict is defined as a state of opposition or hostility between incompatible ideas, desires, aspirations etc. Alternatively, it is defined as the distress resulting from this incompatibilities. Therefore, there are two fundamentally different concepts. One understands conflict as a cause and the other as a consequence. The difference is important in order to developed theoretical explanations. In this study, we understand conflict as a consequence and we will try to formulate the factors that produce it[1].

Before we define conflict from an economic perspective it is worth to note that this is an ambiguous concept that has been approached by many different disciplines and has been given many different interpretations.

Scientist have studied the biological drive to violence in human beings. A

[1]It is also important to say that this concept is influenced by western ideological background which is grounded in the reproduction of mutually exclusive oppositions. The Gandhian[1] concept of peace offers an alternative to the existence of conflict. Some religions for example, see the individual in harmony with their environment or universe. However in our western rational mentality, we see conflict as something inherent to our human condition. From the very moment that we are born we see our personal development as a conflictive process. Conflict may be deeply rooted in our language and cognitive ability. This can be problematic for its definition because 'conflict is the way we think'.

group of scholars under a UNESCO [2] initiative challenged a number of alleged biological findings that have been used to justify violence and war. They alleged that the misuse of scientific theories and data to justify violence and war is not new, but has happened since the advent of modern science. For example, the theory of evolution has been used to justify not only war, but also genocide, colonialism, and suppression of the weak.

Peace studies refuse a narrow definition of conflict because it would perhaps imply a narrow definition of peace. One of the main differences is based on their approach to violence. Peace studies refuse the definition violence as a somatic incapacitation or deprivation of health. The most important of all dimensions of violence for Johan Galtung [3] is who is the subject that perpetrates it. According to Galtung there are two different kinds of violence: personal or direct, when it is carried out by a person, and structural, when nobody is directly committing it.

Based on that, he points at six important dimensions of conflict. The first dimension is the difference between what is *potential* and what is *actual* and what impedes this distance from decreasing.

> ... if a person died from tuberculosis in the eighteenth century it would be hard to conceive of this as violence since it might have been quite unavoidable, but if he dies from it today, despite all the medical resources in the world, the violence is present according to our definition

The great difference of this approach with others, is that conflict can take place without specific agents (subjects) and without specific actions. Conflict is secondary to Peace Studies and the absence of violence is represented in a broad manner. It depends in 'what is' and 'what could be'.

Political Science has studied conflict more extensively than any other discipline. Ted Gurr [4] in his study of political violence defines it as *all collective attacks within a political community against the political regime, its actors – in-*

cluding competing political groups as well as incumbents – or its policies. For Gurr conflict is an action which involves force or the threat of force independently of its actors or their goals.

A great part of political theory has been concerned with the study of power. A variety of explanations have been put forth. Many of those theories focus on major conflicts in general and war in particular. Bremer [5] groups those theories in three main families according to what they think is the basic source of conflict (e.g. the concentration of power, power parity or balance of power).

Conflict is also a common subject in the field of social science. A great part of sociological research has been concerned with this theme. It also covers the political struggles for power as well as other expressions of social conflict, such us revolutions.

Social Scientists define conflict as a process of resolution of incompatibilities. According to Simmel [6, 7] conflict is "a way of achieving some kind of unity, even if it will be through the annihilation of one of the conflicting parties". Another common definition of conflict refers to the scarcity of resources.

Giner [8] defines conflict as:

> a hostile contest for or against domination, control, and self-preservation. Domination may be sought over goods, values, thought or behaviour. It may involve the annihilation of one collectivity or the harmless struggle for first place in a competition where the losers lose nothing or very little. More often, it is aimed at the re-arrangement of the relationships and hierarchies of power and authority in a given social structure.

1.2 Economic definitions of conflict

This thesis is not concerned with analysing the origins of conflict by understanding the sources of incompatibilities. The aim is to isolate some micro-foundations of conflict and look at the strategic behaviour of parties engaged in conflict. Our concern is mainly related to one problem: The process of allocation of resources to conflict. For that, despite the complexity of the subject, we will isolate a given dispute and consequently analyse different conflict–cooperation strategies of rational players. We do not take other problems for granted or try to avoid them. However, at this stage, we think is better to concentrate on understanding the micro-level. Therefore we will provide a general definition and then, propose a specific definition which relates directly to economic issues.

1.2.1 Conflict as a exchange process

We know of all the complications that may be introduced trying to define actors, goals, actions, and different structures. However, all these previous definitions have something in common: the understanding of conflict as a process of resolving a given incompatibility. Whenever two hypothetical actors fail to internalize the distribution of scarce resources, they will allocate efforts to impose a given share. Conflict is a process that takes place to restore equilibrium when both actors disagree between the balance of forces and the shares. This is our starting point.

Sometimes the absence of confrontation seems to be equivalent to the absence of the dispute. This is not true, peace research show us how in many cases a peaceful situation is based upon a violent imposition. We may see a great variety of conflicts and many sources of disputes that lead to different formations of agents, historic subjects, and many different solutions, including the extermination (by an agent's action) or extinction (by structural conditions) of one party.

A more specific definition was used by Peter Wallensteen [9]. He defines conflict as a social situation in which a minimum of two parties strive at the same moment in time to acquire the same set of scarce resources. This confrontation is produced because different agents, given some differences (in all the broad manifestations of culture) cannot internalise the scarcity of resources. This scarcity is the matter of our disputes. There is confrontation when one or more parties dedicate their energies to imposing a settlement to a dispute that cannot be solved by agreement.

At the same time, other activities may lead to the expansion of these resources. Isard [10] defines four main simultaneous activities basic to the understanding of conflict: competitive, curious and inventive, co-operative and self-propagating activities. Economics should deal with the efficient allocation of resources in all these activities.

Since scarcity is the driving force behind conflict it is not surprising that most part of the theories of conflict have a lot to do with economics. However, this scarcity of resources can be expressed in a very wide range of incompatibilities. Therefore, isolating economic conflicts can be a difficult task. For example, conflict can be built into cultural incompatibilities. Some typical conflicts in the history of humanity are the multiple religion wars.

John Rex [11] formulates a paradigm of conflict that has a strong relation to economic theory. The whole point about exchange theory which is essentially an attempt to base sociological theory on the theory of economics is that, although it starts by assuming actors who have conflicting goals, it posits the possibility of a point being reached at which each individual pursues his own ends in a way which is actually beneficial to the other. Although it starts with the potentiality of conflict it ends with what is in effect a process of co-operation.

Conflict theory approaches this problem from the opposite side. It starts with what appears to be a mutually beneficial exchange, but discovers within this relation elements of compulsion and exploitation which appear as normative only because the oppressed and the exploited do not have the power to resist them. It then focuses on the power underlying the relationship and considers the consequences of a change in the balance of power.

Another element further distinguishes conflict theory from exchange theory when the latter is extended to constitute a theory of market behaviour. The peacefulness of the market rests upon the use of the sanction of 'going to another supplier'. This peaceful process, however, is seen to break down in economic theory, and the theory of the free market has to be supplemented by a theory of oligopoly and monopoly. However, what is an embarrassment for exchange theory and economic theory is the essence of the matter for conflict theory. Market sanctions (i.e. the sanction of 'going to another supplier') are used to regulate particular transactions only because there is the possibility of resort to the more fundamental political conflict if there is monopolistic competition on both sides of the market. They represent a convenient way of solving particular problems within a fundamentally political order which rests upon a balance of power.

Jack Hirshleifer [12] defines conflict as an extension of competitive behaviour. It takes place when instead of using the available resources for productive or consumptive purposes, contenders try to hamper, disable or destroy rivals.

According to consumer's choice theory, every individual maximises her subjective status subject to the resources available to her and to her rivals. Therefore, there must be a mechanism to allocate resources optimally to those two activities. It implies two different technologies, one for production and other for destruction and two different main activities, exchange and struggle.

When we look at conflict from an economic perspective, we should take into account the other factors that affect it and assume that they are incorporated in either the environment variables, or the strategic ones.

For our studies we will concentrate on the question of distribution of resources, in colloquial language known as 'sharing the cake'. Then, we can also look at several disputes that can be related to that question, such as crime, litigation, rent-seeking activities in oligopolistic markets, industrial actions, warfare and revolutions.

In all those disputes conflict may take place in two different levels. At one level, the existence of a dispute diverts resources from production to rent-seeking activities, which might produce a higher individual outcome, given that there is no reaction from the other agents. In any case, this is an inefficient outcome compare to the social optimal, when all the resources are dedicated to production and consumption. In many of the disputes that I have mentioned, there is a regulator that could influence agents behaviour to achieve this optimal point. However, we will concentrate in warfare and similar situations where there is no social planner, and therefore we need to derive the conditions for an efficient outcome to take place. This will be always related to a given dispute.

We will focus on the study of two basic problems. First, given some scarce resources and socio-political differences, we will find what is the optimal allocation of those resources between productive and fighting activities. The second is to establish under which circumstances both parties manage to negotiate a peaceful outcome and when they decide to make war.

Following Hirshleifer [12], failure to agree does not necessarily imply actual fighting but it is certainly a precondition. The main determinants of this failure are the preferences of the players, the existence of opportunities to make gains either from co-operation or conflict and perceptions of the outcome in case of conflict.

The relevance of information as a source of conflict is being highlighted by game theory models. If we accept the classic assumptions made by game theory about agents behaviour, we may come to the conclusion that asymmetric information is a necessary and sufficient condition for the existence of conflict.

The assumption of rationality is a main element of game theory. Individuals are said to act rationally when they attempt to maximise their utility given some constrains and some information set. The other important assumption is that individuals know that the rest of players in a game are also rational.

Once we have established that individuals act rationally, we will establish why information is the basic source of conflict. We are going to present a model that distinguishes between two different aspects: optimal allocation of resources and sources of conflict.

In social sciences there is a trend to analyse the multi level properties of conflict. In many cases we can see that warfare is influenced by internal struggles for power. Its source is found in the social question that is also expressed in other disputes, such as high level of criminal activity or high levels of industrial action. If we have a basic understanding of the micro-foundations of conflict, it would be useful to understand the links between other levels of conflict and the relation between disputes.

In economic terms we could classify conflict as a process of allocation of resources that works alongside, against or in spite of the market. We could classify both conflicts and markets according to the level of efficiency that they bring about. With perfect information we see that we have the most efficient process of allocation of resources. This is a perfectly competitive market. In second place we have monopolistic markets, and incomplete markets.

Following this line of thinking, we could go on and say that we can also have disputed allocations, where agents engage in non-productive activities. They spend their resources in coercion or appropriation technologies to force a non-agreed outcome and finally conflict where, on top of the allocation of resources to unproductive activities, we have an extra cost produced by fighting. Conflict is an exchange mechanism which does not require agreement and is also characterized by high transaction costs.

1.3 Other considerations

In our definition we abstract from many important features that can be found in any process of conflict. We do not take into account the complex social and psychological determinants of the initial dispute such us cultural factors or the legitimation processes.

Since we are concentrating in the micro-foundations of conflict, we will postpone the study of other factors that may also be important for a better understanding of economics and conflict. In the meantime, the general question of resources is at the heart of our approach. While conflict is an activity that looks at the distribution of resources, other activities lead to the expansion of them. A clear example is technological progress which can be a double-edged sword. Technology can expand one resource while at the same time undermining another. But in general the outcome is believed to be positive.

We will also consider rational decision processes, given some disputes based in some particular determinants, such us different system values of different actors, and also given a basic source of incompatibility. By "rationality" we mean utility maximizing agents. In many cases, we will simplify this concept and say that utility depends solely on income. In others we will extend the utility function to account for different preference sets. But in general, we are analysing rational decision-making processes.

The structural complexity of conflict presents a few problems for the empirical analysis. When we are analysing a given conflict we have to be sure that the contenders are not involved in any other dispute that may influence their behaviour. In the real world aseptic conflicts happen rarely. Most conflicts take place in complicated networks or nested disputes. International Conflict suffers from the kind of complications that can be found at the internal level of conflict, exponentially increased by the number of countries that are directly or indirectly affected.

I just mentioned that there are several analytical choices in the study of conflict. We also have two basic sources of incompatibilities. One is the existence of scarce resources and the other is the presence of cultural differences (in the broad sense of the term) that do not allow for a process of internalisation of scarcity. We will assume those mechanisms –such as cultural reproduction and assimilation– to be constant over time or not to affect the nature of our analysis. One of the key assumptions about our model is that agents are stable and make rational choices.

The existence of incompatibilities is a necessary condition but not sufficient to explain conflict. There are many theories that explain conflict according to different classifications of it. It can be approached from many different methodologies and every one of them offers some understanding of it. However, no discipline can claim the production of some "General Theory of Conflict". We do not claim that we can give a complete explanation of conflict from the discipline of economics; however we can offer some understanding of the 'economic dimensions of the problem'. We can understand the problem of optimal allocation of resources and derive the micro-foundations of conflict.

Chapter 2

Foundations of Conflict

2.1 Introduction

In the economic literature conflict is explained in different ways. Different branches of economics (Public Choice, Game theory, Trade, etc.) are coming together to produce what we could call an *embryonic* theory of economics and conflict. We identify two main activities.[1] One is the allocation of resources to productive and appropriation activities and the other is the negotiation mechanisms that are usually present in most conflicts. These two activities may take place at the same time, but have been considered to be independent. Therefore, in some cases, researchers have concentrated on one or the other. Only in the last few years, are scholars producing models that integrate these two problems.

Conflict also takes place at different levels, involving two strategic actors and many non-strategic actors.[2] However, for simplicity, models normally consider only the first scenario. We may also focus on a once for all conflict; or we can have a conflict in stages. There are many other variants which will be explained

[1] A question of great importance is the internalisation of scarcity which we assume to be explained by other factors such as cultural incompatibilities, etc.

[2] A non-strategic actor is one that can influence the conflict but cannot make a move. For instance a producer is a non strategic actor in the perfect competition market and a strategic one in a monopoly (only with respect of setting prices).

when we look at the principles of conflict theory in section 2.2.

Since participants must decide whether they engage in conflict activities or not, the decision to fight as well as the production of fighting capabilities must be endogenous. In relation to endogenous conflicts, we will review the rent-seeking literature in section 2.3. With respect to the negotiations that take place to avoid it, we will look mainly at game theory and bargaining models in section 2.4.

The models of rent-seeking activities are based on the concept of optimal allocations between production and fighting activities. Most part of these models have been developed within the branch of Public Policy and Political Economy. We also understood negotiation in its broadest sense. It normally entails some kind of transfer from one participant to another in order to avoid the cost of conflict at the level of hostilities.[3]

In this chapter we are going to review these two paradigms because –in combination– they can produce endogenous conflicts. In some way, we can say that they provide the most simple way of explaining conflict without resorting to the use of exogenous factors –such us malevolent preferences or restricted bargaining options[4].

2.1.1 Other considerations

Evolutionary game theory has been applied in the social and natural sciences to the study of conflict. It has provided interesting applications in the study of biological evolution, animal behaviour, and human behaviour in both primitive and advanced societies.

In a seminal paper Maynard Smith [13]proposed the concept of *Evolutionary*

[3]This is particularly relevant to the relation between trade and war; or other forms of transactions that although apparently peaceful may take place under the shadow of conflict.

[4]Malevolent preferences only means that we may get positive utility out of conflict because we enjoy loses to others as well as we enjoy our gains. Restricted bargaining options are very common in real life. It means that we may not be able to divide or make any small transfer to compensate other parties for the value of the disputed resource.

Stable Strategy ESS. Among the issues addressed by these theories are altruism and selfishness, the dynamics and resolution of conflicts and bargaining processes and the emergence of conventions and social norms.

It is possible to argue that the study of conflict should start at this point, looking at the guiding principles for actors in the light of their evolutionary properties. It requires a shift of focus from the micro-level explanations of conflict –allocation of resources and negotiating processes– to the study the process of generation of incompatibilities and social agents that face the basic problem of acquiring scarce resources.

This constitutes a different paradigm in the political study of international conflicts applied to the study of systemic interactions and theories of hegemony, alliance formation, power struggles, theories of imperialism, etc. However, as we have mentioned in the previous chapter, we will concentrate on dyadic interactions of countries, abstracting from these systemic factors. These dyadic interactions are assumed to be in a evolutionary stable environment. Extending the analysis of the micro-level to account for possible evolutionary influences goes beyond the scope of these thesis. We have therefore, decided to focus in a narrow definition of conflict where economic models can offer some added value to the already vast number of theories of conflict.

2.2 The Strategy of Conflict

Conflict analysts have approached this subject from many different perspectives. We have already defined conflict as a process of solving an incompatibility. Due to the presence of scarce resources, conflict is part of the competition process that takes place in order to acquire them. Conflicts cannot be completely resolved as long as we do not find the way of dealing with the intrinsic scarcity of resources that human beings face in every given environment.

One of the options is to develop more efficient ways of using this scarce resource. For instance, the increase of the price of oil during the early seventies produced a crisis in the industrialized countries. As a result of that, there was a development of new technologies that produce and use energy much more efficiently.

Since conflict is a process of resolving a given incompatibility, in many cases we cannot contemplate reducing it by this sort of supply side long term intervention. For this to happen, we need to have systemic stability. Conflict is a crisis that reestablishes this systemic stability. We understand conflict as a consequence, not as a cause.

There are two main different ways of dealing with incompatibilities in the short run. One is an agreement of all parts involved, the other is by the use of force. This use of force produces a high cost. Conflict analysis would try to promote crisis management by common agreement because this approach is Pareto-superior to any fighting outcome.

We can build paradigms along this line for nearly every known conflict in the world. Conflict models has been develop in Zoology which model fighting in the animal world, both inter and intra-species and for different sorts of scarcity, mainly for food resources and for the right to mate.[5] Trade unions and employers

[5]Animal case is a special case in conflict analysis because conflict as understood by humans needs complicated organization and communication processes that animal lack. However, those

fight for wages. People spend valuable resources in litigation processes over the greatest variety of subjects. Firms engage in a huge variety of rent-seeking activities such as aggressive price policies, advertising campaigns and other measures well known specially in oligopolistic markets.

Since the microeconomic foundations of conflict are at an early stage of development, we can establish a initial parallel regarding the difficulties of the application of the mathematical methods and the necessary limitation of objectives that were laid by the initial researchers in game theory.

Von Neumann and Morgenstern [14] vindicated the mathematical "games of strategy" against some of its usual criticisms. Most of their assertions can be easily applied to our studies in conflict theory.

First, there isn't a general theory of conflict. This gives room, to some degree, for speculation about the validity and applicability of its conclusions. However, this is a question that concerns the methodology of science that could be well applied to most hard-core disciplines. Human thought has been dominated by theories that claim a universal understanding until they were replaced by more advanced alternatives. In trying to established a universal core from the beginning we would probable impede further development in this new field. The nature of human science requires in many cases a more descriptive method. In these cases we have no doubt that a careful compilation, classification and description of the facts around of conflict can be more useful, but in other cases mathematics can be applied with great success.

There is no fundamental reason why mathematics shouldn't be applied to conflict theory. Von-Neumann and Morgenstern said something similar about game theory:

> The arguments often heard that because of the human element, of the psychological factors, etc., or because the is – allegedly – no measure-

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models give a very good insight into the evolutionary dynamics of conflict.

> ment of important factors, mathematics will find no application, can all be dismissed as utterly mistaken. Almost all this objections have been made, or might have been made many centuries ago in fields where mathematics is now the main instrument of analysis...

We can also find some parallels with the early game theory when we try to understand why mathematical techniques have not been applied with more success to the study of conflict. One of them is that it is very difficult to use these methods when there is no clarity of concepts. The very definition of conflict is ambiguous and confusing as we have seen in the previous sections; there are many definitions some of them quite contradictory. Nevertheless, there have been huge advances in conflict resolution which uses techniques of analysis often derived from game theory principles.

Anatole Rapoport [15] writes about the very same question:

> Much of the controversy about the possibility of creating a mathematical social science centres on the issue of "determinism" vs. "free will" and on the applicability of mathematical models to phenomena as complex as those involving human behaviour. Indeed, both deterministic causality, which seems to govern the behaviour of non living matter, and the simplicity of the systems singled out for study in classical mathematized physical science constitute the foundations of that science. Determinism in human affairs remains a metaphysical assumption without convincing evidence. The indefinitely large number of variables that perforce enter the description of any system with human components precludes the use of tractable mathematical models in deriving the behaviour of such systems.

Rapoport argues that the fact that determinism cannot be proved, the absence of it neither can be. At the same time, the assumption of determinism is not necessary since, for instance, probabilistic theories can be as logically compelling as deterministic ones. And the complexity involving human systems is a matter of degree although analogies with physical science cannot be established invoking only differences of degree of complexity with human systems. In human science there are no analogous to physical "laws" governing the systems.

Therefore so far, all mathematical models give some general ideas about the behaviour of different agents, or better, given some environment, what would one expect to be the outcome of a conflict situation if its agents were rational players or rational decision making units.

As the key variables get identified and precisely defined, data collected, computer software developed[6] and other factor necessary for quantitative analysis improved, we will get more and more accurate predictions.

Our project tries to bring together a set of theories and models that help us to understand the process of allocation of scarce resources among two main alternative uses, production and conflict technology. This resulting allocations would in term produce some equilibrium of forces consistent with a peaceful or a fighting outcome. This generates some concepts and relationships that should help us to understand and provide policy advice in conflict resolution, international relations, labour economics and many other processes.

We think about conflict theory as a part of economics that deals with the microeconomic foundations of issues such as arms trade and arms races, military expenditure, defence industries, influence of military expenditure in GDP growth and other macro-economic variables. It shares many features with microeconomic theory such us the individual decision taking as the unit and method of analysis or the concept of rationality.[7]

Rationality is an strong assumption, specially when we refer to conflict. This

[6]Since it is likely unprovable that conflict experiments will be performed, computer simulation is one of the key tools for the development of mathematical conflict theories

[7]See for example Gravelle and Rees [16].

concept is used in the strict economic sense. That is:

1. The decision-maker sets out all the feasible alternatives rejecting anyone that is not feasible.

2. He takes into account whatever information is readily available, or worth collecting, to assess the consequence of choosing each of the alternatives.

3. In the light of their consequences he ranks the alternatives in order of preference where this ordering satisfies certain assumptions of completeness and consistency.

4. He chooses the alternative highest in this ordering. In other words, he chooses the alternative that he prefers the most and no other one.

All these points are consistent with the common meaning of rationality. The controversial part is point 2. Gathering all the relevant information is sometimes very costly and therefore the notion of rationality must take into account these costs. Many actions may seem at first sight irrational until we analyse the informational problems underneath or the costs of information-gathering. Information plays a crucial part in conflict. It may affect environment variables such as the knowledge of previous plays, it might affect the knowledge of available strategies, the payoffs resulting from playing them and other crucial factors that would influence the final outcome. Information plays a crucial part in the strategy of conflict, specially in understanding the rationality of war or any other kind of costly confrontation.

2.2.1 General principles of conflict analysis

We are going to introduce next a general framework for conflict analysis. For this, we will review the methodology established by Isard and Smith [17]. This study compiled a great variety of techniques and models that can be summarised in some general principles. The well-versed reader will find many similarities with the principles of game theory and other mathematical methods in social science research.

The basic concepts defined by this analysis are very similar to the elements of game theory, that is: actions and joint actions, states of the environment, outcomes, preferences and utility, and extent and nature of available information. After the basic elements, we should establish in which forms this elements can be found in the real world producing a conflict situation.

Isard and Smith [17] found two main classifications. Conflicts can be classified according to two main factors. First whether there are a small number of actions or not and second whether participants can only rank outcomes or they have a numeric value for the outcome or a payoff.

Basic elements

In conflict there might be a wide range of different actions, for instance war is a very complex social phenomenon that involves highly organized chains of actions by the participants. The whole set of actions is called, in game theory terms, the pure strategy space S_i for player i.

The other factors that determine the outcome of the game are collected in what is called the environment of the game. Here we collect all the factors that affect the final outcome, but are not under control of the participants.

The combination of actions and environment produces an outcome which can also be simple or complex. For example, war produces a complex outcome that can be evaluated in terms of the number of human and material losses and the gains in political controlled territory, political advantages etc.

Participants are assumed to have some preferences over all possible outcomes. These preferences are described by using a utility function that summarizes all the preference rankings for a given participant. The utility function can be *car-*

dinal, ordinal or *relative.* Cardinal utility functions indicate how much utility participants get from each outcome. If the information provided by the utility function only tells which outcome is preferred with respect to the other outcomes, we represent preferences by an ordinal utility function and finally when the information also includes which outcome is preferred by a certain fixed amount, percentage or ratio, it is a relative utility function.[8]

Once the outcomes are known, we need some guiding principles. These principles imply that all participants are aware of the behaviour considerations of the other players. In economics, the behaviour principle is utility maximisation. This is normally the principle that every player uses in order to choose actions. This might be far too general in some conflict cases. In conflict resolution a more specific principle might be needed. Utility maximisation might be difficult to put into practice in some complex situations and some general guidelines, such us moral codes, cultural practices, standards of fairness etc., are used in its place

Those principles relate to some objectives embodied in the concept of 'optimal state of affairs' considered attainable, which will take one of those two forms; To maximize or to minimize the level of some index reflecting the desirable properties of different sets of possible outcomes.

Finally, information is a key variable in conflict. Rationality of players, the strategies available and the payoffs depend on the perception of participants. Isard and Smith define the many possible states of information:

> ...there is a definition of a statement of knowledge, the distinction between objective and subjective knowledge, the consideration of the degree of belief associated with any piece of knowledge, the distinction of being fully informed of a piece of knowledge and being positive of

[8]The literature in economics also has studied different forms of utility functions. In the study of conflict it might be highly relevant to use reference dependent utility functions. This might deal with the fact that many times participants face some minimum constraints for survival, or there exist some political and cultural influences that might affect their behaviour in some or another way, depending on an initial reference point. See Samuelson and Zeckhauser [18]

> that knowledge, the distinction between knowing a relationship and assuming it, the distinction between certain, probabilistic and uncertain knowledge of the stage of the environment and outcomes, and recognition of the need to specify the different amounts and kinds of knowledge that participants have regarding other participant's preferences and perceptions.

On the issue of information, it is important to explain the idea of 'common knowledge'. Knowledge is what everybody knows about the environment and the payoffs, whereas common knowledge means that everybody knows that everybody knows it. The assumption of common knowledge has strong implications for the equilibrium strategies.[9] Information structures can give place to very different outcomes and this is a question that hasn't been properly explored in conflict theory.

Conflict with small number of actions and ordered preferences

This is a simple assumption that can be applied to real conflict situations. In terms of game-theory, the models that follow from this assumption could be classified as static models of complete information. We have argued in our introduction that under perfect information there is no possibility of conflict. However, the number of actions are limited. Given this limitation, the process might result in a fighting equilibrium. This limitation is exogenous, and therefore, these models can be only used in an ad hoc manner for any given situation. Although they provide a good tool for conflict management, they do not offer a positive explanation of it and it is difficult to make generalisations.

In the second place, for the moment, we are focusing only on static models. This is a contradiction with our definition of conflict as a process of in-

[9]For a survey of implication of common knowledge see the review of Brandenburger and Dekel [19].

compatibility resolution. We must make an extra effort, collecting all dynamic factors in the exogenous variables represented by the environment. Robert Axelrod [20] has already shown the importance of the dynamic factors in shaping the equilibrium outcome in tournaments and games such us the iterated Prisoner's Dilemma [21, 22].

In the next table we can find an example of the Prisoner's Dilemma and how game theory can help in conflict management.

In this example C1 and C2 represent the governments of two countries. C1 is the government of an hegemonic country that has economic interests all around the world. C2 is the government of a developing nation. Both governments are rational and follow the strategy that maximizes their own utility represented by the payoff matrix of Table 2.1.

The strategies available to the local government are 3.

- To take a blind eye to the activities of the hegemonic power in his territory.
- To collaborate with him in the extraction of economic-rent from the exploitation of natural resources and labour
- To fight against foreign control in order to achieve a higher degree of economic and political emancipation

In turn, the hegemonic power has also three different alternatives.

- To use a 'fair' policy in his economic relations with the developing country.
- To use sanctions in order to force the local government to collaborate with his economic policies.
- To engage in military action to remove the government and replace it with a more favourable one.

Table 2.1: The War Game 1

C1 , C2	Blind Eye	People's oppression	War of Liberation
Fair Policy	10 , 10	0 , 20	–10 , 40
Sanctions	20 , 5	30 , –5	–5 , 20
War of Domination	30 , 0	40 , –10	5 , 5

To find the outcome of the game we proceed to the elimination of dominated strategies.[10] The equilibrium is a war outcome with payoffs of $(5, 5)$ (Bottom right cell).

This is an example of how war might take place in an environment of complete information. The war outcome of this game is an ad hoc result. The war scenario takes place due to the fact that both governments have restricted actions and outcomes.

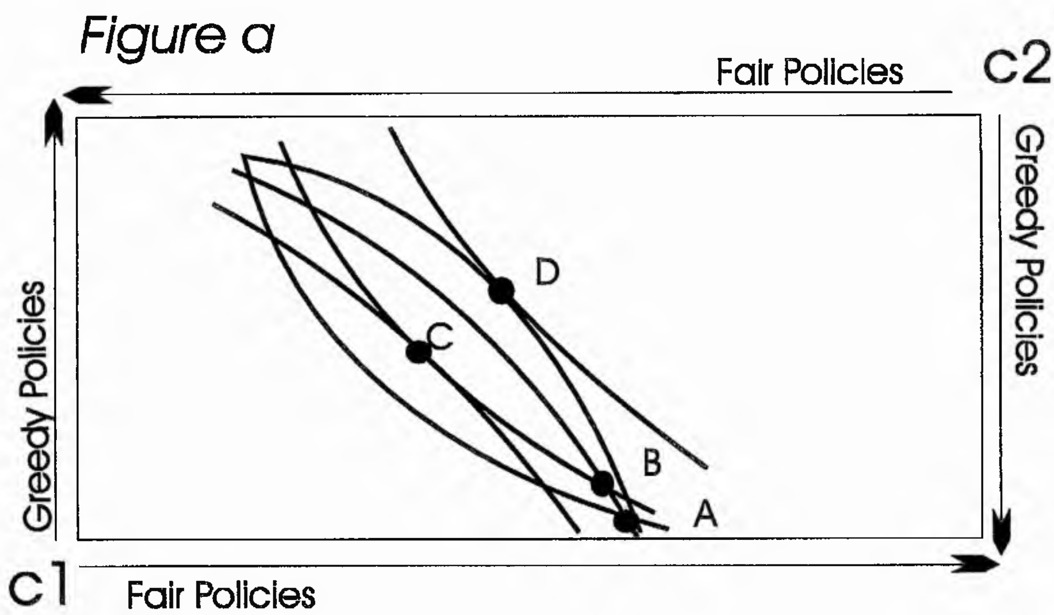
Continuous action spaces

In the previous case, both participants had three options only. We can give them an continuous number of options. In that case C1 might choose a completely fair policy to a completely greedy one whereas C2 might decided from fighting back or help C1 with absolute control of its population.

For those who are not familiar with exchange theory, we proceed to draw an 'Edgeworth box' representing all the efficient allocations for both countries' initial endowments. Each point inside the box in figure 2.1(a) represents a unique outcome for every joint action. Preferences over those outcomes can be represented by a map of indifference curves represented in the graph by the continuous concave lines for C1 and the convex lines for C2.

For instance point A in this figure gives the same utility for C2 as point D because it is on the same indifference curve. Starting at point A, any policy choice that is outside the area delimited by the thick lines would make any of the

[10]A good introduction to statics games of completed information is Gibbons [23]



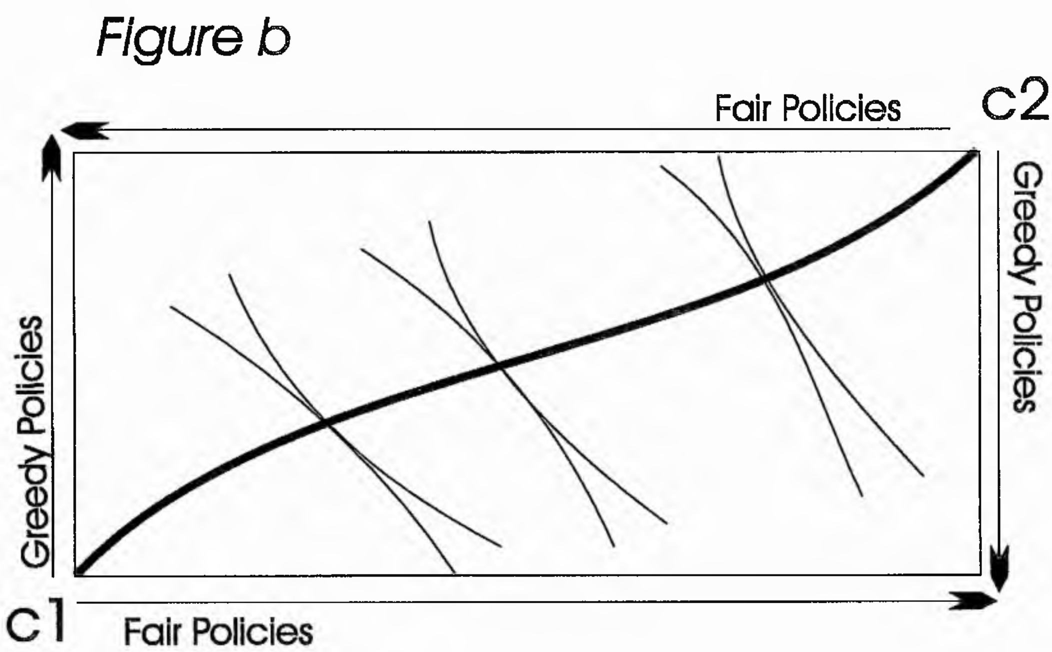


Figure 2.1: The Edgeworth box and efficiency in conflict

two participants worse off, and wouldn't generate an agreement.

The move from A to B would be mutually beneficial but is not an efficient allocation. Only points of tangency between indifference curves can be considered efficient outcomes. For instance a point such as C provides the same utility for C1 and a higher utility for C2 because it is on a higher indifference curve.

As you can see point D is also an efficient allocation.[11] In fact, there can be many efficient allocations, In figure 2.1(b) we can see the locus containing all allocations for which both participants indifference curves are tangent. Each point in this curve is efficient because one cannot make a party better off without making the other one worse off.

Formally the Bargaining Set X is said to be *Pareto-efficient* if there is no other outcome y in X that all the players like at least as much as x and some players like more that x

$$\forall y; y > x \Rightarrow y \notin X$$

This notion of efficiency should not be confused with what is socially desirable. As Binmore [24] explains:

> Sometimes a Pareto-efficient point is said to be *Pareto-optimal*, but this is an unfortunate piece of terminology since it suggests that a Pareto-efficient point cannot be improved on. But it is Pareto-efficient for a mother to give all cookies in her cooky jar to one of her children, leaving the others with nothing. No child can then have its situation improved without making another one worse off. However, nobody would wish to claim that the mother's decision is necessarily socially optimal. [12]

[11]Isard and Smith [17] provide a thorough review of different management procedures to attaining different efficient outcomes according to some general guiding principles.

[12]Some of the techniques in conflict resolution are based in this concept of Pareto-efficiency.

Biology would only consider those strategies that allow for the survival of the species, mainly concerned by a process of adjustment to a changing environment. A priest would only consider strategies that abide by the principles of his religion. Other people would like to eliminate strategies that impose a great damage in one of the participants or produce great inequalities. Since the scope of our study is restricted to the economics of conflict, we focus on Pareto improvements as the guiding principles of our theories. This implies not only the definition of the feasible bargaining set in the manner explained above, but also the definition of individual rationality, common knowledge and the information set in accordance with the rest of economic theories.

Identifying the bargaining set is quite useful for the evaluation of conflict resolution strategies. It says little about the micro-economic foundations of conflict. At this stage, we depart from the standard analysis in conflict management.

Our economic analysis should focus on two main issues. First, a theory of conflict should endogenise the bargaining set. Many conflict theories assume that the set is exogenously determined. However, economists are concerned with optimal allocation of resources. In the case of conflict, participants can allocate their respective resources into two main activities. One leads to the generation of more resources. Investment and trade are typically activities that lead to a more efficient use of some limited resources. The other activity leads to their appropriation. Military expenditure, and wars bring about a higher share of the initial scarce resources for one participant, but reduce the overall use.

Second, we analyse at which conditions exchange takes place by agreement and/or conflict. In situations with perfect information, markets are the most efficient mechanism providing the best allocation. In many circumstances markets

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Groom [25] produced a conceptual review of different principles and paradigms from which the ones classified under the category of Peace Research could be the most critical with the concept of Pareto-efficiency. In many occasions they suggests solutions that would make a party worse-off in support of another party. This doesn't contradict the fact that when Pareto-efficient solutions are found, they should be implemented fail and exchange takes place under the shadow of conflict. In extreme cases, agreement breaks down and the allocation takes place by fighting. This achieves equilibrium without agreement but is very costly.

The microeconomic foundations of conflict should establish some general theories applicable to situations such as wars, strikes, litigation price and trade wars in oligopolistic markets, etc. These theories should be consistent with a process of optimal allocation of resources in conjunction with cooperative and one-side advantage activities. They should take into account that cooperation is in many case Pareto superior but requires agreement. Therefore, some mechanism of negotiation should be incorporated into the previous framework.

Regarding the study of the allocation of resources into productive or fighting activities, we may turn to the study of rent seeking models. There is a great variety of them, specially in the literature of public goods. With respect to the negotiations on how to split the pie, the literature in game theory has already produced extensive research from the Nash equilibrium to complicated equilibrium concepts in dynamic models of imperfect information.

In the next two sections we will be looking both at theoretical research and practical applications of rent-seeking activities, game theory and bargaining and other models that address different aspects of conflict. This will provide the theoretical background for developing our models of conflict in chapters 3 and 4.

2.3 Models of rent-seeking activities

There are several approaches in the recent economic literature dealing with different aspects of conflict – understanding it as those activities that go beyond what is normally described as competition activities. In some cases, participants use the available resources to damage or destroy other participants. But there is a whole range of activities that rather than seeking other participants' destruction, are concerned with the gain of economic rents. These activities, compared to competition activities, are directly unproductive. There are many models of rent seeking activities in the literature. In the last few years, we have seen a convergence between models of conflict and rent-seeking. As research advances, some common features can be identified in these models that may eventually lead to a formulation of a microeconomic theory of conflict.

The economic models of profit-seeking and rent-seeking have concentrated largely in the theoretical analysis of lobbying for protection, import licences and tariffs, Johnson [26], Bhagwati and Hansen [27]. The term *rent-seeking* is attributed to Krueger [28]:

> In many market–oriented economies, government restrictions upon economic activity are pervasive facts of life. These restrictions give rise to rents of a variety of forms, and people often compete for the rents.

These activities were reduced to a subset of what Bhagwati [29] calls *directly unproductive profit-seeking activities (DUP)*. Krueger paper is concerned with the lobbying activities which are triggered by different licensing practices of governments. Some different licensing mechanisms can lead to different lobbying activities. At the same time, her paper studies the welfare implications of these activities in a model where three basic regimes are compared: Free Trade, Tariff or import restrictions without Rent-seeking, and Import Restrictions with Rent-seeking. The introduction of restrictions produces welfare losses since entrepreneurs dedicate resources to gain the economic rent produced by those restrictions. Prevention of that loss can only be achieved by *restricting entry into the activity for which a rent has been created*, which can have many political implications.

Bhagwati [29] makes a taxonomy of rent-seeking activities which are directly related to governmental policies. He classifies these activities into four main categories according to the distortional effect that they entail. These activities can be of a legal or illegal nature and can be grouped according to their analytical similarities.

There are four main categories:

a) The initial and final situations are both distorted.

b) The initial situation is distorted but the final situations is not, as a result of DUP activity.

c) The initial situation is distortion free, but the final situation is distorted and

d) Both the initial and final situations are distortion free.

An example of a legal activity in the first category is premium-seeking which was studied by Krueger [28]. Illegal activities such as tariff evasion and smuggling are also part of these group, Bhagwati and Srinivasan [30], Pitt [31]. In the second category we can mention examples of tariff-destroying lobbying. Two typical activities in the third group are Monopoly seeking and tariff seeking. In the final category we can mention examples such us zero-tariff lobbying and theft.

We can summarize the results of his paper by looking at the consequences of these activities. Categories (a) and (b) produce a beneficial outcome and the activities in groups c and d produce a negative one. The critical difference is that the activities in categories (a) and (b) have initial situations that are distorted, whereas categories (c) and (d) depart from an initial situation of free distortion. The point here is that the diversion of resources from directly productive to directly unproductive activities, when undertaken in a context of initially distorted situations, is fundamentally different from the situations where diversion occurs in distortion-free initial situations.

The classic examples of rent-seeking activities with negative outcomes are monopoly and tariff seeking. In both cases the welfare cost imposed by those activities can be decomposed into two main negative effects.

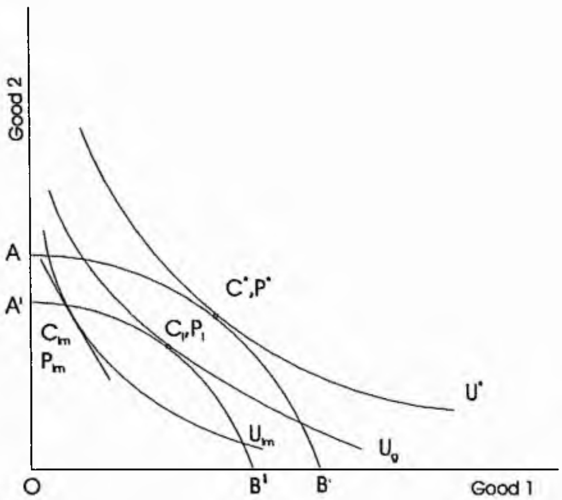
a) the withdrawal of resources into unproductive activities and

b) the welfare effect of imposition of the distortion.

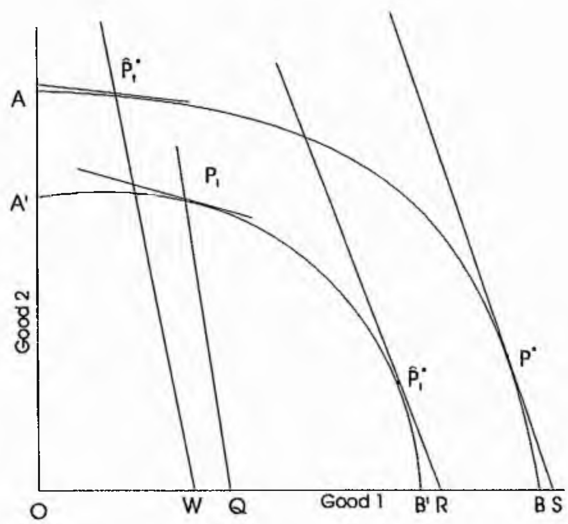
In figure 2.2, we draw two graphs for the monopoly and tariff seeking activities. Consider monopoly seeking in graph A:

> In this small, closed economy which produces at P^* initially, with welfare at U^*, the lobby to secure a monopoly in good 1 production succeeds. The resources expended in securing the monopoly shift the production possibility curve down to $A'B'$, whereas the monopoly itself leads to non-tangency of the goods-price ratio with $A'B'$ in equilibrium. Equilibrium production and consumption therefore shifts to P_{lm}, C_{lm} and welfare declines to U_{lm} from U^*. The total decline in welfare then can be decomposed into (1) the shift from U^* to U_l reflecting only the diversion of resources from directly productive use to the lobbying activity and the resulting move of production and consumption to P_l, C_l, if it is assumed hypothetically that monopoly has not resulted; and (2) the further shift from U_l to U_{lm} coming from the admission of the monopoly into the economy and the resulting move of production and consumption to P_{lm}, C_{lm}, respectively.

Figure 2.2: Welfare effects of rent-seeking activities



Graph A



Graph B

In graph B we have a very similar situation in which a protectionist lobby manages to implement a tariff against free trade:

> If we take only the diversion of resources to lobbying into account, at free-trade prices production would shift from P^* to $\hat{P}_l^*$ on the shrunk-in production possibility curve $A'B'$, which represents therefore a loss of RS measured in terms of good 1. Moreover, the tariff resulting from the successful lobbying shifts the production point further to $\hat{P}_l$, which is the final observed equilibrium under tariff seeking; this is tantamount to a further loss of QR in terms of good 1. These measures are conventional Hicksian equivalent-variational measures, as before, at world prices. Thus the overall loss QS, as already explained, is decomposed into two constituent elements, each of which is unambiguously negative.

2.3.1 Contest success functions

The concern about the welfare effects of rent-seeking was soon captured by what is a classic in this branch of the literature. Gordon Tullock in his article, efficient rent-seeking [32], analyses optimal strategies from players that participate in a lottery that resembles very much a two party conflict. They are asked to buy as many tickets as they wish at one dollar each. The lottery tickets are collected together and only one is pulled out, and whoever owns the ticket wins the prize. The probability of success for participant A depends on the amount of tickets that he/she has and the total amount of tickets, $\Lambda_1 + \Lambda_2$.

$$P_{\Lambda_1} = \frac{\Lambda_1}{\Lambda_1 + \Lambda_2}$$

For any prize of this lottery, we could calculate the expected value of the lottery for each player given the amount of tickets that each will buy. Since the value of this tickets is not added to the prize of the lottery, this is equivalent to the waste of resources that takes place in every conflict situation.

There is nothing to indicate that the lottery production function should be linear. Tullock also considered other cases with exponential functions for more than two participants.

$$P_{\Lambda_1} = \frac{\Lambda_1^r}{\Lambda_1^r + \Lambda_2^r, \ldots, +\Lambda_N^r}$$

Thus Tullock introduces the first 'contest success functions'. Many other authors follow the analysis of this kind of game, introducing different assumptions. Corcoran [33] considered a long-run setting, finding that rents would completely disappear if there was free entry. Hillman and Riley [34] allowed the players to value the prize differently, Linster [35] presented this analysis in a cooperative context.

The above lottery could be considered a Nash game in which participants spend resources in order to increase their probabilities of winning. Economists have studied such games in many different situations. Dasgupta and Stiglitz [36] examined rivalry in R&D. Nabeluff and Riley [37] studied wars of attrition; and Lazear and Rosen [38], Nabeluff and Stiglitz [39] and Rosen [40] dealt with conflict from the perspective of incentive design.

We should try to understand the incentives to manipulate the effort level to increase the probabilities of winning, given some strategic considerations such us the resource-holding potential, the technology of conflict or the symmetry (or asymmetry) of initial resources.

In Tullock's formula each party's success is a function of the ratio of the respective input levels Λ_1^r and Λ_2^r. There is another possibility. The probability of success may also depend on the difference between these inputs. Hirshleifer [41] shows how the outcome can change considerably according to the choice of one or another variant of CSF (contest success function).

The generalization of the ratio CSF for any number of players N was provided by Tullock.

$$P_{\Lambda_i} = \frac{\Lambda_i^m}{\Lambda_1^m + \Lambda_2^m + \cdots + \Lambda_N^m} = \frac{\Lambda_i^m}{\sum_i \Lambda_j^m} \tag{2.1}$$

Another option for modelling 'contest success functions' is based on the difference in fighting efforts that a participant enjoys with respect to the other participants. The probability of success in the difference version takes the following form for two participants:

$$P_A = \frac{1}{1 + \exp\{k(\Lambda_2 - \Lambda_1)\}}$$

and for N Participants

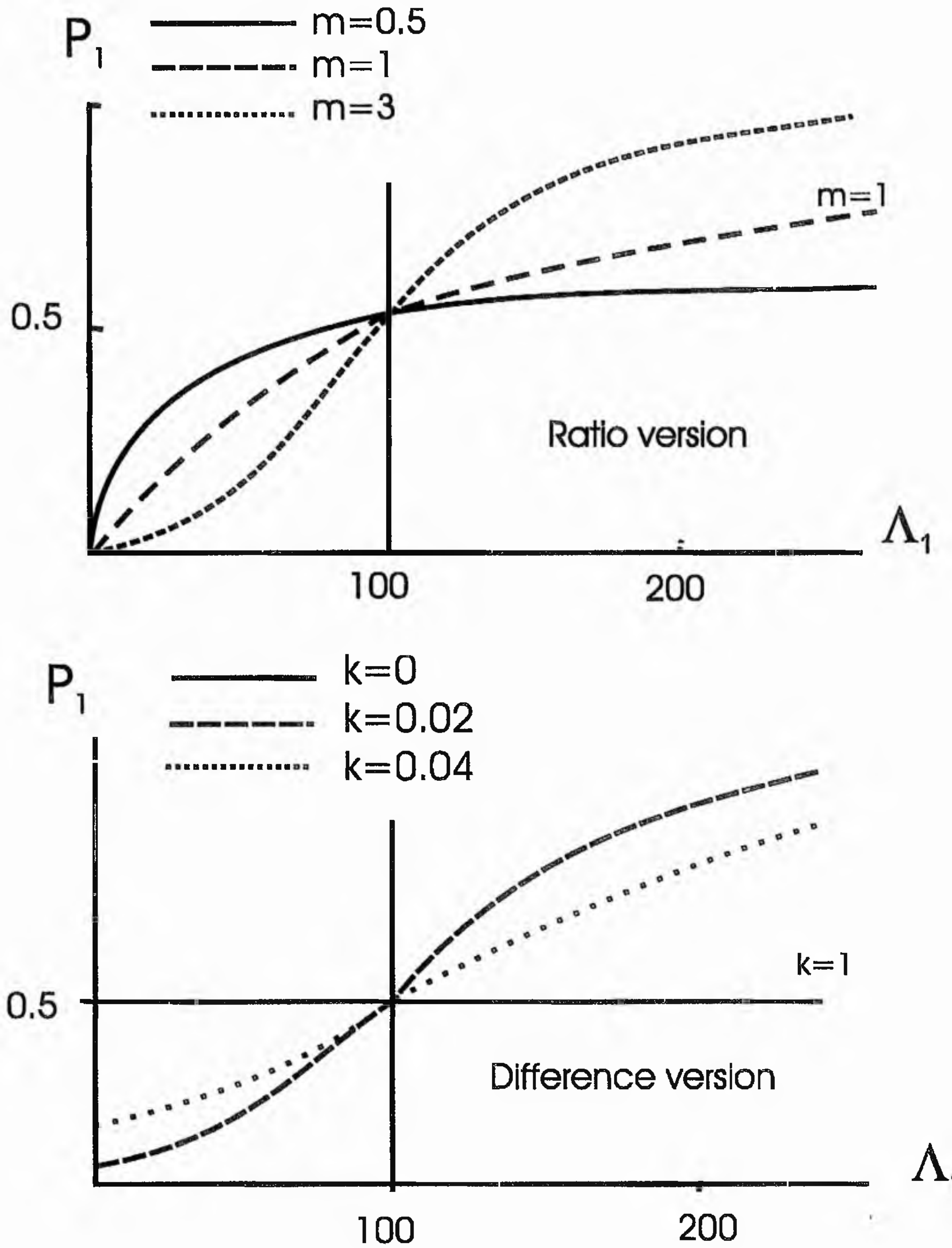
$$P_{\Lambda_i} = \frac{\exp\{k\Lambda_i\}}{\sum_j \exp\{k\Lambda_j\}}$$

In the ratio form of the CSF equation 2.1, the parameter m is the effectiveness parameter, or decisiveness parameter. High values of this parameter produce higher increases in the probability of success. In the difference version, the parameter k has a similar effect. The effects of these parameters in the shape of the CSF are displayed in figure 2.3. The resources for the other participant have been fixed to $\Lambda_2 = 100$. In the ratio version, regardless of m, we see that $P_1 = P_2 = 0.5$ when $\Lambda_1 = \Lambda_2$. If $m \leq 1$ diminishing returns to fighting effort hold for any combination of Λ_i. But if $m > 1$ there is an initial range of increasing returns. This indicates that, as long as one participant has much less expenditure on arms than the other participant, it enjoys increasing returns, which disappears when both military expenditures are similar.

In many occasions, it is recommended to assume that the appropriation technology follows the ratio version. But there are some implications that would make this form quite unreasonable in some circumstances. For example, if one side employs zero resources to fighting effort, the probabilities of success for the other party go to infinity as long as they commit some finite amount to fight, no matter how little. Hirshleifer gives a brief guide on how to apply these versions

Figure 2.3: Contest Success functions.

In the top figure we see the plot of the ratio version of CSF. There are three loci, corresponding with three different values of the effectiveness parameter m. The figure at the bottom plots the difference version of CSF.



of CSF.

> In a military context we may expect the *ratio form* of the Contest Success Function to be applicable when clashes take place under close to "idealized" conditions such us: an undifferentiated battlefield, full information, and unflagging weapons effectiveness. In contrast, the *difference form* tends to apply where there are sanctuaries or refuges, where information is imperfect and where the victorious player is subject to fatigue or distraction. Given such "imperfections of the combat market", the defeated side need not loose absolutely everything.

2.3.2 Resource allocation and distribution of income

There is no unique model of allocation of resources between fighting and appropriative activities. These models have to be selected with a certain types of application in mind. However the process of allocation of resources by using the concept of lotteries introduced by Tullock is a common feature in the models of conflict.

These models try to reconcile two different branches of economic thought. as Hirshleifer says [12]:

> A first aim of conflict analysis is therefore to provide an underlying micro-theory that would be applicable to all topical areas of application such as warfare, litigation, etc. More sweepingly, *exchange theory* and *conflict theory* constitute two equal branches of economic analysis: the former based upon two-sided advantage and contract, the latter upon one-sided advantage and struggle.

A. Dixit [42] considers the effects of precommitment in contests where the rivals expend effort to win a prize. These players can have both symmetric or asymmetric power (favourite and underdog), resulting in over-commitment of resource in a Stackelberg equilibrium. This is one of the first models showing that the order of play is very important in the economic theory of contests. Alternatively, Baik and Shogreh [43] found that the favourite will never over-commit effort. Given endogenous order of moves, the favourite finds advantageous to wait until the underdog moves while the under-dog's best strategy is not to wait, but to make the first move. Hirshleifer [44] uses the same scenario to show under which conditions there is system stability when one has no exogenous mechanisms of enforcing property rights. (e.g.: the system of international relations). Skaperdas [45] analyses the conditions under which cooperation is possible in long term relationships. In "full cooperation" none of the players invest in arms. Other things being equal, an agent's use of power is inversely related to its resources when these are valued according to marginal-productivity theory.

Skaperdas and Syropoulos [46] produced an interesting framework to analyse the key elements of these conflict models, comparing the determinants of distribution of income under conflict exchange to a market with perfect competition.

> Except for the introduction of appropriative activities ('arms', for short), we retain a structure frequently used in many areas of economics and we are thus in a position easily to compare the equilibrium allocations in the presence of appropriation with those that would emerge under competitive conditions. We find a general tendency for those who have a comparative advantage in useful production to receive smaller shares, whereas those who have a competitive advantage in arms production receive a larger share. Moreover, improvements in an agent's efficiency of useful production or factor–augmenting technical progress reduce the agent's share of income. In contrast, improvements in an agent's efficiency of arms production and increases in an agent's endowment raise the agents share of income.

Let's introduce a basic model of conflict where participants try to maximize their income given some initial resources. They must allocate these initial resources R_i (that are inalienable) into production of a single good E_i from which utility is derived or into arms F_i which secure the output of production.

The production function contains constant returns to scale: $f(E_1, E_2)$; $f_{1i} > 0$, $f_{11i} < 0$ and $f_{12} > 0$. The participants can transform the initial resources into factors of production according to the resource holding constraint:

$$R_i = a_i E_i + b_i F_i \tag{2.2}$$

where $a_i > 0$ and $b_i > 0$ $(i = 1, 2)$. Once the resources are allocated, they cannot transform them back to initial resources. The parameters a_i and b_i indicate how many units of initial resources are needed to produce one unit of F_i and E_i.

The resources dedicated to arms by each participant determine the probabilities of winning according to a contest success function. Alternatively, assuming risk neutral preferences, it gives a share of the final output. This scenario also assumes no cost of conflict, not cost of trade and perfect divisibility of the common good. Once produced, this good can be shared perfectly without loss of utility. This scenario, although seemingly unrealistic, helps us to understand the most important strategic features of efficient allocation of resources under conflict.

For any given allocation of resources to arms, the winning probabilities are $p(F_1,\, F_2)$ for player 1 and $1 - p(F_1,\, F_2)$ for player 2. The contest success function has the following properties:

$$\begin{aligned} &0 < p(F_1,\, F_2) < 1\; \forall (F_1,\, F_2);\; p(F_1,\, F_2) = 1 - p(F_2,\, F_1)\; \forall (F_1,\, F_2) \\ &\text{and } p_1 \equiv \partial p(F_1,\, F_2)/\partial F_1 > 0,\; p_2 \equiv \partial p(F_1,\, F_2)/\partial F_2 < 0\; \forall (F_1,\, F_2). \end{aligned} \tag{2.3}$$

The first line indicates that the probabilities of winning for both players are between 0 and 1, $p_1 + p_2 = 1$. In the second line we have a typical effect of arms spending. A positive increase in one's own arsenal increases one's probability of winning whereas an increase in the other participants arsenal decreases it.

The payoffs are a function of the final production and the probabilities of winning:

$$\Pi^1(F_1,\ F_2) = p(F_1,\ F_2)f(E_1, E_2) =$$
$$= p(F_1,\ F_2)f((R_1 - b_1F_1)/a_1,\ (R_2 - b_2F_2)/a_2);$$

$$\begin{aligned}\Pi^2(F_1,\ F_2) &= (1 - p(F_1,\ F_2))f(E_1, E_2) = \\ &= (1 - p(F_1,\ F_2))f((R_1 - b_1F_1)/a_1,\ (R_2 - b_2F_2)/a_2)\end{aligned} \tag{2.4}$$

Szidarowszky and Okuguchi [47] provided a formal proof of the existence and uniqueness of a symmetric pure Nash equilibrium assuming that there is a limited number of player $j = 1, 2, \ldots n$. The contest success functions are identical and take the form:

$$p_i = \frac{f_i(F_i)}{\sum_{j=1}^{n} f_j(F_j)}$$

The *elasticity of substitution* between inputs of production is a crucial concept defined as:

$$\sigma \equiv \delta(E_1/E_2)\omega/\delta\omega(E_2/E_1))$$

where $\omega = -\mathcal{F}_1/\mathcal{F}_2$[13]

Let the *factor share* of player i be $\theta \equiv F_i\mathcal{F}_i/\mathcal{F},\ \theta_1 + \theta_2 = 1,$ and $\theta \in (0, 1)$. It can be show that :

$$\sigma = \mathcal{F}_1\mathcal{F}_2/\mathcal{F}_{12}\mathcal{F}$$

and

$$\frac{\partial\theta_i}{\partial F_i} \lesseqgtr 0 \text{ as } \sigma \lesseqgtr 1$$

Which means that a participant's factor share is increasing in that participant's contribution only if the elasticity of substitution is greater than 1. Then we can see the Nash equilibrium distribution of resources by maximizing the payoffs

[13]for simplicity we call $f(E_1, E_2) = \mathcal{F}$ and $\mathcal{F}_i = \frac{\partial \mathcal{F}_i}{\partial E_i}$

with respect of arms expenditure.

$$\begin{aligned} \partial\Pi^1/\partial F_1^* &= p_1^*\mathcal{F}^* - p^*\mathcal{F}_1^*(b_1/a_1) \leq 0 \\ \partial\Pi^2/\partial F_2^* &= -p_2^*\mathcal{F}^* - (1-p^*)\mathcal{F}_2^*(b_2/a_2) \leq 0 \end{aligned} \tag{2.5}$$

If the marginal quantity of arms that players can produced is the same $R_1b_1 = R_2b_2$, their respective share is inversely related to their initial resources, when these resources are valuated at the efficient point of production. Thus the higher is a player's marginal product, the lower is his equilibrium share. This result contradicts the postulates of marginal theory under perfect competition and it was coined the *Paradox of Power* by Hirshleifer.[14] We do not want to extend further our analysis in this direction because we are also concerned with the effects that cost and imperfections in the negotiation process may have in the final equilibrium of this models.

However it is worth mentioning briefly some of the consequences of changes in the main parameters of the model. According to Skaperdas and Syropoulos:

1. *The role of useful productivity*

    - The effect of changes in the technology of production in the final income can be of two different types. One is the nature of the production function $f(E_1, E_2)$ and the second is the effects of the parameter a_i which determine the efficiency with which the initial resources can be converted into inputs of productions.

    - For the case of the production function they studied the share equilibrium under an alternative production function $g(E_1, E_2) \equiv f(\lambda E_1, E_2)$ where $\lambda > 1$ for all $\sigma > 1$ and $\lambda < 1$ for all $\sigma < 1$. With some restrictions in the elasticity of substitution σ and the production functions

[14]Note that in the absence of other influences, this should produce a convergence between rich and poor.

they show that increases in player 1 productivity produces a lower share of income for himself.

- The effects of changes in a_i (the efficiency parameter in 2.2) in the equilibrium shares operate in an identical way to increases in the parameter λ in the production function. Thus an improvement in player 1's input production through an increase of a_i reduces the allocation of resources into arms of player 1, reducing its power and final share.

2. *The role of factor endowments*

    - Player 1 increases her levels of arms investment F_1 for any given F_2 as a result of an initial increase in her initial resources R_1. An increase in R_1 increases player 1's marginal benefit from investing in arms because it increases production, but also reduces the opportunity cost by forcing her marginal product to decline owing to diminishing returns.

3. *Change in the efficiency of arms production*

    - The effect of changes of b_i in 2.2 is ambiguous. For high enough elasticities of substitution, increased efficiency of arms production is favourable to the player who undertakes it.

We can see that the elasticity of substitution plays a crucial role in determining the changes in final shares brought about by changes in the key parameters of the model. The exception being the case when players experience an increase in their initial resources, which produces unambiguously increases in the final share.

Risk aversion

Different attitudes can have a drastic impact in the outcome of conflict. There are two kind of attitudes that should be considered with respect to the economics of conflict.[15] These are expressed by risk aversion and reference-dependent utility functions. There is abundant empirical evidence of both cases in the literature.[16]

Risk aversion is a common feature in economic models that have some degree of uncertainty. In wars and other types of conflict it is very difficult to predict with certainty who will be the winner. Therefore one would expect that risk averse preferences would affect the allocation of resources to arms investment.

Skaperdas [49])looks at the effect of risk aversion on winning probabilities under two different settings, conflict and settlement. We look at the particular scenario when settlement is possible. We suppose that the two parties are able to communicate and can divide the prize in two different shares s and $(1-s)$, $0 < s < 1$.

If we assume that both parties are risk averse [17], the two parties will face a bargaining problem with the threat point represented by the outcome of conflict and a bargaining possibility frontier represented by the share interval

$$[\underline{s}, \overline{s}] \text{ such that } \begin{cases} U(\underline{s}f(E_1, E_2)) = pU(f(E_1, E_2) \\ k[U((1-\overline{s})f(E_1, E_2))] = (1-p)k(Uf(E_1, E_2)). \end{cases}$$

Where $\underline{s}$ corresponds to the minimum utility expected by player 1 and $\overline{s}$ to the minimum level of utility expected by player 2.

Under the assumption that both players divide the prize according to their respective winning probabilities, attitudes toward risk do not have any influence on the strategic choices of the two parties. Deriving the first order conditions

[15]This is an important question. When we model economic conflict, by definition we are concentrating in parties that only care about economic gains or losses. In real world conflicts, preferences of actors can included not only economic factors but also political, social, and cultural influences. Preferences might be hard to capture by simple utility functions, however the strategy of conflict shouldn't be affected by it.

[16]See Tversky and Kahneman [48].

[17]Assuming that a party is risk loving and the other is risk neutral may reduce the bargaining set until it disappear, not having any possibility of settlement

that maximize income we get:

$$U'(pf(E_1, E_2)[p_1(E_1, E_2) - p(E_1, E_2)] = 0$$

Which does not depend on the utility function. However, in case of conflict, both parties (being more risk averse) invest more in arms. In the settlement case they behave as if they were risk neutral, which makes settlement more efficient even in the absence of cost. The effects of risk-loving–aversion preferences is ambiguous. It can induce participants to spend more or less in arms according to the type of scenario.

Konrad and Schlesinger [50] examined the effects of risk aversion for two types of expenditures in rent-seeking contests.

- *Rent–seeking expenditures*, which improve the probability that the rent is obtained, and
- *Rent–augmenting expenditures*, which increase the size of the rent that a player might be awarded.

They found that, in the first case, the effect of risk aversion on rent–seeking expenditures is indeterminate. They also follow the marginal theory to found their propositions.

In a reference-dependent utility function, the utility is determined not only by the final outcome, but also by the relationship between this outcome and a reference point. Experimental results on a wide range of games show clearly that a large proportion of players offer "fair" allocations, and "unfair" allocations are systematically rejected. The reference point affects directly the *disagreement* or *threat point* which is relevant to the outcome of the problem.

These reference-dependent utility functions may be successful in explaining many deviations from the equilibrium outcome of other models. Shalev [51] analyses the relation between loss aversion and reference-dependent utility functions.

> The most striking result of the investigation of reference-dependent utility functions is the existence of loss aversion. Experimental works in both the psychological and the economic literature suggest that people are motivated to minimize losses (relative to a reference point) much more than they are motivated to maximize gain.

A reference-dependent utility function contains as additional elements the *loss–aversion* coefficient of the players $\lambda_i \in \Re_+$ and a reference point $r_i \in \Re$. Given a basic utility value $x_i \in \Re$, the reference-dependent utility function takes the following form:

$$v_i(x_i, r_i) = \begin{cases} x_i & \text{if } x_i \geq r_i \\ x_i - \lambda_i(r_i - x_i) & \text{if } x_i < r_i. \end{cases} \tag{2.6}$$

if $\lambda_i = 0$, player's i utility function is not reference dependent. Otherwise it retains the main aspects of the risk averse utility functions which is steeper for losses (relative to a reference point) than for gains.

Huck and Oechssler [52] offered an explanation of this behaviour using the "indirect evolutionary approach" which is based on the assumption that players behave rationally for given preferences but that their preferences change through an evolutionary process. They showed that a preference for punishing participants that make unfair offers is an evolutionary stable strategy despite anonymous interaction.[18]

The cost of conflict

The cost of conflict also affects the final outcome of the conflict game by changing the *threat point*. It also introduces a fundamental difference in the concept of expected utility. When conflict is costly (even if we have symmetric risk-neutral

[18]A more developed approach into the concept of fairness and game theory can be found in Binmore [53]

players), the partition of the final outcome cannot be done by agreeing to take a share equal to the respective probability of winning. Since conflict is costly, a participant could increase (decrease) its share by exploiting (being exploited by) the high (low) cost that conflict would produce in the other side.

In many of the previous models of conflict, there is an opportunity cost, which is the consumption foregone by allocating resources into unproductive activities. But few models contemplate one of the most crucial facts of appropriating technologies: their destruction power.[19]

The cost of conflict can be decomposed into two main components, the opportunity cost and the destruction cost. [20] Neary [54] argues that the opportunity cost may be in fact much higher than the actual cost of fighting.

> However, while actual wars and conflict are costly, it is not war itself, as (Hobbes [55] observed, but the disposition to war and to the use of force, that is the main concern. Losses incurred in actual conflict are the tip of an iceberg; it is the vast stock of otherwise productive resources held frozen in the world's arsenals that is the more relevant pointer to the social cost of self–enforcement in a world without Leviathan. For example, world–wide military expenditures in 1992 equalled the income of 49% of the world's population (UNDP), [56].

There is a great variety of studies that look at the opportunity cost of military expenditure. They have proliferated in recent years and this effect is also known in the literature as the *peace dividend*.[21]

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[19]The literature of the nuclear arms race tries to offer a rational explanation to why the destruction power exceeds the value of the resources in contest. Otherwise, it is normally consider to be less than the value of the initial resources

[20]There is also an extra cost (considered part of the destruction) that is very important but difficult to account for. This is human cost. It is very difficult to put a price to a human life, to historical patrimony that may be irrecoverable, to the cultural trauma for a generation and the benefits of a peaceful society.

[21]See Gleditsch et al. [57].

However, the behaviour and equilibrium outcome in a model with destructive power differs qualitatively from previous models. According to Neary, [58] costly conflict creates a region of strategy space in which it is not individually rational for players to fight. If participants allocate resources into arms in stage 1 and redistribute in stage 2, when conflict is cost-less they will always try to use force. Consequently, conflict always occurs and normally initiated by the poorer player. [22]

Destructive conflict has also implications for the interpretation of private property:

> A second area in which the costly conflict model extends its predecessor concerns the interpretation of property. In both models the players' consumption stocks are aggregated into a common pool that is at risk of being redistributed by force. Since force is always used in the cost-less conflict case the model can be interpreted as a theory of the right access to common property, but not as a theory of private property. In contrast, in the costly conflict model, whenever conflict does not occur the players do not in any sense share an aggregate consumption stock; rather, each one consumes the consumption stock that he has individually created. In this way the possibility of private property exists because conflict is destructive.

There are several ways to introduce cost in models of conflict. The most straight forward case is by deleting a fraction $(1-\gamma)$ of the final payoffs for available consumption $(1-\gamma)U_i f(E_i, E_j)$. The conflict best reply functions are independent of the value of γ and therefore the destructive power of conflict does not affect the allocation of resources but introduces a deadweight loss. [23] The cost parameter γ can also be made endogenous by making it dependent on the

[22] This is another case of the conflict paradox; Hirshleifer [12].

[23] In models where players can bargain over conflict this scenarios changes radically because participants can gain (lose) bargaining power.

total arms stock but given the complexity of thoroughly accounting for all the factors, we will assume that it is a fraction of the final outcome.

In our models of conflict cost will feature as one of the key variables. This is a distinguishing factor between political models of conflict and economic models. While politics concentrates in the study of power and the probabilities of success, from an economic perspective, we concentrate in the expected value of every possible strategy. The cost of fighting may have a greater influence than the power to impose a given outcome by military victory. This will bring some drastic changes to our models.

2.4 Models of Bargaining

Game theory is becoming one of the most common methods of analysis in social sciences. It is no longer confined to economics. There are plenty of examples of game theory models of conflict in political science.[24] For the study of the micro-foundations of conflict, bargaining theory seems to be one of the most useful tools. In this section we are going to review some of the models of bargaining with a clear application to conflict modelling.

When two parties are confronted with the problem of splitting a resource (more often known as splitting the pie), negotiations increase the welfare in comparison to any non-agreed outcome. This happens independently of the complications proposed by the parties or how much energy they spend in analysing the other's side offers. This is especially true, in those case where disagreement can lead to costly conflict.

To the extent that agreement is not immediately reached in some models of bargaining, this process is socially wasteful. Rasmusen [60] models a negotiation as a two period auditing game concluding that negotiations raise welfare, rather than reduce it. According to Rasmusen, much deal-making, rather than concentrating in the classic problem of splitting the pie, is about the process of setting the terms of agreement. This is certainly true, regarding international conflicts, where setting the agenda of negotiations is the most important part of any settlement process, Burton [61].

2.4.1 Rubinstein Model

The basic bargaining model is studied following Rubinstein [62] strategic approach:

Two participants have to reach an agreement in the partition of a homoge-

[24]see Peter Ordeshook [59].

neously divisible pie of size 1. Each participant has to make a proposal of how it should be divided in turns. If they agree on a partition of the pie at time t, $t = 1, 2, 3, \ldots$, each receives the share of the pie agreed at that time.

The game is played following these rules. The first player is Country 1,c_1. He proposes a share, s_{1c1} of a given Income ,I, which gets accepted or refused. In case of refusal Country 2, c_2, proposes another share s_{2c2}. The game goes on until one of the two countries accepts.

The participants preference relations are defined by the pair (s, t), where $0 < s < 1$. This preference relation is assumed to be complete, reflexive, and transitive, on the set $S \times T \cup \{(0, \infty)\}$, where T is the set of natural numbers and S is the set of all possible offers.

These preferences satisfy five assertions described in appendix B.1.

The equilibrium

In this model a *Nash Equilibrium* is a weak concept. The bargaining equilibrium sharpens the Nash equilibrium using the concept of subgame perfectness. This equilibrium concept is based on the ordering of moves and the moves along the equilibrium path without accepting equilibria which deviates from this path. The game has a unique solution if the players are impatient and prefer to receive a given share at time t rather than $t+1$. If there is no discounting, Country 1 (c_1) gets all the income I (assuming that it makes the last offer). It offers $s_{t1} = 1$ at every round and Country 2 (c_2) can accept or reject at any round.

Suppose that there is a discounting factor $\delta \in (0, 1)$. The total value of I in the first period is 1 . In the second period it is δI, and so on.

First, we consider a finite bargaining horizon with 3 periods $T = 3$. The bargaining ends with an imposed settlement $(s, 1-s)$. For simplicity let's assume that both participants have the same discounting factor $\delta = \delta_1 = \delta_2$. We can derive the equilibrium strategy by backward induction. In period 2 c_2 offers a share $(s_2, 1-s_2)$ that is accepted by c_1 iff $s_2 > \delta s$. Country 2 would offer $s_2 = \delta s$ iff $(1-\delta s) \geq \delta(1-s)$. Since $1 > \delta$, this is true and the equilibrium offer for period 2 is:

$$(s_2^*, 1-s_2^*) = (\delta s, 1-\delta s)$$

In period 1 c_1 offers $(s_1, 1-s_1)$. For c_2 is acceptable iff $1-s_1 \geq \delta(1-s_2^*) = \delta(1-\delta s)$. Subsequently, the lowest offer that c_1 can make is $1-s_1 = \delta(1-\delta s)$. Since $1-\delta(1-\delta s) \geq \delta^2 s$ the equilibrium offer is:

$$s_1^* = 1-\delta+\delta^2-\delta^2 s$$

For $2n+1$ periods is

$$s_1^* = 1-\delta+\delta^2-\delta^3+\cdots+\delta^{2n}-\delta^{2n}s \tag{2.7}$$

and when $n \to \infty$:

$$s_1^* = \frac{1}{1+\delta};\ 1-s_1^* = \frac{\delta}{1+\delta}; \tag{2.8}$$

Proof of this equilibrium is in appendix B.2.

In this model c_1 has a clear advantage by moving first. There are two possible solutions. Countries could agree to choose who starts the game by a random draw. Alternatively, we can show that as the time in between periods goes to zero the advantage disappears. This is best illustrated by the next model.

2.4.2 The Nash bargaining solution in economic modelling

Binmore, Rubinstein and Wolinsky [63], developed the previous model in two directions. On the one hand, they established the relation between the static axiomatic theory of bargaining and the sequential strategic approach. On the other hand, they introduced two source of incentives to reach agreement. One is the time preference and the other is an exogenous risk of breakdown in negotiations.

The Nash bargaining solution

In any bargaining problem, by definition, all payoffs in the bargaining set are plausible. This is far too general. Nash [64] derived some properties that should be satisfied in any bargaining problem so the equilibrium can be clearly analysed. He established five axioms that any payoff should satisfy in any bargaining problem. Then he proved that only one payoff satisfies those conditions.

Binmore [24] expressed these criteria as follows

1. The final outcome should not depend on how the players' utility scales are calibrated.
2. The agreed payoff pair should be always in the bargaining set.
3. If the players sometimes agree in the payoff pair s_i when s^* is feasible, then they never agree on s_i when s^* is feasible
4. In symmetric situations both players get the same share.

In figure 2.4, the shaded region, X, is the bargaining feasible set. The threat point is $\overline{U} = (\overline{U}_{c1}, \overline{U}_{c2})$. The Nash Bargaining solution $U^* = (U^*_{c1}, U^*_{c2})$ depends on the threat point and the bargaining set.

We can also define the five axioms as:

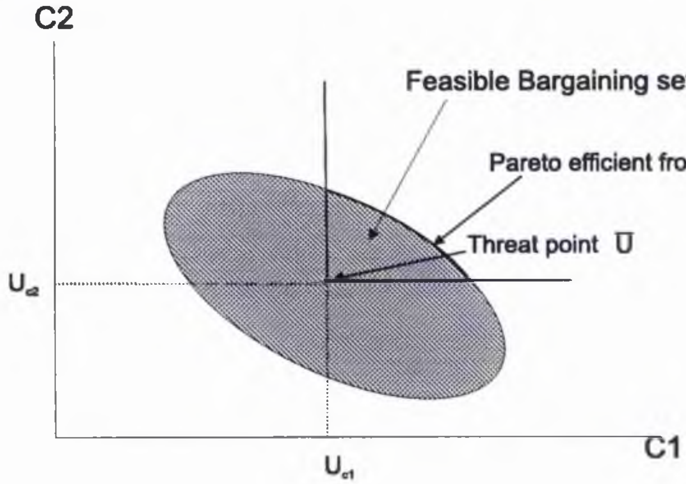
1. To say that the final outcome does not depend on the utility scale can be expressed, given any strictly increasing utility function $\mathcal{F}$ by this equality:

$$U^*[\mathcal{F}(\overline{U}), \mathcal{F}(X)] = [U^*(\overline{U}, X)]$$

2. If the agreed payoffs are always in the bargaining set then,

$$(U_{c1}, U_{c2}) > U^* \Rightarrow (U_{c1}, U_{c2}) \notin X$$

Figure 2.4: The bargaining feasible set.
The bargaining feasible set is limited by U_{c1} on the x axis , U_{c2} on the y axis and the threat point $\overline{U}$.



3. If we drop some possible payoffs pairs, reducing the set X to Y, then, if U^* wasn't one of the dropped pairs, U^* does not change.

$$U^*(\overline{U}, X) \in Y \subseteq X \Rightarrow U^*(\overline{U}, X)$$

4. This axioms indicate that for the bargaining solution it doesn't matter which participant is labelled c_1 and who is labelled c_2. Let ρ be a function defined as $\rho(U_{c1}, U_{c2}) = (U_{c2}, U_{c1})$ that swaps the participants' payoffs. Then:

$$\mathcal{F}(\rho(\overline{U}), \rho(X)) = \rho(\mathcal{F}(\overline{U}, X))$$

If U^* satisfies these four conditions, then $\mathcal{F}$ is a generalised Nash bargaining solution such that:

$$U^* =^{U \in X, U > \overline{U}}_{\arg\max} (U_{c1} - \overline{U}_{c1})(U_{c2} - \overline{U}_{c2})$$

This approach describes the bargaining problem by using only the information contained in a pair of utility functions, (U_{c1}, U_{c2}), which represent both participants preferences over all the feasible pairs of payoffs in the set X and a pair of utility levels that is the threat point $\overline{U} = (\overline{U}_{c1}, \overline{U}_{c2})$.

There is some additional information depending on the modelling choices. An example is the choice of the utility functions (risk aversion, absolute or relative risk, reference dependent, etc). Another modelling choice is concerned with the threat point and deciding what happens in case of no agreement. Whether participants bargain for ever or there is an imposed settlement should be reflected in the threat point. In this way any other element such as the environment or history of the game is abstracted in the axiomatic approach. In this article they study the insights of this approach in selecting the appropriate static representations of the bargaining games in strategic forms.

This Nash product can represent the underlying strategic approach in many bargaining situations. In order to demonstrate this problem Binmore et al. describe first the main bargaining features explicitly and then show that the Nash solution coincides with the limiting perfect equilibrium outcome of the dynamic strategic model when the length of the bargaining periods approaches zero.

They presented two types of model according to the underlying force that drives participants to reach an agreement. In the first model the parties' incentive to agree is the discounting factor. Both give more value to present resources. The second incentive is an exogenous probability of breakdown of negotiations, and in this case, both participants will lose the benefits of an agreement. In their original model, this takes place because they could lose the resources in contest to a third party. In a model of internal or interstate conflict, these benefits are clear, since war is costly.

In each period of bargaining there is an exogenous risk of ending the process in war. Geopolitical models of conflict show certain areas of the world where this exogenous risk tends to be higher. Certain places like Switzerland would have a very low risk but others , like the Middle East, are areas of high tension.

Given the difficulty of accounting for all the factors that might influence conflict situations, we might think of this as a example of exogenous risk.

At each period of time of length Δ, there is a positive probability of breakdown of negotiations.

$$p = p(\Delta) = 1 - e^{-\lambda\Delta}$$

In that case the outcome will be the threat point for each participant $w = (\overline{U}_{c1}, \overline{U}_{c2})$.

Assume that the preference orderings satisfy the following assumptions:

Assumption 1 *There is a conflict of interest*

Assumption 2 *There are mutual beneficial agreements*

Assumption 3 *The preference orderings can be represented by the expected values of continuous utility functions $u_i : X \to R$.*

These are the von Neumann-Morgensten [14] assumptions

Assumption 4 *The participants are risk averse. For every*

$$x, y \in X, \alpha \in [0,1], \alpha x + (1-\alpha)y \tilde{>} \alpha x \oplus (1-\alpha)y$$

where $\alpha x \oplus (1-\alpha)y$ is a lottery with outcomes x and y with probabilities respectively α and $1-\alpha$.

Suppose that the bargaining process does not break before time $t\Delta$, concluding with agreement x. Thus $(t\Delta, x)$ is the lottery:

$$(t\Delta, x) = (1-p)^t x \oplus (1-(1-p)^t)w$$

and the preferences of both countries are represented by imposing this lottery in the strategic approach:

$$(1-p(\Delta))^t U_{ci}(x) + [1-(1-p(\Delta))^t] U_{ci}(w) \quad i = 1, 2$$

The preferences can be also represented by:

$$(1 - p(\Delta))^t [U_{ci}(x) - U_{ci}(w)]$$

When the length Δ is sufficiently small, the probability $p(\Delta)$ of breakdown is also small $\Delta \to 0$ and its solution will maximise:

$$x^* = \arg_{x \in X} \max[(U_{c1}(x) - U_{c1}(w)][U_{c2}(x) - U_{c2}(w)]$$

Which is equivalent to the Nash Bargaining solution given by

$$\max_{u_{c1}, u_{c2} \in S} (U_{c1} - \overline{U}_{c1})(U_{c2} - \overline{U}_{c2})$$

Proof: See Binmore [65]

In the literature of conflict, the strategic approach has been used more often than the axiomatic one. One of the reasons is that war produces high cost and therefore, the dynamic aspects of conflict are considered very important. The Nash Bargaining solution is used more often in models of power politics and reallocation of resources. We will see some applications of these models in the next chapter.

2.4.3 Models that include no agreement equilibria

In this section we are going to review some models of bargaining with incomplete information. The breakdown of hostilities in conflict processes can be explained by this type of model. Given that agreement produces a superior outcome, in order to explain the lack of it, we have to look closely at possible information asymmetries. These information asymmetries must be, directly or indirectly, related to the payoffs of participants.

It is not difficult to see the informational asymmetries at the source of conflict in the real world. Societies are complex hierarchical systems of decision making. This decisions must be decentralized which in turn leads to what Isard and Smith [17] call *a hierarchical pattern of decision making nodes.* These patterns depend, amongst other things, on several informational problems:

1. Information Collection Cost

2. Information Processing Cost

3. Information Transmission Cost

To model the mechanism by which this informational problems are translated into the perceived payoffs of different participants in a bargaining game is one of the most challenging parts of the research in the microeconomics of war. Many of the following models have been designed in a rather general form. The applications of those models have been directed into explaining problems of buyers and sellers and conflicts between unions and firms. Therefore, some of the assumptions might not be the most appropriate. Bearing this in mind, these models give a handful of techniques and principles that can be translated into the field of defence economics and in particular into the study of *rational wars.*

A first model of bargaining with incomplete information

In a seminal paper Fudenberg and Tirole described a simple two-period two-person bargaining game with incomplete information. This captures one of the main facts about bargaining: the participants do not know the value to others of reaching an agreement. In their model, there is a buyer and a seller. They try to agree the price of a good. In this scenario the seller makes the offers and the buyer accepts or rejects. Therefore there are not alternating offers as in the Rubinstein (1982) model. We will use the first part of their paper in our approach of bargaining and war and also discuss the implications of these assumptions for the equilibrium. For the moment we are concerned with the techniques they use, in as much as they prove that when the buyer knows the valuation of the seller but has private information about its own valuation, there is a set of unique equilibria that is fully characterised in their paper.

Both players are risk neutral and the good is non-divisible. The value of the good is s for the seller and b for the buyer. The buyer knows both b and s. The seller doesn't know the valuation of the buyer –but he has some probability distribution of it, which is also common knowledge.

In this model the uninformed player, the seller, moves first. He makes an offer and if it gets rejected, updates his believes over the distribution of the valuation of the buyer by a Bayesian updating process.

Bargaining takes place in two stages only. If at the second stage the seller refuses the offer, there is no sale. Both players have a discounting factor defined as δ_s and δ_b. The valuation of the seller is s, and we assume that there are two kinds of buyers[25], a tough buyer with a low valuation ($\underline{b}$), and a soft buyer with a high one ($\overline{b}$). The seller's prior probability distribution over $\{\overline{b}, \underline{b}\}$ is assumed to be $(\frac{1}{2}, \frac{1}{2})$.

We must define the actions of the seller and the buyer at every period. Let $p_1(s)$ be the offer of seller in period 1 and $p_2(s, p_1)$ the offer in period 2. The actions of the buyer are either accept or reject. He accepts p_1 with probability $r(p_1, b)$. Since there are only two periods the only relevant conditional probability arises in case of a rejection in the first period. Given the strategies of a soft buyer and a tough buyer $[\overline{r}_1(p_1), \underline{r}_1(p_1)]$, in case of rejection we compute the posterior probability by Bayesian updating $[\overline{q}_1(p_1), \underline{q}_1(p_1) = 1 - q_1(p_1)]$.

In order to find the Perfect Bayesian Equilibrium we proceed as usual, eliminating non-equilibrium strategies by backwards induction. First we find out the

[25]Another interpretation is that the bargaining takes place between a seller facing a continuous distribution of buyers. The two types of buyers can be interpreted as different fractions of the population.

strategies for a one period game. It is clear that the seller will make offers only in the set $\{\bar{b}, \underline{b}\}$. Note that the buyer will accept any price $p_2 < \underline{b}$ regardless of his type, but this price is dominated by $\underline{b}$, which will be also accepted. Any price $p_2 \in]\ \underline{b}, \bar{b}\ ]$ is only accepted by a buyer type $\bar{b}$ and this price is also dominated by $\bar{b}$.

The solution to the two period model can be derived from the solution to a one period one. There are two scenarios. When $s > \underline{b}$, the seller sets the price always to be equal to $\bar{b}$. When $s < \underline{b}$, he offers $\underline{b}$ if he is soft and $\bar{b}$ if he is tough.

We calculate the expected payoffs for both strategies (offer $\underline{b}$ or $\bar{b}$) and then, the seller uses the one that maximises his payoff. If he offers $\bar{b}$, the expected payoff will be $\frac{1}{2}\underline{b} + \frac{1}{2}\underline{b} = \underline{b}$. With offer $\bar{b}$ the expected payoff is $\frac{1}{2}(\bar{b} + s)$. Then if:

$$\underline{b} > \frac{1}{2}(\bar{b} + s)$$

the seller offers[26] $\underline{b}$.

In the two period game, the second period strategies are identical to the one period game described above. A tough buyer will accept any offer in period 2 which is smaller or equal to her valuation, $p_1 \leq \underline{b}$, with probability $\underline{r}_1(p_1) = 1$. Therefore, only the strategy of a soft buyer, $\bar{r}_1(p_1)$, needs to be consider.

The Bayesian Equilibrium of the second period subgame is as follows:

1. For the Seller

   $p_2 = \underline{b}$ with probability 1 if $\underline{b} > \bar{r}_1(p_1)\bar{b} + \underline{r}_1(p_1)s$

   $P_2 = \bar{b}$ with probability 1 if$\underline{b} < \bar{r}_1(p_1)\bar{b} + \underline{r}_1(p_1)s$

2. For the Buyer

   She accepts any offer providing that $p_2 \leq b$ (of each type)

[26]For simplicity they ignored the borderline case where $\underline{b} = \frac{1}{2}(\bar{b} + s)$

The next step is to calculate the Bayesian Perfect Equilibrium for the two period game. First we have to define $\tilde{b}$ as the highest first period offer that buyer $\bar{b}$ will accept if she knows that in the second period the buyer will offer $\underline{b}$.

$$\tilde{b} \equiv (1-\delta_b)\bar{b} + \delta_b \underline{b}$$

And we must also define $\bar{r}$ as the value of the probability of accepting first period offer by a soft buyer. This leaves the seller indifferent between offering $\bar{b}$ or $\underline{b}$ in the second period.

Assume that the seller is soft. When the first offer is rejected, he updates the probability of facing a tough buyer, $\underline{q}(p_1) \geq \frac{1}{2}$ which implies that seller will play soft in the second period. (see appendix 4.7 for a proof).

$$\underline{b} > \frac{1}{2}\bar{b} + \frac{1}{2}s \Rightarrow \underline{b} > \bar{q}(p_1)\bar{b} + \underline{q}(p_1)s$$

A soft buyer, anticipates the second period offer and accepts the first period one only if

$$\bar{b} - p_1 \geq \delta_b(\bar{b} - \underline{b}) \Leftrightarrow p_1 \leq \delta_b \underline{b} + (-\delta_b)\bar{b} = \tilde{b}$$

If the seller is tough, he can offer $\bar{b}$ in the second period. Regardless of this offer, a soft buyer will accept any offer in period 1 that is greater or equal to $\tilde{b}$.

For any offer exceeding $\tilde{b}$, if a soft buyer, $\bar{b}$, accepts with a probability bigger that $\bar{r}$, the seller will play $\underline{b}$ in the second period, because $\bar{r}$ is the probability that makes him indifferent between $\underline{b}$ and $\bar{b}$. In that case, the buyer would be better off waiting for the second period.

If the buyer accepts $\bar{b}$ with probability $\bar{r}_1(p_1) < \bar{r}$, then, the seller will play tough in the second period and a soft buyer $\bar{b}$ will be better of accepting the offer in the first period. Therefore the only equilibrium strategy for the soft buyer is to play $\bar{r}$.

Finally, in order to play $\bar{r}$, the seller must be indifferent in the second period between proposing price $\bar{b}$ and $\underline{b}$. If $\sigma_2(p_1)$ is the probability of the seller playing soft in the second period, he would be indifferent if it takes the value:

$$\sigma_2(p_1) = \frac{\bar{b} - p_1}{\delta_b(\bar{b} - \underline{b})}$$

Once that we know $\bar{r}$ and σ, we can calculate the final payoffs.

$$\begin{cases} \pi(\underline{b}) = \underline{b} \\ \pi(\bar{b}) = \frac{1}{2}\bar{b} + \frac{1}{2}\delta_s\underline{b} \\ \pi(\bar{b}, \bar{r}) = \frac{1}{2}\bar{b}, \bar{r} + \left(\frac{2-\bar{r}}{2}\right)\delta_s\underline{b} \end{cases}$$

This model with one-side incomplete information shows that for any set of initial parameters, there is a unique equilibrium that is described by any of the above strategies.

However, the models contradicts the case of complete information in some important aspects. Bargaining does not necessarily stop at the first period and there is a probability of disagreement when the seller is tough. The solution is not necessarily Pareto-Optimal. Other differences with the model of perfect information derives from the fact that if the buyer discount factor decreases, this can lead to a higher payoff for him, because the probability of accepting $\bar{b}$ increases. This is the contrary to what happens in the complete information case.

General topics in bargaining with incomplete information

Rubinstein's (1982) model of bargaining reflecting a dynamic process of offers and counter offers under complete information has been modified to reflect many different bargaining situations. These models introduce inefficiencies into the original model using different assumptions about the bargaining process. Rather than a complete theory of bargaining, they represent a collection of different modelling choices, giving a broad picture of the different equilibria that can arrive in non cooperative games.

This highlights two problems concerning the strategic approach of bargaining. First, in both complete and incomplete information models, the equilibrium outcome is very sensitive to the choice of extensive form. The shares obtained by bargaining change if we change the form of bargaining. For instance, there is an strategic advantage to the country that makes the first offer. This is even more crucial when we add some cost of fighting. Second, incomplete information games may produce multiple equilibria, thus bargaining does not offer a unique solution and therefore sometimes it might be impossible to predict the final outcome. At the moment, we would like to say that this problem can be tackled using some prior assumptions or equilibrium refinements. There is a vast literature in game theory about these refinements.

There are some alternatives to the two period models with one-sided asymmetric information in strategic form games. Fudenberg and Tirole analyse the problem of two-sided incomplete information. Myerson and Satterthwaite [66] analyse the general conditions in which the equilibria of a bargaining game is inefficient when the valuation of both participants is unknown.

There is no clear explanation why both parties should stop bargaining when they reach the end of second period without agreement. Fudenberg & Tirole , [67] explained the existence of a finite horizon by introducing a fixed bargaining cost per period, or by giving the participants the opportunity to bargain with someone else if they become too pessimistic about the gains from trade with the current partner.

Sobel and Takahashi [68] presented a multistage model of bargaining with one-sided offers between a seller and a buyer. The seller's ability to make commitments affect the outcome and there are different equilibria with and without commitment. Cramton [69] presented also an infinite horizon model with two-sided information uncertainty. He explored how timing and information affect the behaviour or rational agents when they cannot commit to a given strategy.

Incomplete information may not be the only source of inefficiency in bargaining. Fernandez and Glazer [70] addressed this question. Their model is based in a wage negotiation between a union of workers and a firm. The union has to decide whether or not to strike in each round for the length of the negotiations in that period. They show that there exists subgame-perfect equilibria in which the union engages in a succession of strike periods until agreement is reached at time T.

In previous models, strikes or delays in agreement worked as signalling devices. But in this model, there is an old wage that the union will loose if it chooses to strike, and the firm will loose its profits. Thus the decision to strike is costly for both parties. Some of the equilibria are Pareto-inefficient. This occurs when the union starts with a very high wage demand, to which the firm responds with a very low wage offer. Both parties reduces their claim in successive periods until they reach an agreement. In every period the union strikes, however, even if both know the outcome, and would be willing to avoid the cost of industrial action, no one can deviate from the equilibrium path. Otherwise, any attempt to reach an early agreement would affect negatively the deviating party.

The Folk Theorem

The case above is an example of how conflict can occur without informational asymmetries. It is worth to say that, somehow, the model implies an infinitely repeated game with no discount factor after agreement is reached. Although we concentrate in the effect of asymmetric information in the likelihood of conflict, it is worthwhile to look briefly at the effect of infinite repetitions in the equilibrium outcome of these games.

Suppose that we repeat infinitely a game such as the Prisoner's Dilemma and all the actions from previous stages are observed in the present stage. Summing all the discounted payoffs does not provide a useful equilibrium concept because any number of initial rounds can be easily sacrificed in order to ensure future cooperation.

The implications of this problem are better explained by the Folk Theorem, Rasmusen [71].

> *In an infinitely repeated n-person game with finite action sets at each repetition, any combination of actions observed in any finite number of repetitions is the unique outcome of some subgame perfect equilibrium given*
>
> **Condition 1:** *The rate of time preference is zero, or positive and sufficiently small: and*
>
> **Condition 2:** *The probability that the game ends at any repetition is zero, or positive and sufficiently small; and*
>
> **Condition 3 (Dimensionality):** *The set of pay-off combinations that strictly Pareto-dominate the minimax payoff combinations in the mixed extension of the one-shot game is n-dimensional.*

This may have many implications for the study of conflict. If participants perceive the conflict as an infinitely repeated game, the expectation of future gains may encourage players to take some costly actions in the present. It can be also used to explain the origins of cooperation since the fear of future retaliation can deter a player from using other than cooperative strategies.[27]

2.4.4 Bargaining with cost

One of the main characteristics of conflict or war is that it entails very high costs. Rubinstein refers to two sub-families of models in his' original model of bargaining.

[27] We have already mentioned some papers where punishing greedy players can be viewed as an evolutionary stable strategy.

1. *Fixed bargaining cost*: i's preference is derived from the function $y - c_i t$, i.e. every player bears a fixed cost for each period.

2. *Fixed discounted factor*: i's preference is derived from the function $y \cdot \delta_i^t$, i.e. every player has a fixed discounting factor.

Most developments in bargaining don't include the cost per period. These models of bargaining have normally one main type of cost which is the cost of delaying agreement. War could be understood as a costly negotiation process. In this case the Coasian properties of these models would cease to work. We have also seen in the previous models that Pareto-inefficient outcomes are possible. We would expect (as will be shown in following chapters) that introducing a new type of cost would affect these strategies.

Accounting for the cost of conflict is a task that has largely escaped to economists. It is difficult to account empirically for all the factors that can be classified as absolute or relative; direct or indirect. Accounting for the indirect cost of war is the most problematic issue. During war, the population suffers not only from casualties in battle but also from diseases during mobilisation, war-induced famine, epidemic and decline of birth rate. There is a direct destruction of capital and labour by military action, destruction of markets and foreign trade, displacement of resource, hysteresis and other supply side effects. Finally there is a loss of public liberties and an accumulation of power in the government that may have long term effects in the internal politics.

However, this problem shouldn't be difficult to solve in theory. We are going to review a couple of models that have introduced cost in a bargaining game scenario.

Bargaining and Destructive Power

Bargaining procedures tend to ignore the possibility that participants may take actions to affect the bargaining by destroying part of the feasible set. Dasgupta and Maskin [72] explore the destruction power in a model of bargaining between a union and a firm, in which each party is capable of inflicting some damage to the other party. The workers can neglect the firm's equipment and in turn, the firm can replace its technology for a technology less favourable to the workers. This is an interesting point. In damaging the other party, one does not need to damage oneself. Conflict can take many shapes, and the technology of conflict should take into account this kind of destructive power. Unfortunately, most part of the research in conflict success functions has been directed to the formulation of some formal relations between inputs into conflict activities and output expressed as a probability of winning. The ability to direct the cost asymmetrically has been neglected by most part of the literature on 'contest success functions'.

The model of Dasgupta and Maskin considers a union of workers and a firm have to negotiate over hours of work (L) and wages (W). The vN-M utility function for the union is:

$$U = W^{\frac{1}{2}}(1-L)^{\frac{1}{2}} \text{ where } 0 \leq L \leq 1 \text{ and } W \geq 0,$$

and the firm utility is:

$$V = \pi - W$$

Where π is the firm's revenue. It is determined by labour hours and installed capital ($\overline{K}$) such that

$$\pi = \begin{cases} L \text{ if } L \leq \overline{K} \\ \overline{K} \text{ if } L > \overline{K} \end{cases}$$

where $\overline{K}$ is given initially. If $\overline{K} \geq 1$, the set of efficient pairs (U, V) is given by the straight line

$$V = 1 - 2U$$

If no agreement is reached after the negotiations between the union and the firm, each party earns zero utility. This is the threat point which is represented at the origin of figure 2.5. The horizontal axis corresponds to the utility of the union (U) and the vertical axis to the firm (V). The bargaining set is delimited by the straight line **(A-B)**.

Both the union and the firm, can engage in destructive activities that affect the bargaining set by deleting a part of it, where it is more favourable to the other side. For example, by not maintaining equipment, the value of $\overline{K}$ can fall to a level where $L > \overline{K}$. The result of this action is the deletion of the top corner of the bargaining set in figure 2.5. The firm can also engage in similar activities by replacing its technology which will delete an area favourable to the union. The bargaining set reduces and the new efficient frontier is delimited now by the line **(C-D)**.

Call R_0 the initial bargaining set delimited by $v = 1 - 2U$, R_t the feasible set of utility pairs before move $t(t = 0, 1, 2 \ldots)$ and (u, v) a pair of utilities that belong to R_0.

The players move alternately. A move consists of deleting a portion of R_t of any size or shape up to a maximum area $\overline{\delta}$, and simultaneously proposing a point in the remaining set of utility pairs. The proposal is either accepted or rejected by the other party. Negotiations end when one of the participants accepts and offer or when the bargaining set reduces to zero utility.

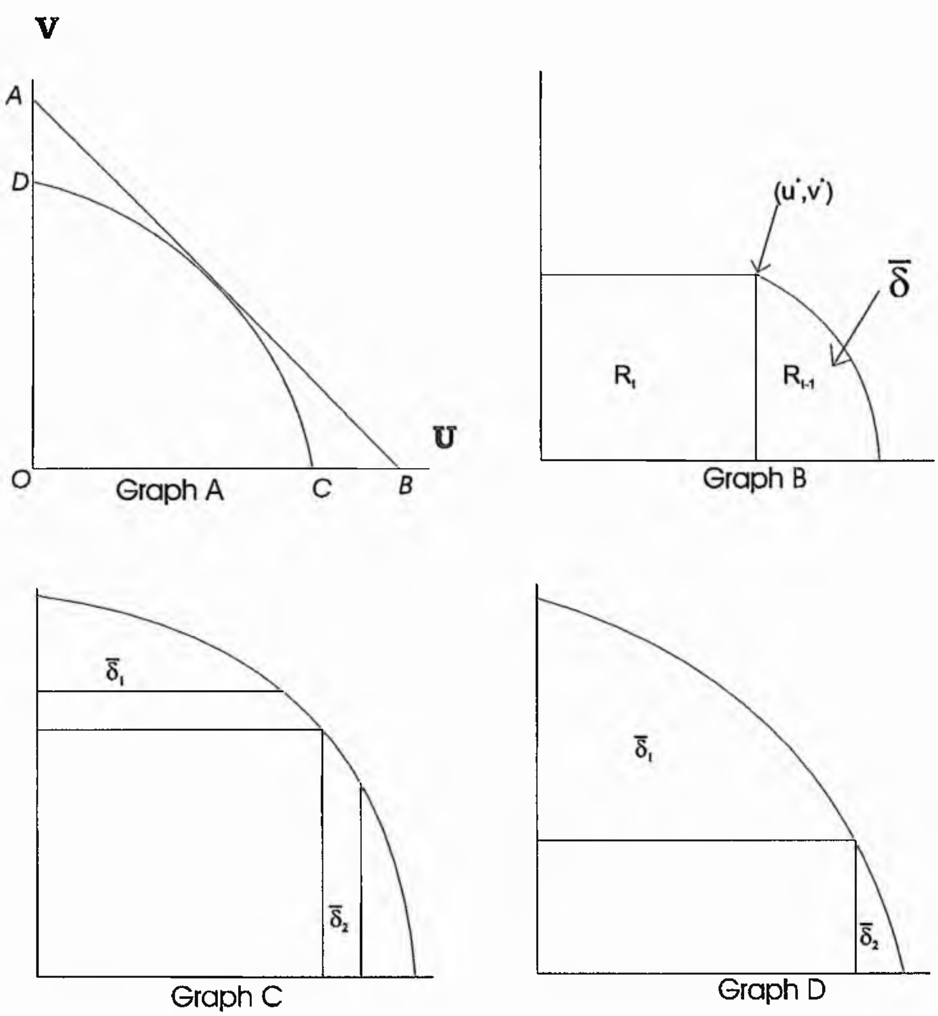
Under perfect information, this game has a unique equilibrium in $t = 0$. Moreover, when the capacity to delete a portion of the bargaining set $\overline{\delta}$ is relatively small in relation to the set, the equilibrium offer (u^*, v^*) divides R_0 in two equal areas.

Since we have seen some of the characteristics of the subgame perfect equilibrium of the strategic approach in the previous models, we turn our attention to the axiomatic development.

Given a utility representation for each participant that satisfies the von Neumann-

Figure 2.5: Bargaining with Cost.

The agreement between both participants is a point in the feasible bargaining set (Graph A) in the triangle (OAB). Both parties can reduce this set by imposing cost into the other side. This changes the efficient set from (AB) to (DC). Participants can take a portion up to $\overline{\delta}$ of the bargaining set in each successive round. The bargaining set reduces to the rectangle R_T in Graph B where R_{T-1} is the last deletion. The bargaining equilibrium is at the top right hand side corner of the set, (u^*, v^*). In Graph C we present a symmetric solution where $\overline{\delta}_1 = \overline{\delta}_2$. If participants have different deletion power $\overline{\delta}_1 > \overline{\delta}_2$ the solution would depend on the relative power of each participant (Graph D).



Morgensten axioms [28], let (u_0, v_0) be the status quo or threat point and a feasible set of utility pairs called R. The solution will be a function F of $[R,\ (u_0, v_0)]$ that gives a point in R.

$$F[R,\ (u_0, v_0)] = (\overline{u}, \overline{v}).$$

This solution requires several axioms. We have seen already in section 2.4.2 the criteria that should be satisfied in order to guarantee the existence of a Nash bargaining solution. In this model of bargaining with cost, the third axiom – also called the *independence of irrelevant alternatives* – is substituted by another axiom called the *deletion axiom.*

The four axioms in a model with costly conflict that guarantee the existence of a Nash equilibrium are as follows:

A1 Invariance: Let $[R,\ (u_0, v_0)]$ and $[R',\ (u_0', v_0')]$ be two versions of the same bargaining game; that is, they differ only in the units and origins of the utility function. Then $[R,\ (u_0, v_0)]$ and $[R',\ (u_0', v_0')]$ are related by the same utility transformations.

A2 Weak Pareto Efficiency: There is no $(u, v) \in R$ such that $u > \overline{u}$ and $v > \overline{v}$.

A3 Symmetry: Suppose that $[R,\ (u_0, v_0)]$ satisfies the properties:

1. $u_0 = v_0$
2. $(u, v) \in R$ if and only if $(v, u) \in R$

Then $\overline{u} = \overline{v}$.

A3 Deletion: Let $[R, (0, 0)]$ be a bargaining game. If $\overline{R}$ is what remains of R when horizontal and vertical strips[29] of equal area

[28] See appendix B.4

[29] Horizontal and vertical strips refer to a specific deletions of the bargaining set that have the properties of affecting the shape and size of the cake directing the destructive power towards the other participants.

have been deleted, then neither component of $F[\overline{R}, (0,0)]$ exceeds the corresponding component of $F[R, (0,0)]$.

The solution that satisfies axioms (A1-A4) bisects the bargaining set in two equal parts. It can be viewed as a generalisation of the conclusion of the strategic game. This suggests that there are many negotiation procedures that can support the same solution as long as the axioms are satisfied.

If the size of the cake($\overline{\delta}$) that can be deleted at any time is small enough, and there are simultaneous demands, there is a symmetrical solution. The solution is quite robust to the timing of the game. This is based on the fact that strategic deletions reduce the feasible utility set to a rectangle. This is also responsible for the uniqueness of the game. Otherwise, weak discounting wouldn't be a sufficient condition to guarantee the uniqueness of equilibrium.

Finally, on many occasions parties differ in their destructive power. The relative power to damage the interest of its rival at each player's disposal influences the outcome of the negotiation moving the equilibrium outcome toward the area that benefits the participant with more destructive power.

In Dasgupta and Maskin the destructive power is exogenous. They don't consider either the destructive activities that would damage the bargaining set by an overall reduction of its size. And finally, although the intuition is different, the bargaining procedure and its features are quite similar to Rubinstein's bargaining model. This could be eventually exploited by adapting the discounting factors of Rubinstein to account not only for impatience but for the extra resources destroyed at each period.

Markov perfect equilibrium in a model with destructive power

A condition for the use of destructive strategies by rational players is that this activity leads to increases in the expected payoffs of the players using it. In the next model, although the decision to harm is endogenous, harming power is given exogenously. This is one of the missing features of this model. As far as conflict modelling is concerned, we should also design a mechanism for the allocation of resources to the increase or decrease of destructive power. The following model applies to wage negotiations between firms and unions, where unions can reduce the profits by striking. This kind of activity does not require any investment in *appropriation technology.*[30]

Manzini [73] proposes a model of bargaining between a union and a firm over the firm's profits. The union can affect the size of those profits by engaging in destructive activities. The union commitment to a specific destructive action affects the equilibrium outcome increasing its payoff according to its harming power.

Manzini focuses on Markov strategies. These imply that that at each node a player *chooses the action independently of the history of the game except for the immediately preceding action and depend only on the state of the game.*

In this model the state of the game is defined as the size of the pie π_n after a number of deletions n. If c_n is the amount of damage that can be imposed at any n time, then $\pi_n = 1 - \sum_{i=0}^{n-1} c_{i+1}$. The state of the pie reveals how many times the union has struck in the past.

> *In a bargaining game of alternating offers in which the worker can harm after every rejection of the firm, if $c_n \leq \delta \sum_{i=0}^{\infty} c_{n+i+1} \delta^{2i} \; \forall \; n$, there exist two m.p.e, which I will call "harming equilibrium" and "Rubinstenian equilibrium", respectively, in which agreement is reached immediately either on the "harming" equilibrium partition:*

[30]Alternatively, both parties can engage in costly activities that require different levels of investment. Typically the firm could resort to bribing local police and politicians, use the Mafia to intimidate workers and run expensive advertising campaigns to influence public opinion and workers against their representatives. The union can in term set up picket lines, demonstrations and actions of sabotage. The legal framework normally prevents this kind of activity which would produce such high cost of society. However, in these thesis we have decided not to tackle situations with exogenous commitment

$$x^* = (x_w^*,\, x_f^*) = \left(\frac{1}{1+\delta} \left(1 + \delta \sum_{i=0}^{infty} c_{i+1} \delta^{2i} \right),\ \frac{\delta}{1+\delta} \left(1 - \sum_{i=0}^{infty} c_{i+1} \delta^{2i} \right) \right)$$

or on the "Rubinstenian" equilibrium partition:

$$x^* = (x_w^*,\, x_f^*) = \left(\frac{1}{1+\delta};\ \frac{\delta}{1+\delta} \right)$$

where in each partition the first entry is the share received by the union, and the second entry is the share received by the firm

There are two equilibria, In the *"Rubinstenian equilibrium"* the union chooses not to harm. However, in the *"harming equilibrium"* the union achieves its maximum payoff which is directly proportional to its harming power. The harming equilibrium must satisfy two conditions: a) rule out deadline effects and b) a credibility constraint. In order to satisfy the first condition the union can alternate any sequence of harms of any size as long as it does not exhaust the whole pie. The second condition is more restrictive since –in order to be credible– any harming structure at any time must be less than the present discounted sum of all future damage.

Given the existence of a credible harming structure, this model shows that the ability to destroy part of the pie can be exploited in a negotiation process since it increases the cost of rejection to the other participant. Compared to the bargaining model of Rubinstein, the harming power acts as if we increase the other participant's rate of time preference. Compared to repeated bargaining games, which predict a continuum of equilibria, this models produces only two equilibria.

This shows how the ability to inflict cost on the other participant affects bargaining power. This should be taken into consideration when looking at models of rent seeking activities. That is to say, given that the ability to damage the size of the pie produces some advantage in bargaining, how much power should we have, and how many resources should we dedicate to it? These are the questions that we will try to answer in the next chapter.

2.5 Conclusion

In this chapter we have explained the background theories that constitute the main focus of this thesis. That it to say, the allocation of resources to conflict (destructive and non productive activities) and the negotiation process that takes place in order to distribute these resources.

In section 2.2 we have presented the main strategic considerations and principles guiding behaviour in conflict. In section 2.3, we looked at the allocation of resources to non-productive activities and in section 2.4, we studied models of bargaining and negotiation. The papers reviewed are found mainly in the literature of political economy, public choice and game theory. Although this is not an exhaustive review of the latest literature in these fields, when brought together, they lay the foundations for the theories of conflict that we are going to present in the next chapters.

Chapter 3

Conflict with complete information

Game theory models have been used to explain the strategic interactions of two countries involved in arms races and other strategic situations. They have also been used in political science to explain the causes and determinants of war. However, in most part of these studies, security doesn't have micro foundations. It depends directly upon the amount of weapons or military capability that each country in dispute have. For example, In Levine and Smith [74] security for a single country depends directly on the excess of weapons that this country has in relation to what is the minimum amount needed to successfully defend from a potential attack. Combining both security functions with the respective budget constraints for each country, we can draw a feasible mutual security region. Two reactions functions can also be drawn. Those functions are derived from the optimal allocation between consumption and arms given the budget constraint and the security functions that we have already mentioned. If the resulting equilibrium lays on the security region there is a peaceful outcome.

However, although security is one of the key factors in conflict analysis, it will be a concept of marginal relevance in our approach because the decision to fight will be derived endogenously.

In this chapter we examine the strategic interactions of two countries that try to maximise their respective incomes and the subsequent equilibrium of resource allocation between conflictive and productive activities. We introduce bargaining theory into a well know framework of conflict whose initial development was produced by Haavelmo [75] and has been further developed by the work of Hirshleifer [44] and Skaperdas [45, 46].[1]

Assume that two countries are confronted by a basic incompatibility which is the partition of a desirable scarce or limited resource. In order to have some understanding of the strategic interactions of those two agents other assumptions about the characteristics of the conflict must be made.

We shall concentrate in the economic aspects of conflict in order to simplify our models. Assume that the contesting states, despite being subject to internal struggles react in a uniform way under external threat.[2]

Every game will have some basic elements. The first element is **the players.** We are going to refer to independent countries not involve in any sort of alliances. The fundamental assumption about the players in a game concerns rationality and common knowledge. Rationality in this context means that players maximise their utility and all players are aware of the rationality of the others. In many economic models of conflict this utility is given in terms of income or consumption only. For some models of arms races utility depends on consumption and another variable called security. This is a perfect example of how those assumptions can be modified in order to explain some form of conflict. We can extend our model introducing many other variables such as power, glory or revenge into our utility function. However, since it is our purpose is to study the general strategic considerations of a model of conflict, it is better to keep the assumptions as simple as possible. Our countries will only be concerned by maximisation of

[1]See chapter 2.2.

[2]For a model of internal and external linkages see Simon and Starr [76]

their respective incomes.

The **number of players** in a conflict is normally two. Game theory models can cope with more than two players, however empirical evidence suggests that before a conflict takes place, there is always a complex system of alliance formation that typically produces a two player situation.

The second element of a game is the **strategy space**. Players facing different strategies choose one that strictly dominates the others. In a model of conflict the most basic strategy space consists of a war strategy and/or a deterrence strategy. This is a principle by which players allocate resources into the production of goods and services and military procurement.

The third element is a **payoff function**. This function gives a utility value for each combination of strategies.

Finally, there is the **environment** and structure of the game. Games can be one-shot game played at the same time or not. There can also be different moves before the payoffs take place and it can have a dynamic structure or be a repetition of the same game. A very important element of the environment is the **information set**. This is the knowledge available to each player.

Different combinations of these elements will result in a **equilibrium outcome.**

According to the kind of environment game theory models have been classified as:

- Static games of complete information
- Dynamic games of complete information
- Static games of incomplete information
- Dynamic games of incomplete information.

Most part of the literature describes conflict as a contest in which non pre-play agreement can be taken. Therefore some of these games are also described as non-cooperative games. In this article we will not distinguish between cooperative or non cooperative situations. We will use, instead, the concept of cooperation developed in Skaperdas work. We start with a non cooperative situation and find the necessary conditions that make cooperation the optimal strategy.

Information plays a crucial role in conflict. Due to the assumption that player's utility comes only from income maximisation, in a game of complete information, there is no possibility of war since it is dominated by any negotiated settlement. The rationale is that war imposes such a high cost in the loosing side that it would be better to surrender and obtain some income that the winning side will give away in order to also avoid its cost. However, we can always construct a model similar to the Prisoner's Dilemma where given the payoffs and strategies, the equilibrium outcome is to fight even if both players could gain from committing themselves to a peace strategy. It is important to understand the distinction between the concept of equilibrium of a game, when both players have no incentive to deviate from their strategies, and crisis stability. These crises can have a variety of forms such as war and arms races which might be the equilibrium outcome of a game.

3.0.1 Skaperdas concept of full cooperation

Assume that players do not have any specific structure to coordinate their play or in other words, there are no pre-play binding agreements. But before strategies are chosen players may communicate costlessly. In many cases, it is rational for players to cooperate by coming to an agreement. This is one of the main features of conflict in human societies[3] .

Skaperdas [45, 49] shows that cooperation is possible in a model of resource allocation in the absence of property rights. The main aspect of this model is that it doesn't assume any specific functional form for conflict technology.

[3] See Ridley [77] or Axelrod [20].

Skaperdas restricts his model to the analysis of the signs of the derivatives of conflict technology and production technology. In that situation the result can be applied to many different circumstances.

In his model we have two player that must divide a unit of resource between production and contest.

$$1 = x_1 + y_1 \quad \text{and} \quad 1 = x_2 + y_2 \ ;$$

The resources dedicated to productive activities and to conflict are respectively, x_i and y_i. By allocating more resources to conflict we might increase the probability of winning, but we are also decreasing the amount produced with those resources. Therefore, the payoffs are a function of common production and probabilities of winning:

For player 1;

$$V^1(y_1, y_2) = p((y_1, y_2)C(1 - y_1, 1 - y_2)$$

and player 2;

$$V^2(y_1, y_2) = [1 - p((y_1, y_2)]C(1 - y_1, 1 - y_2)$$

where $p(\cdot)$ is the appropriation technology and $C(\cdot)$ is the production technology. The production function has constant returns to scale and is twice differentiable. The signs of the derivatives for production function are:

$$C = C(x_1, x_2); \ c_i > 0; \ c_{ii} < 0; \ \frac{c_1}{c_2} \neq 0 \text{ and finite.}$$

The context success function gives the probability of winning related to players respective fighting efforts.

$$p(y_1, y_2) + p(y_2, y_1) = 1 \quad \text{or} \quad p(y_1, y_2) = 1 - p(y_2, y_1)$$

where p is the probability of success. And p_1 and p_{11} are respectively the first and second order derivatives of the winning probabilities of player 1.

$$p_1(y_1, y_2) = -p_2(y_2, y_1);$$
$$p_{11}(y_1, y_2) = -p_{22}(y_2, y_1);$$
$$p_{12}(y_1, y_2) = -p_{12}(y_2, y_1);$$

There are some necessary assumptions about the signs of the derivatives for the existence of a unique equilibrium other than $(y_1, y_2) = (1, 1)$, which means that players spend all their resources in fighting. Those assumptions are based on the following form that the first and second derivatives should take:

1) $0 < p_i < \infty$ and $-\infty < p_2 < 0;$

2) $p_{11} > 0$ as $y_1 \leq y_2$ and $p_1 1 \leq 0$ as $y_1 \leq 0$

3) $p_{11} < p_1^2$

4) $p(1-p)p_{12} + (2p-1)p_1 p_2 = 0\ ;$

This not only guarantee the uniqueness of equilibrium but also is consistent with some conventional beliefs about the importance of strategic advantage in various contexts. For example, the winning probability increases in each player's own strategy and decreases in the other player's strategy. Also, it is easy to increase one's power when is lower than the opponent's and more difficult otherwise.

A point of full cooperation happens whenever $s^*(y_1, y_2) = (0, 0)$, is the optimal strategy. It means that neither party allocates resources to conflict technology. By maximisation of the payoff function he derives the following condition for full cooperation or in another words the condition for a non armed equilibrium,

$$\frac{2p_1(0,0)}{1-2p_1(0,0)} \leq \frac{C_1(0,0)}{C_2(0,0)} \leq \frac{1-2p_1(0,0)}{2p_1(0,0)}$$

This condition states that a non armed equilibrium is more plausible the closer the ratio of marginal products of two countries and the more inefficient is the conflict technology.

In many cooperative games it is taken for granted that players can write whatever contracts they choose and that these contracts will be totally binding on the players (this is a basic condition for a cooperative game). In International Relations there are only a few cases in which such contracts may take place. For instance, a hegemonic country may take the function of sanctioning and monitoring those agreements. For this article we assume that countries do not have a precommitment mechanism, these contracts do not take place, and therefore we use the idea of cooperation suggested by Skaperdas.

3.0.2 The model of Hirshleifer as a long run equilibrium model

One of the most simple and at the same time elucidating models of conflict is Hirshleifer 1995. He introduces a framework to model conflict with micro foundations and specific functional forms.

We have already said that games consist of some basic elements. Following this classification Hirshleifer presents the following analytical choice: There are two players, Country 1 and Country 2, each contesting a common resource that it is normalised to unity. The decision facing each country is how much of that resource should they convert into fighting effort and how much into productive effort. The difference in fighting effort determines the probability of each party winning the conflict. The function that relates fighting effort to the probability of winning the conflict is called the contest success function (CSF). This is central in an economic theory of conflict. The CSF together with the production technology generate the set of payoffs from which the players make their choices. In order to maximise income we have to decide how many resources we dedicate to production and how many to conflict.

In a more general framework the optimising strategies can be of two kinds: A war strategy, in which we have a competitive advantage from maximising our fighting effort, or a deterrence strategy in which we maximise the production given a security constraint. In equilibrium the marginal profit of capital investment and the marginal profit of fighting effort have to be equal to their marginal costs. This is the utility maximising condition when utility is only generated by income.

According to Hirshleifer there are only two forms of CSFs. In the first one the outcome depends upon the ratio of fighting efforts.

$$\frac{p_1}{p_2} = \left(\frac{F_1}{F_2}\right)^m$$

i.e., since $p_1 + p_2 = 1$, we have:

$$p_1 = \frac{F_1^m}{F_1^m + F_2^m} \quad \text{and} \quad p_2 = \frac{F_2^m}{F_1^m + F_2^m} \tag{3.1}$$

Equivalently, in the second form of CSF, the outcome depends upon the difference

$$p_1 = \frac{1}{1 + \exp[k(F_2 - F_1)]} \quad \text{and} \quad p_2 = \frac{1}{1 + \exp[k(F_1 - F_2)]} \tag{3.2}$$

This can be interpreted as a resource control rule. The proportions of a common resource that each party achieves is given by p_1 and p_2 . The aggregate resource is $R = R_1 + R_2$ and the resource partition equation is $R_i = p_i R$. Therefore, each contender divides the available resources between fighting and productive effort $R_i = a_i E_i + b_i F_i$

The a_i and b_i can be interpreted as unit conversion cost (assumed constant) of transforming resources into productive effort or into fighting effort, respectively. Hirshleifer (1995) works out the fighting and productive intensities using the first form of CSFs.

Normalising with respect to resources.

$$e_i = \frac{E_i}{R_i}, \quad f_i = \frac{F_i}{R_i} \tag{3.3}$$

Using first form of CSF, then from 3.1 and 3.3 and $R_i = p_i R$ we have:

$$\frac{p_1}{p_2} = \left(\frac{f_1}{f_2}\frac{R_1}{R_2}\right)^m = \left(\frac{f_1}{f_2}\frac{p_1}{p_2}\right)^m$$

$$\frac{p_1}{p_2} = \left(\frac{f_1}{f_2}\right)^{m/(1-m)} \quad \text{or} \quad p_1 = \frac{f_1^M}{f_1^M + f_2^M} \tag{3.4}$$

where, for convenience, define $m/(1-m) = M$.

The player i chooses the fighting and producing intensities in order to maximise income Y, given the production function,

$$Y_i = E_i^h = (e_i R_i)^h; \quad h \leq 1 \tag{3.5}$$

and subject to

$$a_i e_i + b_i f_i = 1 \tag{3.6}$$

For Country 1 we then have:

$$\max Y_1 = E_1^h = (e_1 R_1)^h = (e_1 p_1 R)^h = \left(\frac{e_1 R f_i^M}{f_1^M + f_2^M}\right)^h \tag{3.7}$$

The reaction curve for Country 2 is calculated in the same way. Assuming that $h = 1$ (constant returns to scale), the first order conditions for this optimization problem lead to:

$$\text{Reaction Curve for Country 1 } RC_1 = \frac{f_1^M}{f_2^M} = \frac{M}{b_1 f_1} - (M+1)$$
$$\text{Reaction Curve for Country 2 } RC_2 = \frac{f_2^M}{f_1^M} = \frac{M}{b_2 f_2} - (M+1) \qquad (3.8)$$

The solution of f_1 and f_2 for a symmetrical conflict equilibrium is :

$$f_1 = f_2 = \frac{M}{b(M+2)} = \frac{m}{b(2-m)}. \qquad (3.9)$$

and

$$e_1 = e + 2 = \frac{1}{a}(1 - bf) = \frac{1}{a}\left(1 - \frac{m}{2-m}\right) = \frac{2}{a}\left(\frac{1-m}{2-m}\right) \qquad (3.10)$$

In the symmetrical conflict both probabilities are equal to a 1/2. Direct substitution leads to:

$$Y_i = (e_i p_i R)^h = \left(\frac{1-m}{a(2-m)} R\right)^h \qquad (3.11)$$

From the result of equation 3.9 we conclude that the fighting equilibrium depends upon two parameters m and b. For an interior stable equilibrium the values of m must be between $0 < m < 1$. The larger is the decisiveness coefficient and the smaller is the logistics cost coefficient the bigger are the fighting intensities. An increase in technology that affects both a, the production cost coefficient and b, the logistic coefficient produces a higher income and a higher fighting intensity.

There are two important assumptions. The first one is that resources are constant. And the second one is that there is perfect information so conflict never takes place.[4] Rather than explaining conflict, this model explain which are the necessary but not sufficient conditions for conflict not to occur. If we relax

[4]Hirshleifer establishes two sufficient but not necessary conditions for sustainability. The first one is dynamic stability: $m < 1$. The second one is the minimum amount of resources required to sustain life or integrity for an individual or a group.: $Y_i \geq y,\ i = 1, 2$

those assumptions - constant resources and perfect information - we will have to deal with situations in which actual fighting occurs. According to Hirshleifer fighting takes place in three circumstances.

First, whenever we have some utility in conflict. This can be a very realistic situation, nevertheless, more appealing to sociological or political science than to economist. Second, conflict may arise when there is incomplete or imperfect information about the pay-offs of the game. This is a very important case and we will dedicate to it a specific chapter of this article.

The third circumstance happens where there is a possibility of conflict arising from disharmonious opportunities . For instance when no sharing is allowed, the winning player takes the whole prize, and therefore there can be more incentives for war. There is nothing intrinsic in the structure of the game that induces to believe that this is the case. However we can always build a model of a specific game contemplating this kind of scenario.

The equilibrium outcome produced in Hirshleifer's model can be understood as the steady state or long run equilibrium of allocation of resources between fighting and productive efforts. We are going from now on to be concerned with what happens in the short run, before a given allocation between fighting effort and productive effort can be altered.

Hirshleifer tries to describe a non-cooperative game without formalising the negotiation procedure. This is our starting point for our model of bargaining. We use the methodology developed by Rubinstein [62] and Binmore [65].

3.1 Bargaining

Our model of conflict is a development of these models of Hirshleifer and Skaperdas describing the optimal allocation of resources between fighting a producing activities. However, we make a distinction between two different situations. The first case is war. Any party can start a war without a previous agreement with the other party. But this scenario produces a high cost. (We also incorporate cost in a different way). The second scenario is agreement which requires a bargaining mechanism.

We tried to introduce bargaining theory into conflict in the simplest possible manner. In order to simplify we have to introduce some assumptions. Once the allocation of resources is produced, this creates a commitment so during the bargaining period parties cannot change this allocation. We assume that resources cannot be transfer from production to conflict technology once they have been allocated. Given this irrevocable allocation, the Nash static equilibrium provides a solution for this game. The situation is irrevocable until the next game.

Another element of our bargaining process is that agreement may take some time. (In some cases agreement may never take place). Neither Hirshleifer nor Skaperdas contemplate this possibility. But there is some historical evidence that in the case of war there are some previous negotiations. It is very normal that before war each country sends its ambassadors just before hostilities begin. There are also a number of spontaneous wars, generally more often in the case of internal war. But we do not know of two countries agreeing to share resources without a negotiation process. Diplomacy is a discipline that has been developed with the specific purpose of dealing with this issue in a rational manner.

Hirshleifer and Skaperdas never explain how the negotiation process takes place. For those authors, the winner of the context receives the total product as prize, or both players divide the prize according to their respective probabilities of winning.

In order to introduce a model of bargaining, we assume that there is a difference between winning the whole pie with probability "p" and taking a part of it that corresponds to the proportion expressed by the probability of winning. The reason is that to get a partition of the pie, agreement of both parties is necessary. A party can delay agreement as long as he pleases. On the contrary, in order to get the whole pie with probability "p", agreement is not required. However the pie reduces in size due to the consequences of fighting. The reduction is mainly due to a externality cost imposed by fighting.

In a model of bargaining conflict the two players must agree to share the unit of resource as in the previous models . In periods 1,3,5,... player one proposes shares $(s_1, 1 - s_1)$, $(s_3, 1 - s_3) \ldots$ and player two can accept or reject. In the second period player two makes and alternative offer. The game continues until one of the players accepts one offer.

Following Binmore, Rubinstein and Wolinsky [63], we think of two basic motives that may induce parties to reach an agreement instead of bargaining indefinitely. The first is that parties are impatient to reach the fruits of an agreement. So they prefer to have a given share now, rather than in next period. Each player has to weight the consequences of waiting for a possible better offer against accepting the existing one now.

The second motive for an agreement is that parties might prolong the period of negotiations indefinitely. It might be impossible to reach and agreement at all. In that case the unit of resource could be either enjoy by a third player or players might end up loosing the opportunity of jointly exploit the resource . This is a situation where there is a exogenous risk of breakdown of negotiations.

We are going to present a model of bargaining with fixed externality cost of conflict and a utility function that reflects preferences over time.

3.1.1 Time preference model

In the time preference model we construct a static problem that reflects a bargaining situation. The status quo or threat point correspond with Hirshleifer model. But we have a choice of utility functions u_1 and u_2 that reflects countries impatience. The set of possible agreements is given by

$$X = \{(s_1, s_2) | s_1, s_2 \geq 0,\ s_1 + s_2 \leq 1\}.$$

This game cannot be represented only by the choice of X. We need some utility function with extra information about the preferences (impatience or risk aversion). For instance, in a international environment where countries have considerable ongoing economic transactions conflict might also stop those activities. This might be a reason to be impatient. Later we will see how a country that has an economic advantage also has a stronger bargaining position by exploiting the others country's time preference.[5]

$$S = \{(u_1(s_1), u_2(s_2)) : s \in X\}$$

We can specify the preferences over agreements and their timing. Each player can have a discount factor . The next task is to define the players bargaining strategies that take place in a determined way. A strategy f for a country is a sequence of rules where each rule ft describes each player's move at time t. Moves are made at points in time $t = 1, 2, 3 \ldots n$. As we mentioned above at any period $1, 3, 5 \ldots$ player 1 makes the offer so player 1 is starting the game.

The strategy might also depend on the entire history of the game up to period t. In period 1 player one offers a share of the pie $(s_1, 1 - s_1)$, Player two might accept or reject. In period two player two makes an offer and so on.

Finally, there are also a basic set of assumptions for the existence of a unique equilibrium:

1. There is a conflict of interest
2. There are mutually beneficial agreements
3. There are time indifferent agreements. For each player exists an agreement that represents the status quo agreement

[5]Note that in this model countries preferences are neutral towards other countries. For friend or enemies, the preferences could be represented as $S = \{(u_1(s_1, s_2), u_2(s_2, s_1)) : s \in X\}$

4. There is stationarity and monotonicity in time

5. Compensation is concave.

First, we are going to establish an arbitrary time limit of three periods. This idea of the time limit can have two alternative explanations. The first one is that there is a time preference frontier. This frontier could be established by discounting the value of expected income of a negotiated outcome up to the value of the expected income in case of fighting. Parties cannot expect to negotiate so long that they loose more utility than the externality cost of war.

A more simple approach is to think of an exogenous risk of war. Countries can look at past history and calculate the probability of war. We suppose that bargaining takes place for three periods with a settlement imposed by the respective probabilities of winning the war. Remember that dedicating resources to war is a long term commitment. If the bargaining period is long enough to change this commitment we will have to model another game.

The general model of bargaining by Rubinstein will take the following form:

Period three settlement $(s_1, 1-s_1)$

Period two. Country 2 offers a settlement $(s_2, 1-s_2)$. For player 1 s_2 is acceptable iff $s_2 \geq \delta s_1$, where δ is the discounting factor. So the equilibrium settlement for period 2 is: $(s_2^*, 1-s_2^*) = (\delta s_1, 1-\delta s_1)$.

Period one. Player 1 offers a settlement $(s, 1-s)$. For player 2 $(1-s)$ is acceptable iff:

$$1 - s \geq \delta(1 - s_2^*) = \delta(1 - \delta s_1)$$

Then for three periods:

$$s^* = 1 - \delta + \delta^2 s_1 \tag{3.12}$$

So far there is no exogenous probability of breakdown. Therefore, we can extend the model to an infinity bargaining process.

For player 1

$$s_1^* = \frac{1}{1+\delta};\ 1 - s_1^* = \frac{\delta}{1+\delta}. \tag{3.13}$$

The final partition of the pie is independent of the fighting effort.

Does it mean that players should not expend any resources in fighting? The discounted value of getting s_1 at any time cannot be smaller than the expected value of a war strategy. It is this assumption that means conflict will not take place.

The problem now is that the solution is not a Nash bargaining solution. If player 1 does not invest any resources into fighting, player two can improve his expected payoff by building up an army and making a war that he will win for sure, getting the whole pie as a prize. Realising this problem, player one must invest some resources into fighting effort in order to prevent player two from attacking.

Consider, now the 3 period bargaining model followed by a conflict similar to the one explain by Hirshleifer. This model is more realistic because it encompasses two kind of models, on the one hand we have a period of negotiations, if a negotiated settlement does not take place, then , we will have to divide the resources by going into war.

This is a simplifying assumption. It can be argued that countries set up their expected last period of bargaining based on past history for similar conflicts. Gleditsch and Hegre [78] constructed a table of probabilities of breakdown of negotiations between different types of regimes based on the percentage of dyad years at war. [6]

Those correspond in our model to the probabilities of breakdown in a single period providing that we normalise the length of the period to be equal to a year.

[6] We will review in the last chapter some models for estimating these probabilities

The expected length of the bargaining would be equal to :

$$\sum_{\tau=0}^{\infty} \tau(1-q)^{\tau-1}q = \frac{1}{q} \tag{3.14}$$

for simplicity we assume that the probability of breakdown, q, is 1/3, and therefore the expected length of bargaining is 3 periods.[7]

The sequence of events is:

- Period 0: Players commit resources to fighting and productive efforts. (E_1, F_1) and (E_2, F_2) independently.
- Period 1: Player 1 proposes shares s_1 and $1 - s_1$ for herself and player 2 respectively . Player 2 either accepts ending the game or rejects.
- Period 2: Player 2 proposes s_2 and $1 - s_2$. Player 1 accepts or rejects.
- Period 3: War takes place due to lack of agreement leading to incomes $p_1 I(1-\gamma)$ and $(1-p_1)I(1-\gamma)$.

In this case p_1 corresponds to Hirshleifer Contest Success Function 3.1 for player 1. The externality cost, γ, is expressed as a proportion of resources I destroyed by fighting. There reason of calling it a externality cost is that normally wars are not only problematic because of material destruction, but also because the human tragedy they bring about. Therefore we use Hirshleifer (1996) context success functions, production function and resource constraint.

$$I = A\left(E_1^{1/\alpha} + E_2^{1/\alpha}\right)^{\alpha} \tag{3.15}$$

Comparing to equation 3.5 income is jointly generated and there is no cost of trade or transactions. This function exhibit 'constant returns to scale' and contains

[7]We could make the model more realistic by using the probabilities of real conflict calculated by Gleditsch and Hegre

other well known production functions according to the values of the parameter α.[8] The contest success function is:

$$p_1 = \frac{1}{1 + \left(\frac{F_2}{F_1}\right)^m} \tag{3.16}$$

and the resource constraint: $R_i = E_i + F_i$; $i = 1, 2$ Where F_1 and F_2 are military expenditure and E_1 and E_2 production effort.

The model is solved by backwards induction ;

Period 3: Expected outcome imposed by the probability of winning the conflict.

$$s_1 = p_1 I(1-\gamma) \text{ and } s_2 = p_2 I(1-\gamma)$$

Period 2: Player 1 accepts $s_2 I$ iff

$$s_2 I \geq \delta p_1 I(1-\gamma)$$

Player 2's optimal offer is $s_2^* = \delta p_1 I(1-\gamma)$ iff

$$1 - s_2^* \geq \delta(1-p_1)(1-\gamma)$$

Period 1: Player 2 accepts $1 - s_1$ iff

$$1 - s_1 \geq \delta(1 - s_2^*)$$

$$s_1^* = 1 - \delta(1 - s_2^*) = 1 - \delta(1 - \delta p_1(1-\gamma))$$

Therefore the settlement is $(s_1^*, 1 - s_1^*)$.

In period 0 player 1 will commit some levels of E_1 and F_1 that maximises $s_1^* I$

[8]In the range $-\infty \leq \frac{1}{\alpha} \leq 0$, as $\frac{1}{\alpha}$ approaches $-\infty$ the production function get closer to the Leontief technology. In the case that $\frac{1}{\alpha}$ approaches 0 the production function gets closer to a Cobb-Douglas technology. For most part of the examples we will assume that $\alpha = 1$, so both inputs of production (E_1, E_2) are perfect substitutes.

. We now derive the first order conditions for players 1 and 2. Both players try to maximize their income given the probability of winning the conflict p_i and the resource constraint R_i.

$$\max_{w.r.t.E_1,F_1} s_1^* I = (1-\delta)A\left(E_1^{1/\alpha}+E_2^{1/\alpha}\right)^\alpha + \delta^2(1-\gamma)\frac{1}{1+\left(\frac{F_2}{F_1}\right)^m}A\left(E_1^{1/\alpha}+E_2^{1/\alpha}\right)^\alpha + \lambda(R_1-F_1-E_1) \tag{3.17}$$

$$\max_{w.r.t.E_2,F_2} 1-s_1^* I = \delta A\left(E_1^{1/\alpha}+E_2^{1/\alpha}\right)^\alpha - \delta^2(1-\gamma)\frac{1}{1+\left(\frac{F_2}{F_1}\right)^m}A\left(E_1^{1/\alpha}+E_2^{1/\alpha}\right)^\alpha + \lambda(R_2-F_2-E_2) \tag{3.18}$$

which we can solve for the reaction curves RC_1 and RC_2

$$Rc_1 = \frac{F_1}{F_2^m} = \frac{mE_1\left(1+\left(\frac{E_2}{E_1}\right)^{1/\alpha}\right)F_1^m}{\left[\frac{1-\delta}{\delta^2(1-\gamma)}(F_1^m+F_2^m)+F_1^m\right](F_1^m+F_2^m)} \tag{3.19}$$

$$Rc_2 = \frac{F_2}{F_1^m} = \frac{m\delta(1-\gamma)E_2\left(1+\left(\frac{E_1}{E_2}\right)^{1/\alpha}\right)F_2^m}{\left[(F_1^m+F_2^m)-F_1^m(\delta(1-\gamma))\right](F_1^m+F_2^m)} \tag{3.20}$$

There are no analytical solutions for the asymmetrical case. We can obtain an analytical solution only for the symmetric case . This requires some prior assumptions. Rubinstein model of bargaining is asymmetric in nature. The player moving first has always some advantage. Therefore, even if initial resources are equal, the outcome will always be different.

In order to arrive to an analytical solution some kind of symmetry must be introduced. This is done by assuming that both parties have equal chances of starting the game. In this case player 1 maximises:

$$\frac{1}{2}s_1^* + \frac{1}{2}(1-s_1^*)$$

Then, we get two identical reaction curves with the following form:

$$F_1 = F_2 = \frac{1}{2} mE \left[\frac{\delta^2(1-\gamma)}{1(1-\delta)+\delta^2(1-\gamma)} + \frac{\delta(1-\gamma)}{2-\delta(1-\gamma)} \right] \tag{3.21}$$

where $E = E_1 = E_2$. Lets call for simplicity

$$\left[\frac{\delta^2(1-\gamma)}{1(1-\delta)+\delta^2(1-\gamma)} + \frac{\delta(1-\gamma)}{2-\delta(1-\gamma)} \right] = \eta$$

Then, putting $E = R - F$,

$$F = \frac{mR\eta}{2+m\eta}$$

You can easily check that if we make the time preference parameter $\delta = 1$ and the ratio of resources destroyed by conflict $\gamma = 0$, we get the same solution as Hirshleifer(1996). i.e., $\eta = 2$ and

$$F = \frac{mR}{1+m}$$

Also we can do some analysis of the influence of δ and γ. Taking the derivatives of F with respect to both parameters we get

$$\frac{\partial F}{\partial \delta} = \frac{4mR}{(4+2m\eta)^2} \left[\frac{2}{[2(1-\delta)+\delta^2(1-\gamma)]^2} + \frac{2}{[2-\delta(1-\gamma)]^2} \right]$$

which is > 0, always positive

$$\frac{\partial F}{\partial \gamma} = \frac{4mR}{(4+2m\eta)^2} \left[\frac{-2\delta^2(1-\delta)}{[2(1-\delta)+\delta^2(1-\gamma)]^2} + \frac{-2\delta}{[2-\delta(1-\gamma)]^2} \right]$$

which is < 0, always negative

A very simple conclusion of this model can easily be explained from the symmetrical case: the smaller is the discounting factor, the smaller the expenditure in arms. In other words, the more we value future consumption the more we invest in weapons. And also the higher is the fixed cost of war the lower is expenditure in fighting power. Thus, military expenditure has many similarities with capital investment. We do not get utility out of it, but we can get future consumption.

In this model we have introduced a fixed cost of fighting and a time preference mechanism that enforces agreement. If there is no time preference, people would be bargaining forever.

Now we turn to the asymmetric case, where one of the players enjoys the advantage of playing first. In order to consider the behaviour of the key variables, we run a few simulations using different values for a specific parameter while holding the rest of the parameters constant. These simulations use different versions of the Matlab program set out in appendix C.1.

By setting δ(the discount factor) and γ(the externality cost) equal to zero and one respectively, we produce the same Nash equilibrium as in the Hirshleifer model.

If players have 100 units of initial resources the allocation to military capability will be $F_1 = F_2 = 50$ and the corresponding income for both parties also equals 50. We report simulations varying some of the parameters in the model. We plot different values on the x axes of the following parameters: δ, the time preference, or discounting factor: m the combat decisiveness parameter, and γ the cost of conflict expressed as a proportion of resources.

Resources are expressed on the y axes. We perform simulations with symmetric resources; i.e, initial resources for player 1 = player 2 = 100, and with asymmetric resources, player 1 = 100, player 2 = 200.

The plotted values correspond to the optimal allocations in fighting F_1 and F_2, and the resulting equilibrium incomes.

Figure 3.1 shows how increases in δ, the time discounting factor, affect expenditure in arms positively and so reduce the equilibrium income of both parties. However the relative ratios of income of both players are constant.

Table 3.1: Baseline Parameter Values

Country 1	Country 2
$\alpha = 1$	
$m = 1$	
$R_1 = 100$	$R_2 = 100$
$0 < \delta_1 < 1$	$0 < \delta_2 < 1$
$\gamma = 0.2$	$\gamma = 0.2$

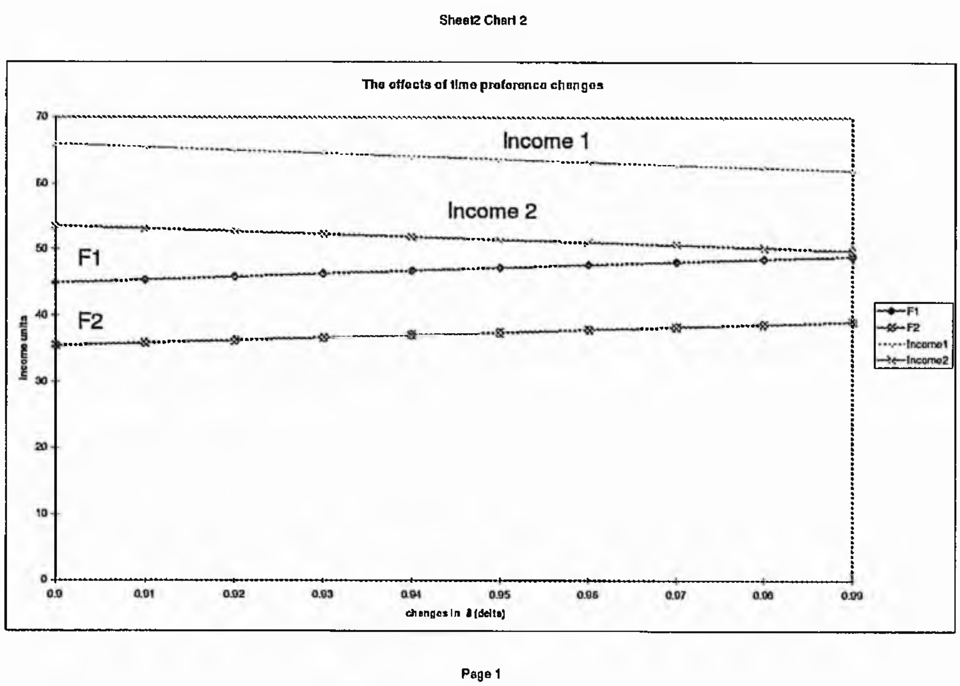


Figure 3.1: Equilibrium outcomes: The effect of the discounting factor (delta)δ.

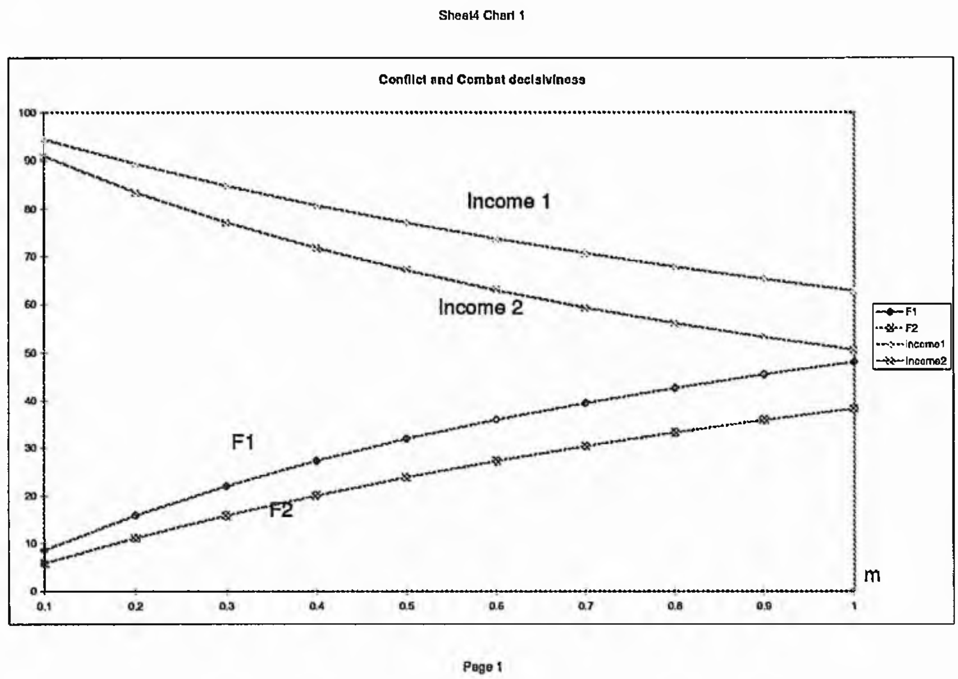
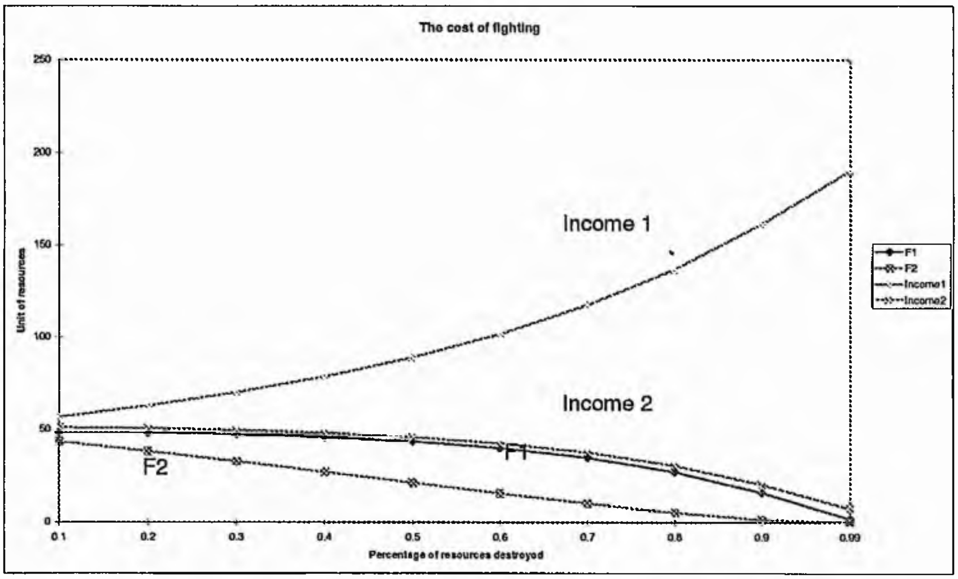


Figure 3.2: Equilibrium outcomes: The effect of combat decisiveness $0 < m < 1$.

In figure 3.2 combat decisiveness has no ambiguous interpretation. For income maximising agents the bigger the difference needed in fighting capability to win a war, the lower is the investment in weapons and the higher is the resulting income. This result might have some implications in order to analyse efficient procurement policies. Those policies that invest heavily in R & D might be biased toward a military concept of security in detriment of economic efficiency and security. Figures 3.3 and 3.4 show the effect produced by an increase in

Sheet5 Chart 1



Page 1

Figure 3.3: Equilibrium outcomes: The effect of γ

cost. This gives very interesting results. If the externality cost is very high, the party that has the advantage of starting will take most part of the resulting income. According to this graphs we need to reconsider the role of context success functions in the allocation of resources.

The strategies consistent with conflict are of two kinds:

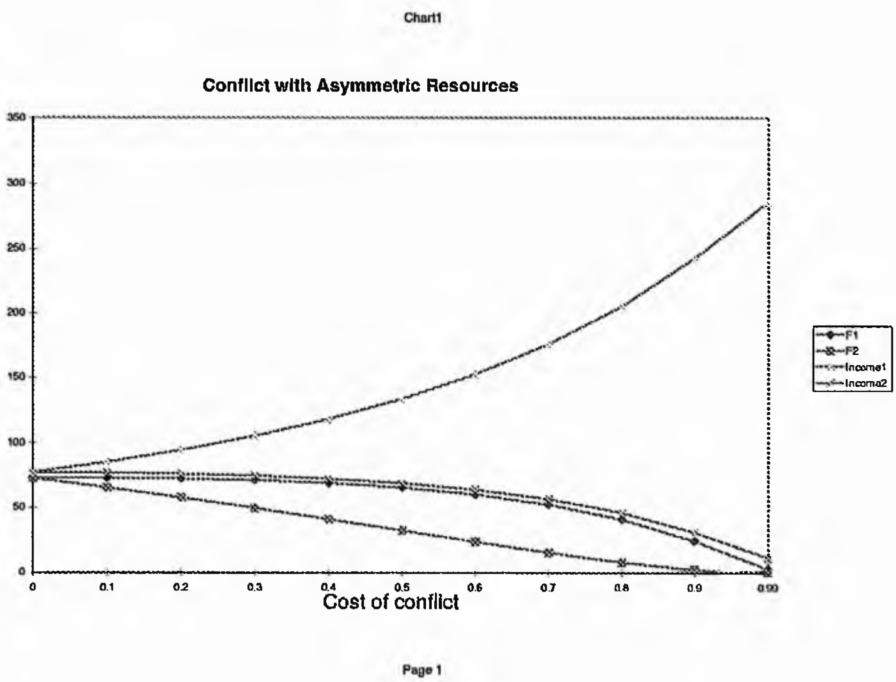


Figure 3.4: Asymmetric Resources. Player 1 starts the game with twice as much resources than Player 2. $R_1 = 100$ and $R_2 = 50$.

- Bargaining
- War

CSF map expenditure in fighting resources with the probability of winning a contest. However, the partition of resources not only depends on the outcome of conflict but also in the cost of it. When we focus on the economic aspects, the key question about conflict might not be what is the probability of winning, i.e., deterrence is important for defence. Instead, it could be how much damage can I inflict on my enemy and how much bargaining power can I derive from it. This is fundamental in every conflict and we should pay more attention to it.

We should use a new concept of security that is directly related to economic optimisation or to a cost–benefit analysis.

3.1.2 Models with risk of war

We have already introduced the idea of the existence of exogenous risk of breakdown when we decided the expected length of the bargaining period. The bargaining process can be also be modelled so this exogenous risk is the driving force towards settlement. The model is essentially the same. We complement the time preference parameter δ by a constant probability of breakdown which is equal to:

$$q(\Delta) = 1 - e^{-\lambda\Delta}$$

Settlement takes place determined by their respective probabilities of winning the war $p_i I$. This game can be represented by the following lottery:

$$(p_i, \Delta t) = (1 - q(\Delta))^t p_i u_i(I) \oplus \left[1 - (1 - q(\Delta))^t\right] p_i u_i(I)(1 - \gamma)$$

We have to developed a model where the final stage is not predetermined. The procedure is exactly the same, however, this time we introduce the constant probability of negotiation breakdown into the model. In the first simulation we excluded γ, the proportion of the pie lost due to the externality cost. Then we redo the models including both the cost of fighting and an exogenous risk of breakdown..

We calculate the outcome of the game using backwards induction for two hypothetical cases, when the game finish in period three and period four and then we extend it to an undetermined number of periods. In the table 3.2 we can see the share in a three period game.

To solve for the equilibrium of this game we proceed by backward induction. Starting with:

Time	Player 1 Share	Player 2 Share	Offers
Period 3	s	$1-s$	imposed settlement
Period 2	s_2	$1-s_2$	offer by player 2
Period 1	s_1	$1-s_1$	offer by player 1

Table 3.2: The bargaining process

Period 3, The outcome is imposed by war or settlement $(s, 1-s)$

Period 2 is equivalent to Rubinstein's bargaining game. We follow the same procedure. Player 2 will offer no more than $s_2 \geq \delta s$. This offer, $s_2 = \delta s$, will be acceptable for Player 2 iff $1-\delta s \geq \delta(1-s)$. This is true because $1 \geq \delta$. Therefore $s_2^* = \delta s$. There is no probability of breakdown q because they know for sure that period 3 is the last one.

Period 1 corresponds to an offer by Player 1. She wouldn't offer more than the discounted valued of what player 2 can get in period 2, therefore:

$1-s_1 \geq (1-q)\delta(1-s_2)+q\delta(1-s)$. Or, $1-s_1 = (1-q)\delta(1-s_2)+q\delta(1-s)$ It is acceptable to player 1 iff $s_1 \geq (1-q)\delta s_2 + q\delta s$. Since

$$s_1 = 1-(1-q)\delta(1-s_2)-q\delta(1-s)$$

Then, the condition becomes:

$$1 \geq (1-q)\delta(1-s_2+s_2)+q\delta(s+1-s)$$

That is,

$$1 \geq (1-q)\delta + \delta q$$

which is true because $1 \geq \delta$

The bargaining equilibrium offer equals to:

$$s_1^* = 1-(1-q)\delta(1-\delta s) - q\delta(1-s) \tag{3.22}$$

It is easy to check that:

$$\left.\begin{array}{l} \text{if } q = 0; \\ s_1^* = 1-\delta(1-\delta s) \end{array}\right\} \text{taking us back to Rubinstein's solution}$$

We now extend the game to 4 Periods. Table 3.3 shows the offers at every period.

Time	Player 1 Share	Player 2 Share
Period 4	s	$1-s$
Period 3	$s_3 = 1-\delta(1-s)$	$1-s_3 = \delta(1-s)$
Period 2	$s_2 = (1-q)\delta s_3 + q\delta s$	$1-s_2$
Period 1	$s_1 = 1-[(1-q)\delta(1-s_2)+q\delta(1-s)]$	$1-s_1$

Table 3.3: The bargaining process

The optimal offer in period 1 from player 1 is :

$$\begin{aligned} s_1^* &= 1-[(1-q)\delta(1-s_2)+q\delta(1-s)] \\ &= 1-(1-q)\delta(1-(1-q)\delta(1-\delta(1-s))) - (1-q)q\delta^2 s - q\delta(1-s) \\ &= 1-(1-q)\delta+(1-q)^2\delta^2-(1-q)^2\delta^3-q\delta(1-s)+(1-q)q\delta^2 s \end{aligned} \tag{3.23}$$

Now we can extend the number of periods to an infinite number of periods as follows:

Period 1:

$$s_1 = 1 - [(1-q)\delta(1-s_2) + q\delta(1-s)]$$

Period 2:

$$s_2 = (1-q)\delta s_3 + q\delta s$$

Period 3:

$$s_3 = 1 - [(1-q)\delta(1-s_4) + q\delta(1-s)]$$

by substituting period 2 offer s_2 into period 1 equation, we get:

$$s_1 = 1 - [(1-q)\delta(1 - \{(1-q)\delta s_3 + q\delta s\}) + q\delta(1-s)]$$

and for any given period t, we get the general expression

$$s_t = \alpha + (1-q)^2\delta^2 s_{t+2} \tag{3.24}$$

Where

$$\begin{aligned} \alpha &= 1-(1-q)\delta - q\delta(1-s) + (1-q)q\delta^2 \\ &= 1-(1-q)\delta - q\delta(1-s-(1-q)\delta s) \end{aligned} \tag{3.25}$$

Then, we can solve forwards for period 1:

$$s_1^* = \alpha + (1-q)^2\delta^2\alpha + (1-q)^4\delta^4\alpha + \cdots$$

$$s_1^* = \frac{\alpha}{1-(1-q)^2\delta^2} = \frac{1-(1-q)\delta - q\delta(1-s-(1-q)\delta s)}{1-(1-q)^2\delta^2} \tag{3.26}$$

Equation 3.26 is the equilibrium partition of resources. We can check that if we make the probability of breakdown equal 0 we go back to Rubinstein equilibrium.

$$\begin{cases} \text{if } q = 0 & s_1^* = \frac{1-\delta}{1-\delta^2} = \frac{1}{1+\delta} \text{ as required} \\ \text{if } q = 1 & s_1^* = 1 - \delta(1-s) \text{ as required} \end{cases}$$

3.1.3 Proof of SPE for finite horizon.

To prove that

$$s_1^* = \frac{\alpha}{1-(1-q)^2\delta^2} = \frac{1-(1-q)\delta - q\delta(1-s-(1-q)\delta s)}{1-(1-q)^2\delta^2}$$

is a unique perfect equilibrium we follow Fudenberg and Tirole (1995)

We obtain an upper bound and a lower bound on each player's equilibrium payoff using the stationarity of the game. Then we show that those two bounds are equal. In order to exploit the stationarity of the game, we define the continuation payoffs of a strategy profile in a subgame starting a t to be the utility in time t units of the outcome induced by that profile. This corresponds with the time discounting factor and the probability of breakdown. In other words, waiting one more period each player is confronted with $(1-q)\delta x + q\delta s$ lost of utility. Where x is the expected payoff and s is the outcome in case of conflict breakdown.

The lower and higher bounds for player 1 and player 2 are respectively

$$\underline{v}_1,\ \overline{v}_1,\ \underline{v}_2,\ \overline{v}_2$$

$$\text{For player 1 at any subgame starting with his offer} \begin{cases} \overline{v}_1 \\ \underline{v}_1 \end{cases}$$

Also;

$$\text{for player 1, at any subgame starting with player 2 offer} \begin{cases} \overline{w}_1 \\ \underline{w}_1 \end{cases}$$

The upper and lower bounds for player 2 are equally defined,

$$\text{For player 2 at any subgame starting with his offer} \begin{cases} \overline{v}_2 \\ \underline{v}_2 \end{cases}$$

$$\text{for player 2, at any subgame starting with player 1 offer} \begin{cases} \overline{w}_2 \\ \underline{w}_2 \end{cases}$$

The minimum that player 1 can expect in each subgame started by player 1 is,

$$\underline{v}_1 \leq 1 - [(1-q)\delta\overline{v}_2 + \delta q(1-s)] \tag{3.27}$$

$$\underline{v}_2 \leq 1 - [(1-q)\delta\overline{v}_1 + \delta qs] \tag{3.28}$$

and for player 1 when player 2 offers is: $\overline{w}_1 = [(1-q)\delta\overline{v}_1 + \delta qs]$. A similar equation can be found for player 2.

Player 1 highest equilibrium offer is:

$$\overline{v}_1 \geq \max[1 - \{(1-q)\delta\underline{v}_2 + \delta q(1-s)\};\ \delta(1-q)\overline{w}_1 + qs]$$

$$\overline{v}_1 \geq \max[1 - \{(1-q)\delta\underline{v}_2 + \delta q(1-s)\};\ \delta(1-q)\{(1-q)\delta\overline{v}_1 + \delta qs\} + \delta qs]$$

as neither δ nor $\overline{v}_2$ can exceed 1:

$$\max \quad [1 - \{(1-q)\delta \underline{v}_2 + \delta q(1-s)\};\ \delta(1-q)\{(1-q)\delta \overline{v}_1 + \delta qs\} + \delta qs] = 1 - [(1-q)\delta \underline{v}_2 + \delta q(1-s)]$$

The upper bound for player 1 is therefore:

$$\overline{v}_1 = 1 - [(1-q)\delta \underline{v}_2 + \delta q(1-s)] \tag{3.29}$$

by similar reasoning, the upper bound for player 2 is:

$$\overline{v}_2 = 1 - [(1-q)\delta \underline{v}_1 + \delta q(1-s)] \tag{3.30}$$

We have already establish the upper and lower bounds corresponding to both players. Combining the four equations (3.27) (3.28) (3.29) (3.30) we can solve for all the upper and lower bounds.

For player 1:

$$\underline{v}_1 \geq 1 - [(1-q)\delta\{1 - [(1-q)\delta \underline{v}_1 + \delta qs]\} + \delta qs]$$

$$\begin{cases} \overline{v} \leq \frac{1-(1-q)\delta-\delta q[1-s-(1-q)\delta s]}{1-(1-q)^2\delta^2} \\ \underline{v} \geq \frac{1-(1-q)\delta-\delta q[1-s-(1-q)\delta s]}{1-(1-q)^2\delta^2} \end{cases}$$

$$\text{Since } \underline{v}_1 \leq \overline{v}_1 \Rightarrow \text{Then } \underline{v}_1 = \overline{v}_1 \quad \square \tag{3.31}$$

3.1.4 The Reaction Curves

Finally, we proceed to find the reactions functions of both players by maximising with respect to expenditure in production effort and fighting effort, For Player 1

$$\max \frac{1-(1-q)\delta-\delta q[1-s-(1-q)\delta s]}{1-(1-q)^2\delta^2} I$$
$$\text{subject to } I_1 = R_1 + E_1 \tag{3.32}$$

Where

$$s = \frac{F_1^m}{F_1^m + F_2^m}; \quad I = A(E_1^{1/\alpha} + E_2^{1/\alpha})^\alpha$$

and Player 2

$$\max \left(1 - \frac{1-(1-q)\delta-\delta q[1-s-(1-q)\delta s]}{1-(1-q)^2\delta^2}\right) I$$
$$\text{subject to } I_2 = R_2 + E_2 \tag{3.33}$$

Where

$$s = \frac{F_2^m}{F_1^m + F_2^m} \quad I = A(E_1^{1/\alpha} + E_2^{1/\alpha})^\alpha$$

The solution to the maximisation exercise is given by Matlab in figure 3.5:

We can see that the omission of cost produces a completely different result. When no cost occurs, and there is no fixed last period, the higher is the probability of breakdown the closer we get to the Hirshleifer's symmetrical equilibrium. This happens because the first party can exploit second players' discounting factor. The game becomes, de facto, an ultimatum game.

Finally, we reproduce this model introducing the cost of war. We follow backward induction as before. In this case, for any period t, the offer from player 1 will be:

$$s_t = 1-(1-q)\delta+(1-q)^2\delta^2 s_{t+2}+(1-q)q\delta^2 s(1-\gamma)-q\delta(1-s)(1-\gamma) \tag{3.34}$$

Let

$$a = 1-(1-q)\delta - q\delta(1-\gamma) + s(1-\gamma)\left((1-q)q\delta^2 + qd\right)$$

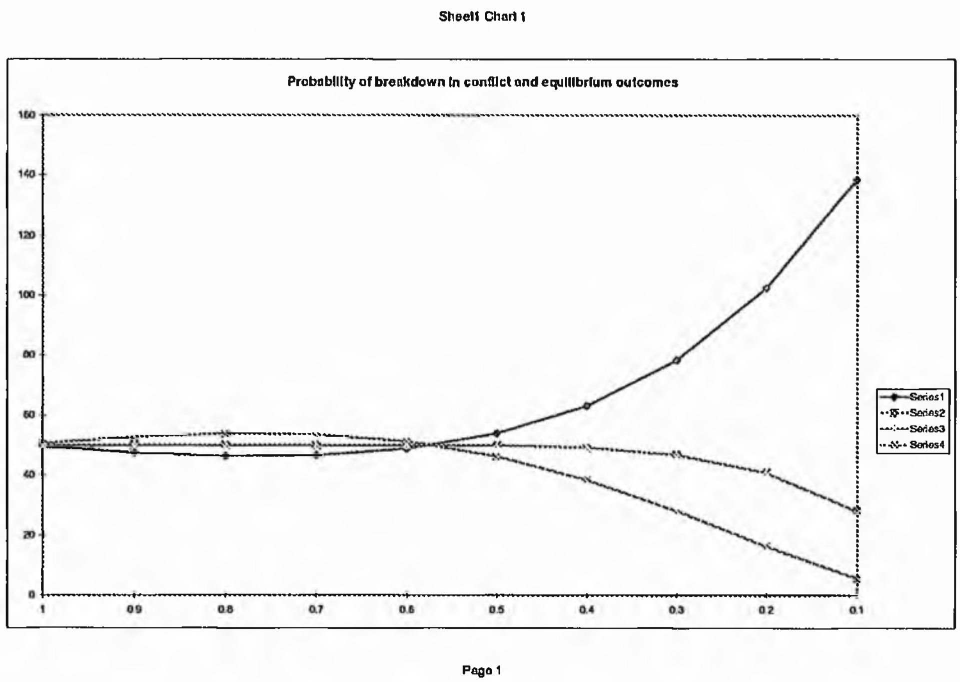


Figure 3.5: Exogenous risk of conflict.

Then,

$$s_t = a + (1-q)^2\delta^2 s_{t+2} \tag{3.35}$$

Solving forward,

$$s_1^* = a + a(1-q)^2\delta^2 + a(1-q)^4\delta^4 + \cdots$$

The equilibrium offer from player 1 in period $t = 1$ is therefore:

$$s_1^* = \frac{a}{1-(1-q)^2\delta^2} \tag{3.36}$$

The reaction curves for Country 1 and 2 are found by maximizing:

For Country 1

$$\max_{w.r.t:E_1,F_1} \left(\frac{a}{1-(1-q)^2\delta^2}\right) I \quad \text{subject to } R_1 = F_1 + E_1 \tag{3.37}$$

and for Country 2

$$\max_{w.r.t:E_2,F_2} \left(1 - \frac{a}{1-(1-q)^2\delta^2}\right) I \quad \text{subject to } R_2 = F_2 + E_2 \tag{3.38}$$

This gives the following reaction curves:

$$F_1 = \frac{mF_1^m F_2^m E^{1-\alpha}\left(E_1^\alpha + E_2^\alpha\right)}{\left(\frac{b}{c} + \frac{F_1^m}{F_1^m + F_2^m}\right)\left(F_1^m + F_2^m\right)^2} \tag{3.39}$$

$$F_2 = \frac{mF_1^m F_2^m E^{1-\alpha}\left(E_1^\alpha + E_2^\alpha\right)}{\left(\frac{a-b}{c} - \frac{F_1^m}{F_1^m + F_2^m}\right)\left(F_1^m + F_2^m\right)^2} \tag{3.40}$$

Where:

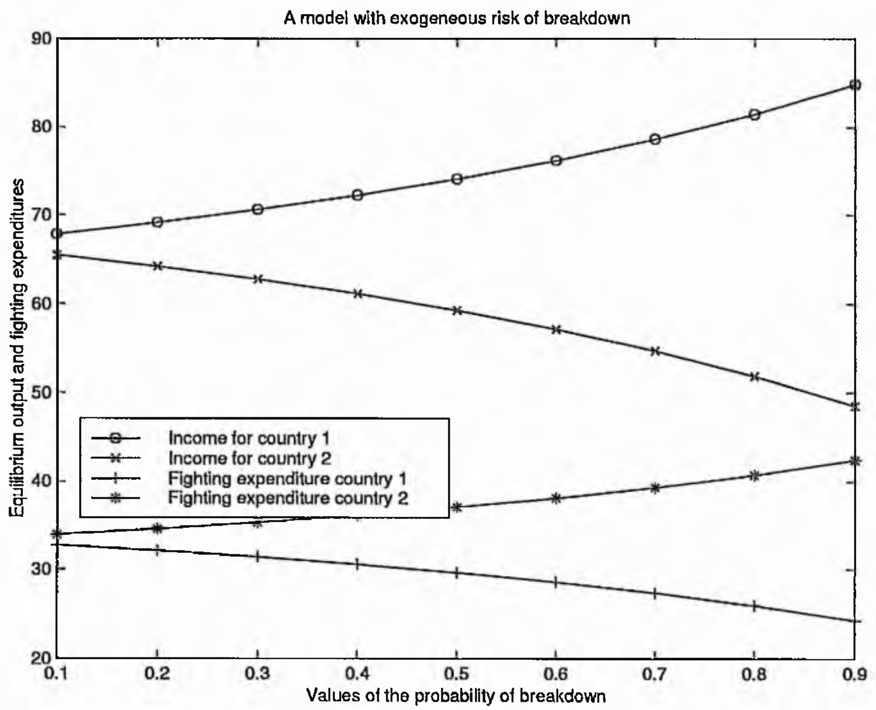
$$a = 1 - (1-q)\delta - q\delta(1-\gamma) + s(1-\gamma)\left((1-q)q\delta^2 + qd\right)$$

$$b = 1 - (1-q)\delta - q\delta(1-\gamma)$$

$$c = (1-\gamma)\left((1-q)q\delta^2 + q\delta\right)$$

we also solved this problem numerically. The result of the simulation is presented in figure 3.1.4. The risk of breakdown has a similar effect to the effects produced by the cost of fighting. But if we compare this results to figure 3.1.1, we can easily appreciate that cost has a stronger impact on the final outcome. Basically, the strategic advantage of moving first is less important when there is some exogenous probability of breakdown of negotiations. Therefore, while most part of the literature concentrates on the probability of success, we still maintain that the cost of conflict is the parameter with the strongest influence on the strategic decisions of rational players.

Figure 3.6: Exogenous risk with cost of fighting



3.2 Conclusions

Introducing a model that allows parties to negotiate over the possible outcomes of an underlying conflict gives us some interesting results. The possibility of negotiations introduces a completely new scenario into the study of conflict, arms trade, and efficient allocation of resources.

When parties are allowed to negotiate, the corresponding equilibrium of military expenditure is much lower compared to situations where negotiations cannot take place. The traditional idea that military security is a public good that depends on the respective stocks of arms of two potential enemies may not apply to every conflict. In these cases where the cost of negotiating is very low, and countries have perfect or near perfect information, the concept of military security should be replace by economic security.

This model also introduces some interesting developments into Hirshleifer framework. The possibility of negotiating gives countries different allocation of resources than the ones predicted by the original models. This happens due to the fact that in the earlier models the cost of conflict has no strategic effect on the optimal allocation of resources. However, when countries are allowed to negotiate, the country that has an strategic advantage can exploit the amount of potential damage in his favour, producing different optimal strategies.

Finally, despite the obvious problems with the classification of the stylised facts, this model is consistent with some of the regularities that other authors have found.

- The decline of Major Wars, and the revolution in telecommunications together with the massive increase in productivity that some countries have experienced in the last decades is consistent with the optimising behaviour predicted by the model.

- The stylised facts about the probabilities of war between different kind of regimes, classified according to the free circulation of information is also consistent.

- The fact that not a single war has broken out between nuclear powers (due to the high cost) can also be explained if we introduce some negotiation procedure into our models of conflict. So far, the closest that we have been to the nuclear holocaust has been always due to information-related problems.

- Some of the regularities about conflicts is that countries never go into war because of new militarized disputes. This indicates that countries try to avoid the cost of conflict by finding a negotiated settlement.

- During the last period of high uncertainty in the international system, conflict seems to be at its peek.

- The amount of ethnic conflict, identity struggles, religious war and other problems that cannot be negotiated easily seems to be higher than conflicts related to disagreements about the partition of scarce resources. All these facts are consistent with the model that we have just introduced, and the one that will be presented in the next chapter.

We have presented a model with perfect information and optimal allocation of resources to appropriative and productive activities. At the moment, we have only presented an outlined picture of the importance of imperfect information. In the next chapter we are going to present a development that takes into account this problem. We are going to produce a model with endogenous conflict.

There are two obvious developments in relation to the study of conflict. First, we can introduce some changes to our production function

$$I = A(E_1^{1/\alpha} + E_2^{1/\alpha})^{\alpha}.$$

as explained in section 3.1.1. We did our simulations fixing the parameter "α" to be equal to 1. That give us an Income Frontier that takes the shape of a straight line. By increasing "α" we can easily introduce a concave Income Frontier. That is the scenario that Hirshleifer calls "complementary and harmonious opportunities".

The other aspects are players' preferences. In our model players utility comes strictly and uniquely from the level of income obtained. People do not get any positive or negative utility from other players' income. That is represented in our model by two straight lines crossing the equilibrium point. Negatively sloped indifference curves reveal a degree of benevolence on each side. Also, positively sloped indifference curves would give a degree of malevolence, a country could get some disutility from increases in other country's income.

Our extension of Hirshleifer model that includes a bargaining process complements the model in some aspects. First, we have allowed for an externality cost of fighting. Hirshleifer's analysis derives mainly from the analysis of asymmetric resources. His *Paradox of Power* is clearly a result of analysing asymmetric resources. If we introduce bargaining we arrive at very different results using a very similar model. Basically asymmetric resources do not explain at all the final outcome, whereas the externality cost of conflict plays a much more important role.

Chapter 4

Conflict with incomplete information

4.1 Introduction to conflict and information

In this chapter we are going to present asymmetric information as the source of conflict. We must assume that players are rational. Some of the most deadly conflicts can be portrayed as highly irrational if we use the common or popular meaning of the term. We know that this concept depends in the information available. Therefore, the study of information and rationality are strongly linked.

There are two different kinds of explanations of war offered by political science rational models [1]. This first one studies a scenario in which leaders are rational but overestimate their chances of military victory. The second argues that states may lack information about their adversary's willingness to fight. This is directly linked to possible informational asymmetries in the cost of fighting. Our approach is closest to this second strand. We confront the problem from an economic perspective. The literature from which we draw includes public choice models of economics and conflict which focus on directly unproductive activities.

[1]See Fearon [79].

We have already established the optimal allocation of resources between fighting effort and productive effort. We have also shown how the model behaves when we introduce some changes to some of the parameters.

In graph 4.1 we have several figures representing different situations that may explain the sources of conflict from different forms of asymmetry. Figure

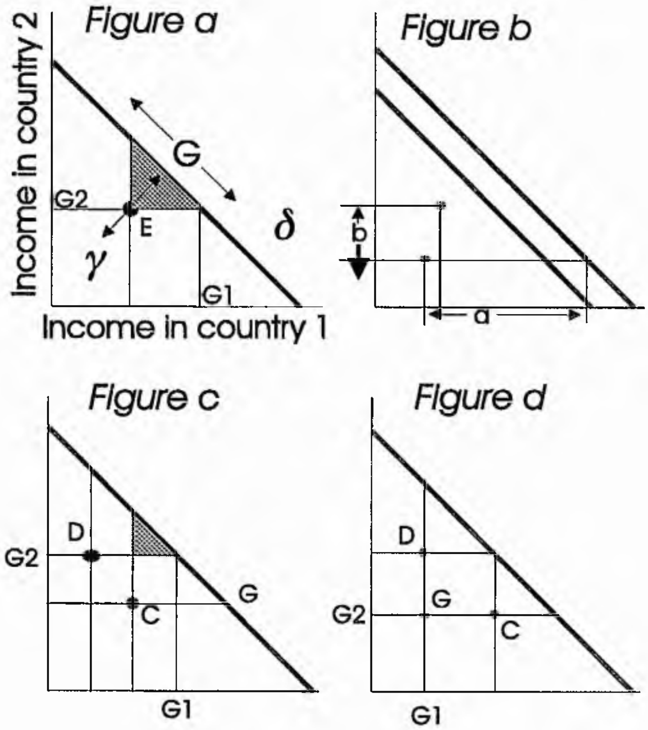


Figure 4.1: Asymmetric information:1

(a) shows settlement when both parties have perfect information. The thick line shows all the possible combinations of outcome in case of settlement. It is the maximum outcome that can be obtained when both countries allocate their resources optimally. We will call it the *Income Frontier.* As we increase F_1 and F_2 , this line shifts to the left. Income for Country 1 is represented on the x' axis and income for Country 2 on the y's.

We have already shown how much the size of the triangle changes according to some different values of the parameters that we have studied in previous diagrams.

The point E is the threat point which depends directly on γ, a parameter not present in Hirshleifer model. Here it is fixed and exogenous. This point reflects the externality cost of war. It is not only the value of destruction of material things but also lives, and other non material things. However, it also influences the amount of resources that both parties dedicate to war, reducing or augmenting F_1 and F_2 , the initial resources dedicated to fighting effort.

Increases in γ -the externality cost of fighting- are shown in figure a as perpendicular movements in relation to the Income Frontier. Increases or decreases in δ correspond to movements along this line. Finally, we simplify figure a by assuming that point G_1 already takes into account the influence of δ, the time preference parameter..

Using this kind of diagram we can explain some cases of asymmetric information. In our analysis we considered only asymmetric information about the cost of conflict, but there are other sources of conflict such us asymmetric preferences, opportunities, malevolent preferences or limited bargaining sets.

In figure (b) we have represented a special case of asymmetry. Country 1 perceives a bigger externality cost that Country 2. That is represented by another threat point, closer to the origin of the diagram. There is also a second Income Frontier to the right of the one perceived by Country 2. The bigger is the difference of distance "a" to distance "b" the high the prospect of achieving a peaceful solution.

Figure (c) and Figure (d) show a different kind of asymmetry. They do not perceive different magnitudes of γ, but they think that cost is not symmetrically distributed. In this scenario one country perceives that the opponent bears much higher cost.

In figure c the asymmetry is not big enough to eliminate the Potential Settlement Region. Both countries can adjust their shares on account of the potential gains of an agreement. In figure d there is not possibility of agreement; both countries will fight.

We must introduce incomplete information. However, one of the problems of games with asymmetric information is that the equilibrium outcome can be a result of the chosen assumptions regarding the bargaining procedure and informational asymmetries. The equilibrium concept changes whether there is a single country that makes offers or both. The equilibrium will change if the bargaining process is extended to more than two periods. The valuation of the cost of fighting can be introduced in many different ways. Countries can be represented either as tough with low externality cost or soft with high cost. These factors feature prominently in our model.

Fudenberg and Tirole [80] draw the following conclusions about the effects of valuations and discount factors on the strategic behaviour of buyers and sellers in a model of bargaining with incomplete information.

> First, a decrease in the buyer's discount factor may make him better off in spite of the fact that, being more impatient, he becomes more vulnerable to a high demand. We explained this phenomenon by the impossibility of commitment to take given actions (here to accept a compromising offer) which is required by the concept of perfect equilibrium. Second, increasing the contract zone (E.g., by making the seller more eager to sell) may increase the possibility of disagreement. Third, if the buyer has complete information about the seller, the seller may charge a higher price in the second period than in the first; the buyer may nevertheless refuse such a first-period offer since there is the possibility that the seller is soft and will charge less in the second period. Fourth, increasing the number of periods may have surprising welfare effects: it can decrease efficiency even when the one-period game has an inefficient solution.

The message from Fudenberg and Tirole (henceforth F&T) is clear. We have to be very careful about general assertions about parameter changes and the effect on conflict. The solutions have to be treated within a specific context and the conclusions of our work should be interpreted carefully. Despite these caveats, models of game theory and conflict can give us important insights into the decision making process leading to the outbreak of war. They can also give us some indication of what would be consider optimal policies if we were facing an ideal world of rational players, even if rational behaviour is accompanied by problems of incomplete information.

The rest of the chapter is organised as follows. Section 4.2 describes the model and the set up for the game. Section 4.3 sets out the Perfect Bayesian Equilibrium for a given total income and a given end-of-bargaining settlement. This section is a generalisation of F&T to allow for any priors on the part of the less informed country. Section 4.4 closes the model by making output dependent on the allocation of resources to production and the settlement after bargaining (i.e. following a conflict) dependent on the allocation of resources to fighting (as in Hirshleifer [12]. Analysis in this section relies on numerical simulations. Section 4.5 provides conclusions and suggestions for future work.

4.2 The Model

Our model integrates two other models. The model of efficient allocation of resources of Hirshleifer [12] the model of bargaining with incomplete information by Fudenberg and Tirole [80]. It will set out to explain the foundations of conflict behaviour assuming rational income-maximising agents.

We consider two abstract players that have to divide a common scarce resource. We call those players Country 1 and 2, but it could be any other abstract players. We assume some change in the environment that breaks the previous equilibrium of forces. For example, assume that a new resource is found and it is arbitrarily distributed. Country 1 has found the resource and takes possession.

Country 2 thinks that there is some potential to make some economic gain and makes a claim for a new redistribution of resources. Since Country 1 would be happy maintaining the status quo it has some strategic advantage. Therefore we give Country 1 the advantage of having the initiative (making the offers).

Both countries are rational and before fighting takes place, they will try to find a settlement avoiding the cost of war. Prior to the bargaining process, they have to decide in advance how to allocate their respective initial resources into production effort and fighting effort. This allocation is irreversible until the discovery of new resources.

After the allocation of resources, a new equilibrium takes place either by war or by settlement. Countries share the resources obtained in a common production process and the output is distributed according to this new situation.[2] Each participant balances between productive exploitation of the current resource base and acquisition of the results of production. Correspondingly, there are two separate technologies: *a technology of production* and a *technology of appropriation, conflict and struggle.*

Consider first the technology of production. We adopt a common CES function and the technology of conflict follow exactly Hirshleifer Production and Contest Success Functions.

$$I = A(E_1^\alpha + E_2^\alpha)^{1/\alpha} = f(E_1, E_2) \tag{4.1}$$

say, where I is the jointly generated output, A is a technological constant and E_1 and E_2 are the inputs of production. This CES production function has an elasticity of substitution $\epsilon = \frac{1}{1-\alpha}$, and takes a variety of shapes depending

[2]Countries at war do not normally trade with each other. Some authors have studied the relation between trade and war. They found that countries with very high levels of trade do not normally make war, but once the war takes place it is normally longer and more devastating. For a review of those issues see Mansfield [81] and Gowa [82].

on its value.[3] In the case where $\alpha = 1$ the production function is a simple linear function. As α approaches to zero, ϵ tends to unity and we have a Cobb-Douglas production function. As α approaches $-\infty$, it gradually transforms into a Leontief technology.

The participants have an initial *resource constraint* given by:

$$\bar{R}_i = F_i + E_i; \quad i = 1, 2 \tag{4.2}$$

for Country i where F_i is devoted to fighting and E_i to production. A distinctive feature of our set-up is the inclusion of *costs of war* which are quantified differently by different countries. For Country 1 the cost of war in monetary terms is a proportion γ of output I and γ is common knowledge. Country 2 may be one of two types, a 'tough' country for whom the utility loss $\underline{\gamma}$ is relatively low, and a 'soft' county for whom the utility loss is $\bar{\gamma}$ where $\overline{\gamma} > \underline{\gamma}$. Country 1 does not know which type it faces and adopts priors v_0 and $1 - v_0$ that Country 2 is soft and tough respectively. In a Perfect Bayesian Equilibrium (PBE, these probabilities are revised as bargaining proceeds in accordance with Bayes Rule.

The two types of Country 2 will in general choose different allocations of total resources between production and fighting effort. Let the soft and tough countries choose $\overline{E}_2$ and $\underline{E}_2$ respective. Total output is then given by the production functions $\overline{I} = f(E_1, \overline{E}_2)$ and $\underline{I} = f(E_1, \underline{E}_2)$ if Country 2 turns out to be soft or tough respectively, where $f(E_1, E_2)$ is given by (1).

Now let us turn to the *technology of appropriation* which takes the form of Hirshleifer's *Contest Success Functions* (CSF)3.4. Let the probability of Country 1 winning the war be denoted by $\overline{p} = g(F_1, \overline{F}_2)$ and $\underline{p} = g(F_1, \underline{F}_2)$ respectively for the two types of Country 2. Country 1 does not know for certain which type it is confronting and adopts an initial probability $p = v_0\overline{p} + (1 - v_0)\underline{p}$. The functional

[3]See Varian [83].

form $g(F_1, F_2)$ of the appropriation technology follows *the ratio variant*,

$$p_1 = \frac{F_1^M}{F_1^M + F_2^M}; \quad p_2 = \frac{F_2^M}{F_1^M + F_2^M} \tag{4.3}$$

The parameter M[4] is called the *decisiveness coefficient* and affects the degree to which greater fighting effort translates into battle success. The function captures how expenditure in fighting effort, (F_1, F_2), translates into a certain probability of victory. For Country 2 the probability of victory is given by $1 - \overline{p}$ and $1 - \underline{p}$ for the soft and tough respectively.

The *sequencing of events* is first, countries commit to the allocation of their factors of production to output or fighting. This then determines the probabilities of winning a war for the two types of Country 2 and for Country 1 conditional on the type of Country 2 it faces. Second, bargaining takes place. As we have mentioned, we assume that Country 1 is the less informed player and makes the offers. After two offers either agreement is reached or a war ensues which Country 1 can win with an updated probability p. In detail, the game in extensive form is given by the following sequence of events.

1). Country 1 has prior v_0 that Country 2 is soft. Countries simultaneously commit themselves to the resource allocation (F_i, E_i); $\quad i = 1, 2$ which determines expected output for Country 1, $E[I|v_0] = v_0\overline{I} + (1 - v_0)\underline{I}$, actual output $\overline{I}$ and $\underline{I}$ for the two types of Country 2, and probabilities p, $\overline{p}$ and $\underline{p}$.[5]

2). It is convenient to consider offers in the form of *shares* of output given by $I = \overline{I}$ if Country 2 is soft and $I = \underline{I}$ if Country 2 is tough. At the first stage of the bargaining process, Country 1 claims an amount $s_1\overline{I}$ which corresponds to an offer of $(1 - s_1)\overline{I}$ to the soft type of Country 2 and of $\underline{I} - s_1\overline{I}$ to the hard type. Alternatively we can write the claim as an amount $s_1\underline{I}$ which corresponds to an offer of $\overline{I} - s_1\underline{I}$ to the soft type of Country 2 and of $(1 - s_1)\underline{I}$ to the hard type.

[4]This parameter is defined for compactness as $M \equiv m/(1 - m)$ following Hirshleifer [44]

[5]$\overline{F}_2$ and $\underline{F}_2$ are not observed by Country 1.

(We confine ourselves to pure strategies for Country 1).

3). Country 2 accepts the offer with probabilities $\underline{r}_1(s_1)$ and $\overline{r}_1(s_1)$ for types $\underline{\gamma}$ (tough) and $\overline{\gamma}$ (soft) respectively. Acceptance ends the game with single-period payoffs $s_1\overline{I}$ for Country 1 and $(1-s_1)\overline{I}$ or $\underline{I} - s_1\overline{I}$ for Country 2 of types soft and tough respectively. Otherwise we proceed to:

4). Country 1 updates its probability from v_0 to v_1.

5). Country 1 claims an amount $s_2\overline{I}$ which corresponds to an offer of $(1-s_2)\overline{I}$ to the soft type of Country 2 and of $\underline{I} - s_2\overline{I}$ to the hard type.

6). Country 2 accepts the offer with probabilities $\underline{r}_2(s_2)$ and $\overline{r}_2(s_2)$ for types $\underline{\gamma}$ and $\overline{\gamma}$ respectively. Acceptance ends the game with single-period payoffs $s_2\overline{I}$ for Country 1 and $(1 - s_2)\overline{I}$ or $\underline{I} - s_2\overline{I}$ for Country 2 of types soft and tough respectively. Otherwise we proceed to:

7). War ensues which Country 1 wins with probability $p = v_1\overline{p} + (1 - v_1)\underline{p}$. Soft Country 2 expects to win with probability $1-\overline{p}$ and the tough country with probability $1-\underline{p}$. Expected single period payoffs are

$$(1-\gamma)[v_1\overline{p}\overline{I} + (1-v_1)\underline{p}\,\underline{I}]$$

for Country 1, and $(1-\underline{p})(1-\underline{\gamma})\underline{I}$ and $(1-\overline{p})(1-\overline{\gamma})\overline{I}$ for tough and soft Country 2 respectively.

4.3 The Equilibrium for a Given Allocation Between Output and Fighting Effort

The full equilibrium is described by $(F_1, \overline{F}_2, \underline{F}_2, s_1, r_1, s_2, r_2)$ and depends upon cost of war parameters γ, $\overline{\gamma}$, $\underline{\gamma}$, the prior v_0, discount factors δ_1, $\overline{\delta}_2$, $\underline{\delta}_2$ and functional forms $f(\cdot)$ and $g(\cdot)$. In this section we solve for the equilibrium of events 3 to 6 above for a given allocation $(F_1, \overline{F}_2, \underline{F}_2)$ which then determines $E[I|v_0]$

and the probabilities of winning for a given v_0. This corresponds closely to the game with one-sided incomplete information described in F&T, except that we generalize the game to one with any prior v_0. The PBE of this game is based on the following five elements:

Range of Possible Offers

This is based on the solution to the *complete information game.* In bargaining period 2, a soft country is guaranteed a payoff of say $(1-\overline{\gamma})(1-\overline{p})\overline{I} = (1-\overline{s}_2)\overline{I}$. It follows that any claim by Country 1 $s_2\overline{I} \leq \overline{s}_2\overline{I}$ will be accepted by the soft country. Similarly any claim $s_2\underline{I} \leq \underline{s}_2\underline{I}$ will be accepted by the tough country where $\underline{s}_2 = 1-(1-\underline{\gamma})(1-\underline{p})$. Therefore only claims in the interval $[\underline{s}_2\underline{I}, \overline{s}_2\overline{I}]$ or $[\overline{s}_2\overline{I}, \underline{s}_2\underline{I}]$, depending on whether $\underline{s}_2\underline{I} < \overline{s}_2\overline{I}$ or $\underline{s}_2\underline{I} > \overline{s}_2\overline{I}$, will be accepted by Country 2 in bargaining period 2. In the rest of this section we assume that $\underline{s}_2\underline{I} < \overline{s}_2\overline{I}$;[6] ie, if the claim by Country 1 is less if the Country 2 is tough. This may in fact not be the case if $\overline{F}_2 > \underline{F}_2$; that is if the soft country devotes more resources to fighting activity. However the details of the equilibrium are very similar in both cases.

Under complete information, Country 1 facing a tough adversary will only claim $\underline{s}_2\underline{I}$ and agree to a settlement without war if

$$(1-\gamma)\underline{p}\underline{I} < \underline{s}_2\underline{I} \tag{4.4}$$

Using $\underline{s}_2 = 1-(1-\underline{\gamma})(1-\underline{p})$, condition 4.4 holds if $\underline{p} < 1$, which, of course, is always the case. Similarly, Country 1 facing a soft adversary will only claim $\overline{s}_2\overline{I}$ and agree to a settlement without war if $\overline{p} < 1$, which again always holds. In bargaining period 1, soft Country 2 will accept any offer

$$(1-\overline{s}_1)\overline{I} \geq \overline{\delta}_2(1-\overline{s}_2)\overline{I}.$$

[6] Appendix A considers the case where $\underline{s}_2\underline{I} > \overline{s}_2\overline{I}$.

Thus Country 1 can claim up to $\overline{s}_1\overline{I}$ where $\overline{s}_1 = 1 - \overline{\delta}_2(1 - \overline{s}_2)$ if it confronts the soft country. Similarly Country 1 can claim up to $\underline{s}_1\underline{I}$ where $\underline{s}_1 = 1 - \underline{\delta}_2(1 - \underline{s}_2)$ if it confronts the tough country. As for period 1, we assume that $\underline{s}_1\underline{I} < \overline{s}_1\overline{I}$. Then, only claims in the interval $[\underline{s}_1\underline{I}, \overline{s}_1\overline{I}]$ will be considered by Country 2 in bargaining period 1. By similar reasoning as before, under complete information Country 1 will offer $(1 - \underline{s}_1)\underline{I}$ to the tough country and $(1 - \overline{s}_1)\overline{I}$ to the soft country, and the offer will be accepted.

Tough, Soft Country 1 Distinction

Consider a one-shot game where Country 1 has priors v_0 that Country 2 is soft. Define a 'soft' Country 1 that prefers to make a low claim $\underline{s}_2\underline{I}$ with certain agreement to making an offer $\overline{s}_2\overline{I}$ which is only accepted by a soft Country 2. The condition for this is

$$\underline{s}_2\underline{I} > v_0\overline{s}_2\overline{I} + (1 - v_0)(1 - \gamma)\underline{pI} \tag{4.5}$$

Otherwise the country is 'tough'.

A Threshold Intermediate Share $\tilde{s}$

Define $(1-\tilde{s})\overline{I}$ as the lowest offer that soft Country 2 will accept in the first period of bargaining when it expects a generous low claim $\underline{s}_2\underline{I}$ in the second period. If $\overline{\delta}_2$ is the discount factor of soft Country 2 then $\tilde{s}$ is given by

$$(1 - \tilde{s})\overline{I} = \overline{\delta}_2(\overline{I} - \underline{s}_2\underline{I}) \tag{4.6}$$

Bayesian Updating of v_0 to v_1

By Bayes Rule

$$v_1 \quad = \quad \text{prob}(\gamma = \overline{\gamma} \mid s_1 \text{ refused})$$

$$= \frac{\text{prob}(s_1 \text{ refused} \mid \gamma = \overline{\gamma})\text{prob}(\gamma = \overline{\gamma})}{\text{prob } (s_1 \text{ refused})}$$
$$= \frac{(1 - \overline{r}_1(s_1))v_0}{(1 - \overline{r}_1(s_1))v_0 + (1 - \underline{r}_1(s_1)))(1 - v_0)} \tag{4.7}$$

A Threshold Probability $\tilde{r}$

Define $\tilde{r}$ as the value of $\overline{r}_1$ which makes a tough Country 1 just indifferent between playing soft and tough in the second period when $\underline{r}_1(s_1) = 0$. Then

$$v_1 = \text{prob}(\gamma = \overline{\gamma} \mid s_1 \text{ refused})$$
$$= \frac{(1 - \overline{r}_1(s_1))v_0}{(1 - \overline{r}_1(s_1))v_0 + (1 - v_0)} \tag{4.8}$$

and

$$\underline{s}_2\underline{I} = v_1\overline{s}_2\overline{I} + (1 - v_1)(1 - \gamma)\underline{p}\,\underline{I} \tag{4.9}$$

Hence from (5)

$$\tilde{r} = \frac{v_0 - v_1}{v_0(1 - v_1)} \tag{4.10}$$

where

$$v_1 = \frac{\underline{s}_2\underline{I} - (1 - \gamma)\underline{p}\,\underline{I}}{(\overline{s}_2\overline{I} - (1 - \gamma)\underline{p}\,\underline{I})} \tag{4.11}$$

Lemma 1 *If Country 1 is tough then* $0 \leq \tilde{r} \leq 1$.

proof. Using 4.10 and 4.11 we have that

$$\tilde{r} = \frac{v_0\overline{s}_1\overline{I} + (1 - v_0)(1 - \gamma)\underline{p}\underline{I} - \underline{s}_2\underline{I}}{v_0(\overline{s}_2\overline{I} - \underline{s}_2\underline{I})} \tag{4.12}$$

Since $\overline{s}_2\overline{I} > \underline{s}_2\underline{I}$ the denominator of 4.10 is positive. From the definition of a tough Country 1 the numerator is also positive. Hence $\tilde{r} > 0$.

After some algebraic manipulation of 4.10, we find that $\tilde{r} < 1$ iff

$$(1-\gamma)\underline{pI} < \underline{s}_2\underline{I} \tag{4.13}$$

which is precisely the condition 4.4 above. □

Proposition 1 *Consider the strategies in table 1. In a PBE, a soft Country 1 chooses from strategies 1 or 2 depending on which strategy maximises the expected payoff given in table 2; a tough country 1 chooses from all three. If strategy 3 is chosen, then, if Country 2 turns out to be tough, war ensues*

Proof. When equation 4.5 holds, Country 1 is soft, and will play $\underline{s}_2\underline{I}$ in the last period. Recall that Country 2 (tough) will refuse any offer in the first period smaller than $(1-\underline{s}_1)\underline{I}$. If the first period offer is rejected Country 1 will update its beliefs according to B.3 . Since $\underline{r}_1(s_1) = 0$ that implies that $v_0 > v_1$. Knowing what Country 1 will offer in the second period, Country 2 will accept any offer in period 1 if $\tilde{s}$ is lower than:

$$\tilde{s} = 1 - \frac{\overline{\delta}_2(\overline{I} - \underline{s}_2\underline{I})}{\overline{I}} \tag{4.14}$$

Now, we assume that Country 1 is tough. It will play any strategy in table 4.1. A soft country will accept any offer bigger than $(1-\tilde{s})$. If any offer smaller than $(1-\tilde{s})$ is made by Country 1 in period 1 and country soft would accept it with probability $\overline{r}_1(s_1) > \tilde{r}$, then – using equations 4.10 and 4.11– it can be shown easily that

$$\underline{s}_2\underline{I} > v_2\overline{s}_2\overline{I} + (1-v_2)(1-\gamma)\underline{pI}$$

Table 4.1: Three Strategies for Country 1

Strategy	Period	Country 1	Country 2 (Soft)	Country 2 (Tough)
1	1	$\underline{s}_1\underline{I}$	$\overline{r}=1$	$\overline{r}=1$
	2	-	-	-
2	1	$\tilde{s}\overline{I}$	$\overline{r}=1$	$\underline{r}=0$
	2	$\underline{s}_2\underline{I}$	-	$\underline{r}=1$
3	1	$\overline{s}_1\overline{I}$	$\overline{r}=\tilde{r}$	$\underline{r}=0$
	2	$\overline{s}_2\overline{I}$	$\overline{r}=1$	$\underline{r}=0$

Table 4.2: Expected payoffs for the Three Strategies

Strategy	Country 1	Country 2 (Soft)	Country 2 (Tough)
1	$\underline{s}_1\underline{I}$	$\overline{I}-\underline{s}_1\underline{I}$	$(1-\underline{s}_1)\underline{I}$
2	$v_0\tilde{s}\overline{I}+(1-v_0)\delta_1\underline{s}_2\underline{I}$	$(1-\tilde{s})\overline{I}$	$\underline{\delta}_2(1-\underline{s}_2)\underline{I}$
3	$v_0[\tilde{r}\overline{s}_1\overline{I}+(1-\tilde{r})\delta_1 r\overline{s}_2\overline{I}]$	$\tilde{r}(1-\overline{s}_1)\overline{I}+$	$\underline{\delta}_2(1-\underline{\gamma})(1-\underline{p})\underline{I}$
	$+(1-v_0)(1-\gamma)\delta_1\underline{p}\underline{I}$	$(1-\tilde{r})\overline{\delta}_2(1-\overline{s}_2)\overline{I}$	

That is, Country 1 will be soft in period two, which is a contradiction, since Country 2 (soft) will be better off refusing first period offer $\overline{r}_1(s_1)=0$. Simultaneously, if Country 2 (soft) will accept an offer in period 1 with probability $\overline{r}_1(s1)<\tilde{r}$ then Country 1 will be tough in the second period and Country 2 will be better off accepting offer in period 1. Thus the equilibrium strategy for Country 2 is to play $\overline{r}_1(s1)=\tilde{r}$. [7]

[7]For Country 1, in order to be indifferent between playing tough or soft in second period, Country 2 must play a mixed strategy. It must also be indifferent between accepting and refusing first period offer. Therefore, Country 1 first period offer must satisfy this condition. The probability of making a tough offer in second period must be:

$$\sigma_2(1)=\frac{(1-\underline{s}_2)\underline{I}-(1-\overline{s}_2)\overline{I}}{\overline{s}_2\overline{I}}$$

which makes Country 2 (soft) indifferent between the payoff in period one and the expected outcome of period two. According to Rasmunsen [60], strategy 3 in this models have one equilibrium offer but multiple equilibria of acceptance probabilities. This is due to the fact that Country 2 must mix between accepting and rejecting and Country 1 must also mix between playing tough and soft in second period. We assume that country soft will accept when $\overline{r}_1(s1)=\tilde{r}$ without ruling out the multiple equilibria because at this value is the largest probability of Country 2 accepting immediately avoiding the loss of utility in delay.

Table 4.3: Possible equilibrium outcomes

No war	Strategy 1	Acceptance in first period
	Strategy 2	Soft country accepts in the first period and tough in the second.
Possible war	Strategy 3	Soft country accepts sometimes in period 1, always in period 2. Tough country never accepts

Consequently, there is a unique perfect equilibrium in this game with three possible strategies which depend on the particular values of the parameter of this model. According to these parameters, Country 1 can play any of the strategies in table 4.2. The result of the choice of strategies by Country 1 is summarised in table 4.3.[8] □

4.3.1 Numerical computation of the 3 strategies

In order to analyse the equilibrium properties of the model we first carry out a numerical computation allowing for some variation of the key factors, whilst holding others constant. In this section we hold $(F_1, \overline{F}_2, \underline{F}_2)$ constant thus keeping fixed the probability of winning a potential war after disagreement in period 2. We choose $F_1 = \overline{F}_2 = \underline{F}_2$ thus setting the probability of either country winning at $p = 1/2$. Other parameters are as given in table 4.4 unless stated otherwise.

Figures 4.2 and 4.3, show graphs of expected payoffs given in table 2 associated with changes in parameters α and v_0 from their baseline values. All the results are intuitive. First, a war equilibrium in our model might happen when Country 1 finds optimal to play strategy 3. This is more likely to happen when Country 1 thinks it is facing a soft country; i.e. when v_0 is high. In figure 4.2 we choose

[8] In our simulations we considered the case $\underline{s}_2\underline{I} > \overline{s}_2\overline{I}$ and/or $\underline{s}_1\underline{I} > \overline{s}_1\overline{I}$ which occurs when the soft Country 2 allocates much higher resources to fighting than the tough Country 2. The table of payoffs for this case is given in appendix D.1.

Figure 4.2: The production function parameter α and the resulting outcome when investment in war is fixed.

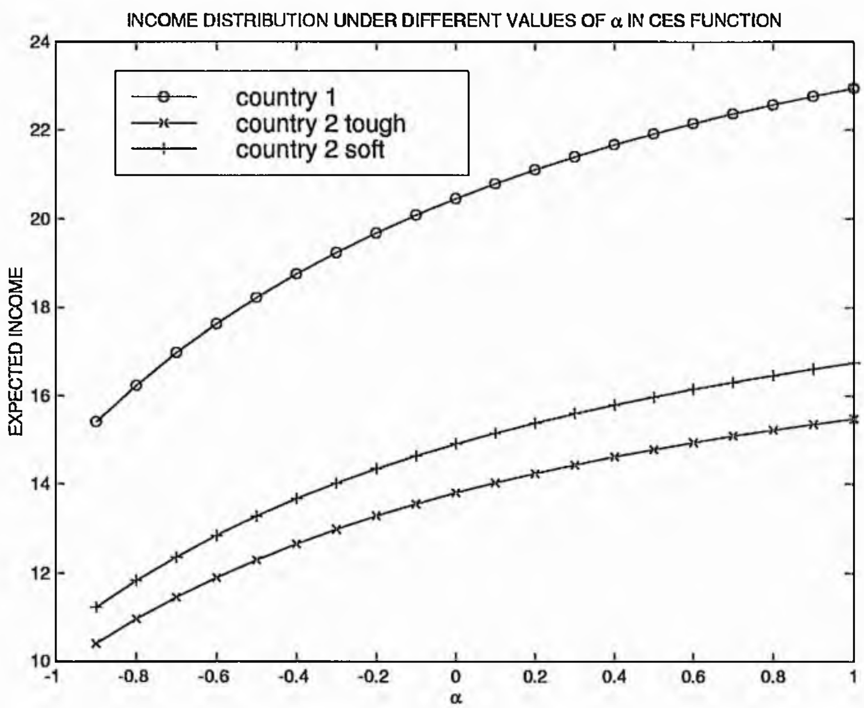
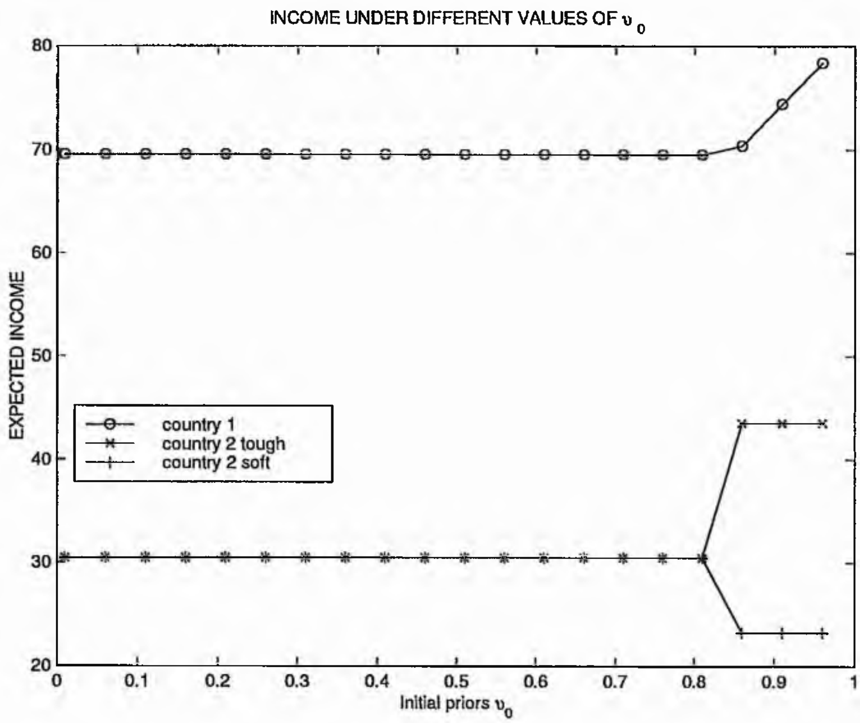


Figure 4.3: The change in the initial priors υ and the resulting outcome when investment in war is fixed



$v_0 = 0.5$ and vary α in the range [-1,1]. Then for the parameter values shown strategy 1 is chosen and settlement always occurs in period 1. The parameter α is a measure of interdependence and consequently as it increases so does the payoffs for both countries.

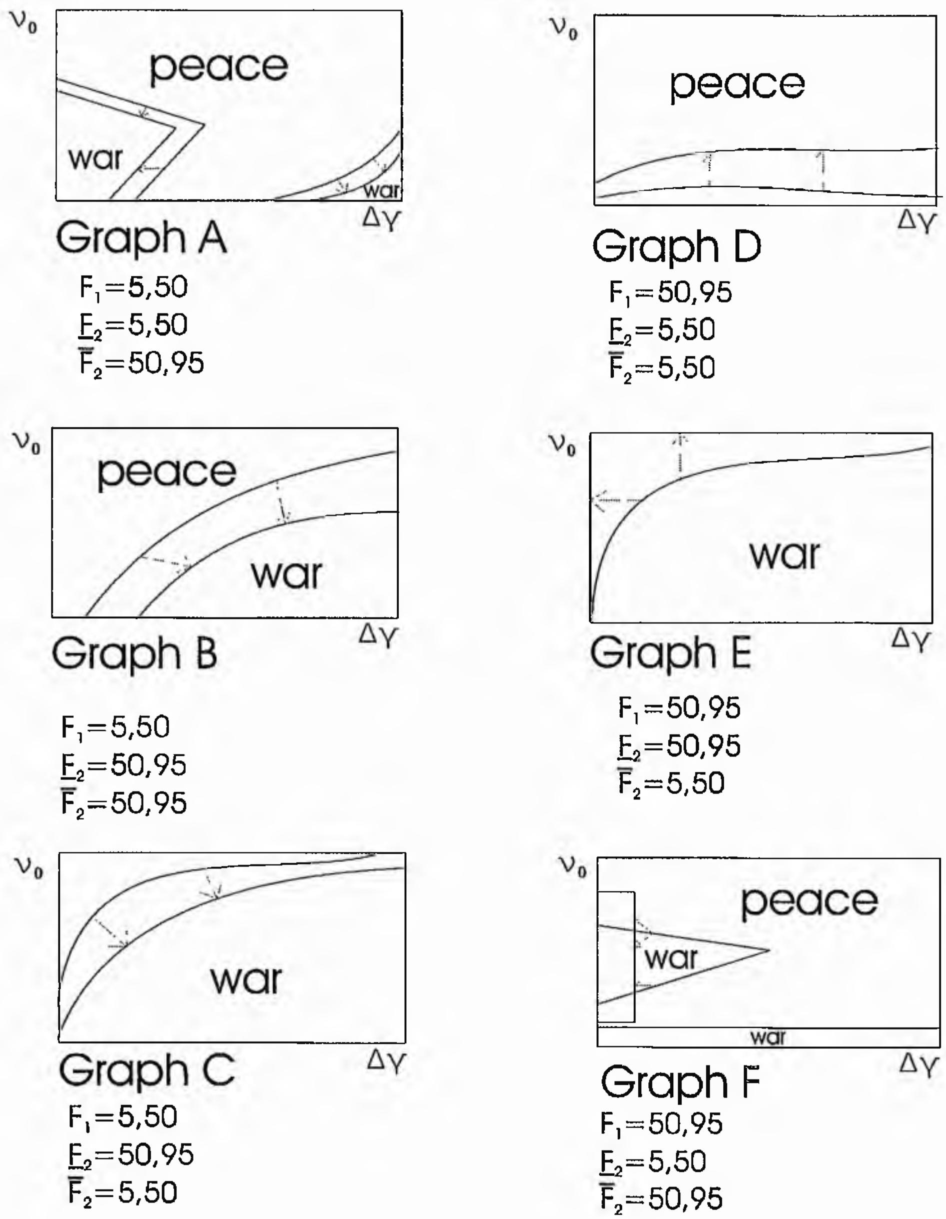
Table 4.4: Baseline Parameter Values

Country 1	Country 2 soft	Country 2 tough
	$v_0 = 0.5$	
	$M = 1$	
	$\alpha = 1$	
$\delta_1 = 0.9$	$\delta_2 = 0.9$	$\underline{\delta}_2 = 0.9$
$\gamma = 0.2$	$\overline{\gamma} = 0.15$	$\underline{\gamma} = 0.1$
$R_1 = 100$	$\overline{R}_2 = 100$	$\underline{R}_2 = 100$

A more interesting result is obtained by varying the initial priors, see figure 4.3. When Country 1 believes that there is a probability of facing a soft country, $v_0 > 0.8$, then it uses strategy 3. Although, there is a general loss of income, given the high probabilities of facing a soft country, the expected income is increasing toward what would be the perfect information optimal offer. There is an increase in the income of Country 1, since $E[I|v_0]$ increases with the probability of facing a soft country only when it plays strategy 3. For Country 2 (soft), there is a loss of income and a gain for Country 2 (tough).

Under imperfect information the resulting outcomes are more realistic than the models of costly conflict with perfect information presented in chapter 3. Country 2 enjoys some strategic advantage from its private information and this diminishes the strategic advantage that Country 1 has by making the offers. Another interesting result is that a high prior can produce a war outcome independently of the resource allocation process. Given certain values of v_0, Country 1 will find optimal to play strategy 3. However this threshold value depends also on the magnitudes of $\overline{\gamma}$ and $\underline{\gamma}$ as the next result shows.

Figure 4.4: Conflict outcomes with fixed allocations.
In graphs **A,B,C,D,E,F** the difference between $\overline{\gamma}$ and $\underline{\gamma}$ is located in the horizontal axe (**cost of war** $\Delta\gamma$) and the prior belief about facing one of the two types of Country 2, v, in the vertical axe (**Probability of facing soft**). Different combinations of v and $\Delta\gamma$ produce two scenarios (PEACE and **possible** WAR). The investment in fighting ($F_1, \overline{F}_2\underline{F}_2$) is fixed.



In figure 4.4, there are several graphs, all of them, with fixed allocations to fighting efforts, representing the relation between the informational problem measured as the difference perceived by Country 1 about Country 2 cost of war, $\Delta\gamma = \overline{\gamma} - \underline{\gamma}$, and the prior v_0. It produces two regions: the region 'PEACE' corresponds to the use by Country 1 of strategies 1 or 2 in table 4.3. This gives a settlement without war. The region call 'WAR' is the result of Country 1 playing strategy 3 in table 4.3. The probability of war is conditional on Country 1 choosing strategy 3. It means that Country 1 prefers to face a situation of war with some probability rather than using strategy 1 that is always accepted in the first round.

In graphs **A**,**B**, and **C** Country 1 has less fighting power that Country 2. In graphs **D**,**E** and **F**, Country 1 has a higher fighting power. In order to test the effect of arms races, the same simulation was carried out for low and high levels of arms expenditure. In each graph there should be two loci that plot the frontier between peace and war strategies. One for low fighting effort and one for high. In Graph A there are two war areas. The one on the left correspond to the scenario were $\overline{s}_2\overline{I} > \underline{s}_2\underline{I}$ and the one at the bottom corner corresponds with the inverse situation. This occurs since we exogenously fixed $\overline{F}_2 > \underline{F}_2$.

We also contrasted cases where Country 1 is less likely to win the war than Country 2, represented by the graphs on the left, and simulations where Country 1 has a greater probability of winning the war, on the right side of figure 4.4. An overall increase in arms expenditure has a negative impact on peace for those cases where Country 1 had a disadvantage in military power with respect of Country 2. Alternatively, increases in fighting expenditure has a positive effect when Country 1 had a positive advantage in fighting capability. These results are consistent with the empirical results about arm races and war. This seems to support the conventional conclusions about the effect of deterrence, risk uncertainty and ambiguous effect of arms races in political models of conflict, specially the case were arms races have a positive effect on peace when they increase the advantage of the leader and negative otherwise.[9]

In figures 4.5 and 4.6, the allocation of resources is fixed as in figure 4.3.1 and 4.3.1. In the first case there is a large difference in the perception of the externality cost that Country 1 has about Country 2 ($\overline{\gamma}$ = soft and $\underline{\gamma}$ = tough), and in the second case the externality cost of war is small for all types (γ = 0.2, $\overline{\gamma} = 0.15$, $\underline{\gamma} = 0.1$). In figure 4.5, the probability of facing a soft country is relatively small for all types ($v_0 = 0.3$). Given some equal probabilities of winning the war, the effect of asymmetric information is quite significant in this figure. For values of $\overline{\gamma}$ higher that 0.5, Country 1 is better off playing strategy 3, which is reflected by the jump in the payoffs curves. Figure 4.6 is a special case. This figure shows how unstable the equilibrium can be when investment is fixed and the soft country has a higher expenditure in arms that the tough one. Fixing allocations to fighting capabilities can have a roller-coaster effect in conflict. When countries can allocate resources optimally, the soft Country 2 never spends more resources in fighting capabilities, producing a greater system stability. The war probabilities in figure 4.6 are more complex. although it has the same probabilities of winning the war than Country 2 soft, the fact that it faces a tough country (low cost of war) with a military disadvantage, creates many opportunities for conflict.

[9]However we must be careful with the fact that our model chooses Country 1 and the private information of Country 2 in an arbitrary manner. So far there is no a consistent empirical study of the influence of asymmetric information about the cost of war. Regarding the effects of arms races and war that provide similar stylized facts to the ones found by our simulation read S.Sample [84], Huth *et al.* [85] and Huth *et al* [86].

Figure 4.5: The effect of changes in the cost of a soft Country 2 when investment in war is fixed

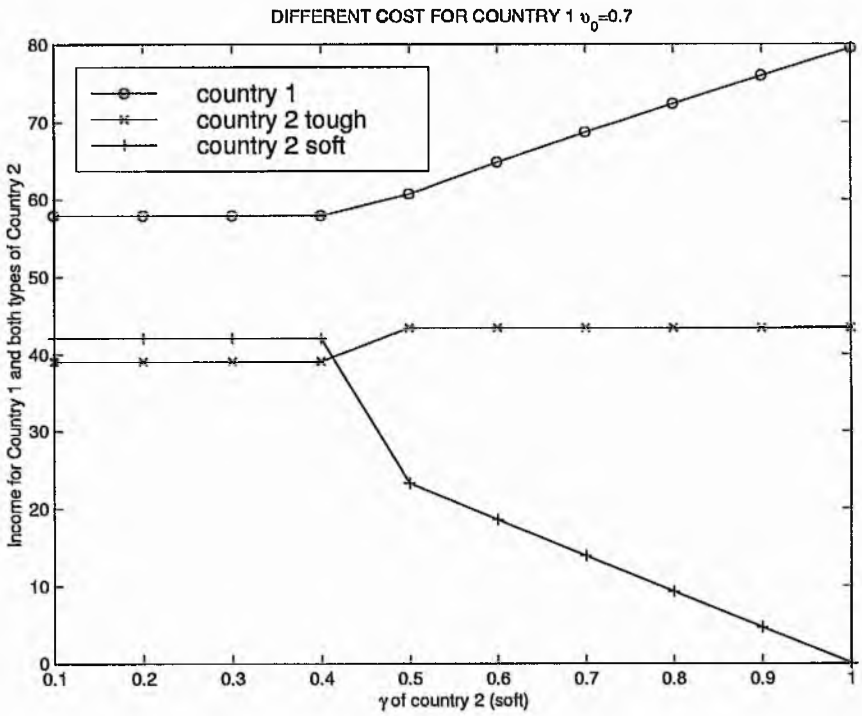
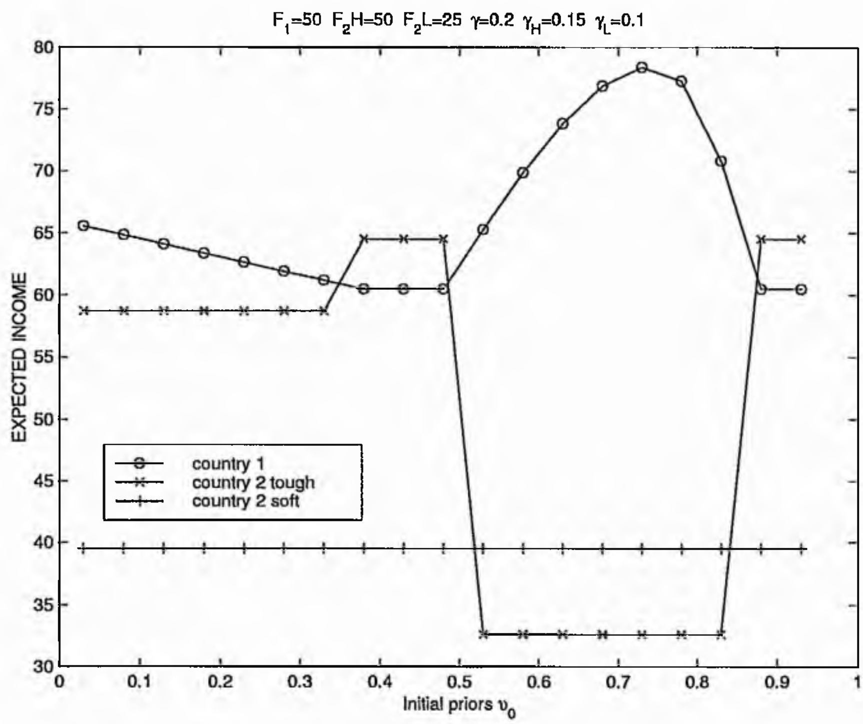


Figure 4.6: The conflict roller-coaster.

When the tough country invest significantly less than the soft, the condition $(1-\underline{\gamma})(1-\underline{p}) > (1-\overline{\gamma})(1-\overline{p})$ does not hold. Consequently, Country 1 plays strategies in appendix D.1. when $0 < \upsilon_0 < 0.4$ Country 1 uses strategy 2. From $0.4 < \upsilon_0 < 0.5$ it uses strategy 1. From $0.5 < \upsilon_0 < 0.85$ it uses strategy 3 and for $\upsilon_0 > 0.85$ it uses strategy 1 again. Under this circumstances, the probabilities of war increases when Country 1 is not sure about what type of adversary is facing.



4.4 The Full Equilibrium

The previous section studied out-of-equilibrium strategies with respect to the initial allocation of resources. The next step is to decide what would be the optimal allocation of resources given the nature of the game. One important remark is that in our model we assume a tie-breaking rule in strategy 3 by which Country 2 (soft) plays $\overline{r}(s_1) = \tilde{r}$. Otherwise we will have scenarios where participants update their beliefs in an arbitrary fashion which causes multiple equilibria. Therefore, the possibility of finding which is the optimal allocation of resources in this framework should be ruled out. This could be tackled introducing some equilibrium refinements, which would be more interesting in a two-sided asymmetric information framework.

In order to find out the optimal strategy, we form a three player Nash game for any of the three strategies.

4.4.1 The Nash equilibrium in Initial Allocations

Consider first the case where Country 1 adopts strategy 1. Then Country 1 chooses F_1 according to:

$$\max_{wrtF_1} \underline{s}_1\underline{I} \;\; s.t.\, R_1 = F_1 + E_1 \tag{4.15}$$

Country 2 (soft) chooses $\overline{F}_2$ according to:

$$\max_{wrt\overline{F}_2} (\overline{I} - \underline{s}_1\underline{I}) \;\; s.t.\, R_2 = \overline{F}_2 + \overline{E}_2 \tag{4.16}$$

and Country 2 (tough) chooses $\underline{F}_2$ according to:

$$\max_{wrt\underline{F}_2} (1 - \underline{s}_1)\underline{I} \;\; s.t.\, R_2 = \underline{F}_2 + \underline{E}_2 \tag{4.17}$$

We substitute the linear constraint into the objective function and take the derivatives with respect of F_1 $\overline{F}_2$ $\underline{F}_2$. This maximisation problem produces 3 *reaction curves* that can be solved simultaneously in order to find the Nash equilibrium.

$$\text{RC1: } \frac{\partial \underline{s}_1}{\partial F_1}\underline{I} + \frac{\partial \underline{I}}{\partial F_1}\underline{s}_1 = 0 \tag{4.18}$$

$$\text{RC2 (soft): } \frac{\partial \overline{I}}{\partial \overline{F}_2} - \frac{\partial \underline{s}_1}{\partial \overline{F}_2}\underline{I} - \frac{\partial \underline{I}}{\partial \overline{F}_2}\underline{s}_1 = 0 \tag{4.19}$$

$$\text{RC1 (tough): } \frac{\partial (1-\underline{s}_1)}{\partial \underline{F}_2}\underline{I} + \frac{\partial \underline{I}}{\partial \underline{F}_2}(1-\underline{s}_1) = 0 \tag{4.20}$$

For the other strategies, every player maximizes its payoff according to table 4.2. It also produces three reaction curves in each strategy which every country solves simultaneously. Then, Country 1 compares the outcomes of these 3 strategies and chooses the one that has the highest payoff.

4.4.2 Numerical Results

Once Country 1 knows what will be the equilibrium allocation of resources, it can play the game in the same manner that we presented in section 4.3. We developed this game in a Matlab algorithm that finds the unique equilibrium for any parameter in the game. In figures 4.7, 4.8 and 4.9 we present the outcomes of this simulation.

In figure 4.7 the externality cost for Country 1 was $\gamma = 0.7$ and for Country 2 (soft and tough) was $\overline{\gamma} = 0.3$ and $\underline{\gamma} = 0.2$. The strategy 3 was played for $v_1 > 0.9$ which gives a very low probability of war. When the values of the cost of fighting were low ($\gamma = 0.2$, $\overline{\gamma} = 0.15$, $\underline{\gamma} = 0.1$), the probability of war nearly disappears and tends to the perfect information solution (see figure 4.8). The difference between the cost of Country 1 and Country 2 affects the expected income, but not the probability of war. The highest probability of war occurred when we

Figure 4.7: The payoffs effect of changes in v_0 when investment in war is optimal: Case a.

In this figure the difference between $\overline{\gamma}$ and $\underline{\gamma}$ is not high, ($\gamma = 0.7, \overline{\gamma} = 0.3, \underline{\gamma} = 0.2$) and the cost of conflict for all players is very low. This produces war with a small probability represented by the jump in the three curves for values of $v_0 > 0.9$. Naturally, the three curves are horizontal as long as Country 1 does not choose strategy 3, which is the only one that depends on v_0.

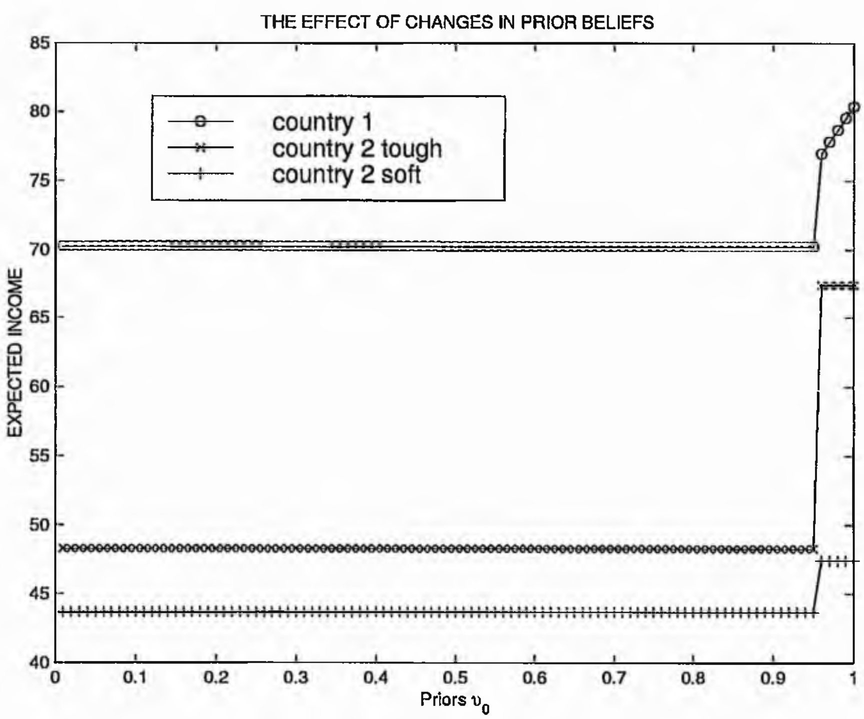


Figure 4.8: The payoffs effect of changes in v_0 when investment in war is optimal:b.
In this figure, the cost of war was very low for Country 2,($\overline{\gamma} = 0.3, \underline{\gamma} = 0.2$) Country 1 chooses always strategy 1, which produces peace. Compared to the previous graph, we can see that asymmetric cost between Country 1 and 2 has little effect in the choice of strategy (but affects the expected income).

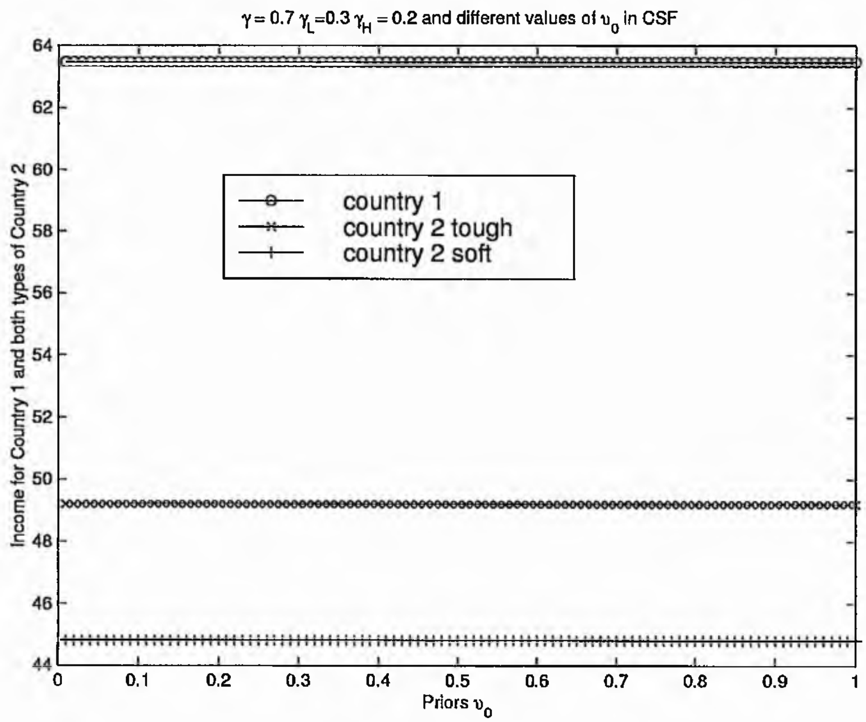


Figure 4.9: The payoffs effect of changes in v_0 when investment in war is optimal :c.

When the difference between $\overline{\gamma}$ and $\underline{\gamma}$ is very large ($\gamma = 0.7, \overline{\gamma} = 0.6, \underline{\gamma} = 0.1$) , the probability of war increases drastically. Unless we have strong believes that a country is soft, we cannot get a war scenario which in this models is represented by the decision of Country 1 to use strategy 3 in table 4.2

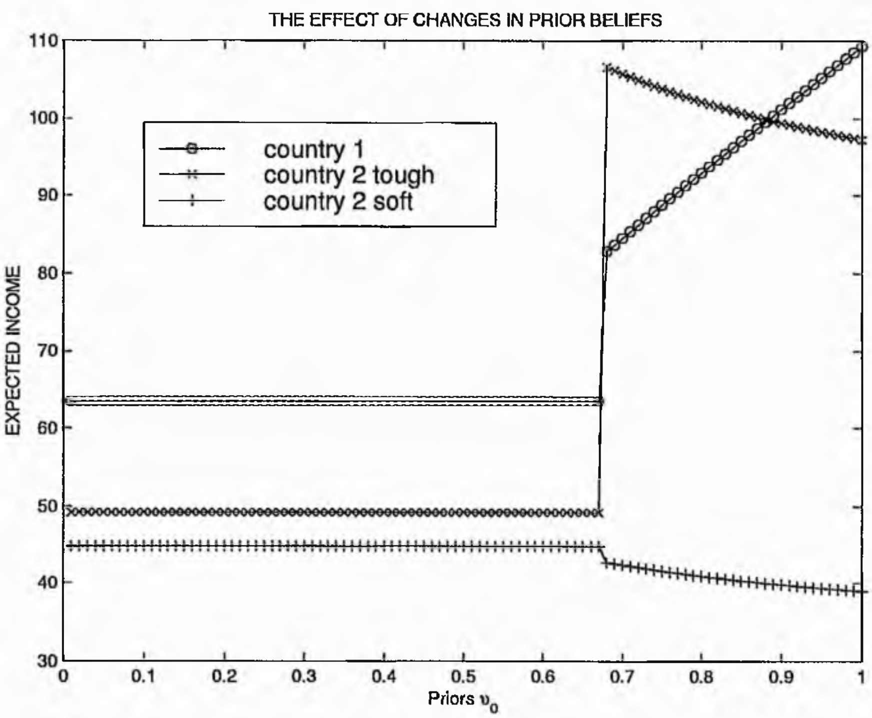


Table 4.5: Values for the baseline calibration of optimal allocations

Country 1	Country 2 soft	Country 2 tough
$\alpha = 1$		
$M = 1$		
$\delta = 0.9$	$\overline{\delta}_2 = 0.9$	$\underline{\delta}_2 = 0.9$
$\gamma = 0.7$	$\overline{\gamma} = 0.3$	$\underline{\gamma} = 0.2$
$R_1 = 100$	$\overline{R}_2 = 100$	$\underline{R}_2 = 100$

introduced a high difference between $\overline{\gamma}$ and $\underline{\gamma}$.

The rest of the parameters in the model took the values as in the table 4.5

These results indicate that priors and cost of war work in a complementary manner. In a model of one-sided incomplete information war outcomes are the result of a combination of high probabilities of facing a soft country with large differences between the cost faced by soft and tough participants. Finally, unless Country 1 decides to follow strategy 3, the expected income is constant in relation the priors v_0. The point where the income curve jumps for all the countries in figures 4.8 and 4.9 corresponds to a change from strategy 1 to 3.[10]

Finally we show in figures 4.10 and 4.11 the ratio of resources dedicated to fighting effort under optimal allocations.

In 4.10 we plot the ratio of resources against the probability of facing a soft country v_0. When this probability is larger than 0.7 Country 1 plays tough. Paradoxically, despite facing some probability of war, it reduces its military expenditure. This is due to the change of strategy. In this scenario, the probability of facing a strong adversary is so low that Country 1 decides to take a gamble, plays tough and reduces its expenditure in arms at the same time. While both types of Country 2 are forced to increase the expenditure in arms to compensate for the risky behaviour of Country 1.

Figure 4.11 is also interesting. In most part of our simulations, the expendi-

[10]High values of δ ruled out strategy 2.

ture dedicated to fighting effort experiences little changes with changes in other parameters. However, differences in the cost of war have a great impact in the optimal allocation of resources. This is also the case in previous models with perfect information where the cost of fighting is one of the main factors in determining optimal allocations. In this graph, the probability of facing a soft county is: $v_0 = 0.5$. The cost of fighting increases with the asymmetric cost. This is due to some constrains that we put previously in the model.[11] Therefore, figure 4.11 shows that the tougher is the adversary, the more one should spend in fighting effort.

Figures 4.12, 4.13 and 4.14 represent the relation between war(or peace) and the informational problem –the difference perceived by Country 1 about Country 2's cost of war, $\Delta\gamma = \overline{\gamma} - \underline{\gamma}$– and the initial probability (v_0) of facing one type or the other. It produces two regions. As in figure 4.4, the region PEACE corresponds with the use of strategies 1 or 2 in table 4.3 by Country 1 and WAR is conditional on Country 1 playing strategy 3 in table 4.3. For instance, in figure 4.12, when $\Delta\gamma$ is between 0.3 and 0.4, the conditional probability of war equals 0.2. (The probability of war was never bigger than 0.5 per cent when participants allocate their resources optimally).

The peace area in the graph produces always a peaceful outcome. However, the war area should be read carefully. For any combination of $\Delta\gamma$ and v_0 that falls in the area, there is a probability of war, which will take place only if Country 2's true type is tough $p = (1 - v)$.

In figure 4.13, the simulation was carried out with very low values of the discounting factor for tough and soft participants. After an initial increase in the probability of war, it decreases for values of $\Delta\gamma > 0.5$ in the horizontal axe. The reason is that since the discounting factor is very low, facing the probability of

[11] $\gamma \geq \overline{\gamma} \geq \underline{\gamma}$. We have already shown that for the case where $\gamma \leq \overline{\gamma} \leq \underline{\gamma}$ Country 1 does not need to care about asymmetric information and plays the as if it had perfect information regarding only its own cost.

Figure 4.10: Resources dedicated to fight and prior beliefs

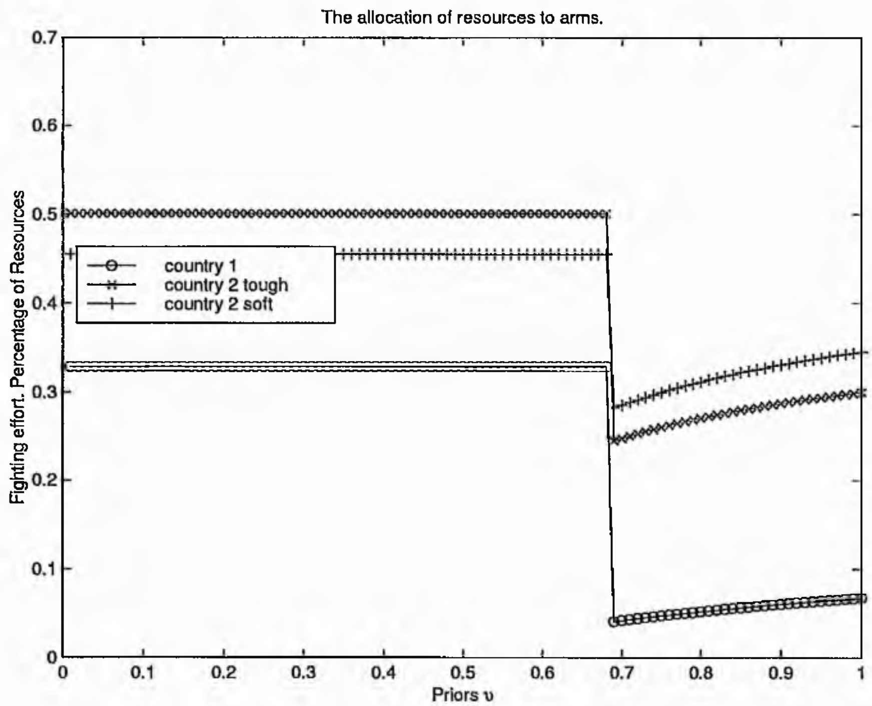


Figure 4.11: Resources dedicated to fight and cost asymmetry

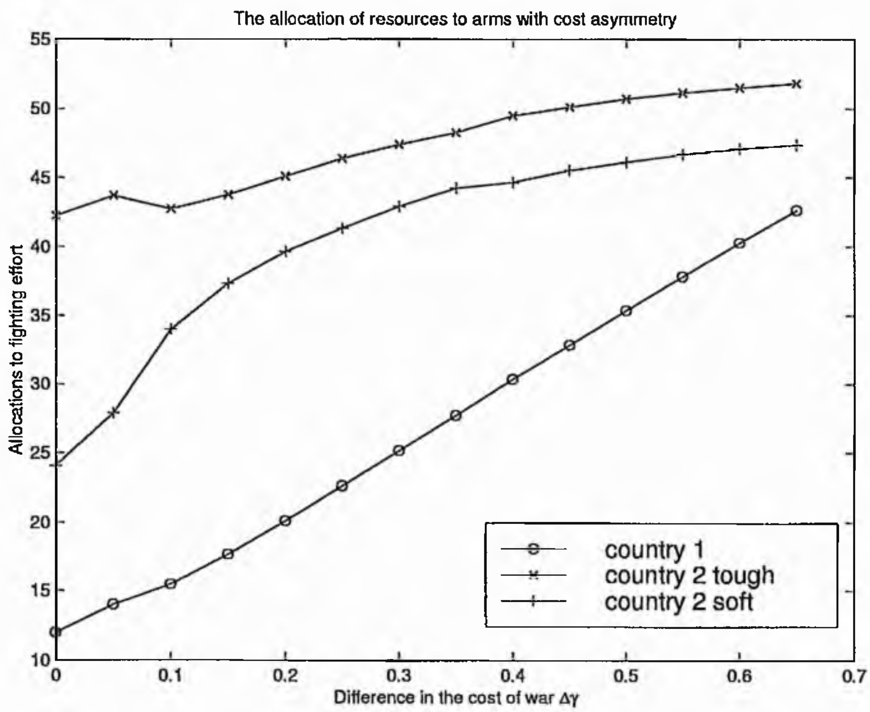
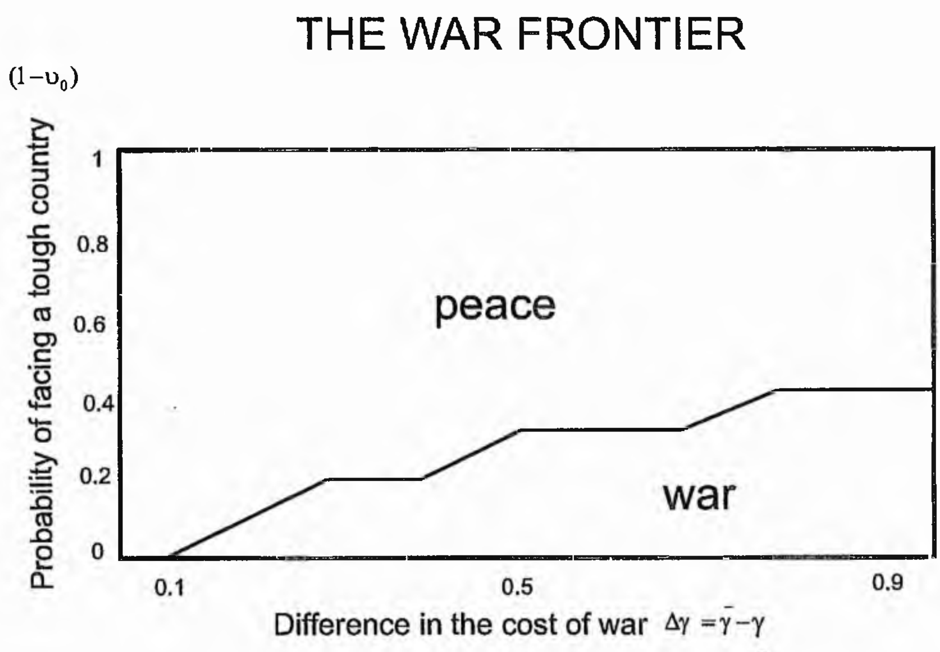


Figure 4.12: The probability of war: a

This graph is similar to the graphs in figure 4.4. But we have used optimal allocations The probability of using strategy 3 is positively related with increases in $\Delta\gamma$ and υ.We can see that the probability of war is positively related to the probabilities of facing a soft country and to the magnitude of the information asymmetry

Country 1	Country 2 soft	Country 2 tough
$M = 1$		
$\alpha = 1$		
$R_1 = R_2 = 100$		
$\delta = 0.9$	$\bar{\delta} = 0.9$	$\underline{\delta} = 0.9$



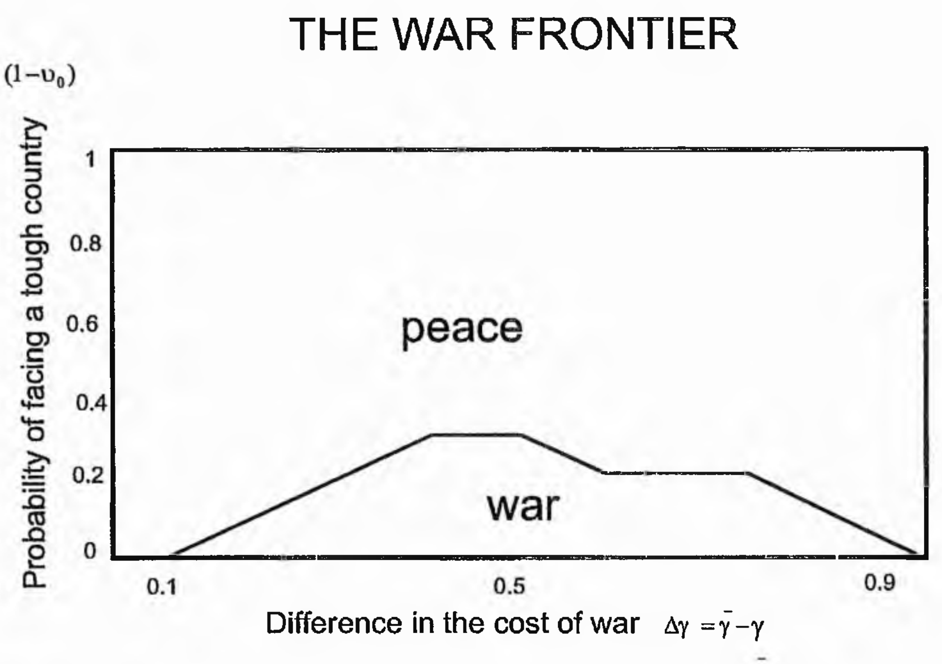


Figure 4.13: The probability of war: Case b

In this figure we show the contradictory effects of the discounting factor. The values of the discounting factors were set to a extremely low level. $\bar{\delta} \approx \underline{\delta} \approx \delta \approx 0.2$. There is a point where the effect of impatience is more important that the possible gains of war. The more likely is Country 1 to face a tough country, the more likely is to play strategy 1.

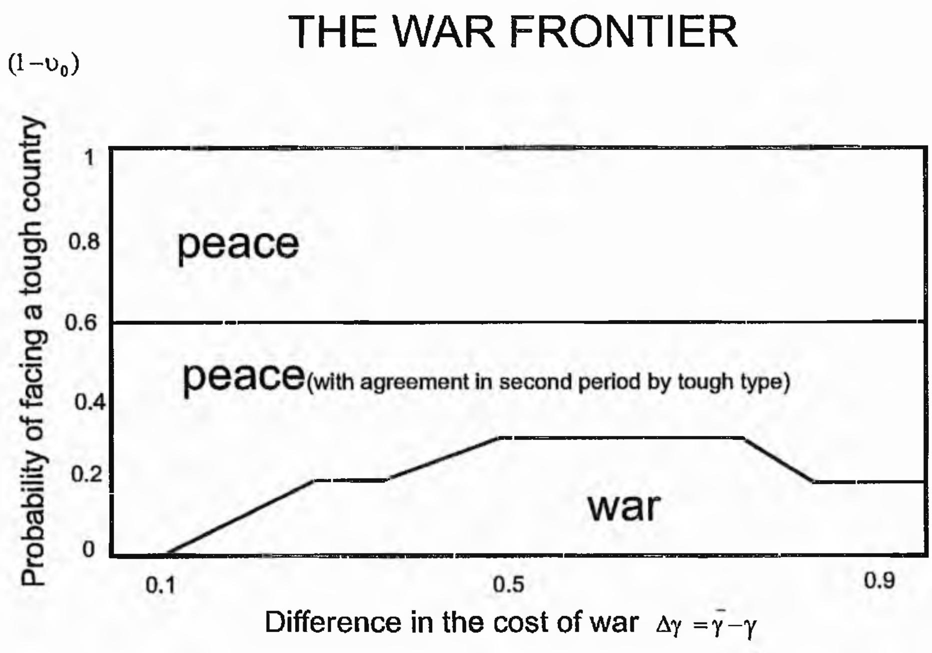


Figure 4.14: The probability of war: Case c

After several simulations, we found out that according to our model, strategy 2 is rarely played. For that to take place, we had to give low values to $\overline{\delta} = 0.05$ and high values to $\underline{\delta} = 0.95$

war makes strategy 1 more attractive in relative terms because both countries are very eager to reach an agreement in period one. Remember that in order to face a war, we must wait for period 2. So, with high probabilities of facing a soft Country 2, $v_0 > 0.4$, waiting becomes very expensive. But there is a point where the benefits obtained from the discounting factor, outweigh the benefits of exploiting the difference in the cost of war, producing a substitution effect in strategies.

In figure 4.14, we have three areas. In this occasion, the three strategies in table 4.2 are played by Country 1. When Country 1 believes that there is less than 0.6 probability of facing a soft country, it plays strategy two. This only happens when the difference between $\overline{\delta}$ and $\underline{\delta}$ is considerably big. For situations where a tough and soft country have the same degree of impatience, Country 1 never plays strategy 2.

Although we cannot generalized, from our simulations we can concluded that first, those situations where Country 1 has a strategic advantage in both negotiation and military terms, a peaceful settlement is more likely to occur — that is to say, when the status quo is not challenged we have more probabilities of seeing a peaceful outcome. Second, an optimal allocation of resources is also optimal in producing peaceful outcomes.

There is an abundance of evidence in the literature of conflict showing that leaders are subject to limited information, which produces misconception and bias in the choice of policy. Uncertainty about the cost of war can bring together some aspects from the rationalist explanations of war and alternative theories that argue that leaders are sometimes irrational. Asymmetric information about the cost of war could produce a pattern of behaviour from leaders that, although being rational, could act as if they would neglect the costs of war or enjoy the benefits without paying the costs.

Optimal allocation of resources doesn't guarantee a more peaceful situation. However, since changes in some of the factors can produce ambiguous results, it seems more obvious, from the perspective of the public choice, that interventions directed to reduce the informational gap, are likely to have a better result than those directed to control the allocation of resources into fighting activities.

4.5 Conclusions and Developments

This paper presents a model of conflict that differs from other models of war in that we combine incomplete information with an endogenous probability of winning the war, which for most part of the literature of bargaining over conflict is exogenous. We allow participants to establish a negotiation process over the possible consequences of endogenous allocations to production and fighting efforts, absent in the old literature of conflict. Participants always try to avoid costly conflict which takes place in some extreme circumstances.

The main conclusions of the model are:

- The likelihood of war with endogenous optimal allocation of resources were significantly lower than the probability of a war under a fixed initial allocation of resources. This indicates that the optimal prior allocation of fighting and productive activity on the negotiation process has a more significant impact in the outcome of war than other factors traditionally considered more important such us the military strategic advantage or the probabilities of victory in contest.

- We have experimented with variations in all exogenous parameters. The parameters that have the most influence are a) prior beliefs, v_0 and b) the difference between the externality cost of the soft and tough countries $(\overline{\gamma}-\underline{\gamma})$. This difference moves the break point of strategy 1 to strategy 3 for Country 1 to a lower value of v_0, increasing the difference between the final payoffs of Country 1 and Country 2 (soft and tough). The war decisiveness parameter m and the degree of integration α affect the expected income for both countries but have little role in determining the outbreak of war.

- In this model, the probabilities of winning the war and the size of the final outcomes are endogenous. Although war is not an efficient outcome, it is sometimes a PBE due to the effect of asymmetric beliefs about the cost of fighting. We conclude that if *the first casualty of war is truth* we should add that *the ultimate victim of asymmetric information is peace.*

A number of directions for future research are suggested by our results. The obvious question is what happens when both countries have incomplete information. F&T found that not strictly separating equilibrium can exist when the two participants are uniformed. Bayes rule places no restriction on Country 2 posteriors. The choice of conjectures and additional restrictions may lead to pooling or separating equilibria. Separating equilibria works as a restriction to the optimal allocation of resources, which will provide a higher probability of war outcomes.

It would also be interesting to study the effect of long-term war. Our model assumes a once-for-all war. There are some models where participants can decided in each period if the take part in war or not. This would allow us to design new conjectures and signalling mechanisms, increasing the probability of war in the short run but reducing the long run-cost of war.

Chapter 5

Empirical Analysis

5.1 Introduction

In the previous chapters we have studied the foundations of conflict from an economic perspective. It takes place at two distinct but interrelated levels. On the one hand, conflict exists because resources are limited. Given this limitation, different actors competing for these resources will allocate their efforts into a) appropriation of these scarce resources and b) transformation of those resources into goods and services. On the other hand, given this allocation of appropriative and productive efforts, actors need a mechanism of distribution.

In this chapter, we look at the second level of conflict with application to inter-state conflicts. We focus on conflict versus trade as means of distribution.[1] Conflict is an exchange mechanism for restoring the equilibrium in the absence of agreement. The main characteristic of this mechanism is that induces high transaction costs. Therefore trade and agreement are always influenced by the

[1]Given some balance of power, preferences and a specific environment (institutional constraints, access to information etc.), countries will eventually find an equilibrium between capabilities and share of resources. When some external factors (say technological innovations) break this equilibrium, countries will reallocate their resources by two different mechanisms: one is trade, based on common agreement, and the other is conflict, which does not require agreement.

shadow of conflict and both activities can be regarded as the two faces of the same coin. This rather theoretical concept will be helpful in understanding the relations between two different traditions [2]of empirical research on conflict.

We will focus on inter-state war for several reasons. Wars are the most costly and significant form of conflict. In the absence of an economic taxonomy of conflict, we choose war because there is a great number of theories and data against which we can contrast our theories. And finally, although states are members of an international system, they mainly interact in a purely strategic manner [3] in the sense that they have full sovereignty and they are not subject to the control of higher institutions.

One of the characteristics of empirical research about war is that data is grouped according to different levels of actor interaction. The level of analysis in international politics was first raised by Waltz [90] and Singer [91]. According to these authors, we can derive explanations of war based on the analytical level of the unit of observation from the level of the state, dyad[5], region and finally, international system. Since our study focuses on the micro-economic foundations of conflict we will concentrate on the state and dyad level.

[2]Of course conflict can take place in many different situations and it is also affected by non-economic factors. It is a very heterogeneous phenomena that can be approached in many different ways. There is an absence of a scientific taxonomy of conflict that clearly establishes the stylized facts, although several researchers have approached its classification from different perspectives. Any scholar that reviews the empirical studies of war will face a great disparity of studies, based also on a great disparity of theories, disciplines and assumptions that go with them. It has also produced a great disparity of results. After a careful review of the literature, the impression that one gets is that, as far as conflict analysis is concerned, there is a lack of consensus.

In the words of Geller and Singer [87] ...*for every investigation devoted to the search for war's correlates, there are thousands of studies that hope or claim to identify its causes or origins or roots.*

See appendix E.1 for other studies of conflict.

[3]Although we will mainly focus on the empirical analysis of rational choice models, there are also some implications of our theories for system analysis.[4] In this chapter we don't review these main families of empirical studies. However we will mention some of the main conclusions in order to establish whether our theories about optimal allocation of resources and the role of information are compatible with these models or not.

[5]A dyad is an interaction between two countries for a given period of time

As well as the different levels of actor interaction, we have other caveats. The empirics of war show that there are also different levels or intensities of conflict. In our models we assume a once-and-for all war, whose intensity depends on the level of militarization. But we can also assume non-optimal allocations which will produce out of equilibrium levels of conflict. Barringer [92] investigates the factors that contribute to the transition from one stage of conflict to a higher level. He defines a dispute as a *"felt grievance by a party capable of waging war"*. But every party can engage in different levels of hostility, starting from military mobilisation to full use of its military power. However we are concerned in our study with the outbreak of hostilities.[6]

Therefore, from all the set of possible conflicts we will concentrate in dispute dyad interactions which are closer to the strategy of bargaining procedures. Gochman and Maoz [93] define Militarized Interstate Disputes as "A set of interactions between or among states involving threats to use force, display of military force, or actual uses of military force". Once we have defined a set of disputes, we hope to find some empirical evidence of the significance of our variables to the outbreak of hostilities, notwithstanding the level of hostilities which we assume to be endogenous.

In the next section we will focus on the liberal peace paradigm and expected utility theories of war, which are closely related to our models. In section 5.3, we review the available data on conflict, especially data sets and variables. Finally in section 5.4 we carry out some empirical analysis, taking into account some of the features of both traditions. We will interpret the results in the light of our simulations of bargaining and conflict of previous chapters. Given the difficulties to capture the true cost of conflict, our empirical analysis won't estimate an econometric model but will concentrate in one aspect: We will use some modifications suggested by our theoretical models to develop our own explanation of

[6]We have already mention in our models the possibility of introducing some level of war which will function as a signalling process in a bargaining procedure

the trade–conflict debate. Thus, we will defend the empirical applicability of our previous research.

5.2 A survey of empirical research

Given the complexity and heterogeneous nature of conflict, we can find many different empirical approaches. During the eighties, the main theories concentrated at the systemic level of conflict and power, as the force driving international relations. This is more closely related to what we called the first level of conflict which is related to the distribution of resources and capabilities.

In the nineties the research has concentrated mainly on the dyad level. At this level there are two main approaches that can be regarded as complementary. The first one is relevant to the democratic peace theories. It looks at the effects of democracy and interdependence while controlling for systemic and realist factors. On the other hand we have the rational or utility models of war which look at the relation between status quo, balance of power and the utility derived by different kind on international interactions, while controlling for factors such as the democracy and the levels of trade. In section 5.4 we will try to analysed how these two branches are highly interconnected when accounting for the microeconomic foundations of conflict as set up in the theoretical models of conflict of previous chapters.

5.2.1 Testing the Liberal Peace Paradigm

The relation between democracy and peace has been studied thoroughly in political science. There is a broad consensus that at dyadic level, more democratic dyads are less likely to get involved in wars. In fact, there are no examples of two democratic countries[7] fighting each other. At the systemic level, Gleditsch and Havard [78] analyse the impact of democratisation of the international system. The numbers of wars at the global level is a parabolic function of the increase in numbers of democratic countries in the system. A democratic development in the

[7] We refer to a definition of democracy in terms of high scores in the democracy index by the POLITY III [94] project.

global system produces a reduction of wars only at higher levels of democratisation. There is otherwise, considerable consensus amongst researchers about the positive effects of democracy.

Oneal and Russett [95], [96], [97], [98] expand the analysis of the democratic peace incorporating the influence of trade and joint membership in international organizations.[8]

The hypotheses maintained by these authors are:

- Democracies will use force less frequently, especially against other democracies
- Economically important trade creates incentives for the maintenance of peaceful relations
- International organizations constrain decision makers by promoting peace in a variety of ways.

Their statistical methods pooled BCSTS (Binary Cross Section Time Series) data of dyads observed annually. They determine the likelihood of conflict as a function of differences across thousands of pairs of states.

On the dependent variable side they use the MID (Militarized Interstate Disputes) data set. Disputes are defined as those interactions in which one or both states threatened the other to use force, made a demonstration of force, or actually use it. The variable equals 1 if the dispute was ongoing and 0 if not. They include all disputes whether they are the initial one or not, with all independent variable lagged one year to ensure that they are not affected by the dispute to be explained.

On the right-hand side of the equation they include three main categories of variables. Variables related to measuring the effects of democracy, trade and

[8]Since have the intention of concentrating in measurement problems of different hypothesis, we refer to this articles for a comprehensive review on the subject.

alliance membership; variables to control for rational explanations of war and variables to control the effect of system interaction. It is important to control for realist variables. Take, for example, countries that are so far apart that they cannot reach each other with military effectiveness, it seems reasonable to include a variable for contiguity. System variables are important since power theorists argue that prior to 1945 the world was a multi-polar system, bipolar during the cold war and presently is understood to be multi-polar. The effects of the *Kantian*[9] variables are considered under these three types of regimes.

In Russett and Oneal(1999)(Henceforth R&O) the variables that correspond to the liberal democracy hypothesis are several indices of democracy as measured by the POLITY III project. Economic interdependence is measured by dividing a country's sum of exports and imports with its partner by its GDP. International organizations membership counts the number of these organizations in which a country takes part as reported by the *Yearbook of International Organizations.*

Although the levels of democracy and alliance membership seem to have a undisputed positive effect on peace, the effects of trade are more contradictory. Beck et al. reassess the liberal peace paradigm taking temporal interdependence into account. Their analysis corrects for duration dependence by using a set of dummy variables or fitting a natural cubic spline in a variable capturing the number of peace years. It accounts for the number of peace years before the current observation. For observations with no previous disputes, this variable is simply $t - 1$: following a disputes this variable is $t - t0$ where $t0$ is the time index of the most recent dispute. The results contradict the hypothesis that trade has a significant influence in the reduction of conflict [10].

R&O [99] took this problem into consideration and reanalysed the model using different theoretical specifications and measurement for trade variables. They used GEE methods for controlling temporal dependency, introduce alternative

[9] Also known as the democratic liberal peace hypothesis.
[10] See Appendix E.5

measures of interdependence and proximity and estimate some observations for unreported trade. They reported significant effects from trade dependency for the set of all politically relevant dyads. However, the case is not so clear under the logit estimation method for the set of all dyads as reported in the appendix table E.2.

Therefore, there is a general belief that democracy has a positive impact on peace. Although there has been extensive research on the effects of trade, and new estimation methods have been developed, there is no unquestionable evidence of a significant relationship.

5.2.2 Testing utility models of War

Rational choice applications to war initiation begin with the assumption that states interact with each other in order to maximize the utility generated by these interactions. Decision makers are assumed to evaluate the costs and benefits of their actions and choose those strategies that produce higher utility. Utility models of war assume that these decision makers behave in similar manners when faced with an interstate crisis.

The origin of expected utility theories of war can be traced to Bueno de Mesquita's *The War Trap* [100]. In posterior work, Bueno de Mesquita and Lalman [101] expand the strategic nature of decision makers interactions to a game theoretic framework.

> The course of events leading to or away from international conflict is always diverse and complex in its details. Thus we would not expect any two historical events to be identical, certainly not in their specifics. No account of an event, no model, no history, is complete in its representation. Learning from an understanding of general phenomena, whether for the historian or for the social scientist, requires a concentration on essential features – in this case, the structure rather

than the specifics of interstate actions. Our model is an attempt to delineate interrelated decisions around international military crises, highlighting the opportunities for peaceful relations, which are juxtaposed against the sometimes great dangers imposed by negotiating in the presence of a potential resort to arms. We incorporate a number of aspects of international disputes, ranging from concern over the military costs if an action precipitates an attack by an antagonist to concerns over domestic political opposition. In addition, once we have explored the non-obvious, unanticipated implications of our model under the assumption that both players are fully informed about the preferences and the intentions of the opponent, we turn to an analysis of the game under the condition of imperfect information.

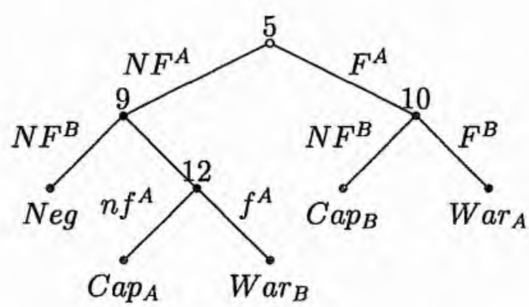


Figure 5.1: The conflict subgame

Figure 5.1 represents the crisis subgame of *War and Reason.* In node 5 Country A faces the choice of whether it uses force against B or not, F^A and NF^A respectively.[11]

At node 9, Country A did not resort to arms, and Country B can in turn, choose from using force or not. If it doesn't use force it leads to a negotiated

[11] We concentrate only in the dispute outcomes of the international interaction game. This games starts by a nature move by which Country A makes the offers in node 1. There are other equilibrium outcomes— such as making no demands or acquiescence to other's player demands— which, for simplicity, are not shown here.

outcome. If it uses force, it can expect to be counterattack or not, which leads to two possible outcomes, Capitulation by A or War initiated by B. At node 10, Country B is attacked by A. It faces again two options, retaliate or capitulate.

The game in itself is of a simple nature. The complications start when one tries to assess the equilibrium outcome by evaluating the preferences of both players over all the possible outcomes. Bueno de Mesquita and Lalman made a series of assumptions in this respect.

They assume common rationality in a similar fashion to game theory. The outcome of war is uncertain and therefore the evaluation of war outcomes are based in expected values. The probability of winning the war for country i is P_i. On the other hand, the outcome of capitulation is certain. Both countries prefer to settle the dispute by negotiation rather than conflict. Measures from the status quo (SQ) are $U^i(\Delta_i)$ —the utility from obtaining one's demands— and $U^i(\Delta_j)$ — the utility from acceding to the other country's demands.

The concern about the cost of conflict is one of the fundamental parameters in the evaluation of all the violent outcomes. We have already mentioned in section 5.3.2 the different costs assumed in M&L's models.

> Each outcome has a set of potential benefits and/or costs appropriately associated with it. We make restrictions on the various costs such that $\alpha, \tau, \gamma, \phi > 0$; and $\tau > \alpha$.

Given these assumptions, the formulas of the expected values are provided in table 5.1. Bueno and Lalman provide a series of restrictions concerning the preferences, payoffs, the status quo and the informational structure of the game.

There are two main problems with the empirical evaluation of these models. One is of a theoretical nature: Critics of rationality have highlighted a series of problems that must be taken into consideration. [12] The other problem concerns

[12]Many authors have reviewed extensively the rationality assumption in relation to interstate

Table 5.1: Outcomes and expected utilities for nation i
SQ (Status Quo); Acq_j (Acquiescence to i's demands by j) ,Acq_i (Acquiescence to j's demands by i) ,Neg (Negotiated Outcome) , $Cap\ A$ (Capitulation by Country A), War_B (War initiated by Country B), $Cap\ B$ (Capitulation by Country B) and War_A (War initiated by Country A).

SQ	$U^i(SQ)$
Acq_j	$U^i(\Delta_i)$
Acq_i	$U^i(\Delta_j)$
Neg	$P^i[U^i(\Delta_i)] + (1 - P^i)[U^i(\Delta_j)]$
Cap_j	$U^i[\Delta_i - \phi_i(P^i)]$
War_j	$P^i(U^i[\Delta_i - \phi_i(P^i) - \alpha_i(1 - P^i)])+$ $(1 - P^i)(U^i[\Delta_j - \phi_i(P^i) - \alpha_i(1 - P^i)])$
Cap_i	$U^i[\Delta_j - \gamma_i(1 - P^j)]$
War_j	$P^i(U^i[\Delta_i - \phi_i(P^i) - \tau_i(1 - P^i)])+$ $(1 - P^i)(U^i[\Delta_j - \phi_i(P^i) - \tau_i(1 - P^i)])$

Source: Bueno and Lalman

the utility calculations. Even if international decision makers are rational, it is very difficult to know the correct functional forms to calculate the payoffs of different actions for players that may have highly heterogeneous preferences and perceptions.

If we control for the utility of conflict, we should be able to estimate rational choice models. We have already mentioned that one of the main obstacles is the absence of data about the cost of conflict. Nevertheless, many authors have found considerable empirical support for this kind of models.

Bennett and Stam [102, 103, 104] test the utility theory of war. They ran some test for different time periods and sets of dyads. They found that the set of politically relevant dyads produce more robust results that the set of all dyads. They also explore the effects on preferences and decision structures of regional differences in culture, learning and domestic policies finding substantial differences across regions and time on how expected utility correlates with conflict

conflict. In this chapter we only address the empirical difficulties.

occurrence.

Their expected utility calculations are based on the eight possible outcomes from the international interaction game (IIG). Given state A as a potential challenger and state B as a target these outcomes are:

1. A status quo outcome
2. A challenge resolved by negotiation
3. A challenge resolved by state A acquiescence (Giving in to B's demands)
4. A challenge resolved by state A capitulation (Giving in to B's demands after the threat or use of force)
5. A challenge resolved by state B acquiescence
6. A challenge resolved by state B capitulation
7. A war initiated by A
8. A war initiated by B

Utilities are unique to each member of the dyad and each interaction AB is distinct from BA which leads to different equilibria.[13] Thus, there are 16 relevant utilities: $U^i(SQ)$, $U^j(SQ)$, $U^i(Acq_i)$, $U^i(Acq_j)$, $U^j(Acq_i)$, $U^j(Acq_j)$, $U^i(Neg)$, $U^j(Neg)$, $U^i(Cap_i)$, $U^i(Cap_j)$, $U^j(Cap_i)$, $U^j(Cap_j)$,$U^i(War_i)$, $U^i(War_j)$, $U^j(War_i)$ and $U^j(War_j)$.

These utilities include all the parameters in the *War and Reason* equilibrium although there is no way to distinguish between τ, γ and ϕ. (See appendix E.6). The states' utilities and preferences are operationalised using risk attitudes and similarity scores from their alliance portfolios. The subjective probability of

[13]In previous chapters we have already explain the theoretical differences in outcomes from assuming different initiators

winning is estimated using the Correlates of War's national capabilities index and estimations of the probability of intervention by third parties.

Finally, under the realpolitik variant[14] war is never expected under perfect information so it is not analysed further.

Then, the expected equilibrium can be estimated and compared with the actual events to assess how well the prediction correlates with the actual behaviour. Bennett and Stam[103] use the computer program *Eugene* to generate the data for each directed-dyad-year from 1816 to 1993. The dependent variable was reclassified in an index that captures different disputes levels which approximate the equilibria of the war game. Level 1 represents the status quo. Level 2 is a threat. Level 3 represents a display of force from one state. Level 4 indicates the actual use of force by one state and level 5 is the mutual use of force or war.

Their model consists of a single decision amongst 5 different not ordered[15] choices. On the independent variable side, they include 3 dummies, covering the 4 possible[16] outcomes of the IIG. The model is estimated by multinomial logit. They also corrected for temporal dependence using Beck et al method[105, 106] of including a set of 4 spline variables that take into account the time that has passed in a dyad since a prior dispute.

The multinomial logit estimates the effects of the equilibrium predictions on whether the disputes ends up at each of the five previous outcomes. In order to assess how the game's equilibria correlate with the initiation of disputes we can test if they predict any outcome but the status quo, level 1. However, it is difficult to predict whether the variables are statistically significant and what are

[14]The realpolitik variant treats foreign policy demands as emanating from a realist or neorealist perspective of international affairs. The magnitude of these demands depend on the structure of the international system. National leaders are seen as professional decision makers that select policy goals by examining the external constraints and opportunities that arise from international interactions.

[15]Whether, for example, mutual use of force was chosen or not, doesn't say anything about the intensity. However, most part of conflict follow an escalation process and it may be well justified to assume that these choices are in fact ordered.

[16]The logical restrictions of the game eliminates some outcomes.

their specific effects. All the coefficients in each equation are given in relation to a base outcome. Individual t-tests indicate only whether the variable has a significant effect in distinguishing between the base category and the category in question.

Any statistically significant coefficient in any equation suggests that a variable is important in differentiating between base case and equation. This indicates in turn that the variable in question has some significant effect in the outcome. They use the likelihood ratio test to assess whether a variable has any significant effect on the overall model.[17]

Finally, they assess their model against different sets of countries and time periods. They found that their predictions fit substantially better in some areas and some times than others. In general, they interpret those results by suggesting that most part of this variation is due to potential differences in preferences or in how costs and benefits may be weighted by different societies. This is an idea supported by our simulations of conflict where asymmetric information about the costs of conflict and prior beliefs play a crucial role in the equilibrium outcome.

[17] See Bennett and Stam[103] for a complete description of the tests.

5.3 Data

Any empirical analysis begins with the systematic collection of data. Unfortunately, economists haven't dedicated a lot of effort to producing a comprehensive dataset of conflict and economic variables. There are two main problems. On the one hand, no one has systematically collected those events where the nature of conflict was primarily economic. We have to deal with data sets that may include disputes where the underlying factor may be economic or not. This is particularly relevant to our models of expected utility because they do not normally include non-economic factors in the utility functions. On the other hand, the cost of conflict has never been carefully collected. We have recorded military expenditure. But conflict incurs an extra cost that goes beyond the opportunity cost of moving resources away from directly productive activities to unproductive ones. Conflict is an exchange mechanism with high transaction costs. But these costs have never classified and recorded systematically.

Since we rely on data produced by different projects on the field of politics, it is helpful to mention how this data is collected. The application of scientific methods to political events has been aided by the development of *event data.* According to Rummel [107], *event data* is "data for which the rules of inclusion and exclusion of political events are clear and consistently applied to all events." Since data is collected, according to some political behavioural rules for a number of states, we can establish different profiles, and comparisons can be made between these states.

Data can also be collected in three other forms. First, *behavioural flows*, that is statistical aggregates measuring many kinds of uniformly occurring transactions such us trade, economic aid, tourists, migrants, and the like. Second, there are *behavioural structures*, "These are existing, formal behavioural relationships, such us a treaty, alliance, or common membership in an international organization". And third, there are *attribute data*, "which measures or define the magnitude of a nation on some characteristic."

In appendix E.3 we provide a review of the most important sources of data for the study of major conflicts. We focus on how much information is available in these data sets to establish some stylised fact of conflict and the importance of asymmetric information.

In the next two sections we are going to mention some of the problems that affect the collection of data for testing the models of rational decision making and war that we have developed in previous chapters.

5.3.1 Data on Expected Utility of War

We have seen that there are many ways of approaching the empirical study of conflict. It would be an overstatement to say that there are as many approaches as conflicts recorded, but the list goes certainly a long way. Bueno de Mesquita argues that the strategic approach developed in *War and Reason* should have no systematic cross–regional cross-temporal variation.

Bueno de Mesquita and Lalman's [101] theory of expected utility of war is some of the most widely cited research about international conflict that can be traced to the publication of *The War Trap* [100]. Bueno de Mesquita's measurements of interest, risk and utility are widely accepted in the literature. This is one of the most interesting developments because these theories are also decision-theoretic in nature and rely on the use of game theory models.

The process of deriving utility data can be quite complicated because it involves many options regarding the representation of the initial parameters of these models. Testing strategic models requires a specific approach to generate the utility data. One of the most interesting efforts in this field has been undertaken by Bennett and Stam [108] in the development of the EUGene project. EUGene is a program that serves as a data management tool for creating data sets. It creates output data sets with directed-dyad year, country year, and directed-dispute dyad units of analysis. It is particularly appropriate to test so-called expected utility theory of war and models of games–theoretic interactions between states.

The variables are described in appendix E.6 according to the unit of analysis of relevance. Some of these data come from well-known data sets that are described in this section. But most part of the data is calculated following the models in *The War Trap* or further extensions developed by Bennett and Scam [102, 103].

5.3.2 The cost of war

In order to test expected utility models, it is crucial to understand all possible benefits and costs of a given action or strategy. However, when we talk about international relations, these calculations might be hard to perform. This is especially true when it comes to the cost of conflict. In *War and Reason* the cost takes the same range of values, $[0, 1]$, as in our models of bargaining and conflict. Bueno de Mesquita and Lalman distinguish four different costs:

> α is the cost born by the attacker for fighting away; τ is the cost borne by the target in a war; γ is the cost borne by a state that gives in after being attacked; and ϕ is the domestic political cost associated with the use of force.

There are several assumptions about these costs. They differ between initiators and target nations in terms of expected losses in life and property. It is justified because normally the initiator have a greater control over the venue of fighting. The expected cost is also a function of the probability of success and therefore, weak states facing strong rivals expect larger losses, all things being equal. The cost of giving in after being attacked, apart from the costs in life and property, includes psychological effects such as loss of face and loss of credibility and reputation. And finally the domestic political costs is assumed to be larger for big developed nations, since they have more instruments of international interaction and their populations expect their politicians to find other means of solving international disputes.

The large diversity of factors that may affect the expected value of the cost of war creates one of the main problems of any empirical analysis. Mostly, scholars use approximations for it. Assessing the effects of the value given to the status quo, Bueno de Mesquita defines the cost of war as *the duration of the war in dyads and in terms of battle deaths per million population for each participant.* This is based on the data found in the COW data set.

However, trying to estimate the expected cost for all the factors mention about is impossible since data would be difficult to collect and appropriate testing hasn't been developed yet. The domestic cost, ϕ in *War and Reason* is estimated using the probability of winning and the value of the status quo. The cost of life is assumed to be an inverse function of relative power. However, they do not devise a way to estimate α, γ and τ.

> Clearly, our estimation of expected cost only scrapes the surface of the problem. These very limited approximations may introduce considerable measurement errors into our analyses. Assuming that our operational procedures are not systematically biased on way or the other, we expect that, on average, the crudity of our estimation of costs suppresses rather than inflates our results. But we cannot be confident of this claim until better indicators are developed and tested in the future.

After a careful consideration, we can clearly see that the cost of conflict is a factor very difficult to estimate and the data sets on conflict contain weak approximations to it. Basically, we only have duration, the battle deaths and the value of the status quo before and after the war.

If data sets contain weak approximations of this factor, we face the same problem when we turn our attention to case studies. Most part of the time we rely on qualitative research. One of the few projects that tries to establish a comprehensive evaluation of all the cost of conflict was carried out by Michael Cranna et al. [109].

The book consists of seven studies of individual conflicts. The conflicts have been selected to represent the different kinds of conflict that occur, from wars between nations to guerrilla campaigns.

The costs incurred by the countries involved in these particular conflicts are analysed both quantitatively and qualitatively. The costs of development are studied looking at the impact of conflict in education and health. The economic costs is analysed by looking at macroeconomic indicators such us production, debt, inflation, etc. Other non-economic costs are analysed such the human costs of conflict and the cost to the environment. A general problem was finding the data to account for the impact of these factors, especially regarding the internal conflict. Cranna explains the limitations and assumptions of the project:

> It has been difficult to find contemporary data about some of the conflicts. This is particularly so for the intra-state conflicts, like those in Kashmir and Sudan, where regions rather than nations are involved, and regional statistics are unavailable.

If we look at post-World War II patterns of conflict we can see a decline in international conflict and an increase in intra-state conflict. A possible factor to explain it might be the increase/decrease of uncertainty at the national/international level. This can influence the cost–benefit analysis of conflict strategies by different actors. In our previous models we show that asymmetric information about the cost of conflict may be one of the most important determinants of the occurrence of conflict. Unfortunately, for the time being, this claim is only speculative due to the lack of data on these factors in the empirical literature.

5.3.3 Other factors to take into consideration

For our purposes, which is the study of conflict from a economic perspective, we find several obstacles. Although there is a wide range of studies that produce a systematic compilation of all conflicts, both at the level of hostilities (the aspect of conflict that is normally recorded) and the dispute level. There is no a systematic compilation of all the factors that may affect conflict. At this level there might be diplomatic measures such as threats of military intervention, international litigation's or arms races. We would like to know all those circumstances in which despite the presence of incompatibilities of any kind, parties resolve their problems without the use of violence. Few data sets can provide a good description of the information structure. Since we assume that actors are rational and try to maximise expected utility, in war or peace, the problem is to establish what is the set of information in relation to what Bueno de Mesquita calls *the relevant sources of uncertainty for a decision maker*:

1. Marginal advantage or disadvantage in war capabilities of his nation as compared with the potential opponent,

2. how much he values the policies adopted by his own country in comparison with those of the potential enemy,

3. the capabilities of each other nation that might become involved in the war and,

4. the relative value or utility that these other nations may contribute to his nation, as compared with the value that they may contribute to his potential enemy.

In the presence of incomplete information problems one of the solutions is to assume that actors make rational choices based on subjective estimations of other actors military capabilities and expected utility. We can use rational expectations models and other method to deal with uncertainty. In this context there are several models or arms races. Since we cannot observe directly military capability (quantity and quality of weaponry and personnel[18]) we based our estimation on the level of military expenditure of the other country. Simon and Starr [76] developed a simulation model in which they calculate the probabilities of other actors having both the opportunity and willingness to initiate war or escalate rebellion.

The problem of unobservable factors is a fundamental one. There are many hidden influences that may precede the outburst of violence. How well those can be approximated by a few variables in a statistical model is a question yet to be addressed. As we mentioned before, scholars have put great effort in recording all different categories of violent events and disputes, but there has been little work done to account for all the relevant factors.

Bloomfield and Moulton [110] in a qualitative study of the causes of conflict produce a taxonomy[19] of all the factors that may have been of influence in all types of conflict. They represent conflict as a dynamic process in the sense of passing through some or all of a sequence of distinctive and identifiable stages or phases. Only at the highest level– hostilities involving combat among organized military forces– they accounted for 10 different categories and more than two hundred factors of influence.

> Within each phase there are a variety of influential events and conditions called factors, such as personalities, relationships, actions, events, perceptions, and other conditions. Some of these factors can generate pressures moving the situation toward "worsening," that is, increased violence or its threat; or, conversely, pressures to move the situation in a more benign direction, that is, away from violence. In

[18]Not many countries keep a public and up to date register of arms and military personnel
[19]See Appendix E.7

> other words, "factors" combine so as to worsen or improve the conflict and, thus, move the conflict towards or away from "thresholds" between phases in the direction of greater or less violence. The factors in the model are all from the specific facts that have been identified as influential in particular historical conflicts. These case-specific factors were then grouped and restated in generalized terms to permit comparison across cases.

This affects seriously the statistical analysis of conflict. It is specially relevant with regard to international disputes and wars. Some of the problems are also related to the methodology commonly used. Widely used statistical procedures such us logistic regression need certain characteristics on the dependent variable. But wars are very rare events. This has lead to very inefficient collection strategies. International conflict data sets contain more than a quarter million dyads with only very few wars. This has produced data with a huge number of observations and normally few and poorly measured explanatory variables.

King and Zeng [111] address the problem of logistic estimation and efficient variable selection in rare event data.

> ...we use all dyads (pairs of countries) for each year since World War II to generate a data set below 303,814 observations, of which only 0.3%, or 1042 dyads were at war. Data sets of this size are not uncommon in international relations, but the make data management difficult, statistical analyses time–consuming, and data collection expensive. (Even the more common 5,000–10,000 observations data sets are inconvenient to deal with if one has to collect variables for all the cases.) Moreover, most dyads involve countries with little relationship at all(say Burkina Faso and St. Lucia), much less with some realistic probability of going to war, and so there is a well founded perception that much of the data is "nearly irrelevant" (Maoz and Rus-

set, 1993:627). Indeed, most of it has very little information content, which is why we can avoid collecting the vast majority of observations without much efficiency loss. In contrast, most existing approaches in political science designed to cope with this problem, such as selecting dyads that are "politically relevant" (Maoz and Russett, 1993), are reasonable and practical approaches to a difficult problem, but they necessarily change the question asked, alter the population to which we are inferring, or require conditional analysis (such us only contiguous dyads or only those involving major powers). Less careful uses of these types of data selection strategies by others, such as trying to make inferences to the set of all dyads, are biased. With appropriate easy–to–apply corrections, nearly 300,000 observations with zeros need not be collected or could even be deleted with only minor impact on substantive conclusions.

This data problem should be considered especially when we try to interpret the results of econometric models.

5.4 A synthesis of the liberal peace hypothesis and utility theories of war

In the last two sections we have reviewed two well known paradigms: the utility theories of war and the liberal peace. The utility models of war provide a better theoretical framework for the study of conflict but they are difficult to test because data is hard to collect and the basic components of these theories (such us the number of strategic actors, utility functions or information environment) may vary between conflicts.

Therefore we concentrate on the *liberal peace* hypothesis (section 5.2.1). But these models lack a theoretical explanation of why democracy and trade should have an impact on conflict. We rely on game theory models to explain the importance of these factors through asymmetric information about cost, restrictions in the bargaining set, the role of the status quo and other environmental factors that influence expected outcomes. But the amount of trade, should not influence the final decision to go to war. Of course, if trade represents a large proportion of a country's GDP, the loss of trade should be added to the overall estimation of the cost. But it would only represent a proportion of the total cost and it doesn't affect the informational problem.

At this stage we make a clear distinction. Trade is important because it affects the bargaining set. For instance, two countries involved in a sovereignty question will have problems finding an agreement because sovereignty is not a divisible concept. However, if both countries have mutual trade interest, the dispute could be solved by altering the terms of trade. Whether the terms of trade are beneficial for a country or not is a different question. It may be the result of an imbalance in military power as well as a result of compensation from another dispute. In order to clarify this question, we propose a development of *Russet's liberal peace* that incorporates some aspects from the *utility models of* *war*.

We follow R&O specifications and methodology to study the effects on disputes of democracy and trade. But we make two important changes. We change the dependent variable to account for actual fighting. We believe that it is an important change. Game theory or strategic models explain the outbreak of war. But the variable that previous models use accounts for every level of dispute, from a simple threat of use of force to a savage war to the end. Threats and displays of force should not be recorded as disputes[20] as they indicate that some kind of bargaining is taking place. Secondly, we introduce the war equilibrium from the war trap (section 5.2.2) to account for those cases where beneficial trade can be a result of a given military advantage. Apart for these two changes, we use the same variables and estimation method as R&O's paper.

We take all dyads from 1950 to 1992 focusing on the first year of a militarized dispute. Instead of using all disputes, we use the variable war for those disputes that involve the use of physical force using the data from Gochman & Maoz [93] MID set. The variable War takes the value 0 if the level of dispute is 3 or lower and 1 it it is 4 or higher. All the independent variables are lagged one period in order to assure that they are not affected by the dispute to be explained.

The set of all dyads can give spurious result due to the fact that the number of peaceful dyads with low level of interdependence is sharply increased. Therefore we include several measures to control for proximity between countries. The effect of distance have been studied by several authors. Some of the studies (Siverson & Starr 1991 [112] Goertz & Diehl 1992 [113] and Kocs 1995 [114]) show that proximity produces opportunities to fight while other authors also show that it produces opportunities for trade and alliances, (Tinbergen 1962 [115], Deardorff 1995 [116] and Bliss & Russett 1998 [117])

In order to control for distance we use two measures that are common in

[20]We concentrate on the outburst of hostilities only.

the literature: distance between countries and distance between capitals. We introduce the variable *eq_wara* which is the result of the Bennett & Stam [108] calculations of the international interaction game. Eq_wara is the war equilibrium started by 'State A'. The war equilibrium started by state b is meaningless in this case because we coded state a is the initiator. By definition, there cannot be occurrences of war started by b.

The rest of variables follow exactly R&O. *Allies* takes the value 1 if states were linked by a mutual defence treaty or neutrality pact. The capability ratio *lncaprt* is the natural logarithm of ratio of COW's [118] capabilities index. Joint democracy, *jntdem*, uses the POLITY [94] data set scales for autocracy and democracy,

$$JNTDEM_{ij,t-1} = [(DEM_{i,t-1} + 10)(DEM_{j,t-1} + 10)].$$

The variable DEM ranges from -10 to +10, where +10 represents the higher possible level of democracy and -10 the lowest.

Economic interdependence is calculated using the IMF's *Direction of Trade Statistics* [119]. The economic importance of trade is calculated by assessing the sum of exports and imports relative to their national incomes. Let $X_{ij,t}$ be the exports to country j from country i at time t and $X_{ij,t}$ the imports. Then, country i's dependency on trade with j in year $t-1$ is:

$$DP_{i,t-1} = (X_{ij,t-1} + M_{ij,t-1})/GDP_{i,t-1}$$

Scales for small and large dependency, *smldepnp* and *lrgdepnp* respectively, are constructed using either the lower of higher trade-to-GDP ratio for each dyad. This kind of data manipulation takes into account the fact that primarily, the likelihood of conflict is a function of the degree to which the less constrained state is free to use force, in other words, the state that finds a higher utility in war.

5.4.1 The logit and GEE estimations

R&O found a significant relationship between conflict, trade and democracy.[21] We produced a similar analysis, but using those disputes where actual fighting took place. We also introduced the war equilibrium from War and Reason for the same set of dyads. We evaluated again the effects of democracy and trade on the

Table 5.2: Variable definitions

Variable name	Definition
$war_{ij,t}$	The onset of war between countries i and j (time t)
$jntdem_{ij,t-1}$	Index of Joint democracy (time $t-1$)
$smldepnp_{ij,t-1}$	Trade, smallest dependency of countries i and j ($t-1$)
$lrgdepnp_{ij,t-1}$	Trade, largest dependency of countries i and j ($t-1$)
$contgkb_{ij,t-1}$	Contiguity between states ($t-1$)
$majdyds_{ij,t-1}$	Major power involved ($t-1$)
$allies_{ij,t-1}$	Countries were formal allies ($t-1$)
$lncaprt_{ij,t-1}$	Log of capabilities ratio ($t-1$)
$eqwar_{ij,t-1}$	War equilibrium from the IIG ($t-1$)
$lgdstab_{ij,t-1}$	Distance between capitals ($t-1$)
$peaceyr^*$	peace years ($t-1$)

likelihood of military disputes using logistic regression on pooled BTSCS data. We estimated the following two logit equations corrected for temporal dependence and GEE [22]respectively,

$$\begin{aligned} war_{ij,t} &= \beta_1 jntdem_{ij,t-1} + \beta_2 smldepnp_{ij,t-1} + \beta_3 lrgdepnp_{ij,t-1} + \\ &\quad \beta_4 contgkb_{ij,t-1} + \beta_5 lgdstab_{ij,t-1} + \beta_6 majdyds_{ij,t-1} + \\ &\quad \beta_7 allies_{ij,t-1} + \beta_8 lncpart_{ij,t-1} + \beta_9 eqwar_{ij,t-1} \beta_{10} peaceyr1_{ij} + \end{aligned}$$

[21]Reported in table E.2

[22]General Estimating Equation (GEE) is an estimating approach that specifies within group correlation structure of panel data which is comparable to random effects regressions. See appendix E.4.

$$\beta_{11} peaceyr2_{ij} + \beta_{12} peaceyr3_{ij} + \beta_{13} peaceyr4_{ij} \quad (5.1)$$

$$\begin{aligned} war_{ij,t} &= \beta_1 jntdem_{ij,t-1} + \beta_2 smldepnp_{ij,t-1} + \beta_3 lrgdepnp_{ij,t-1} + \\ &\quad \beta_4 contgkb_{ij,t-1} + \beta_5 lgdstab_{ij,t-1} + \beta_6 majdyds_{ij,t-1} + \\ &\quad \beta_7 allies_{ij,t-1} + \beta_8 lncpart_{ij,t-1} + \beta_9 eqwar_{ij,t-1} \end{aligned} \quad (5.2)$$

Equation 5.1 was estimated using standard logit analysis, corrected for temporal dependency. We assume autocorrelated errors in our choice of GEE estimator plus robust standard errors to correct for heteroscedasticity.

Table 5.3: P.(war), trade and democracy. All wars 1950-1992.

	Logit			GEE		
warl	coef.	Std.Err.	p	coef	Std.Err	p
jntdem	-.0045408 *	.000987	.000	-.0043274 *	.0009711	.000
smldepnp	-21.43396	35.13748	.542	-184.326	103.2138	.074
lardepnp	.8349878	3.075665	.786	.098418	5.10245	.985
contigkb	2.98077 *	.2912681	.000	2.533498 *	.2824621	.000
lgdstab	-.5709842 *	.0953958	.000	-.8128447 *	.1080023	.000
majdyds	1.617365 *	.2872776	.000	2.269161 *	.2967918	.000
allies	-.5181032 *	.2181081	.018	-1.067208 *	.2429119	.000
lncaptr	-.203596 *	.0600283	.001	-.3245599 *	.2967918	.000
eqwar	.7322062 *	.2308517	.002	.7441603 *	.2692713	.006
peaceyr1	-.2777619 *	.0301813	.000	—	—	—
peaceyr2	.3018692 *	.0580222	.000	—	—	—
peaceyr3	-.052879	.0833488	.526	—	—	—
peaceyr4	.7449912 *	.2457548	.002	—	—	—
	Pseudo R^2 = 0.384			—		
	log likelihood=-1846.4209			—		
	P of $\chi^2 < .001$			P of $\chi^2 < .001$		
	Number of obs.= 270937			Number of obs.= 213338		

The coefficient estimation of equation 5.1 is reported in column 2 of table 5.3. The robust standard errors are reported in column 2 and the probability in column

3. In columns 4, 5 and 6 we have reported the results of the GEE estimation.

The index of joint democracy was significant in both estimations. Contiguity and distance between capitals were also significant and have the expected signs. The inclusion of a Major Power is significant but has an opposite sign to the results reported by R&O. The sign of the capabilities ratio was also negative and opposite to R&O . The two variables —Lower Dependence and Higher Dependence— that account for dyadic trade levels were not significant. Whereas the variable that corresponds to the *War and Reason* war equilibrium was significant in the logit and GEE methods.

Those changes can be explained if we distinguish between the two levels where conflict takes place that we mentioned in the introduction. We think about conflict from an economic perspective. A conflict is also a sort of trade that takes place by force, rather than two-side agreement. Compare to market exchange, redistribution by conflict is an inefficient exchange because it produces externalities and high transaction costs.

We would like to think that trade and conflict are part of the same process. That is why, when we change the dependent variable to record only actual fighting and not threats, the trade variables becomes insignificant. But when R&O use all levels of dispute they are significant. Obviously, actual fighting makes a difference and this is somehow explained in our theoretical models. It doesn't mean that trade and conflict are not related. Both trade and conflict are similar relations between countries. But when we include actual fighting trade becomes insignificant. Therefore, what we should be asking ourselves is not whether trade has positive effects on peace, but under which conditions, trade will take place by force or agreement.[23]

The inclusion of the war equilibrium and the change in signs of the *majdyds*

[23]This is of course highly relevant for economic theories of conflict. The number of conflicts in the World whose source hasn't got a strong economic component, such us identity conflicts, religion or ethnic, shouldn't be underestimated and this statements shouldn't be taken out of our main economic perspective unless one takes considerable care to account for other factors.

and the *lncaprt* variables —the presence of a Major Power and the Capability Ratio respectively— is also consistent with our theoretical models. For any given war equilibria, the increase in power in the initiator can only help to have a deterrence effect. The effect of a major power is more difficult to assess. However, the stylized facts reported by Rupesinghe [120] section E.1 confirms this result. Moreover, if we assume that major powers play in favour of the status quo, then they should have a definite positive effect.

All the results are supported by both methods of estimation and to some extent, consistent with the equilibrium outcome predicted by strategic theories.[24]

[24]Many of these articles can be found in the bibliography section

5.5 Conclusions

We have reviewed some of the recent empirical work on conflict with specific application to the *utility theory of war*. We have developed Russet and Oneal's work introducing a new dependent variable and using data and variables from Bennett and Stam's utility generation program. The results obtained are encouraging.

We found no significant evidence of the effect on conflict from trade dependency when we use the set of all disputes that record actual confrontation between states. We also found that the war equilibrium from *War and Reason* is significant which gives some support for our rational theories of conflict. These theories can be also applied to the interpretation of the different results in several empirical models.

All these results should be considered carefully. Testing rational models of war lags well behind its theoretical development. There are several caveats that shouldn't be forgotten. First of all, calibrating the utility of war is a difficult task. One of the main problems is the lack of data about the costs and perceptions of decision makers which are fundamental in determining the equilibrium outcome. While Bennett and Stam acknowledge that considerable bias can be introduced in these calculations, they hope that negative and positive effects cancel out. This should not be taken for granted.

Other problems arise from game theoretic constraints. Different equilibria are highly susceptible to the assumptions chosen. We have to be careful to allow for the possible effects in the outcome of a game introduced by one or another mechanism of negotiation. So far we haven't constructed yet any model of war as a signalling mechanism, models of preventive war – or a similar variant which is known as the *window of opportunity*. We haven't considered either any dynamic set up that take into account decisions makers that may base their actions in the calculations of long-term trends.

On the data collection front, although considerable effort has been put into recording all disputes in the international system, there are few explanatory variables which allows the testing of more ambitious hypotheses. The size of the data set is a great constraint and its development would require a considerable amount of time and resources. It might be worth considering smaller data sets, case studies and sampling techniques, in order to introduce more explanatory variables.

Finally, this work is related to those factors that may have a positive or negative effect on the initiation of conflict. We haven't said anything about its intensity. At the individual level, one may not be better or worse off if one gets killed in a small guerrilla campaign or in the nuclear holocaust, but at the social level is a question of great importance.

Despite the above problems, we consider that the amount of empirical evidence in favour of models of rational conflict behaviour is sufficient and further research should be taken in this direction.

5.5.1 Developments

Our simulations of conflict show that the main variable explaining the outbreak of hostilities is the asymmetric perception of the cost of conflict. We also give some advantage to the status quo and show that a scenario with optimal allocation of resources between production and appropriation is less war-prone than a non-optimal one. Some of these findings are consistent with the empirical regularities mentioned in the literature of conflict. It shouldn't be difficult to see the relation between democracy and information; between average ratios of military expenditure to GDP and optimal allocation of resources. We can also find in the literature some support for the design of our bargaining procedures. Geller and Singer wrote about the importance of the status quo:

> The orientation of a state's decision makers toward the status quo might be expected to have a critical impact on the probability of its

> initiation of, or engagement in, foreign conflict and war. Specifically, a nation that is satisfied with the status quo would be expected to engage in war only if attacked, and to initiate war solely under pre-emptive or preventive circumstances. In contradiction, a state which is dissatisfied with the status quo, might be expected to initiate the use of force whenever circumstances are favourable and nonviolent means for change prove inadequate. The logic is found in various studies focusing on the behaviour of major powers (e.g., Oerganski 1958; Organsky and Kugler 1980; Gilpin 1981; Modelski 1983), but it applies to minor powers as well.

However, there are some problems related to testing the utility models of war. First of all, it is very difficult to account for the utility of war since we don't have reliable estimates of the costs of war. On the other hand, game theory models' equilibria are highly influenced by the initial assumptions. For example, in a bargaining model, the equilibrium changes completely with changes on the assumptions about the bargaining extensive form. All these questions will require careful consideration.

Chapter 6

Conclusions

The existence of incompatibilities is a necessary condition but not sufficient to explain conflict. There are many theories that explain conflict according to different classifications of it. It can be approached from many different methodologies and every one of them offers some understanding of it. However, no discipline can claim the production of some "General Theory of Conflict". Therefore, we do not claim that we can give a complete explanation of conflict.

Given the complexity of the subject we have decided to abstract from the influence of cultural and political factors that have great influence in the existence of incompatibilities. We have focused on the strategy of conflict and concentrated on the study of optimal allocation of effort when resources are scarce. This process is complemented by a mechanism of negotiation which takes place in order to avoid the cost produced by conflict.

From all the types of conflict, we have dedicated our attention to wars or interstate conflict. Although the basic ideas should be applicable to many conflict situations, there were two main reasons to concentrate in this type of conflict. First, we considered the cost of conflict as one of the most important factors to understand its occurrence; it is apparent that wars are the most costly type of conflict. Second, the fact that nation-states are fully sovereign, allowed us to concentrate on strategic interactions in the absence of higher coercive institutions.

The political study of international conflicts applies to the study of systemic interactions and theories of hegemony, alliance formation, power struggles, theories of imperialism, etc. However, as we have mentioned in the previous chapter, we concentrated on dyadic interactions of countries, abstracting from systemic factors such as the number of alliances or the concentration of power. These dyadic interactions are assumed to be in evolutionary stable environment. Extending the analysis of the micro-level to account for possible evolutionary influences goes beyond the scope of this thesis. We have therefore, arbitrarily decided to focus in a narrow definition of conflict where economic models can offer some added valued to the already vast number of theories of conflict.

Introducing a model that allows parties to negotiate over the possible outcomes of an underlying conflict gave us some interesting results. The possibility of negotiations introduces a completely new dimensions into the study of conflict, arms trade, and efficient allocation of resources.

When parties are allowed to negotiate, the corresponding equilibrium of military expenditure is much lower compared to situations where negotiations cannot take place. The traditional idea that military security is a public good that depends only on the respective stocks of arms of two potential enemies may not apply to every conflict. In these cases where the cost of negotiating is very low, and countries have perfect or near perfect information, the concept of military security should be replaced by economic security.

This model also introduced some interesting developments into Hirshleifer's framework. The possibility of negotiating gives countries different allocations of resources from the ones predicted by the original models. This happens due to the fact that in the earlier models, the cost of conflict has no strategic effect on the optimal allocation of resources. However, when countries are allowed to negotiate, the country that has a strategic advantage can exploit the amount of potential damage in his favour, producing different optimal strategies.

This is one of the first conclusions of this thesis. In the face of conflict, the status quo has a great value and the parties who have a strategic advantage can exploit it in a negotiation process. The cost of conflict seems to play a crucial part in the allocation of resources and division of income. The models in chapter 3 simply demonstrate the benefits of incorporating the cost of conflict into analyses of wars: there is a great scope for extending research in this field.

Most theories of conflict are based on the assumption that countries decide their war and peace strategies based on calculations of the probabilities of winning a potential war. These models are heavily biased towards the idea of military security and pay little attention to economic security. Rather than concentrating on the probability of winning (as if accuracy about the probability of victory was the only rational consideration) these models put more emphasis in understanding the willingness to fight, a concept based on cost-benefit analysis.

For example, the model of bargaining with complete information in chapter 3 provided a good analysis of the strategic allocations of fighting and productive efforts in terms of the marginal utility obtained by moving initial resources between those activities. It differed from other models of war in that we combined a bargaining model with an endogenous probability of winning the war, whereas most of the literature assumes bargaining over conflict to be exogenous. In other words, we provided a model of bargaining with an endogenous threat point and bargaining set. But it did not provide an endogenous explanation of conflict. In this framework, actual fighting never takes place.

Chapter 4 presented a model of conflict that differed from the previous models because we assumed asymmetric information. Participants always try to avoid costly conflict which takes place in some extreme circumstances. For some large enough probabilities of facing a 'soft' enemy, combined with a large gap in the perception of the cost of fighting, there is a positive probability of war.

We compared two main bargaining situations under asymmetric information: bargaining with fixed threat points and with optimal allocation of resources. The likelihood of war with an endogenous optimal allocation of resources is significantly lower than the probability of a war under a fixed one. This indicates that the optimal prior allocation of fighting and productive activities on the negotiation process has a more significant impact in the outcome of war than other factors traditionally considered more important, such as strategic military advantage or the probabilities of victory in the contest.

The parameters that have the most influence are a) prior beliefs, v_0 and b) the difference between the externality cost of the soft and tough countries $(\overline{\gamma}-\underline{\gamma})$. These have a direct influence on the expected value of different bargaining strategies.

The war decisiveness parameter m and the degree of integration α – parameters that relate to the efficiency of appropriative and productive technologies– affect the expected income for both countries but have little role in determining the outbreak of war. [1]

There are also a few conclusions in terms of the empirical support for our models, bearing in mind that the development of empirical tests of rational models of war lags well behind its theoretical formulation. Therefore, rather than a robust statistical test we present a series of interpretations.

The decline of Major Wars is suggested by our theories, given the increase of productivity of civilian technology and the improvements in telecommunications that many countries have experienced in the last decades. All this factors are consistent with the optimising behaviour predicted by the model. The probabilities of war between different kind of regimes, classified according to the free circulation of information is also consistent. The fact that not a single war has broken out between nuclear powers can also be explained if we introduce some negotiation procedure into our models of conflict (Since destruction would be total, there are no miss-perceptions about it). The work of Bennett and Stam

[1] See the simulations that we carried out in chapter 4.

found considerable support for the effects of perceptions and calculations of the benefits and costs of war.

Some of the regularities about conflicts are that countries never go into war [2] because of new militarized disputes. This indicates that countries try to avoid the cost of conflict by finding a negotiated settlement. During the last period of high uncertainty in the international system, conflict seems to be at its peak . The amount of ethnic conflict, identity struggles, religious wars and other problems that cannot be negotiated easily seems to be higher than conflicts related to disagreements about the partition of scarce resources.

The importance of optimal allocation of resources remains one of the less developed questions. We have also reported in appendix E.1 the positive relation between arms races and conflict — that is a clear example of inefficient allocation of resources leading to more conflict.

We found ample evidence of the crucial role that the status-quo plays in conflict. Many of the factors that have been recorded by Barringer[3] show the importance that the status-quo plays in the occurrence of conflict.

The empirical regularities presented in appendix E.2 are consistent with the models that we have just introduced. We have also reviewed some of the recent empirical work on conflict with specific application to testing rational models, obtaining positive results from those estimations.

We found no significant evidence of the effect on conflict from trade dependency when we used the set of all disputes that record actual confrontation between states. We also found that the war equilibrium from *War and Reason* is significant which gives some support for our rational theories of conflict. These theories can also be applied to the interpretation of the different results in several empirical models.

[2]War is here considered when both countries engage in the use of physical violence, whereas a military dispute involves a threat or display of force

[3]See appendix E.7

There are several problems that shouldn't be forgotten. First of all, calibrating the utility of war is a difficult task: one of the main problems is the lack of data about the costs and perceptions of decision makers which are fundamental in determining the equilibrium outcome.

Other problems arise from game theoretic constraints. Given the changes in the strategic situation of the international system, it is difficult to argue that any single game can be applied to every conflict situation, especially when we know that the outcome of a bargaining game can be influenced by the choice of negotiating procedures.

On the data collection front, although considerable effort has been put into recording all disputes in the international system, there are few explanatory variables which restricts the testing of more ambitious hypotheses. The size of the data set is a great constraint and its development would require a considerable amount of time and resources. It might be worth considering smaller data sets, case studies and sampling techniques, in order to introduce more explanatory variables.

Despite the above problems, we consider that the amount of empirical evidence in favour of models of rational conflict behaviour is significant and further research should be taken in this direction.

Finally, despite the obvious problems with the classification of the stylised facts, our models are largely consistent with some of the widely accepted regularities of conflict.

The conclusion for the policy maker that can be derived from this study is that more resources should be put into making the process of negotiation more efficient. The goals and political practices should be more transparent. More resources should be put into conflict management and resolution strategies and less resources into militarized security.

6.1 Developments

This work could be improved in two main directions. One is the theoretical formulation of conflict and the other is its empirical study.

Regarding the possible theoretical developments there are several main avenues to follow. First, we can improve the models of optimal allocation of resources combining them with other developments in game theory. We could apply other ideas such as wars of attrition, arbitration, evolutionary game theory, models of imperfect information with signalling, with learning and other equilibrium refinements.

There is a fundamental difference between the time preference model and the exogenous risk of breakdown, deriving from the source of asymmetry and bargaining power. In many games the equilibrium outcome could be affected by the choice of bargaining procedure. A model of exogenous risk can give some interesting results because it gives the opportunity to introduce some asymmetries by:

- giving different perceptions to the exogenous risk of breakdown of negotiations
- Introducing different degrees of risk aversion in the utility functions.

On the other hand, we can try to merge models of the optimal allocation of resources, with other models that explain the sources of incompatibilities and the complexity of conflict networks, systemic influences and the formation of norms and values that may a) introduce different principles of behaviour or b) affect the utility functions of the decision makers, introducing, for example, reference points or different degrees of risk aversion.

Another possible development of this thesis will be to improve the concept of the cost of fighting. Our models of bargaining introduce the idea that the effect of cost in conflict has been neglected and there is a great scope for extending research in this field.

Most part of theories of conflict are based on the assumption that countries decide their war and peace strategies based on calculations of the probabilities of winning a potential war. Those models are heavily biased on the idea of military security and pay little attention to economic security.

At this stage we have introduced a fixed exogenous cost. We could introduce the cost of conflict as a function of fighting intensities. We could even try to base our contest success functions on the probability of inflicting unbearable costs on the other party. However, the problem will be to introduce a sensible form that describes military technology not only as a relation between fighting intensities and probability of success, but also as a relation with the cost of destruction.

With respect to the development of empirical testing of rational models of war there is a great task ahead. There is no a systematic collection of data about the cost of war. Without it, it is difficult to establish the relation between say, political systems, military forces and perceptions of cost, which will ultimately bring about a peaceful or violent outcome.

Although this research agenda would be complicated, the results in this thesis suggest that it may prove fruitful to pursue.

Appendix A

Introduction

A.1 The Seville Declaration

IT IS SCIENTIFICALLY INCORRECT to say that we have inherited a tendency to make war from our animal ancestors. Although fighting occurs widely throughout animal species, only a few cases of destructive intraspecies fighting between organised groups have ever been reported among naturally living species, and none of these involve the use of tools designed to be weapons. Normal predatory feeding upon other species cannot be equated with intra-species violence. Warfare is a peculiarly human phenomenon and does not occur in other animals.

The fact that warfare has changed so radically over time indicates that it is a product of culture. Its biological connection is primarily through language which makes possible the coordination of groups, the transmission of technology, and the use of tools. War is biologically possible, but it is not inevitable, as evidenced by its variation in occurrence and nature over time and space. There are cultures which have not engaged in war for centuries, and there are cultures which have engaged in war frequently at some times and not at others.

IT IS SCIENTIFICALLY INCORRECT to say that war or any violent behaviour is genetically programmed into our human nature. While genes are involved at all levels of nervous system function, they provide a developmental potential that can be actualised only in conjunction with the ecological and social environment. While individuals vary in their predisposition to be affected by their experience, it is the interaction between their genetic endowment and conditions of nurturance that determines their personalities. Except for rare pathologies, the genes do not produce individuals necessarily predisposed to violence. Neither do they determine the opposite. While genes are co-involved in establishing our behavioural capacities, they do not by themselves specify the outcome.

IT IS SCIENTIFICALLY INCORRECT to say that in the course of human evolution there has been a selection for aggressive behaviour more than for other kinds of behaviour. In all well-studied species, status within the group is achieved by the ability to cooperate and to fulfil social functions relevant to the structure of that group. 'Dominance' involves social bonding and affiliations; it is not simply a matter of the possession and use of superior physical power, although it does involve aggressive behaviours. Where genetic selection for aggressive behaviour has been artificially instituted in animals, it has rapidly succeeded in producing hyper-aggressive individuals; this indicates that aggression was not maximally selected under natural conditions. When such experimentally-created hyper-aggressive animals are present in a social group, they either disrupt its social structure or are driven out. Violence is neither in our evolutionary legacy nor in our genes.

IT IS SCIENTIFICALLY INCORRECT to say that humans have a 'violent brain.' While we do have the neural apparatus to act violently, it is not automatically activated by internal or external stimuli. Like higher primates and unlike other animals, our higher neural processes filter such stimuli before they can be acted upon. How we act is shaped by how we have been conditioned and socialised. There is nothing in our neurophysiology that compels us to react violently.

IT IS SCIENTIFICALLY INCORRECT to say that war is caused by 'instinct' or any single motivation. The emergence of modern warfare has been a journey from the primacy of emotional and motivational factors, sometimes called 'instincts,' to the primacy of cognitive factors. Modern war involves institutional use of personal characteristics such as obedience, suggestibility, and idealism, social skills such as language, and rational considerations such as cost-calculation, planning, and information processing. The technology of modern war has exaggerated traits associated with violence both in the training of actual combatants and in the preparation of support for war in the general population. As a result of this exaggeration, such traits are often mistaken to be the causes rather than the consequences of the process.

They concluded that biology does not condemn humanity to war. But, what is war? Modern wars take place amongst states. However, 10 out of the 13 most deadly wars since the Congress of Vienna in 1815 were internal-conflicts or civil-wars. On the other hand, if there is no break-out of hostilities, does it mean that there is no conflict? Many people live under a status quo which is the result of a violent imposition. It is, therefore, necessary to define the concept of conflict in a way that can eliminate these contradictions.

Appendix B

The Bargaining Models

B.1 Preference Assertions

Rubinstein makes five assertions about the players preferences:
For all $r, s \in S, t, t_1, t_2 \in N$, and $i \in \{1, 2\}$

A 1 *if* $r_i > s_i$, *then* $(r, t) >_i (s, t)$;

A 2 *if* $s_i > 0$ *and* $t_2 > t$, *then* $(s, t_1) >_i (s, t_2) >_i (0, \infty)$;

A 3 $(r, t_1) \tilde{>}_i (s, t_1 + 1)$ *iff* $(r, t_2) \tilde{>}_i (s, t_2, +1)$;

A 4 *if* $r_n \to r$ *and* $(r_n, t_1) \tilde{>}_i (s, t_2)$, *then* $(r, t_1) \tilde{>}_i (s, t_2)$;
if $r_n \to r$ *and* $(r_n, t_1) \tilde{>}_i (0, \infty)$,*then* $(r, t_1) \tilde{>}_i (0, \infty)$;

A 5 *if* $(s + \epsilon, 1) \sim_i ((s, 0), (\overline{s} + \overline{\epsilon}, 1) \sim_i (\overline{s}, 0)$, *and*, $s_i < \overline{s}_i$, *then* $\epsilon_i \leq \overline{\epsilon}_i$.

B.2 Proof of infinite time horizon model

In order to demonstrate the uniqueness of the infinite time horizon model Fudenberg & Tirole [121] follow the proof by Shaked and Sutton [122]. First, we must define an upper and lower bound representing the maximum ($\overline{v}_i$) and minimum ($\underline{v}_i$) payoffs that each player can obtain, and then, show that both payoffs are the same.

Besides, we have to define the expected payoffs of every strategy profile at any point in time. For example, if a strategy leads to obtaining the whole income I in period 3, its expected payoff in period 1, for c_1 is δI in period 2, and $\delta^2 I$ in period 1.

Similarly we define the maximum $\overline{w}_i$ and minimum $\underline{w}_i$ payoff that each country can obtain when a strategy beginning with the other player.

When c_1 makes an offer, c_2 will accept any s exceeding $(1-s) = \delta_2\overline{v}_2$. Hence, $\underline{v}_1 \geq 1 - \delta_2\overline{v}_2$ and by symmetry, $\underline{v}_2 \geq 2\delta_1\overline{v}_1$.

The highest equilibrium payoff $\overline{v}_1$ satisfies:

$$\overline{v}_1 \leq \max(1 - \delta_2\underline{v}_2,\ \delta_1\overline{w}_1) \leq \max(1 - \delta_2\underline{v}_2,\ \delta_1^2\overline{v}_1).$$

and

$$\max(1 - \delta_2\underline{v}_2,\ \delta_1^2\overline{v}_1) = 1 - \delta_2\underline{v}_2.$$

Thus: $\overline{v}_1 \leq 1 - \delta_2\underline{v}_2$ and $\overline{v}_2 \leq 1 - \delta_1\underline{v}_1$

Combining these inequalities:

$$\underline{v}_1 \geq 1 - \delta_2\overline{v}_2 \geq 1 - \delta_2(1 - \delta_1\underline{v}_1).$$

or

$$\underline{v}_1 \geq \frac{1-\delta_1}{1-\delta_1\delta_2};$$

Because $\underline{v}_1 \leq \overline{v}_1$, this implies $\underline{v}_1 = \overline{v}_1$. We can follow a similar procedure for $(\overline{v}_2, \underline{v}_2)$, $(\overline{w}_1, \underline{w}_1)$ and $(\overline{w}_2, \underline{w}_2)$.

This shows that there is a unique equilibrium.

B.3 Bayesian updating

When an offer gets rejected in period one, the seller updates his believes as follows:

We know that the probability of rejection of p_1 in period 1 for the soft buyer is $(1 - \overline{r}(p_1))$ and for the tough $(1 - \underline{r}(p_1)) = 1$.

Then the updated probability of facing a soft buyer in second period will be:

$$\overline{q}(p_1) = \frac{Pr[\overline{b} \text{ rejects } p_1|p_1]}{Pr(\text{rejection})} = \frac{(1-\overline{r}(p_1))Pr(\overline{b})}{Pr(\text{rejection})}$$

and also the total probability of rejection of p_1 is:

$$\begin{aligned} Pr(\text{rejection}) &= (1-\overline{r}(p_1))Pr(\overline{b}) + (1-\underline{r}(p_1))Pr(\underline{b}) = \\ &= \tfrac{1}{2}[(1-\overline{r}(p_1)) + (1-\underline{r}(p_1))] \end{aligned}$$

Therefore:

$$\overline{q}(p_1) = \frac{(1-\overline{r}(p_1))}{(1-\overline{r}(p_1))+1} \leq \frac{1}{2}$$

B.4 The Axioms of von Neumann-Morgensten

Consider a system U of abstract utilities $u, v, w, \cdots$ In U a *relation* is given, $u > v$,and for any number α, $0 < \alpha < 1$, and *operation*

$$\alpha u + (1-\alpha)v = w.$$

These concepts satisfy the following axioms:

A 1 *$u > v$ is a complete ordering of U. This means $u < v$ when $v > u$, Then:*

A1 1 *For any two u, v one and only one of the three following relations holds:*

$$u = v, \quad u > v, \quad u < v$$

A1 2 *$u > b$, $v > w$, imply $u > w$.*

A 2 *Ordering and combining*

A2 1 *$u < v$ implies that $u < \alpha u + (1-\alpha)v$.*

A2 2 *$u > v$ implies that $u > \alpha u + (1-\alpha)v$.*

A2 3 *$u < v < w$ implies the existence of an α with*

$$\alpha u + (1-\alpha)v < w$$

A2 4 *$u > v > w$ implies the existence of an α with*

$$\alpha u + (1-\alpha)v > w$$

A 3 *Algebra combining*

A3 1

$$\alpha u + (1-\alpha)v = (1-\alpha)v + \alpha u$$

A3 2

$$\alpha(\beta u + (1-\beta)v) + (1-\alpha)v = \gamma u + (1-\gamma)v$$

Where $\gamma = \alpha\beta$

Appendix C

Bargaining and conflict

C.1 Matlab program

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%time preference model a la hischl     %
%                                      %
% 7-1-98                               %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
global mm ss r1 r2 kk f1 f2 dd gg

%initial resources are r1 and r2

r1=100;

r2=100;
%
% is the parameters of the production function with CES
%
ss=1;
%
%mm is the decisiveness parameter that must be between 0 and 1
%
mm=1;

% gg is the destruction coefficient between 0 and 1
%
gg= 0;
%
%dd is the time preference coefficient between 0 and 1
%
```

```
dd=1;
%
%kk is a variable that counts the number of iterations
%
kk=0;
%
%ff are the initial roots
%
ff1=50;

ff2=50;

x=fsolve('tnash2',[ff1 ff2]') ;

f1=x(1) ;

f2=x(2) ;
%
%the symmetric solution is
%
pause
%
%The income for Country 1 and 2 is
%
e1=r1-f1 ;

e2=r2-f2 ;

zz= f1.^mm+f2.^mm ;

yy= (e1.^(1./ss)+e2.^(1./ss)).^ss ;

pause

s1=1-dd+dd.^2 ;

s2=1-s1;

IN1=(1-dd.*(1-dd.*(f1.^mm./zz))).*yy

IN2=(1-(1-dd.*(1-dd.*(f1.^mm./zz)))).*yy
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% 28-12-97  tnash2.m                                %
%                                                   %
%Calculation of the time preference Nash equilibrium%
%                                                   %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function q=tnash2(x)

global mm ss r1 r2 dd gg kk

q=zeros(2,1) ;

f1=x(1) ;

f2=x(2) ;

e1=r1-f1 ;

e2=r2-f2 ;

xy= dd.*(1-gg)

xx= (1-dd)/dd.^2*(1-gg) ;

zz= f1.^mm+f2.^mm ;

q(1)=(f1./(f2.^mm.*f1.^mm))-(mm.*e1.* ...

(1+(e2./e1).^(1./ss)))/(((xx*zz)+f1.^mm).*zz)

q(2)=(f2./(f2.^mm.*f1.^mm))-(mm.*xy.*e2.*...

(1+(e1./e2).^(1./ss)))./((zz-(f1.^mm.*xy)).*zz)

kk=kk+1
```

Appendix D

Incomplete Information

D.1 Alternative Payoffs

D.1.1 Payoffs when Country 2 soft plays tough

Strategy	Period	Country 1	Country 2 (Soft)	Country 2 (Tough)
1	1	$\overline{s}_1 I$	$\overline{r} = 1$	$\overline{r} = 1$
	2	-	-	-
2	1	$\tilde{s}\underline{I}$	$\overline{r} = 0$	$\underline{r} = 1$
	2	$\overline{s}_2\overline{I}$	$\overline{r} = 1$	-
3	1	$\underline{s}_1\underline{I}$	$\overline{r} = 0$	$\underline{r} = \tilde{r}$
	2	$\underline{s}_2\underline{I}$	$\overline{r} = 0$	$\underline{r} = 1$

Table D.1: Three Strategies for Country 1

St.	Country 1	Country 2 (Soft)	Country 2 (Tough)
1	$\overline{s}_1\overline{I}$	$(1-\overline{s}_1)\overline{I}$	$\underline{I} - \overline{s}_1\overline{I}$
2	$(1-\upsilon_0)\tilde{s}\underline{I} + \upsilon_0\delta_1\overline{s}_2\overline{I}$	$\overline{\delta}_2(1-\overline{s}_2)\overline{I}$	$(1-\tilde{s})\underline{I}$
3	$(1-\upsilon_0)[\tilde{r}\underline{s}_1\underline{I} + (1-\tilde{r})\delta_1\underline{s}_2\underline{I}]$	$\overline{\delta}_2(1-\overline{\gamma})(1-\overline{p})\overline{I}$	$\tilde{r}(1-\underline{s}_1)\underline{I}+$
	$+\upsilon_0(1-\gamma)\delta_1\overline{p})\overline{I}$		$(1-\tilde{r})\underline{\delta}_2(1-\underline{s}_2)\underline{I}$

Table D.2: Expected payoffs for the Three Strategies

D.2 The Reaction Curves

D.2.1 Strategy1

Country 1

Country 1 using strategy 1 will maximize $\pi_{11} = \underline{s}_1 \underline{I}$

$$\frac{d\pi_{11}}{dF_1} = \frac{\partial \underline{s}_1}{\partial F_1}\underline{I} + \frac{\partial \underline{I}}{\partial F_1}\underline{s}_1$$

where

$$\underline{I} = A\left((R_1 - F_1)^\alpha + (R_2 - \underline{F}_2)^\alpha\right)^{\frac{1}{\alpha}}$$

$$\underline{s}_1 = 1 - \underline{\delta}_2(1 - \underline{\gamma})\left(1 - \frac{F_1^m}{(F_1^m + \underline{F}_2^m)}\right)$$

$$\frac{\partial \underline{s}_1}{\partial F_1} = \underline{\delta}_2(1 - \underline{\gamma})\frac{mF_1^{m-1}\underline{F}_2^m}{(F_1^m + \underline{F}_2^m)^2}$$

$$\frac{\partial \underline{I}}{\partial F_1} = -A(R_1 - F_1)^{\alpha-1}\left((R_1 - F_1)^\alpha + (R_2 - \underline{F}_2)^\alpha\right)^{\frac{1}{\alpha}-1}$$

Country 2 (soft)

Under strategy 1, Country 2 (soft) will maximize $\pi_{12} = \overline{I} - \underline{s}_1 \underline{I}$

$$\frac{d\pi_{12}}{d\overline{F}_2} = \frac{\partial \overline{I}}{\partial \overline{F}_2}$$

$$\frac{\partial \overline{I}}{\partial \overline{F}_2} = -A(R_2 - \overline{F}_2)^{\alpha-1}\left((R_1 - F_1)^\alpha + (R_2 - \overline{F}_2)^\alpha\right)^{\frac{1}{\alpha}-1}$$

Country 2 (tough)

Under strategy 1, Country 2 (tough) will maximize $\pi_{13} = (1 - \underline{s}_1)\underline{I}$

$$\frac{d\pi_{13}}{d\underline{F}_2} = \frac{\partial \pi_{13}}{\partial \underline{s}_1}\frac{\partial \underline{s}_1}{\partial \underline{F}_2} + \frac{\partial \pi_{13}}{\partial \underline{I}}\frac{\partial \underline{I}}{\partial \underline{F}_2}$$

$$\frac{\partial \pi_{13}}{\partial \underline{s}_1} = -\underline{I}$$

$$\frac{\partial \underline{s}_1}{\partial \underline{F}_2} = -\underline{\delta}_2(1-\underline{\gamma})\frac{mF_1^m \underline{F}_2^{m-1}}{(F_1^m + \underline{F}_2^m)^2}$$

$$\frac{\partial \pi_{13}}{\partial \underline{I}} = (1 - \underline{s}_1)$$

$$\frac{\partial \underline{I}}{\partial \underline{F}_2} = -A(R_2 - \underline{F}_2)^{\alpha-1}((R_1 - F_1)^\alpha + (R_2 - \underline{F}_2)^\alpha)^{\frac{1}{\alpha}-1}$$

D.2.2 Strategy 2

Country 1

Country 1 using strategy 2 will maximize $\pi_{21} = v_0 \tilde{s}\overline{I} + (1-v_0)\delta_1 \underline{s}_2 \underline{I}$

$$\frac{d\pi_{21}}{dF_1} = v_0\left(\frac{\partial \tilde{s}}{\partial F_1}\overline{I} + \frac{\partial \overline{I}}{\partial F_1}\tilde{s}\right) + (1-v_0)\delta_1\left(\frac{\partial \underline{s}_2}{\partial F_1}\underline{I} + \frac{\partial \underline{I}}{\partial F_1}\underline{s}_2\right)$$

where

$$\tilde{s} = 1 - \frac{\overline{\delta}_2(\overline{I} - \underline{s}_2 \underline{I})}{\overline{I}}$$

$$\frac{\partial \tilde{s}}{\partial F_1} = \frac{\partial \tilde{s}}{\partial \overline{I}}\frac{\partial \overline{I}}{\partial F_1} + \frac{\partial \tilde{s}}{\partial \underline{I}}\frac{\partial \underline{I}}{\partial F_1} + \frac{\partial \tilde{s}}{\partial \underline{s}_2}\frac{\partial \underline{s}_2}{\partial F_1}$$

$$\frac{\partial \tilde{s}}{\partial \overline{I}} = -\frac{\overline{\delta}_2 \overline{I} - \overline{\delta}_2(\overline{I} - \underline{s}_2 \underline{I})}{(\overline{I})^2} = -\frac{\overline{\delta}_2 \underline{s}_2 \underline{I}_2}{(\overline{I})^2}$$

$$\frac{\partial \tilde{s}}{\partial \underline{I}} = \frac{\overline{\delta} \underline{s}_2}{\overline{I}}$$

$$\frac{\partial \tilde{s}}{\partial \underline{s}_2} = \frac{\overline{\delta} \underline{I}}{\overline{I}}$$

$$\frac{\partial \underline{I}}{\partial F_1} = -A(R_1 - F_1)^{\alpha-1}((R_1 - F_1)^\alpha + (R_2 - \underline{F}_2)^\alpha)^{\frac{1}{\alpha}-1}$$

$$\frac{\partial \overline{I}}{\partial F_1} = -A(R_1 - F_1)^{\alpha-1}((R_1 - F_1)^{\alpha} + (R_2 - \overline{F}_2)^{\alpha})^{\frac{1}{\alpha}-1}$$

$$\frac{\partial \underline{s}_2}{\partial F_1} = (1-\underline{\gamma})\frac{mF_1^{m-1}\underline{F}_2^m}{(F_1^m + \underline{F}_2^m)^2}$$

Country 2 (soft)

Country 2 (soft) using strategy 2 will maximize: $\pi_{22} = (1-\tilde{s})\overline{I}$

$$\frac{d\pi_{22}}{d\overline{F}_2} = \frac{\partial \pi_{22}}{\partial \tilde{s}}\frac{\partial \tilde{s}}{\partial \overline{F}_2} + \frac{\partial \pi_{22}}{\partial \overline{I}}\frac{\partial \overline{I}}{\partial \overline{F}_2}$$

$$\frac{\partial \pi_{22}}{\partial \tilde{s}} = -\overline{I}$$

$$\frac{\partial \pi_{22}}{\partial \overline{I}} = (1-\tilde{s})$$

$$\frac{\partial \tilde{s}}{\partial \overline{F}_2} = \frac{\overline{\delta}_2\overline{I}\frac{\partial \overline{I}}{\partial \overline{F}_2} + (\overline{\delta}_2(\overline{I} - \underline{s}_2\underline{I}))\frac{\partial \overline{I}}{\partial \overline{F}_2}}{(\overline{I})^2}$$

$$\frac{\partial \overline{I}}{\partial \overline{F}_2} = -A(R_2 - \overline{F}_2)^{\alpha-1}((R_1 - F_1)^{\alpha} + (R_2 - \overline{F}_2)^{\alpha})^{\frac{1}{\alpha}-1}$$

Country 2 (tough)

Country 2 (tough) using strategy 2 will maximize: $\pi_{23} = \underline{\delta}_2(1-\underline{s}_2)\underline{I}$

$$\frac{d\pi_{23}}{d\underline{F}_2} = \frac{\partial \pi_{23}}{\partial \underline{s}_2}\frac{\partial \underline{s}_2}{\partial \underline{F}_2} + \frac{\partial \pi_{23}}{\partial \underline{I}}\frac{\partial \underline{I}}{\partial \underline{F}_2}$$

$$\frac{\partial \pi_{23}}{\partial \underline{s}_2} = -\underline{\delta}_2\underline{I}$$

$$\frac{\partial \pi_{23}}{\partial \underline{I}} = \underline{\delta}_2(1-\underline{s}_2)$$

$$\frac{\partial \underline{s}_2}{\partial \underline{F}_2} = -(1-\underline{\gamma})\frac{mF_1^m \underline{F}_2^{m-1}}{(F_1^m + \underline{F}_2^m)^2}$$

$$\frac{\partial \underline{I}}{\partial \underline{F}_2} = -A(R_2 - \underline{F}_2)^{\alpha-1}((R_1 - F_1)^\alpha + (R_2 - \underline{F}_2)^\alpha)^{\frac{1}{\alpha}-1}$$

D.2.3 Strategy 3

Country 1

Country 1 using strategy 3 will maximize

$$\pi_{31} = v_0(\tilde{r}\overline{s}_1\overline{I} + (1-\tilde{r})\delta_1\overline{s}_2\overline{I}) + (1-v_0)\delta_1\underline{p}\underline{I}(1-\gamma)$$

where

$$\tilde{r} = \frac{v_0\overline{s}_2\overline{I} + (1-v_0)(1-\gamma)\underline{p}\underline{I} - \underline{s}_2\underline{I}}{v_0(\overline{s}_2\overline{I} - \underline{s}_2\underline{I})}$$

lets find firstly $\frac{\partial \tilde{r}}{\partial F_1}$

$$\frac{\partial \tilde{r}}{\partial F_1} = \frac{\partial \tilde{r}}{\partial \overline{I}}\frac{\partial \overline{I}}{\partial F_1} + \frac{\partial \tilde{r}}{\partial \underline{p}}\frac{\partial \underline{p}}{\partial F_1} + \frac{\partial \tilde{r}}{\partial \underline{I}}\frac{\partial \underline{I}}{\partial F_1} + \frac{\partial \tilde{r}}{\partial \underline{s}_2}\frac{\partial \underline{s}_2}{\partial F_1} + \frac{\partial \tilde{r}}{\partial \overline{s}_2}\frac{\partial \overline{s}_2}{\partial F_1}$$

$$\frac{\partial \tilde{r}}{\partial \overline{I}} = \frac{v_0\overline{s}_2(v_0(\overline{s}_2\overline{I} - \underline{s}_2\underline{I})) - v_0\overline{s}_2(v_0\overline{s}_2\overline{I} + (1-v_0)(1-\gamma)\underline{p}\underline{I} - \underline{s}_2\underline{I})}{(v_0(\overline{s}_2\overline{I} - \underline{s}_2\underline{I}))^2}$$

$$\frac{\partial \tilde{r}}{\partial \underline{p}} = \frac{(1-v_0)(1-\gamma)\underline{I}}{v_0(\overline{s}_2\overline{I} - \underline{s}_2\underline{I})}$$

$$\frac{\partial \tilde{r}}{\partial \underline{I}} = \frac{((1-v_0)(1-\gamma)\underline{p} - \underline{s}_2)v_0(\overline{s}_2\overline{I} - \underline{s}_2\underline{I}) + v_0\underline{s}_2(v_0\overline{s}_2\overline{I} + (1-v_0)(1-\gamma)\underline{p}\underline{I} - \underline{s}_2\underline{I})}{(v_0(\overline{s}_2\overline{I} - \underline{s}_2\underline{I}))^2}$$

$$\frac{\partial \tilde{r}}{\underline{s}_2} = \frac{-\underline{I}(v_0(\overline{s}_2\overline{I} - \underline{s}_2\underline{I}) + v_0\underline{I}(v_0\overline{s}_2\overline{I} + (1-v_0)(1-\gamma)\underline{p}\underline{I} - \underline{s}_2\underline{I})}{(v_0(\overline{s}_2\overline{I} - \underline{s}_2\underline{I}))^2}$$

$$\frac{\partial \tilde{r}}{\partial \overline{s}_2} = \frac{v_0\overline{I}(v_0(\overline{s}_2\overline{I} - \underline{s}_2\underline{I})) - v_0\overline{I}(v_0\overline{s}_2\overline{I} + (1-v_0)(1-\gamma)\underline{p}\underline{I} - \underline{s}_2\underline{I})}{(v_0(\overline{s}_2\overline{I} - \underline{s}_2\underline{I}))^2}$$

$$\frac{\partial \underline{p}}{\partial F_1} = \frac{mF_1^{m-1}\underline{F}_2^m}{(F_1^m + \underline{F}_2^m)^2}$$

$$\frac{\partial \overline{I}}{\partial F_1} = -A(R_1 - F_1)^{\alpha-1}((R_1 - F_1)^\alpha + (R_2 - \overline{F}_2)^\alpha)^{\frac{1}{\alpha}-1}$$

$$\frac{\partial \underline{I}}{\partial F_1} = -A(R_1 - F_1)^{\alpha-1}((R_1 - F_1)^\alpha + (R_2 - \underline{F}_2)^\alpha)^{\frac{1}{\alpha}-1}$$

$$\frac{\partial \underline{s}_2}{\partial F_1} = (1-\underline{\gamma})\frac{mF_1^{m-1}\underline{F}_2^m}{(F_1^m + \underline{F}_2^m)^2}$$

$$\frac{\partial \overline{s}_2}{\partial F_1} = (1-\overline{\gamma})\frac{mF_1^{m-1}\overline{F}_2^m}{(F_1^m + \overline{F}_2^m)^2}$$

——————— end of $\frac{\partial \tilde{r}}{\partial F_1}$ ———————

$$\frac{d\pi_{31}}{dF_1} = \frac{\partial \pi_{31}}{\partial \tilde{r}}\frac{\partial \tilde{r}}{\partial F_1} + \frac{\partial \pi_{31}}{\partial \overline{s}_1}\frac{\partial \overline{s}_1}{\partial F_1} + \frac{\partial \pi_{31}}{\partial \overline{I}}\frac{\partial \overline{I}}{\partial F_1} + \frac{\partial \pi_{31}}{\partial \overline{s}_2}\frac{\partial \overline{s}_2}{\partial F_1} + \frac{\partial \pi_{31}}{\partial \underline{p}}\frac{\partial \underline{p}}{\partial F_1} + \frac{\partial \pi_{31}}{\partial \underline{I}}\frac{\partial \underline{I}}{\partial F_1}$$

$$\frac{\partial \pi_{31}}{\partial \tilde{r}} = v_0(\overline{s}_1\overline{I} - \underline{\delta}_1\overline{s}_2\overline{I})$$

$$\frac{\partial \pi_{31}}{\partial \overline{s}_1} = v_0\tilde{r}\overline{I}$$

$$\frac{\partial \pi_{31}}{\partial \overline{I}} = v_0(\tilde{r}\overline{s}_1 + (1-\tilde{r})\delta_1\overline{s}_2)$$

$$\frac{\partial \pi_{31}}{\partial \overline{s}_2} = v_0(1-\tilde{r})\delta_1\overline{I}$$

$$\frac{\partial \pi_{31}}{\partial \underline{p}} = (1-v_0)\delta_1\underline{I}(1-\gamma)$$

$$\frac{\partial \pi_{31}}{\partial \underline{I}} = (1 - \upsilon_0)\delta_1 \underline{p}(1 - \gamma)$$

D.2.4 Country 2 (soft)

Country 2 (soft) using strategy 3 will maximize:

$$\pi_{32} = \tilde{r}(1 - \overline{s}_1)\overline{I} + (1 - \tilde{r})\overline{\delta}_2(1 - \overline{s}_2)\overline{I}$$

find $\frac{\partial \tilde{r}}{\partial \overline{F}_2}$

$$\tilde{r} = \frac{\upsilon_0 \overline{s}_2 \overline{I} + (1 - \upsilon_0)(1 - \gamma)\underline{p}\underline{I} - \underline{s}_2 \underline{I}}{\upsilon_0(\overline{s}_2 \overline{I} - \underline{s}_2 \underline{I})}$$

$$\frac{\partial \tilde{r}}{\partial \overline{F}_2} = \frac{\partial \tilde{r}}{\partial \overline{I}} \frac{\partial \overline{I}}{\partial \overline{F}_2} + \frac{\partial \tilde{r}}{\partial \overline{s}_2} \frac{\partial \overline{s}_2}{\partial \overline{F}_2}$$

$$\frac{\partial \overline{I}}{\partial \overline{F}_2} = -A(R_2 - \overline{F}_2)^{\alpha - 1}((R_1 - F_1)^{\alpha} + (R_2 - \overline{F}_2)^{\alpha})^{\frac{1}{\alpha} - 1}$$

$$\frac{\partial \overline{s}_2}{\partial \overline{F}_2} = -(1 - \overline{\gamma})\frac{m\overline{F}_2^{m-1} F_1^m}{(F_1^m + \overline{F}_2^m)^2}$$

———— end of $\frac{\partial \tilde{r}}{\partial \overline{F}_2}$ ————

$$\frac{d\pi_{32}}{d\overline{F}_2} = \frac{\pi_{32}}{\partial \tilde{r}} \frac{\partial \tilde{r}}{\partial \overline{F}_2} + \frac{\pi_{32}}{\partial \overline{s}_1} \frac{\partial \overline{s}_1}{\partial \overline{F}_2} + \frac{\pi_{32}}{\partial \overline{I}} \frac{\partial \overline{I}}{\partial \overline{F}_2} + \frac{\pi_{32}}{\partial \overline{s}_2} \frac{\partial \overline{s}_2}{\partial \overline{F}_2}$$

$$\frac{\partial \pi_{32}}{\partial \tilde{r}} = ((1 - \overline{s}_1) - \overline{\delta}_2(1 - \overline{s}_2))\overline{I}$$

$$\frac{\partial \pi_{32}}{\partial \overline{s}_1} = -\tilde{r}\overline{I}$$

$$\frac{\partial \pi_{32}}{\partial \overline{s}_2} = -(1 - \tilde{r})\overline{\delta}_2 \overline{I}$$

$$\frac{\partial \pi_{32}}{\partial \overline{I}} = \tilde{r}(1-\overline{s}_1) + (1-\tilde{r})\overline{\delta}_2(1-\overline{s}_2)$$

Country 2 (tough)

Country 2 (tough) using strategy 3 will maximize

$$\pi_{33} = \underline{\delta}_2(1-\underline{p})(1-\underline{\gamma})\underline{I}$$

$$\frac{d\pi_{33}}{d\underline{F}_2} = \frac{\partial \pi_{33}}{\partial \underline{I}}\frac{\partial \underline{I}}{\partial \underline{F}_2} + \frac{\partial \pi_{33}}{\partial \underline{p}}\frac{\partial \underline{p}}{\partial \underline{F}_2}$$

$$\frac{\partial \pi_{33}}{\partial \underline{I}} = \underline{\delta}_2(1-\underline{p})(1-\underline{\gamma})$$

$$\frac{\partial \pi_{33}}{\partial \underline{p}} = -\underline{\delta}_2(1-\underline{\gamma})\underline{I}$$

$$\frac{\partial \underline{I}}{\partial \underline{F}_2} = -A(R_2-\underline{F}_2)^{\alpha-1}((R_1-F_1)^{\alpha}+(R_2-\underline{F}_2)^{\alpha})^{\frac{1}{\alpha}-1}$$

$$\frac{\partial \underline{p}}{\partial \underline{F}_2} = \frac{-mF_1^m \underline{F}_2^{m-1}}{(F_1^m+\underline{F}_2^m)^2}$$

D.3 The Matlab Program

D.3.1 The optimal allocation of resources

```
% -----------------------------------------------------------------%
% -----------------------------------------------------------------%
% -----------------------------------------------------------------%
% war1 is a calculation of the equilibrium of a                    %
% bargaining model with endogenous  allocation of fighting effort  %
% in the paper "Rational Wars with incomplete                      %
% information" by P. Levine and F. Moraiz     3 / 2 /99            %
% -----------------------------------------------------------------%
% -----------------------------------------------------------------%
% -----------------------------------------------------------------%

global f1 f2l f2h r1 r2 gg gh gl aa dd dh dl mm A v1

% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% GLOBAL VARIABLES DEFINITION                                        %
% f1 is the expenditure in arms of Country 1                         %
% f2l and f2h are the expenditure in arms of country tough and soft %
% r1 and r2 are the initial resources of Country 1 and two           %
% gg gh gl are the externality costs gamma gamma low and high        %
% dd dh dl are the discounting factors                               %
% v1 is the probability of facing a soft Country 2                   %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


%
% Enter the initial values
%

disp('--------------------------------------------------------------')
disp('Calibration of the model of Rational Wars with incomplete information')
disp('--------------------------------------------------------------')

disp('Enter the value of the externality cost for Country 1 gg')
disp('Enter the value for Country 2 soft gh and tough gl')
disp('gg must be bigger than gh, and gh bigger that gl')

gg = input('Enter gg: ')
gh = input('Enter gh: ')
gl = input('Enter gl: ')
```

```
r1 = 100 ;
r2 = 100 ;
dd = 0.9 ;
dl =  0.9 ;
dh = 0.9 ;
aa = 1 ;
A  = 1
mm=1;

%
% Loop for different probabilities of facing a soft country
%

for i = 1:100,


%  Calculation of the probabilities of winning of Country 1
%
v1(i)=(i/100);
v1=v1(i);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Barwar is a m file that estimates the unique nash equilibrium %
% Barwar returns a unique value for F1, F2l and F2h             %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

barwar21 ;

fff1(i)=f1;
fff2(i)=f2l;
fff3(i)=f2h;
%
% Once we have calculated the optimal allocation to F1, F2l and F2h
% we can proceed to calculate the different equilibrium outcomes for
% different values of gg, v1, mm ...
%

% Probabilities of winning

pl = (f1^mm)/(f1^mm+f2l^mm);
ph = (f1^mm)/(f1^mm+f2h^mm);
```

```
% Calculation of the size of cake

e1 = r1 - f1 ;
e2l = r2 - f2l ;
e2h = r2 - f2h ;
il = A*(e1^(1/aa)+e2l^(1/aa))^aa ;
ih = A*(e1^(1/aa)+e2h^(1/aa))^aa ;

% Offers in period 2

s2l = 1-(1-gl)*(1-pl) ;
s2h = 1-(1-gh)*(1-ph) ;

% Offers in period 1

s1l = 1-dl*(1-s2l) ;
s1h = 1-dh*(1-s2h) ;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%PART I Country soft becomes tough because %
%it has a higher expenditure in arms       %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

if (1-gl)*(1-pl) < (1-gh)*(1-ph) ;

  den1 = (1-v1)*(s2l*il-s2h*ih) ;
nun1 =(1-v1)*s2l*il+v1*(1-gg)*ph*ih-s2h*ih;
rtilda = nun1/den1;
stilda = 1- (dl*(il-(s2h*ih))/il) ;

%
% For Country 1
%

pi1 = s1h*ih ;
pi2 = (1-v1)*stilda*il+v1*dd*s2h*ih ;
pi3= v1*(rtilda*s1l*il+(1-rtilda)*dd*s2l*il)+v1*(1-gg)*dd*ph*ih ;

%
% For Country 2 soft
%

pih1 = (1-s1h)*ih;
```

```
pih2 = dh*(1-s2h)*ih ;
pih3 = dh*(1-gh)*(1-ph)*ih ;
%
% For Country 2 tough
%

pil1 = il-s1h*ih;
pil2 = (1-stilda)*il ;
pil3 = rtilda*(1-s1l)*il+(1-rtilda)*dl*(1-s2l)*il ;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Country 1 calculates the strategy with the highest payoff  %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

if pi1>pi2
   if pi1>pi3
   c1(i)= pi1 ;
   c2t(i)= pil1 ;
   c2s(i)=pih1;
str1(i)=pi1 ;
end
end
if pi2>pi1
   if pi2>pi3
   c1(i)= pi2 ;
   c2t(i)= pil2 ;
   c2s(i)=pih2;
str2(i)=pi2;
end
end
if pi3>pi1
   if pi3>pi2
  c1(i)= pi3 ;
  c2t(i)= pil3 ;
  c2s(i)=pih3;
str3(i)=pi3 ;
end
end

%
%PART II Country tough is tough and soft is soft
%
```

```
else
%
% This is the individual rationality for Country 1, equation 2 in
% first draft of the paper
%
if (1-gg)*pl*il<s2l*il
%
% This is the condition for Country 1 to be play soft
% Equation 3 in the first draft
%
if s2l*il > v1*s2h*ih+(1-v1)*(1-gg)*pl*il
%
% Country 1 is soft
% pi1 pi2  pil1 pil2 pih1 pih2
% Country 1 plays only strategy 1 and 2
%

stilda = 1- (dh*(ih-(s2l*il))/ih) ;

%
% For Country 1
%

pi1 = s1l*il ;
pi2 = v1*stilda*ih+(1-v1)*dd*s2l*il ;

%
% For Country 2 soft
%

pih1 = ih-s1l*il;
pih2 = (1-stilda)*ih ;

%
% For Country 2 tough
%

pil1 = (1-s1l)*il ;
pil2 = dl*(1-s2l)*il ;

%
% Otherwise Country 1 is tough
%
```

```
if pi1>pi2
   c1(i)=pi1;
   c2t(i)=pil1;
   c2s(i)=pih1 ;
   str4(i)=pi1 ;

end
if pi2>pi1 ;
   c1(i)=pi2;
   c2t(i)=pil2;
   c2s(i)=pih2 ;
   str5(i)=pi1 ;

end
else
%
% Country 1 is tough
% CALCULATE THE PAYOFFS FOR THE THREE STRATEGIES
% pi1 pi2 pi3 pil1 pil2 pil3 pih1 pih2 pih3
%

rtilda = ((v1*s1h*ih)+((1-v1)*(1-gg)*pl*il)-...
(s2l*il))/(v1*((s2h*ih)-(s2l*il)));
stilda = 1- (dh*(ih-(s2l*il))/ih) ;

%
% For Country 1
%
pi1 = s1l*il ;
pi2 = v1*stilda*ih+(1-v1)*dd*s2l*il ;
pi3 = v1*(rtilda*s1h*ih+(1-rtilda)*dd*s2h*ih)+(1-v1)*dd*pl*il ;

%
% For Country 2 soft
%

pih1 = ih-s1l*il;
pih2 = (1-stilda)*ih ;
pih3 = rtilda*(1-s1h)*ih+(1-rtilda)*dh*(1-s2h)*ih ;

%
% For Country 2 tough
%
```

```
pil1 = (1-s1l)*il ;
pil2 = dl*(1-s2l)*il ;
pil3 = dl*(1-pl)*il ;

   if pi1>pi2
   if pi1>pi3
   c1(i)= pi1 ;
   c2t(i)= pil1 ;
   c2s(i)=pih1;
str6(i)=pi1  ;
end
end
if pi2>pi1
   if pi2>pi3
   c1(i)= pi2 ;
   c2t(i)= pil2 ;
   c2s(i)=pih2;
str7(i)=pi2;
end
end
if pi3>pi1
   if pi3>pi2
  c1(i)= pi3;
  c2t(i)= pil3 ;
  c2s(i)=pih3;
str8(i)=pi3 ;
end
end
end
else
   c1(i)=v1*((1-gh)*ph*ih)*(1-v1)*((1-gg)*pl*il);
   c2t(i)=(1-gl)*(1-pl)*il;
   c2s(i)=(1-gh)*(1-ph)*ih;
end
end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Plot the results                           %
% Values for payoffs and expenditure in arms %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
x=[0.01:0.01:1];
```

```
plot(x,c1,'o-', x,c2t,'x-', x,c2s,'+-');
xlabel('Priors \upsilon_0')
ylabel('EXPECTED INCOME')
title(' THE EFFECT OF CHANGES IN PRIOR BELIEFS')
legend('c1 = country 1',...
'c2t= country 2 tough',...
'c2s= country 2 soft')
```

The bargaining game subroutine

```
% ------------------------------------------------------------------------%
% barwar21.m is a calculation of the  nash equilibrium of a               %
% the bargaining model war1.m in the paper "Rational Wars with incomplete %
% information" by P. Levine and F. Moraiz     3 / 2 /99                   %
% ------------------------------------------------------------------------%

global f1 f2l f2h r1 r2 gg gh gl aa dd dh dl mm A v1


%
% Enter the initial values
%

f1= 45 ;
f2l =45;
f2h = 45 ;
f1i = 51;
f2li= 53 ;
f2hi =57 ;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% M11 M22 are the result vectors                           %
% M33 is a control vector for accuracy                     %
% The program loops for the solution until the accuracy   %
% indicated in M33 is satisfied                            %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

M11=[f1; f2h; f2l];
M22=[f1i; f2hi; f2li];
M33= [0.05; 0.05; 0.05];

while abs(M11(1)-M22(1))>M33(1);
   while abs(M11(2)-M22(2))>M33(2);
```

```
      while abs(M11(3)-M22(3))>M33(3);

      M11 = M22 ;
   f1=M11(1) ;
   f2h=M11(2) ;
   f2l=M11(3) ;


%
%  Calculation of the probabilities of winning of Country 1
%

pl = (f1^mm)/(f1^mm+f2l^mm);
ph = (f1^mm)/(f1^mm+f2h^mm);

%
% Calculation of the size of cake
%
e1 = r1 - f1 ;
e2l = r2 - f2l ;
e2h = r2 - f2h ;

il = A*(e1^(1/aa)+e2l^(1/aa))^aa ;
ih = A*(e1^(1/aa)+e2h^(1/aa))^aa ;


%
% Offers in period 2
%

s2l = 1-((1-gl)*(1-pl)) ;
s2h = 1-((1-gh)*(1-ph)) ;

%
% Offers in period 1
%

s1l = 1-(dl*(1-s2l)) ;
s1h = 1-(dh*(1-s2h)) ;
```

```
%
%Distinction between a soft always soft
%and a soft becoming tough
%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%PART I corresponds to PART I in war1.m %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
   if (1-gl)*(1-pl) < (1-gh)*(1-ph) ;


%
% Individual rationality constraint for Country 1
% If the constraint is not met country will proceed to war
%
if (1-gg)*pl*il < s2l*il

% Distinction between soft and tough country
%

if s2h*ih > (1-v1)*s2l*il+ v1*(1-gg)*ph*ih

%
% Country 1 is soft
% pi1 pi2 pi3 pil1 pil2 pil3 pih1 pih2 pih3
% Country 1 plays only strategy 1 and 2
%

stilda = 1- (dl*(il-(s2h*ih))/il) ;


%
% For Country 1
%

pi1 = s1h*ih ;
pi2 = (1-v1)*stilda*il+v1*dd*s2h*ih ;

%
% For Country 2 soft
%

pih1 = (1-s1h)*ih;
```

```
pih2 = dh*(1-s2h)*ih ;

%
% For Country 2 tough
%

pil1 = il-s1h*ih;
pil2 = (1-stilda)*il ;

ff1 = f1 ;
ff2 = f2h ;
ff3 = f2l ;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% For every possible choice of strategy we find the nash equilibrium by  %
% solving the system of simultaneous equations in str1.m, str2,m str3.m, %
% str4.m, str5.m, str6.m and strw.m (and strw1.m)                        %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if all(pi1>pi2)
x=fsolve('str4',[ff1 ff2 ff3]') ;
st1=1;
end
if all(pi2>pi1)

   x=fsolve('str5',[ff1 ff2 ff3]') ;
   st2=1 ;
end
else

%Country 1 is tough and plays three strategies

  den1 = (1-v1)*(s2l*il-s2h*ih) ;
nun1 =(1-v1)*s2l*il+v1*(1-gg)*ph*ih-s2h*ih;
rtilda = nun1/den1;
stilda = 1- (dl*(il-(s2h*ih))/il) ;

%
% For Country 1
%

pi1 = s1h*ih ;
pi2 = (1-v1)*stilda*il+v1*dd*s2h*ih ;
pi3= v1*(rtilda*sil*il+(1-rtilda)*dd*s2l*il)+v1*(1-gg)*dd*ph*ih ;
```

```
%
% For Country 2 soft
%

pih1 = (1-s1h)*ih;
pih2 = dh*(1-s2h)*ih ;
pih3 = dh*(1-gh)*(1-ph)*ih ;
%
% For Country 2 tough
%

pil1 = il-s1h*ih;
pil2 = (1-stilda)*il ;
pil3 = rtilda*(1-s1l)*il+(1-rtilda)*dl*(1-s2l)*il ;

ff1 = f1 ;
ff2 = f2h ;
ff3 = f2l ;
if all(pi1>pi2&pi3)
   x=fsolve('str4',[ff1 ff2 ff3]');
   st3=1;
  end
if all(pi2>pi1&pi3)
   x=fsolve('str5',[ff1 ff2 ff3]') ;
   st4=1;
end
if all(pi3>pi1&pi2)
   x=fsolve('str6',[ff1 ff2 ff3]') ;
   st5=1;
end
end
f1i = x(1)
f2hi = x(2)
f2li = x(3)
M22 = [f1i; f2hi; f2li]
end
if (1-gg)*pl*il>s2l*il ;
ff1=50 ;
   ff2=50 ;
   x=fsolve('stratw',[ff1 ff2]');
   y=fsolve('stratw1',[ff1 ff2]');
f1=x(1);
```

```
f2l=x(2);
f1l=y(1);
f2h=y(2);
f1= v1*f1+(1-v1)*f1l;

f1i= f1 ;
f2li= f2l ;
f2hi=f2h ;
st7=1
%
% Put the results back into M22 and start the loop
%
M22 = [f1i; f2hi; f2li]
end

% PART II
else
%
% Individual rationality constraint for Country 1
% If the constraint is not met country will proceed to war
%
if (1-gg)*pl*il < s2l*il ;

%
% Distinction between soft and tough country
%

if s2l*il > v1*s2h*ih+(1-v1)*(1-gg)*pl*il




%
% Country 1 is soft
% pi1 pi2 pi3 pil1 pil2 pil3 pih1 pih2 pih3
% Country 1 plays only strategy 1 and 2
%

stilda = 1- (dh*(ih-(s2l*il))/ih) ;

%
% For Country 1
%
```

```
pi1 = s1l*il ;
pi2 = v1*stilda*ih+(1-v1)*dd*s2l*il ;

%
% For Country 2 soft
%

pih1 = ih-s1l*il;
pih2 = (1-stilda)*ih ;

%
% For Country 2 tough
%

pil1 = (1-s1l)*il ;
pil2 = dl*(1-s2l)*il ;

ff1 = f1 ;
ff2 = f2h ;
ff3 = f2l ;
if all(pi1>pi2)
x=fsolve('str1',[ff1 ff2 ff3]') ;
st6=1;
end
if all(pi2>pi1)

   x=fsolve('str2',[ff1 ff2 ff3]') ;
   st7=1;
end
%
% Put the results back into M22 and start the loop
%

else


%
%Country 1 is tough and plays any of the three strategies
%


den1 = v1*((s2h*ih)-(s2l*il)) ;
nun1 =(v1*s1h*ih)+((1-v1)*(1-gg)*pl*il)-(s2l*il);
```

```
rtilda = nun1/den1;
stilda = 1- (dh*(ih-(s21*il))/ih) ;

%
% For Country 1
%

pi1 = s1l*il ;
pi2 = v1*stilda*ih+(1-v1)*dd*s2l*il ;
pi3 = v1*(rtilda*s1h*ih+(1-rtilda)*dd*s2h*ih)+(1-v1)*dd*pl*il ;

%
% For Country 2 soft
%

pih1 = ih-s1l*il;
pih2 = (1-stilda)*ih ;
pih3 = rtilda*(1-s1h)*ih+(1-rtilda)*dh*(1-s2h)*ih ;

%
% For Country 2 tough
%

pil1 = (1-s1l)*il ;
pil2 = dl*(1-s2l)*il ;
pil3 = dl*(1-pl)*il ;

ff1 = f1 ;
ff2 = f2h ;
ff3 = f2l ;
if all(pi1>pi2&pi3)
   x=fsolve('str1',[ff1 ff2 ff3]');
   st8=1;
  end
if all(pi2>pi1&pi3)
   x=fsolve('str2',[ff1 ff2 ff3]') ;
   st9=1;
end
if all(pi3>pi1&pi2)
   x=fsolve('str3',[ff1 ff2 ff3]') ;
   st10=1;
end
end
```

```
f1i = x(1);
f2hi = x(2);
f2li = x(3) ;
M22 = [f1i; f2hi; f2li]
end
if (1-gg)*pl*il>s2l*il ;
ff1=50 ;
   ff2=50 ;
   x=fsolve('stratw',[ff1 ff2]');
   y=fsolve('stratw1',[ff1 ff2]');
f1=x(1);
f2l=x(2);
f11=y(1);
f2h=y(2);
f1= v1*f1+(1-v1)*f11;

f1i= f1 ;
f2li= f2l ;
f2hi=f2h ;
st7=1
%
% Put the results back into M22 and start the loop
%
M22 = [f1i; f2hi; f2li]
end
%
% The unique outcome is the input for war1.m
%
f1x(i)=f1;
f2lx(i)=f2l;
f2hx(i)=f2h;
end
end
end
end
```

Calculation of the strategies

```
% -------------------------------------------------------%
% The reaction functions of strategy 1                   %
% This finds the nash equilibrium in PART I, barwar21.m  %
% Model war1.m. Rational wars with incomplete information %
% -------------------------------------------------------%
```

```
function q=str1(x)
global v1 mm dl dd dh gg gl gh r1 r2 aa A f1 f2l f2h
q=zeros(3,1) ;
f1=x(1) ;
f2h=x(2) ;
f2l=x(3) ;

%
% Main functions of the program
%

ih=A*((r1-f1)^aa+(r2-f2h)^aa)^(1/aa) ;

il=A*((r1-f1)^aa+(r2-f2l)^aa)^(1/aa) ;

pl=f1^mm/(f1^mm+f2l^mm) ;

ph=f1^mm/(f1^mm+f2h^mm) ;

s2l = 1-(1-gl)*(1-pl) ;

s2h = 1-(1-gh)*(1-ph) ;

s1h = 1-dh*(1-s2h) ;

s1l = 1-dl*(1-s2l) ;

%derivatives

dpld1=mm*f1^(mm-1)*f2l^mm/(f1^mm+f2l^mm)^2 ;
dpll=mm*f2l^(mm-1)*f1^mm/(f1^mm+f2l^mm)^2 ;
dphd1=mm*f1^(mm-1)*f2h^mm/(f1^mm+f2h^mm)^2 ;
dphh=mm*f2h^(mm-1)*f1^mm/(f1^mm+f2h^mm)^2 ;

dIld1=(-1)*A*(r1-f1)^(aa-1)*((r1-f1)^aa+(r2-f2l)^aa)^((1/aa)-1) ;
dIhd1=(-1)*A*(r1-f1)^(aa-1)*((r1-f1)^aa+(r2-f2h)^aa)^((1/aa)-1) ;
dIhdh=(-1)*A*(r2-f2h)^(aa-1)*((r1-f1)^aa+(r2-f2h)^aa)^((1/aa)-1) ;
dIldl=(-1)*A*(r2-f2l)^(aa-1)*((r1-f1)^aa+(r2-f2l)^aa)^((1/aa)-1) ;

ds2hd1=(1-gh)*mm*f1^(mm-1)*f2h^mm/(f1^mm+f2h^mm)^2 ;
ds2ld1=(1-gl)*mm*f1^(mm-1)*f2l^mm/(f1^mm+f2l^mm)^2 ;
ds1hd1= dh*(1-gh)*mm*f1^(mm-1)*f2h^mm/(f1^mm+f2h^mm)^2 ;
```

```
ds2hdh=-(1-gh)*mm*f2h^(mm-1)*f1^mm/(f1^mm+f2h^mm)^2 ;
ds1hdh= -dh*(1-gh)*mm*f2h^(mm-1)*f1^mm/(f1^mm+f2h^mm)^2 ;
ds1ld1=dl*(1-gl)*mm*f1^(mm-1)*f2l^mm/(f1^mm+f2l^mm)^2 ;
ds1ldl=-dl*(1-gl)*mm*f2l^(mm-1)*f1^mm/(f1^mm+f2h^mm)^2 ;
ds2ldl=-(1-gl)*mm*f2l^(mm-1)*f1^mm/(f1^mm+f2l^mm)^2 ;

p13dsl1=(-1)*il ;
p13dil= 1-s1l ;


Rc1=ds1ld1*il+dIld1*s1l ;
Rc2=dIhdh;
Rc3=p13dsl1*ds1ldl+p13dil*dIldl ;

q(1)= Rc1
q(2)= Rc2
q(3)= Rc3

% ---------------------------------------------------------%
% The reaction functions of strategy 2                     %
% This finds the nash equilibrium in PART I, barwar21.m    %
% Model war1.m. Rational wars with incomplete information  %
% ---------------------------------------------------------%

function q=str2(x)
global v1 mm dl dd dh gg gl gh r1 r2 aa A f1 f2l f2h
q=zeros(3,1) ;
f1=x(1) ;
f2h=x(2) ;
f2l=x(3) ;

%
% Main functions of the program
%

ih=A*((r1-f1)^aa+(r2-f2h)^aa)^(1/aa) ;

il=A*((r1-f1)^aa+(r2-f2l)^aa)^(1/aa) ;

pl=f1^mm/(f1^mm+f2l^mm) ;

ph=f1^mm/(f1^mm+f2h^mm) ;
```

```
s2l = 1-(1-gl)*(1-pl) ;

s2h = 1-(1-gh)*(1-ph) ;

s1h = 1-dh*(1-s2h) ;

s1l = 1-dl*(1-s2l) ;

%derivatives

dpld1=mm*f1^(mm-1)*f2l^mm/(f1^mm+f2l^mm)^2 ;
dpll=mm*f2l^(mm-1)*f1^mm/(f1^mm+f2l^mm)^2 ;
dphd1=mm*f1^(mm-1)*f2h^mm/(f1^mm+f2h^mm)^2 ;
dphh=mm*f2h^(mm-1)*f1^mm/(f1^mm+f2h^mm)^2 ;

dIld1=(-1)*A*(r1-f1)^(aa-1)*((r1-f1)^aa+(r2-f2l)^aa)^((1/aa)-1) ;
dIhd1=(-1)*A*(r1-f1)^(aa-1)*((r1-f1)^aa+(r2-f2h)^aa)^((1/aa)-1) ;
dIhdh=(-1)*A*(r2-f2h)^(aa-1)*((r1-f1)^aa+(r2-f2h)^aa)^((1/aa)-1) ;
dIldl=(-1)*A*(r2-f2l)^(aa-1)*((r1-f1)^aa+(r2-f2l)^aa)^((1/aa)-1) ;

ds2hd1=(1-gh)*mm*f1^(mm-1)*f2h^mm/(f1^mm+f2h^mm)^2 ;
ds2ld1=(1-gl)*mm*f1^(mm-1)*f2l^mm/(f1^mm+f2l^mm)^2 ;
ds1hd1= dh*(1-gh)*mm*f1^(mm-1)*f2h^mm/(f1^mm+f2h^mm)^2 ;
ds2hdh=-(1-gh)*mm*f2h^(mm-1)*f1^mm/(f1^mm+f2h^mm)^2 ;
ds1hdh= -dh*(1-gh)*mm*f2h^(mm-1)*f1^mm/(f1^mm+f2h^mm)^2 ;
ds1ld1=dl*(1-gl)*mm*f1^(mm-1)*f2l^mm/(f1^mm+f2l^mm)^2 ;
ds1ldl=-dl*(1-gl)*mm*f2l^(mm-1)*f1^mm/(f1^mm+f2h^mm)^2 ;
ds2ldl=-(1-gl)*mm*f2l^(mm-1)*f1^mm/(f1^mm+f2l^mm)^2 ;

stilda=1-dh*(ih-s2l*il)/ih ;

dstIl= (-1)*dh*s2l*il/(ih)^2 ;
dsts2l= dh*il/ih ;
dstIh= dh*s2l/ih ;
dst1=dstIh*dIhd1+dstIl*dIld1+dsts2l*ds2ld1 ;
dsth=(dh*ih*dIhdh+dh*(ih-s2l*il)*dIhdh)/(ih)^2 ;

p22dst=(-1)*ih ;
p22dIh=1-stilda ;
p23ds2l=(-1)*dl*il ;
p23dIl=dl*(1-s2l) ;

Rc1=v1*(dst1*ih+dIhd1*stilda)+(1-v1)*(ds2ld1*il+dIld1*s2l) ;
```

```
Rc2=(p22dst*dsth)+(p22dIh*dIhdh) ;
Rc3=(p23ds21*ds21dl)+(p23dIl*dIldl) ;
q(1)= Rc1
q(2)= Rc2
q(3)= Rc3

% -------------------------------------------------------%
% The reaction functions of strategy 3                   %
% This finds the nash equilibrium in PART I, barwar21.m  %
% Model war1.m. Rational wars with incomplete information %
% -------------------------------------------------------%

function q=str3(x)
global v1 mm dl dd dh gg gl gh r1 r2 aa A f1 f21 f2h
q=zeros(3,1) ;
f1=x(1) ;
f2h=x(2) ;
f21=x(3) ;

%
% Main functions of the program
%

ih=A*((r1-f1)^aa+(r2-f2h)^aa)^(1/aa) ;

il=A*((r1-f1)^aa+(r2-f21)^aa)^(1/aa) ;

pl=f1^mm/(f1^mm+f21^mm) ;

ph=f1^mm/(f1^mm+f2h^mm) ;

s21 = 1-(1-gl)*(1-pl) ;

s2h = 1-(1-gh)*(1-ph) ;

s1h = 1-dh*(1-s2h) ;

s1l = 1-dl*(1-s21) ;

%derivatives

dpld1=mm*f1^(mm-1)*f21^mm/(f1^mm+f21^mm)^2 ;
dpl1=(-1)*mm*f21^(mm-1)*f1^mm/(f1^mm+f21^mm)^2 ;
```

```
dphd1=mm*f1^(mm-1)*f2h^mm/(f1^mm+f2h^mm)^2 ;
dphh=(-1)*mm*f2h^(mm-1)*f1^mm/(f1^mm+f2h^mm)^2 ;

dIld1=(-1)*A*(r1-f1)^(aa-1)*((r1-f1)^aa+(r2-f2l)^aa)^((1/aa)-1) ;
dIhd1=(-1)*A*(r1-f1)^(aa-1)*((r1-f1)^aa+(r2-f2h)^aa)^((1/aa)-1) ;
dIhdh=(-1)*A*(r2-f2h)^(aa-1)*((r1-f1)^aa+(r2-f2h)^aa)^((1/aa)-1) ;
dIldl=(-1)*A*(r2-f2l)^(aa-1)*((r1-f1)^aa+(r2-f2l)^aa)^((1/aa)-1) ;

ds2hd1=(1-gh)*mm*f1^(mm-1)*f2h^mm/(f1^mm+f2h^mm)^2 ;
ds2ld1=(1-gl)*mm*f1^(mm-1)*f2l^mm/(f1^mm+f2l^mm)^2 ;
ds1hd1= dh*(1-gh)*mm*f1^(mm-1)*f2h^mm/(f1^mm+f2h^mm)^2 ;
ds2hdh=-(1-gh)*mm*f2h^(mm-1)*f1^mm/(f1^mm+f2h^mm)^2 ;
ds1hdh= -dh*(1-gh)*mm*f2h^(mm-1)*f1^mm/(f1^mm+f2h^mm)^2 ;
ds1ld1=dl*(1-gl)*mm*f1^(mm-1)*f2l^mm/(f1^mm+f2l^mm)^2 ;
ds1ldl=-dl*(1-gl)*mm*f2l^(mm-1)*f1^mm/(f1^mm+f2h^mm)^2 ;
ds2ldl=-(1-gl)*mm*f2l^(mm-1)*f1^mm/(f1^mm+f2l^mm)^2 ;


rtilda=((v1*s2h*ih)+((1-v1)*(1-gg)*pl*il)-(s2l*il))/(v1*(s2h*ih-s2l*il)) ;

vy=v1*s2h*ih+(1-v1)*(1-gg)*pl*il-s2l*il ;
vx=v1*(s2h*ih-s2l*il) ;

drts2h=(v1*ih*vx-v1*ih*vy)/vx^2 ;
drts2l=((-1)*il*vx+v1*il*vy)/vx^2 ;
drtIl=(((1-v1)*(1-gg)*pl-s2l)*vx+v1*s2l*vy)/vx^2 ;
drtpl=((1-v1)*(1-gg)*il)/vx ;
drtIh=(v1*s2h*vx-v1*s2h*vy)/vx^2 ;
drt1=drtIh*dIhd1+drtpl*dpld1+drtIl*dIld1+drts2l*ds2ld1+drts2h*ds2hd1 ;

p31Il=(1-v1)*dd*pl*(1-gg) ;
p31pl=(1-v1)*dd*il*(1-gg) ;
p31s2h=v1*(1-rtilda)*dd*ih ;
p31Ih=v1*(rtilda*s1h+(1-rtilda)*dd*s2h) ;
p31s1h=v1*rtilda*ih ;
p31rt=v1*(s1h*ih-dd*s2h*ih) ;

p32rt=((1-s1h)-dh*(1-s2h))*ih ;
p32s1h=(-1)*rtilda*ih ;
p32Ih=rtilda*(1-s1h)+(1-rtilda)*dh*(1-s2h) ;
p32s2h=(-1)*(1-rtilda)*dh*ih ;

p33Il=dl*(1-pl)*(1-gl) ;
```

```
p33pl=(-1)*dl(1-gl)*il ;

drth=drtIh*dIhdh+drts2h*ds2hdh ;

Rc1=p31rt*drt1+p31s1h*ds1hd1+p31Ih*dIhd1+p31s2h*ds2hd1+...
p31pl*dpld1+p31Il*dIld1 ;
Rc2=p32rt*drth+p32s1h*ds1hdh+p32Ih*dIhdh+p32s2h*ds2hdh ;
Rc3=p33Il*dIldl+p33pl*dpll ;

q(1)= Rc1
q(2)= Rc2
q(3)= Rc3


% ---------------------------------------------------------%
% The reaction functions of strategy 4                     %
% This finds the nash equilibrium in PART II, barwar21.m   %
% Model war1.m. Rational wars with incomplete information  %
% ---------------------------------------------------------%

function q=str4(x)
global v1 mm dl dd dh gg gl gh r1 r2 aa A f1 f2l f2h
q=zeros(3,1) ;
f1=x(1) ;
f2h=x(2) ;
f2l=x(3) ;
v1=1-v1;
%
% Main functions of the program
%

ih=A*((r1-f1)^aa+(r2-f2h)^aa)^(1/aa) ;

il=A*((r1-f1)^aa+(r2-f2l)^aa)^(1/aa) ;

pl=f1^mm/(f1^mm+f2l^mm) ;

ph=f1^mm/(f1^mm+f2h^mm) ;

s2l = 1-(1-gl)*(1-pl) ;

s2h = 1-(1-gh)*(1-ph) ;
```

```
s1h = 1-dh*(1-s2h) ;

s1l = 1-dl*(1-s2l) ;

%derivatives

dpld1=mm*f1^(mm-1)*f2l^mm/(f1^mm+f2l^mm)^2 ;
dpll=mm*f2l^(mm-1)*f1^mm/(f1^mm+f2l^mm)^2 ;
dphd1=mm*f1^(mm-1)*f2h^mm/(f1^mm+f2h^mm)^2 ;
dphh=mm*f2h^(mm-1)*f1^mm/(f1^mm+f2h^mm)^2 ;

dIld1=(-1)*A*(r1-f1)^(aa-1)*((r1-f1)^aa+(r2-f2l)^aa)^((1/aa)-1) ;
dIhd1=(-1)*A*(r1-f1)^(aa-1)*((r1-f1)^aa+(r2-f2h)^aa)^((1/aa)-1) ;
dIhdh=(-1)*A*(r2-f2h)^(aa-1)*((r1-f1)^aa+(r2-f2h)^aa)^((1/aa)-1) ;
dIldl=(-1)*A*(r2-f2l)^(aa-1)*((r1-f1)^aa+(r2-f2l)^aa)^((1/aa)-1) ;

ds2hd1=(1-gh)*mm*f1^(mm-1)*f2h^mm/(f1^mm+f2h^mm)^2 ;
ds2ld1=(1-gl)*mm*f1^(mm-1)*f2l^mm/(f1^mm+f2l^mm)^2 ;
ds1hd1= dh*(1-gh)*mm*f1^(mm-1)*f2h^mm/(f1^mm+f2h^mm)^2 ;
ds2hdh=-(1-gh)*mm*f2h^(mm-1)*f1^mm/(f1^mm+f2h^mm)^2 ;
ds1hdh= -dh*(1-gh)*mm*f2h^(mm-1)*f1^mm/(f1^mm+f2h^mm)^2 ;
ds1ld1=dl*(1-gl)*mm*f1^(mm-1)*f2l^mm/(f1^mm+f2l^mm)^2 ;
ds1ldl=-dl*(1-gl)*mm*f2l^(mm-1)*f1^mm/(f1^mm+f2h^mm)^2 ;
ds2ldl=-(1-gl)*mm*f2l^(mm-1)*f1^mm/(f1^mm+f2l^mm)^2 ;

dp12ds1h=-ih;
dp12dIh=1-s1h;

Rc1=ds1hd1*ih+dIhd1*s1h;
Rc2=dp12ds1h*ds1hd1+dp12dIh*dIhdh;
Rc3=dIldl;

q(1)= Rc1
q(2)= Rc2
q(3)= Rc3


% ---------------------------------------------------------%
% The reaction functions of strategy 4                     %
% This finds the nash equilibrium in PART II, barwar21.m   %
% Model war1.m. Rational wars with incomplete information  %
% ---------------------------------------------------------%
```

```
function q=str4(x)
global v1 mm dl dd dh gg gl gh r1 r2 aa A f1 f2l f2h
q=zeros(3,1) ;
f1=x(1) ;
f2h=x(2) ;
f2l=x(3) ;
v1=1-v1;
%
% Main functions of the program
%

ih=A*((r1-f1)^aa+(r2-f2h)^aa)^(1/aa) ;

il=A*((r1-f1)^aa+(r2-f2l)^aa)^(1/aa) ;

pl=f1^mm/(f1^mm+f2l^mm) ;

ph=f1^mm/(f1^mm+f2h^mm) ;

s2l = 1-(1-gl)*(1-pl) ;

s2h = 1-(1-gh)*(1-ph) ;

s1h = 1-dh*(1-s2h) ;

s1l = 1-dl*(1-s2l) ;

%derivatives

dpld1=mm*f1^(mm-1)*f2l^mm/(f1^mm+f2l^mm)^2 ;
dpll=mm*f2l^(mm-1)*f1^mm/(f1^mm+f2l^mm)^2 ;
dphd1=mm*f1^(mm-1)*f2h^mm/(f1^mm+f2h^mm)^2 ;
dphh=mm*f2h^(mm-1)*f1^mm/(f1^mm+f2h^mm)^2 ;

dIld1=(-1)*A*(r1-f1)^(aa-1)*((r1-f1)^aa+(r2-f2l)^aa)^((1/aa)-1) ;
dIhd1=(-1)*A*(r1-f1)^(aa-1)*((r1-f1)^aa+(r2-f2h)^aa)^((1/aa)-1) ;
dIhdh=(-1)*A*(r2-f2h)^(aa-1)*((r1-f1)^aa+(r2-f2h)^aa)^((1/aa)-1) ;
dIldl=(-1)*A*(r2-f2l)^(aa-1)*((r1-f1)^aa+(r2-f2l)^aa)^((1/aa)-1) ;

ds2hd1=(1-gh)*mm*f1^(mm-1)*f2h^mm/(f1^mm+f2h^mm)^2 ;
ds2ld1=(1-gl)*mm*f1^(mm-1)*f2l^mm/(f1^mm+f2l^mm)^2 ;
ds1hd1= dh*(1-gh)*mm*f1^(mm-1)*f2h^mm/(f1^mm+f2h^mm)^2 ;
ds2hdh=-(1-gh)*mm*f2h^(mm-1)*f1^mm/(f1^mm+f2h^mm)^2 ;
```

```
ds1hdh= -dh*(1-gh)*mm*f2h^(mm-1)*f1^mm/(f1^mm+f2h^mm)^2 ;
ds1ldl=dl*(1-gl)*mm*f1^(mm-1)*f2l^mm/(f1^mm+f2l^mm)^2 ;
ds1ldl=-dl*(1-gl)*mm*f2l^(mm-1)*f1^mm/(f1^mm+f2h^mm)^2 ;
ds2ldl=-(1-gl)*mm*f2l^(mm-1)*f1^mm/(f1^mm+f2l^mm)^2 ;

dp12ds1h=-ih;
dp12dIh=1-s1h;

Rc1=ds1hd1*ih+dIhd1*s1h;
Rc2=dp12ds1h*ds1hd1+dp12dIh*dIhdh;
Rc3=dIldl;

q(1)= Rc1
q(2)= Rc2
q(3)= Rc3

% ---------------------------------------------------------%
% The reaction functions of strategy 5                     %
% This finds the nash equilibrium in PART II, barwar21.m   %
% Model war1.m. Rational wars with incomplete information  %
% ---------------------------------------------------------%

function q=str5(x)
global v1 mm dl dd dh gg gl gh r1 r2 aa A f1 f2l f2h
q=zeros(3,1) ;
f1=x(1) ;
f2h=x(2) ;
f2l=x(3) ;
v1=1-v1;
%
% Main functions of the program
%

ih=A*((r1-f1)^aa+(r2-f2h)^aa)^(1/aa) ;

il=A*((r1-f1)^aa+(r2-f2l)^aa)^(1/aa) ;

pl=f1^mm/(f1^mm+f2l^mm) ;

ph=f1^mm/(f1^mm+f2h^mm) ;

s2l = 1-(1-gl)*(1-pl) ;
```

```
s2h = 1-(1-gh)*(1-ph) ;

s1h = 1-dh*(1-s2h) ;

s1l = 1-dl*(1-s2l) ;


%derivatives

dpld1=mm*f1^(mm-1)*f2l^mm/(f1^mm+f2l^mm)^2 ;
dpll=mm*f2l^(mm-1)*f1^mm/(f1^mm+f2l^mm)^2 ;
dphd1=mm*f1^(mm-1)*f2h^mm/(f1^mm+f2h^mm)^2 ;
dphh=mm*f2h^(mm-1)*f1^mm/(f1^mm+f2h^mm)^2 ;

dIld1=(-1)*A*(r1-f1)^(aa-1)*((r1-f1)^aa+(r2-f2l)^aa)^((1/aa)-1) ;
dIhd1=(-1)*A*(r1-f1)^(aa-1)*((r1-f1)^aa+(r2-f2h)^aa)^((1/aa)-1) ;
dIhdh=(-1)*A*(r2-f2h)^(aa-1)*((r1-f1)^aa+(r2-f2h)^aa)^((1/aa)-1) ;
dIldl=(-1)*A*(r2-f2l)^(aa-1)*((r1-f1)^aa+(r2-f2l)^aa)^((1/aa)-1) ;

ds2hd1=(1-gh)*mm*f1^(mm-1)*f2h^mm/(f1^mm+f2h^mm)^2 ;
ds2ld1=(1-gl)*mm*f1^(mm-1)*f2l^mm/(f1^mm+f2l^mm)^2 ;
ds1hd1= dh*(1-gh)*mm*f1^(mm-1)*f2h^mm/(f1^mm+f2h^mm)^2 ;
ds2hdh=-(1-gh)*mm*f2h^(mm-1)*f1^mm/(f1^mm+f2h^mm)^2 ;
ds1hdh= -dh*(1-gh)*mm*f2h^(mm-1)*f1^mm/(f1^mm+f2h^mm)^2 ;
ds1ld1=dl*(1-gl)*mm*f1^(mm-1)*f2l^mm/(f1^mm+f2l^mm)^2 ;
ds1ldl=-dl*(1-gl)*mm*f2l^(mm-1)*f1^mm/(f1^mm+f2h^mm)^2 ;
ds2ldl=-(1-gl)*mm*f2l^(mm-1)*f1^mm/(f1^mm+f2l^mm)^2 ;

stilda=1-(dl*(il-s2h*ih)/il);

dstdIl=(-dl*s2h*ih)/(ih^2);
dstdIh=dl*s2h/il;
dstds2h=dl*ih/il;

dstd1=dstdIh*dIhd1+dstdIl*dIld1+dstds2h*ds2hd1;

dp21dst=v1*il;
dp21dIl=v1*stilda;
dp21ds2h=(1-v1)*dd*ih;
dp21dIh=(1-v1)*dd*s2h;

dp22ds2h=-dh*ih;
dp22dIh=dh*(1-s2h);
```

```
dstdh=dstdIl*dIldl+dstdIh*dIhdh+dstds2h*ds2hdh;

dp23dst=-il;
dp23dIl=1-stilda;

Rc1=dp21dst*dstd1+dp21dIl*dIld1+dp21ds2h*ds2hd1+dp21dIh*dIhd1;
Rc2=dp22ds2h*ds2hdh+dp22dIh*dIhdh;
Rc3=dp23dst*dstdh+dp23dIl*dIldl;

q(1)= Rc1
q(2)= Rc2
q(3)= Rc3


% ----------------------------------------------------------%
% The reaction functions of strategy 6                      %
% This finds the nash equilibrium in PART II, barwar21.m    %
% Model war1.m. Rational wars with incomplete information   %
% ----------------------------------------------------------%

function q=str6(x)
global v1 mm dl dd dh gg gl gh r1 r2 aa A f1 f2l f2h
q=zeros(3,1) ;
f1=x(1) ;
f2h=x(2) ;
f2l=x(3) ;
v1=1-v1;
%
% Main functions of the program
%

ih=A*((r1-f1)^aa+(r2-f2h)^aa)^(1/aa) ;

il=A*((r1-f1)^aa+(r2-f2l)^aa)^(1/aa) ;

pl=f1^mm/(f1^mm+f2l^mm) ;

ph=f1^mm/(f1^mm+f2h^mm) ;

s2l = 1-(1-gl)*(1-pl) ;

s2h = 1-(1-gh)*(1-ph) ;
```

```
s1h = 1-dh*(1-s2h) ;

s1l = 1-dl*(1-s2l) ;

%derivatives

dpld1=mm*f1^(mm-1)*f2l^mm/(f1^mm+f2l^mm)^2 ;
dpll=mm*f2l^(mm-1)*f1^mm/(f1^mm+f2l^mm)^2 ;
dphd1=mm*f1^(mm-1)*f2h^mm/(f1^mm+f2h^mm)^2 ;
dphh=mm*f2h^(mm-1)*f1^mm/(f1^mm+f2h^mm)^2 ;

dIld1=(-1)*A*(r1-f1)^(aa-1)*((r1-f1)^aa+(r2-f2l)^aa)^((1/aa)-1) ;
dIhd1=(-1)*A*(r1-f1)^(aa-1)*((r1-f1)^aa+(r2-f2h)^aa)^((1/aa)-1) ;
dIhdh=(-1)*A*(r2-f2h)^(aa-1)*((r1-f1)^aa+(r2-f2h)^aa)^((1/aa)-1) ;
dIldl=(-1)*A*(r2-f2l)^(aa-1)*((r1-f1)^aa+(r2-f2l)^aa)^((1/aa)-1) ;

ds2hd1=(1-gh)*mm*f1^(mm-1)*f2h^mm/(f1^mm+f2h^mm)^2 ;
ds2ld1=(1-gl)*mm*f1^(mm-1)*f2l^mm/(f1^mm+f2l^mm)^2 ;
ds1hd1= dh*(1-gh)*mm*f1^(mm-1)*f2h^mm/(f1^mm+f2h^mm)^2 ;
ds2hdh=-(1-gh)*mm*f2h^(mm-1)*f1^mm/(f1^mm+f2h^mm)^2 ;
ds1hdh= -dh*(1-gh)*mm*f2h^(mm-1)*f1^mm/(f1^mm+f2h^mm)^2 ;
ds1ld1=dl*(1-gl)*mm*f1^(mm-1)*f2l^mm/(f1^mm+f2l^mm)^2 ;
ds1ldl=-dl*(1-gl)*mm*f2l^(mm-1)*f1^mm/(f1^mm+f2h^mm)^2 ;
ds2ldl=-(1-gl)*mm*f2l^(mm-1)*f1^mm/(f1^mm+f2l^mm)^2 ;

vy=v1*s2l*il+(1-v1)*(1-gg)*ph*ih-s2h*ih;
vx=v1*(s2l*il-s2h*ih);
rtilda=vy/vx;


drtdIl=(v1*s2l*vx-v1*s2l*vy)/vx^2;
drtdIh=(vx*((1-v1)*(1-gg)*ph-s2h)+v1*s2h*vy)/vx^2;
drtdph=(1-v1)*(1-gg)*ih/vx;
drtds2h=(-ih*vx-v1*ih*vy)/vx^2;
drtds2l=(v1*il*vx-v1*il*vy)/vx^2;



drtd1=drtdIl*dIld1+drtdIh*dIhd1+drtdph*dphd1+drtds2h*ds2hd1+...
drtds2l*ds2ld1;
dp31ds1l=v1*rtilda*il;
dp31dIl= v1*(rtilda*s1l+(1-rtilda)*dd*s2l);
```

```
dp31drt=v1*(s1l*il-dd*s2l*il);
dp31ds2l=v1*(1-rtilda)*dd*il;
dp31dIh=(1-v1)*(1-gg)*dd*ph;
dp31dph=(1-v1)*(1-gg)*dd*ih;

dp32dph=-dh*(1-gg)*ih;
dp32dIh= dh*(1-gg)*(1-ph);

drtdl=drtds2l*ds2ldl+drtdIl*dIldl;
dp33drt=(1-s1l)*il-dh*(1-s2l)*il;
dp33dIl=rtilda*(1-s1l)+(1-rtilda)*dh*(1-s2l)*il;
dp33ds1l=-rtilda*il;
dp33ds2l=dh*il;

Rc1=dp31ds1l*ds1ld1+dp31dIl*dIld1+dp31drt*drtd1+dp31ds2l*...
ds2ld1+dp31dIh*dIhd1+dp31dph*dphd1;
Rc2=dp32dph*dphh+dp32dIh*dIhdh;
Rc3=dp33drt*drtdl+dp33dIl*dIldl+dp33ds1l*ds1ldl+dp33ds2l*ds2ldl;

q(1)= Rc1
q(2)= Rc2
q(3)= Rc3

%
% The reaction functions of war between
% Country 1 and Country 2 tough
% Model barwar1.m and war1.m
%

function q=stratw(x)
global gg dl r1 r2 aa mm f1 f2l gl
q=zeros(2,1) ;
f1=x(1) ;
f2l=x(2) ;

f1l= (f1^mm+f2l^mm);
pp1= f1^mm/(f1^mm+f2l^mm);
pp2= 1-pp1;
rr1= ((r1-f1)^(1/aa)+(r2-f2l)^(1/aa));

q(1)=(1-gg)*(mm*(f1^(mm-1))*(f2l^mm)*(rr1^aa)/f1l^2-pp1*...
((r1-f1)^(1/aa-1))*(rr1^(aa-1)));
q(2)=(1-gl)*(mm*(f2l^(mm-1))*(f1^mm)*(rr1^aa)/f1l^2-pp2*...
```

```
((r2-f21)^(1/aa-1))*(rr1^(aa-1)));


%
% The reaction functions of war between
% Country 1 and Country 2 soft
% Model barwar1.m and war1.m
%

function q=stratw1(y)
global gg dl r1 r2 aa mm f1 f2h gh
q=zeros(2,1) ;
f1y=y(1) ;
f2h=y(2) ;

f11= (f1^mm+f2h^mm);
pp1= f1^mm/(f1^mm+f2h^mm);
pp2= 1-pp1;
rr1= ((r1-f1)^(1/aa)+(r2-f2h)^(1/aa));

q(1)=(1-gg)*(mm*(f1^(mm-1))*(f2h^mm)*(rr1^aa)/f11^2-pp1*...
((r1-f1)^(1/aa-1))*(rr1^(aa-1)));
q(2)=(1-gh)*(mm*(f2h^(mm-1))*(f1^mm)*(rr1^aa)/f11^2-pp2*...
((r2-f2h)^(1/aa-1))*(rr1^(aa-1)));
```

The equilibrium with fixed allocations

```
% ---------------------------------------------------------------%
% fixwar is a calculation of the equilibrium of a                %
% bargaining model with exogenous  allocation of fighting effort %
% in the paper "Rational Wars with incomplete                    %
% information" by P. Levine and F. Moraiz     3 / 2 /99          %
% ---------------------------------------------------------------%

global f1 f21 f2h r1 r2 gg gh gl aa dd dh dl mm A v1

%
% GLOBAL VARIABLES DEFINITION
% f1 is the expenditure in arms of Country 1
% f21 and f2h are the expenditure in arms of country touth and soft
% r1 and r2 are the initial resources of Country 1 and two
% gg gh gl are the externality costs gamma gamma low and high
```

```
% dd dh dl are the discounting factors
% v1 is the probability of facing a soft Country 2


%
% Enter the initial values
%

disp('------------------------------------')
disp('Calibration of the fixed allocations')
disp('------------------------------------')

disp('Enter the value of the externality cost for Country 1 gg')
disp('Enter the value for Country 2 soft gh and touth gl')
disp('gg must be bigger than gh, and gh bigger that gl')

gg = input('Enter gg: ')
gh = input('Enter gh: ')
gl = input('Enter gl: ')

r1 = 100 ;
r2 = 100 ;
dd = 0.90 ;
dl =  0.87 ;
dh = 0.93 ;
aa = 1 ;
A  = 1 ;
mm=1;
v1=0.8;
%
% Loop for different probabilities of facing a soft country
%

for i = 1:10,
for j = 1:10,
for h = 1:10,


%  Calculation of the probabilities of winning of Country 1
%
f11(i)=(i/10);
f1=f11(i);
f211(j)=(j/10);
```

```
f2l=f2l1(j);
f2h1(h)=(h/10);
f2h=f2h1(h);


%
% Barwar is a m file that estimates the unique nash equilibrium
% Barwar returns a unique value for F1, F2l and F2h
%



%
% Once we have calculated the optimal allocation to F1, F2l and F2h
% we can proceed to calculate the different equilibrium outcomes for
%  different values of gg, v1, mm ...
%

% Probabilities of winning

pl = (f1^mm)/(f1^mm+f2l^mm);
ph = (f1^mm)/(f1^mm+f2h^mm);

% Calculation of the size of cake

e1 = r1 - f1 ;
e2l = r2 - f2l ;
e2h = r2 - f2h ;
il = A*(e1^(1/aa)+e2l^(1/aa))^aa ;
ih = A*(e1^(1/aa)+e2h^(1/aa))^aa ;

% Offers in period 2

s2l = 1-(1-gl)*(1-pl) ;
s2h = 1-(1-gh)*(1-ph) ;

% Offers in period 1

s1l = 1-dl*(1-s2l) ;
s1h = 1-dh*(1-s2h) ;

%PART I

if (1-gl)*(1-pl) < (1-gh)*(1-ph) ;
```

```
  den1 = (1-v1)*(s2l*il-s2h*ih) ;
nun1 =(1-v1)*s2l*il+v1*(1-gg)*ph*ih-s2h*ih;
rtilda = nun1/den1;
stilda = 1- (dl*(il-(s2h*ih))/il) ;

%
% For Country 1
%

pi1 = s1h*ih ;
pi2 = (1-v1)*stilda*il+v1*dd*s2h*ih ;
pi3= v1*(rtilda*s1l*il+(1-rtilda)*dd*s2l*il)+v1*(1-gg)*dd*ph*ih ;

%
% For Country 2 soft
%

pih1 = (1-s1h)*ih;
pih2 = dh*(1-s2h)*ih ;
pih3 = dh*(1-gh)*(1-ph)*ih ;
%
% For Country 2 tough
%

pil1 = il-s1h*ih;
pil2 = (1-stilda)*il ;
pil3 = rtilda*(1-s1l)*il+(1-rtilda)*dl*(1-s2l)*il ;
if pi1>pi2
   if pi1>pi3
   inc1(i,j,h)= pi1 ;
   inc2l(i,j,h)= pil1 ;
   inch(i,j,h)=pih1;
str1(i,j,h)=pi1  ;
end
end
if pi2>pi1
   if pi2>pi3
   inc1(i,j,h)= pi2 ;
   inc2l(i,j,h)= pil2 ;
   inch(i,j,h)=pih2;
str2(i,j,h)=pi2;
end
```

```
end
if pi3>pi1
   if pi3>pi2
  inc1(i,j,h)= pi3 ;
  inc2l(i,j,h)= pil3 ;
  inch(i,j,h)=pih3;
str3(i,j,h)=pi3 ;
end
end

%PART II
else
%
% This is the individual rationality for Country 1, equation 2 in
% first draft of the paper
%
if (1-gg)*pl*il<s2l*il
%
% This is the condition for Country 1 to be play soft
% Equation 3 in the first draft
%
if s2l*il > v1*s2h*ih+(1-v1)*(1-gg)*pl*il;
%
% Country 1 is soft
% pi1 pi2  pil1 pil2 pih1 pih2
% Country 1 plays only strategy 1 and 2
%

stilda = 1- (dh*(ih-(s2l*il))/ih) ;

%
% For Country 1
%

pi1 = s1l*il ;
pi2 = v1*stilda*ih+(1-v1)*dd*s2l*il ;

%
% For Country 2 soft
%

pih1 = ih-s1l*il;
pih2 = (1-stilda)*ih ;
```

```
%
% For Country 2 tough
%

pil1 = (1-s1l)*il ;
pil2 = dl*(1-s2l)*il ;

%
% Otherwise Country 1 is tough
%
if pi1>pi2
   inc1(i,j,h)=pi1;
   inc2l(i,j,h)=pil1;
   inch(i,j,h)=pih1 ;
   str1(i,j,h)=pi1  ;

end
if pi2>pi1 ;
   inc1(i,j,h)=pi2;
   inc2l(i,j,h)=pil2;
   inch(i,j,h)=pih2 ;
   str2(i,j,h)=pi1  ;

end
else
%
% Country 1 is tough
% CALCULATE THE PAYOFFS FOR THE THREE STRATEGIES
% pi1 pi2 pi3 pil1 pil2 pil3 pih1 pih2 pih3
%

rtilda = ((v1*s1h*ih)+((1-v1)*(1-gg)*pl*il)-(s2l*il))...
/(v1*((s2h*ih)-(s2l*il))) ;
stilda = 1- (dh*(ih-(s2l*il))/ih) ;

%
% For Country 1
%
pi1 = s1l*il ;
pi2 = v1*stilda*ih+(1-v1)*dd*s2l*il ;
pi3 = v1*(rtilda*s1h*ih+(1-rtilda)*dd*s2h*ih)+(1-v1)*dd*pl*il ;
```

```
%
% For Country 2 soft
%

pih1 = ih-s1l*il;
pih2 = (1-stilda)*ih ;
pih3 = rtilda*(1-s1h)*ih+(1-rtilda)*dh*(1-s2h)*ih ;

%
% For Country 2 tough
%
pil1 = (1-s1l)*il ;
pil2 = dl*(1-s2l)*il ;
pil3 = dl*(1-pl)*il ;

   if pi1>pi2
   if pi1>pi3
   inc1(i,j,h)= pi1 ;
   inc21(i,j,h)= pil1 ;
   inch(i,j,h)=pih1;
str1(i,j,h)=pi1 ;
end
end
if pi2>pi1
   if pi2>pi3
   inc1(i,j,h)= pi2 ;
   inc21(i,j,h)= pil2 ;
   inch(i,j,h)=pih2;
str2(i,j,h)=pi2;
end
end
if pi3>pi1
   if pi3>pi2
  inc1(i,j,h)= pi3 ;
  inc21(i,j,h)= pil3;
  inch(i,j,h)=pih3;
str3(i,j,h)=pi3 ;
end
end
end
else
   inc1(i,j,h)=v1*((1-gh)*ph*ih)*(1-v1)*((1-gg)*pl*il);
   inc21(i,j,h)=(1-gl)*(1-pl)*il;
```

```
    inch(i,j,h)=(1-gh)*(1-ph)*ih;
end
end
end

end
end
```

Appendix E

Empirical Analysis

E.1 Other studies of conflict

So far most part of the empirical studies have been applied to the test of theories of conflict from the political science, which has a great influence is the nature of the data available. Whether those theories are consistent with some empirical regularities or not depends very much in the way we define the empirical regularities of conflict.

Geller and Singer produced a list of consistent and cumulative empirical regularities of war based on a review of over 500 empirical studies. They found some of the most significant factors that increase the probability of war. These factors are classified according to the level of analysis (see appendix E.2 and they are mostly associated with issues such as the balance of power, the status quo, the number of alliances, the contiguity between states, etc.

One of the reasons for the abundance of those theories might be the heterogeneity that characterizes conflict. Azar [123] already recognizes the presence of many different approaches and classifications of empirical facts in international relations. This consists of a series of inventories of empirical findings classified according to factors such as:

1. "Limited War" (the intensity of war)
2. the hierarchical structure of the international systems
3. crisis behaviour
4. international interactions
5. biases of information sources available to practitioners
6. significance of alliances in matters of war and peace

7. predominance of "great powers" in the international communications system, and

8. the complex and changeable but understandable underlying dimensionality of international communications (i.e. the rapid changes in telecommunications and its impact in the international community).

Therefore, we should choose the scope of our analysis before we established the stylized facts of conflict. Brito and Intrilligator (1996) mention some of these facts in relation to the assumption of the rationality of war.

1. In one of the most cited studies on conflict, *The War Trap*, Bueno de Mesquita [100] concluded that wars are consistent with rational behaviour.

2. Wallace ,1982 [124, 125] studied the outbreak of war in serious great power disputes and concluded that conflict and disputes accompanied by arms races are much more likely to result in war.

3. Siverson and Tennefoss [126] divided conflict into three levels: threats, unreciprocated military action and reciprocated military actions. They divide nations into four categories: allied major powers, unallied major powers, allied minor powers, and unallied minor powers. They found that major powers and allied minor powers seem to be involved in less hostility that unallied minor powers.

4. Smith [127] focussed on the war proneness of arms races, and found that war is normally consistent with rational behaviour on the part of the initiator; second, an arms race is less likely to lead to war if the status quo power "loses" the arms race; third, conflict between major powers is limited and is less likely to escalate into war than conflict between major and minor powers; fourth, major powers and allied minor powers seem to be involved in less hostility than unallied minor powers; and fifth, unallied minor powers initiated conflict with major powers, but there is no case where unallied minor powers initiated conflict.

These empirical regularities compiled by Intrilligator and others clearly apply to the family of theories that addresses conflict either as a power concentration problem or power parity.

A more interesting classification is that of Kumar Rupesinghe [120]. He enlarges the scope of conflict in order to prove a basic change in its empirical regularity: the emergence of new types of conflict. It might be the case that major wars are disappearing whereas regional and internal conflict if not increasing are not yet decreasing. He established some characteristics based on SIPRI [128] observations. This data set reduced the threshold to less than 1000 deaths. As a consequence the number of conflicts in the World increased significantly.

Using this data Rupesinghe observed the following regularities from World War II:

1. Most armed conflicts take place in the Third World.
2. The basic issues in the armed conflicts in 1989 were related to internal matters.
3. Inter-state conflicts are currently on the decline.
4. External intervention by a regional or international power lead to military withdrawals or negotiations of withdrawals.
5. The UN security council war rarely involved though is likely to play a more active role in the future
6. In most cases, internal conflicts have been fuelled by arms sales.
7. There has been consistent violations of human rights. Civilians account for 74% of official deaths.
8. Conflict has involved cross-border affiliations or networks.
9. Many of the conflicts are identity struggles.

The last classification of regularities regarding conflict and the State are those stylized facts of conflict and economic performance.

There is no systematic compilation of the regularities of growth and conflict together. Barro and Sala I Martin [129] included in their regression for growth a conflict variable called political instability. It was defined as the average over each decade of revolutions per year and political assassinations per million inhabitants per year. The estimated coefficient on political instability is negative and marginally significant, -0.033(s.e.=0.018). In their regressions they also include two other variables. A war dummy for countries that participated at least in an external war, and a variable for defence expenditures. The estimated coefficient of the defence expenditure variable was essentially 0, whereas the war dummy was negative, but not statistically significant, -0.0061 (s.e=.0.00390). They concluded that the failure to isolate important growth effects from external wars results from the poor quality of data, rather than the unimportance of war. Blomberg (1995) analyzed the effects of political instability and the defence burden in growth. In his model defence expenditure works as a sort of insurance policy for the government against the probability of being overthrown. The empirical evidence that he presented supports the idea that political instability reduces growth. Finally, we did not find any study of the empirical regularities of conflict, information environment and economic growth. There is plenty of data in growth and both internal and external conflict. It could be worthwhile to establish the empirical regularities regarding conflict and economic performance by selecting conflicts of an essentially economic nature .

E.2 Empirical Regularities

Level of Analysis: state

- Power status (major power)
- Power cycle (critical point if major power)
- Alliance (alliance member)
- Borders (number of borders)

Level of Analysis: dyad

- Contiguity/proximity (common border/distance)
- Economic development (absence of joint advanced economies)
- Static capability balance(parity)
- Dynamic capability balance(unstable:shift/transition)
- Alliance (unbalance external alliance-tie)
- Enduring rivalry

Level of Analysis: region

- Contagion/diffusion(presence of ongoing regional war)

Level of analysis:system

- Polarity (weak unipolarity/declining leader)
- Unstable hierarchy
- Number of Borders
- Frequency of civil/revolutionary wars

There are also other factors that affect the seriousness of the war once it takes place. These factor are:

Level of Analysis: state

- Power status(major power)

Level of Analysis: system

- Alliance (high polarisation)

E.3 Data

E.3.1 The Correlates of War Project (COW): International and Civil War Data, 1816-1992

This data collection [118] describes international and civil wars for the years 1816-1992. The unit of analysis is the participant in a particular conflict. Each participant is coded, along with battle and total deaths, pre-war population and armed forces, and whether the member in question initiated the conflict. The conflicts are classified in two major categories: international war and civil war.

International war

In order to be considered a nation-participant in the interstate system, certain minimal criteria of population and diplomatic recognition were used (at least 500,000 total population and either diplomatic recognition by at least two major powers or membership in the League of Nations or United Nations). This part of the dataset describes two types of international wars: Interstate wars, in which a nation that qualifies as a member of the interstate system engages in a war with another member of the interstate system. Extra-systemic wars, in which a nation that qualifies as an interstate system member engages in a war with a political entity that is not an interstate system member. Extra-systemic wars are further divided into two sub-types. The first sub-type, the imperial war, involves an adversary that is an independent political entity but does not qualify as a member of the interstate system because of limitations on its independence, insufficient population to meet the interstate system membership criteria or a failure of other states to recognise it as a legitimate member. The second subtype, the colonial war, includes international wars in which the adversary was a colony, dependency or protectorate composed of ethnically different people and located at some geographical distance or, at least, peripheral to the centre of government of the given system member.

Civil war

This dataset is a study of 150 major civil wars involving a total of 204 participants between 1816 and 1988. An internal war is classified as a major civil war if (a) military action was involved, (b) the national government at the time was actively involved, (c) effective resistance (as measured by the ratio of fatalities of the weaker to the stronger forces) occurred on both sides and (d) at least 1,000 battle deaths resulted during the civil war.

E.3.2 Conflict and Peace Data Bank (COPDAB), 1948-1978

COPDAB is an extensive, longitudinal, computer-based library of daily international and domestic events/interactions. As of January 1, 1980, COPDAB holds about 500,000 event records systematically coded from about 70 international sources, covering the period between January 1, 1948 and December 31, 1978. These event records describe the actions of about 135 countries in the world both toward one another and within their domestic environments. The following pages identify the COPDAB nations and sources, and contain a brief description of the procedures used to code, scale, and store the descriptive and analytic events data.

A typical descriptive event record, such as an international border clash or domestic press censorship, which one finds in a public source such as a newspaper, chronology, or some historical account, is coded into eight variables. Specifically a COPDAB event record contains which source reported who did or said what to whom about what issue-area(s) and when. Furthermore, an event record contains the evaluation of the coder regarding the type and the scale value of an event. International events are occurrences between nation-states which are distinct enough from the constant flow of "transactions", (e.g., trade, mail flow) to stand out against this background as "reportable, or newsworthy". Thus, to qualify as a descriptive event, an occurrence has to be actually reported in a reputable and available public source. For example, the conclusion of a trade agreement would qualify as a descriptive event in COPDAB's scheme but the subsequent routine trade flows conducted under its terms would not.

This data set represents an improvement in relation to the analysis of the role of information in the conflict. There are several variables that might be use as approximations to define the information structure such us the ones that involve: information exchanges, scientific cooperation, tourism, exchange of cultural trips or artifacts, etc. It is also an improvement in respect of measuring tensions and other interactions which indicates high level of conflict but do not result in war or violent confrontation.

E.3.3 The MID data set

There is a growing number of studies on international conflict that employ the dyadic interaction between states as the unit of analysis. The most widely used data set for this type of study is the *Militarized Interstate Dispute (MID) dataset*(Bremer, Jones and Singer, 1996 [130]).

The MID data set consists of two types of observations:

1. The dispute level. This level includes general information about the dispute such us starting date, number of participants, level of hostility, etc

2. The individual participant level. This level provides very similar information about each participant. It includes the starting and end date of each participant in the conflict, the side it fought for etc,.

The extrapolation from the individual level to the dyadic level is easy to perform. However, Maoz [131] highlights some of the problems to carry out such transformations when we deal with multilateral conflicts:

> In strictly bilateral disputes, it is easy to transform the participation records for a specific dispute into a dyadic record, including the combination of data from the dispute profile record in the dispute data set. However, performing such a combination on multilateral dispute may cause a great number of errors.[1]

Although the definitions of what constitutes a dispute may vary, considerable effort has been made recording all different interactions between states that can be categorised as under the general concept of conflict. Despite the problems reported by Zaov, the MID constitutes a comprehensive data set on interstate conflict, the dependent variable in our study.

E.3.4 Other data sets of conflict

There are other data sets on conflict most, collected by Inter-university Consortium for Political and Social Research (ICPSR)3. In this data bank there are over 50 different abstracts and datasets about conflict and stability at national and international level. The range of issues covered is quite large: domestic violence, attitudes and justifications of violence, human rights violations, etc. However none of these studies include a systematic approach to the study of information structures and the realization of conflict. All those compilations of data on conflict are in a certain way customised for the sort of theory they attempt to prove or disprove. The Correlates of War Project Data Set was created in order to test a series of hypotheses and theory proposals that have a lot to do with those families of political theories of conflict that I have already mentioned. If we try to prove another point from another perspective we need more variables or different codifications. COPDAB covers a wider range of issues since they attempt not only to explain the source of war but instability, international tension as well as events that lead to peace, integration and improvements of quality of life. Another example is a more recent compilation by Wallensteen and Sollenberg [132] which only covers the armed conflict of the past decade. They define conflict as a contested incompatibility which concerns government and/or territory where the

[1]Many of these errors are especially relevant for the Eugene data set. They included *Non Valid Dyads* and *Inaccurate Levels of Hostility*, absurds such us states fighting themselves or *Distorted Disputes Outcomes*

use of armed forced between two parties, of which at least one is the government of a state, results in at least 25 battle-related deaths. The data that they produced is quite good at proving the relative importance of external and internal conflict.

E.4 Estimation methods for models of conflict

In most statistical models of conflict the dependent variable itself can be dichotomous in nature. Although this variable could take many values, in most studies it ranges between 1 and 0. Normally 1 means the realization of the variable, for instance war, or militarized dispute. This poses some estimating problem.

There is a wide range of estimating methods in the economic literature. The most commonly used approaches are the *logit* and *probit* models. For example, in a logit model the dependent variable Y_i, $(i = 1, \ldots, n)$, can take only the value 1 if for example conflict takes place, or 0 if peace. We review these methods in the appendix, section: E.4.1. In section E.4.2 we continue with some of the problems of logistic analysis. We will also mention some of the econometric methods of sampling data. This is of particular interest for those cases where we have dyad data with very high numbers of peace years and very few realizations of conflict.

Most data sets of conflict provide cross–sectional and time-series observations. Panel data allow us to construct more complicated models than we would if we only use time–series or cross–section data. It is very common to encounter many estimation problems derived from omitted variables for which panel data estimation may offer some solutions. In section E.4.3 we review this methodology. We put especial attention to this methods that are applicable to Binary Time Series Cross Section Analysis in section E.4.4.

These appendices cover the most commonly used methodology in the empirical literature of conflict. In the next section we are going to review two well-known paradigms of conflict analysis, and then use some of our game-theoretic foundations to propose some improvements to theses models in the following section.

E.4.1 The logit and probit models

In a logit model the dependent variable Y_i, $(i = 1, \ldots, n)$, can take only the value 1 if for example conflict takes place, or 0 if peace. This variable is described by a Bernoulli distribution. The only parameter of the Bernoulli distribution is π_i, the probability of conflict.

If $X_i = \{1, X_{1i}, X_{2i} \ldots X_{ni}\}$ is a vector of explanatory variables, the logit model specifies the relationship between π_i and X_i as a linear function such us:

$$\pi_i = E(Y = 1|X_i) = \frac{1}{1 + e^{-X_i\beta}}. \tag{E.1}$$

If π_i is the probability of war, $1-\pi_i$ is the probability of peace.

$$1-\pi_i = 1 - E(Y=1|X_i) = \frac{1}{1+e^{X_i\beta}}$$

Then the *odds ratio* is simply $\frac{\pi_i}{1-\pi_i}$. Taking natural logs, the odds ratio can be expressed as:

$$L_i = \log\left(\frac{\pi_i}{1-\pi_i}\right) = X_i\beta \tag{E.2}$$

These models have the following properties: *a)* As X_1 increases π_i increases, but never out of the range [0,1] and *b*) Although L is linear with respect of X_i, the relationship between π_i and X_i is non linear.[2]

However, if we have data only at the *individual level* we cannot estimate equation E.2. If we have observations of a single country which will be either at war $\pi_i = 1$ or in peace,$\pi_i = 0$, the values of the log function will be:

$$L_i = ln\left(\frac{1}{0}\right) \text{ if country is in war}$$

$$L_i = ln\left(\frac{0}{1}\right) \text{ if country is in peace}$$

which makes no sense.

Gujarati [133] shows how to use the relative frequencies to obtain the estimated logit. However, most part of econometric packages provide methods for estimating the logit models by maximum likelihood.

The derivation of the logit log-likelihood function is quite straight forward. For simplicity we follow Green's [134] notation. Each observation is treated as an independent event from a Bernoulli distribution. The model with success probability $F(X_i\beta)$ leads to the likelihood function:

$$\Pr(Y_1 = y_1, Y_2 = y_2 \ldots, Y_n = y_n) = \prod_{y=0}[1 - F(X_i\beta)] \prod_{y=1} F(X_i\beta) \tag{E.3}$$

where

$$F(X_i\beta) = \frac{e^{X_i\beta}}{1+e^{X_i\beta}}$$

[2]That the probability of an event must range between 0 and 1 is obvious by definition. An obvious case is the changes in probability of winning the war as a result of a marginal increment in military expenditure. If the ratio of arms is say, 1 to 10, a marginal increase in arms will produce a small change in the probabilities of winning but if the ratio is 1 to 1, a marginal increase will bring about a huge change in the probabilities of winning (other things equal).

and taking logs we get

$$\ln L = \sum_i \left[y_i \ln\left(\frac{e^{X_i\beta}}{1+e^{X_i\beta}}\right) + (1-y_i)\ln\left(1 - \frac{e^{X_i\beta}}{1+e^{X_i\beta}}\right)\right] \tag{E.4}$$

The first order conditions are then

$$\frac{\partial \ln L}{\partial \beta} = \sum_i (y_i - \frac{e^{X_i\beta}}{1+e^{X_i\beta}})X_i = 0 \tag{E.5}$$

and the second order derivatives are

$$\mathbf{H} = \frac{\partial^2 \ln L}{\partial\beta\partial\beta'} = -\sum_i \frac{e^{X_i\beta}}{(1+e^{X_i\beta})^2} X_i X_i' \tag{E.6}$$

The variance of β is computed using the Hessian and equation E.1; we get

$$V(\hat{\beta}) = \left[\sum_{i=1}^{n} \pi_i(1-\pi_i)X_iX_i'\right]^{-1} \tag{E.7}$$

The F.O.C. can be solved by Newton's Method. Most part of econometric software provide easy option to estimate logit and probit models. There can also with a variety of options to test for the significance of the model. Test for omitted variables, heteroscedasticity and goodness of fit are commonly available.

The probit model is quite similar to the logit, but assumes a normal cumulative distribution function.

$$\pi_i = \Pr(Y-1) = \Pr(I_i^* \leq I_i) = F(I_i) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{T_i} e^{-t^2/2} dt \tag{E.8}$$

Logistic and probit models are quite similar and have been used interchangeably in the literature. It is very difficult to choose one on simple analytical grounds and, in many cases, the choice is a question of convenience more than any other thing.

However, we have to be more careful in the case of conflict studies. We should expect different estimates if a) our sample contains very few realizations of $y = 1$, or $Y = 0$ and b) if there is a wide variation in one important independent variable. Which is the case for most data sets of conflict.

E.4.2 Sampling data on conflict

The usual strategies are *random sampling* or *exogenous stratified sampling.*

When one of the values of Y is rare in the population, we can either select on $Y = 1$ randomly or take all the available cases and then select a random sample of $Y = 0$. This requires normally previous knowledge of the fraction of 'ones' in the population. In the case of conflict, this is normally available since considerable effort has been put into recording all international disputes since the Congress of Vienna in 1815. The selection on Y can be efficient but are only valid with the appropriate statistical corrections.

There are many methods for statistical corrections. *Prior correction* and *weighting* are frequently used in the empirical research on conflict as reported by King and Zeng [111].

> Prior correction involves computing the usual logistic regression and correcting the estimates based on prior information about the fraction of ones in the population τ, and the observed fraction of ones in the sample (or sampling probability), $\overline{y}$.

In the logit model Hseih et al [135] and Amemiya and Vuong [136] show the statistical properties of variable selection. For the most general formulation presented by King and Zeng prior correction is consistent, fully efficient and easy to apply. The MLE $\hat{\beta}_1$ is a statistically consistent estimate of β_1 and the following corrected estimate is consistent for β_0

$$\hat{\beta}_0 - \ln\left[\left(\frac{1-\tau}{\tau}\right)\left(\frac{\overline{y}}{1-\overline{y}}\right)\right] \tag{E.9}$$

There are also different approaches to prior correction when information on τ is not available. One of the clear advantages of prior correction is that it is easy to compute. However if the model is misspecified, we can obtain more robust estimates by weight selection. It compensates for differences in the sample ($\overline{y}$) and population (τ) fractions of 'ones' induced by choice-based sampling. The resulting MLE by Manski and Lerman 1977 [137].

$$\ln L_w(\beta|Y) = w_1 \sum_{\{Y_i=1\}} \ln(\pi_i) + w_0 \sum_{\{Y_i=0\}} \ln(1-\pi_i) \tag{E.10}$$

where $w_1 = \tau/\overline{y}$ and $w_0 = (1-\tau)(1-\overline{y})$

Weighting can be more appropriate than prior correction when a large sample is available and the functional form is misspecified, but it is asymptotically less efficient than prior correction.

Finally, there are several problems to avoid selecting on the dependent variable. Sampling design for prior correction and weighting requires independent random selection. Other non-random methods of selection require a different statistical approach. And most important, when selecting on Y, we have to be careful not to select differently on X. Most part of empirical studies pay a lot of attention to this point. A clear example for this problem is the choice of all politically relevant dyads. If we select all the dyad war years $Y = 1$ from a comprehensive data sets and then choose a number of $(Y = 0)$ from the whole population we are implicitly selecting on X, which will pick up the influence of the higher disposition to war that those countries experience.

E.4.3 Panel data methods

A panel data model will take the form:

$$y_{it} = \alpha_i^* + \beta_i' x_{it} + u_{it} \tag{E.11}$$

where x is a scalar exogenous variable $(k_1 = 1)$ and u_{it} is the error term with mean zero and variance σ_u^2. The parameters α_i^* and β_i are assumed in this model to differ cross-section but to remain constant over time. [3]. If heterogeneous coefficients are disregarded the results of least square regression on all NT(cross-section x time-periods) observations can be seriously misleading.

Testing for homogeneity of regression slopes and coefficients can be done in three steps. Since parameters are constant over time we can run a separate regression in each individual. Then three types of restrictions can be imposed.

H_1: Regression slope coefficients are identical, and intercepts are not

$$y_{it} = \alpha_i^* + \beta' x_{it} + u_{it} \tag{E.12}$$

H_2: Regression intercepts are the same, and slope coefficients are not.

$$y_{it} = \alpha^* + \beta_i' x_{it} + u_{it} \tag{E.13}$$

H_3: Both slope and intercept coefficients are the same

$$y_{it} = \alpha_i^* + \beta' x_{it} + u_{it} \tag{E.14}$$

Under the assumption that the u_{it} are independently normally distributed over i and t, this hypothesis can be tested by using the F test based on sums of squared residuals from linear restrictions.

For example, the hypothesis of heterogeneous intercepts but homogeneous slopes (H_1) can be formulated as a series of $(N-1)$ linear restrictions in equation E.11:

$$H_1 : \beta_1 = \beta_2 = \cdots = \beta_N$$

A similar analysis can be produced if we assume that coefficients my vary over time. In general, heterogeneity produced by the effects of omitted variables can be driven by these three categories:

- Individual varying but time period invariant

[3]The possibility of all the coefficients varying over time may be also considered

- Time period varying but individual invariant
- Individual varying and time period varying

A general variable intercept specification for fitting regression models using panel data can take the form:

$$y_{it} = \mu + \beta_1 x_{1it} + \cdots + \beta_K x_{Kit} + v_{it} \tag{E.15}$$

The classic procedure is to assume that the effects of omitted variables are independent of x and are independently distributed. All observations are random variations of a representative individual. This isn't the case for many panels. Ideally, the individual and time effects should be introduced such us:

$$v_{it} = \gamma z_i + \lambda r_t + u_{it} \tag{E.16}$$

Unfortunately there usually are no observation on z_i and r_t. A natural alternative is to consider the effects on the products $\gamma z_i = \gamma_i$, and $\lambda r_t = \lambda_t$. Then, the model can be estimated as:

$$y_{it} = \gamma_i + \lambda_t + \beta x_{it} + u_{it} \tag{E.17}$$

In this specification the effects of omitted variables have been absorbed into the intercept term. These effects can be considered fixed or random. In the case of fixed effects we assume λ_t constant over all the individuals and γ_i constant over time. These models are calculated by introducing a set of i dummy variables for each individual in the panel and a set of t dummy variables for each year in the panel.

In the random effects models the effects across time and individuals are treated as random. The residuals consist of three elements:

$$v_{it} = \alpha_i + \lambda_t + u_{it} \tag{E.18}$$

where

$$E\alpha_i = E\lambda_t = Eu_{it} = 0, \quad E\alpha_i\lambda_t = E\alpha_i u_{it} = E\lambda_t u_{it} = 0,$$

$$Var(\alpha_i) = \sigma_\alpha^2 \quad Var(\lambda_t) = \sigma_\lambda^2 \quad Var(u_{it}) = \sigma_u^2$$

The residuals in the random models are correlated. and most part of the econometric packages estimate these models by Generalized Least Squares methods (GLS).

Discrete data

For a random sample of N individuals, the likelihood function for the linear probability model, the logit and the probit models is the following:

$$L = \prod_{i=1}^{N} F(\beta' x_i)^{y_i}[1 - F(\beta' x_i)]^{y_i} \tag{E.19}$$

We can proceed to take the first and second order derivatives to find the MLE estimator of β and the variance.(Amemiya, Maddala)

Most part of static binary panel data analysis assume for simplicity that the heterogeneity across cross–sectional units is time–invariant. Thus the individual specific effects are capture by the error term as in:

$$v_{it} = \alpha_i + u_{it} \tag{E.20}$$

with

$$Var(v_{it}|\alpha_i) = Var(u_{it}) = \sigma_u^2 \quad \text{For fixed effects}$$
$$E\alpha_i = Eu_{it} = 0 \;\; ; Var(u_{it}) = \sigma_u^2 + \sigma_\alpha^2 \quad \text{For random effects}$$

If the individual specific effect, α_i, is assumed to be fixed, then both α_i and β are estimated for the model $\Pr(y_{it} = 1) = F(\beta' x_{it} + \alpha_i)$. When T tends to infinity, the MLE is consistent. In the case of the logit function, there is a consistent estimator for small T size based on the Neyman and Scott [138] principle. Since there are only a limited number of observations α_i suffers from *incidental parameter* estimation problems. Unfortunately, the MLEs for α_i and β are not independent of each other for the binary models. When T is fixed, the inconsistency of $\hat{\alpha}_i$ is transmitted to β even if N tends to infinity. Hopefully, conflict data sets have a large number of observations, so we may not consider alternatives for consistent estimation for small sample size.[4]

For the random effects models the computation of the MLE produce consistent estimators but is computation is more demanding. Hsiao explains the procedure:

> An alternative is to assume that the incidental parameters α_i are independent of x_i and are a random sampling from a univariate distribution H, indexed by a finite number of parameters δ. The log–likelihood function becomes
>
> $$\log L = \sum_{i=1}^{N} \log \int \prod_{t=1}^{T} F(\beta' x_{it} + \alpha)^{y_{it}}[1 - F(\beta' x_{it})]^{1-y_{it}} dH(\alpha|\delta)$$

[4]Chamberlain [139], otherwise.

Which provides consistent estimators of β as N goes to infinity.

E.4.4 Binary-Time-Series-Cross-Section Analysis

There is a great number of studies of conflict that use logit or probit estimations in conjunction with cross section analysis. If the observations are temporally related the results my be misleading. Nathaniel Beck and Jonathan Katz address this problem in several papers. Beck [140] Beck and Katz [141], [142], Beck et al. [143]. These Authors consider a series of relevant issues: First, there is a *critical distinction between TSCS data* and Panel data and secondly, there is some concern about the use of lagged dependent variables to correct for temporal dependency.

TSCS data is often considered as panel data with a large number of observations and small number of countries.

> When we think of panel data, we are thinking of repeated sample surveys of a large number of respondents, with the number of repetitions typically being small. In panel data there is no interest in the sample *per se*, with all the inferences of interest being to the underlying population.
>
> TSCS data typically consists of annual observations on some countries (perhaps 15-20 OECD nations or 50 states or 100 or so nations) observed annually (say) for some reasonable length of time, say 20-50 years. Inferences of interest are to the observed countries, which are never thought of as a sample from a larger population of countries.
>
> This distinction is quite important when we try to deal with the problem of temporal dependence. Some schoolers such as Russet and Bonnet (1999) proposed to use the *general estimating approach* GEE. The theoretical justification for the GEE is in terms of asymptotics in N. However, in TSCS data the number of countries N is fixed. Any asymptotic work of TSCS must assume that $T \to \infty$. Therefore, we need T to be reasonably large. Fortunately most panel data has an appropriate size of N's and T's.

General Estimating Equation (GEE) is an estimating approach that specifies within group correlation structure of panel data which is comparable to random effects regressions. The GEE is based on quasi-maximum likelihood and is one of the most commonly used approaches in conflict data analysis. It has good asymptotic properties in N but there is no evidence on how well it performs with small samples.

Beck and Katz proposed an alternative to GEE in order to deal with temporal dependence. This is based on the fact that BTSCS and event history or grouped duration data are equivalent. Any history method is potentially suitable for BTSCS data and they allow corrections for censoring, heterogeneity and duration dependence.

Event history analyse the probability of a spell (a duration variable) will end in the following interval of time given that it has lasted for until time t.

$$l(t, \Delta) = \Pr(t \leq T \leq t + \Delta | T \geq t).$$

And the hazard rate is the rate at which spells are completed after duration t. They are the most common approach in even history analysis. Cox (1975) models a continuous time duration hazard rate as:

$$h(s|x_{is} = h_0(s)e^{x_{is}\beta} \tag{E.21}$$

where $x_{i,s}$ is the vector of independent variables at time s. In this model the hazard of exit depends both on the independent variables $\left(e^{x_{is}\beta}\right)$ an the length of time that the unit has been at risk.

For discrete time Beck and Katz use a variant which incorporates logit link.

$$\Pr(y_{it} = 1|x_{it}) = h(t|x_{it}) = \frac{1}{1 + e^{-(x_{it}\beta + \kappa_{t-t0})}} \tag{E.22}$$

This equation differs from the ordinary logit by the inclusion of temporal dummies, κ_{t-t0}. Omitting these variables is equivalent to assuming that the baseline hazard is constant and therefore the model shows no temporal dependence.

If we have temporal independent data, these dummies would produce inefficiency and incorrect standard errors, and in some cases inconsistent parameter estimations and multicollinearity. The test of whether the temporal dummies should be included or not is a standard likelihood ratio test with the hypothesis that all the $\kappa_{t-t0} = 0$.

Finally there are some problems that should be taken into consideration. First BTSCS data allows for multiple failures per unit. Ordinary logit assumes that the probability of failure in any year is the same as any other year (depending only on exogenous variables). Since the only relevant information about κ is time since the most recent event the second and subsequent events are independent of the number and timing of previous events.

> If conflicts really are multi-year, we should simply drop all but the first year of the conflict from the analysis. If we have a theory about the duration of peace, we should not include spells of conflict in testing that theory. However, since we can observe different conflicts in consecutive years, this would be tantamount to discarding new, but very short, spell of peace. A decision on how to proceed should be made on theoretical grounds.

There is another complication regarding left-censoring. Spells are left-censored if we don't know when they started. In data sets there are different criteria to establish the starting point. We can either take the beginning of a new security regime. For example, it is quite common to make the departure at the end of the II World War, taking this point in time as the beginning of a new security regime.

Finally, other problems may arise from missing data and fixed variables across units. In practice, although they may pose some problems at system level analysis, they arise seldom given the particular structure of the dyadic data sets.

E.5 Tables

Column I in table E.1 replicate Russet and Oneal results while columns II-IV show different developments introduced by Beck et al. to correct for dependency. The likelihood ratio test for of I versus II, III and IV respectively, indicates strong duration dependence.

E.6 Variables of the expected utility generation program

These is the list of variables produce by Eugene.

E.6.1 Available for Country-Year, Directed-Dyad-Year, and Dispute Dyad Output

CCode: Output will list the COW country code for the individual country (if country-year output is selected) or for the two members of the dyad (if dyad-year output is selected) for whom the rest of the data correspond to. It is highly recommended that this variable be included in the output file, or the user will not know which data goes with which country.

Year: Output will list the year that the rest of the data corresponds to. It is highly recommended that this variable be included in the output file, or the user will not know which data goes with which year.

Capabilities: Output will include the values of the national capabilities index.

Major Power Status: Output will include a "1" if the country is a major power in the given year, or a "0" if it is not.

Table E.1: Comparison of Ordinary Logit and Grouped Duration Analyses

	Ordinary Logit	Grouped Duration		
		Logit Dummy[a]	Logit Spline	Cloglog Dummy[b]
Variable	I	II	III	IV
Democracy	-0.55	-0.54	-0.49	
	(0.07)	(0.08)	(0.08)	(0.07)
Economic Growth	-2.23	-1.15	-1.15	-0.81
	(0.85)	(0.92)	(0.92)	(0.76)
Alliance	-0.82	-0.47	-0.47	-0.43
	(0.08)	(0.09)	(0.09)	(0.08)
Contiguous	1.31	-0.30	0.69	0.55
	(0.08)	(0.09)	(0.09)	(0.08)
Capability Ratio	-0.31	-0.30	-0.30	-0.30
	(0.04)	(0.04)	(0.04)	(0.04)
Trade	-66.13	-12.67	-12.88	-12.50
Constant	-3.29	-0.94	-0.96	-1.11
	(0.08)	(0.09)	(0.09)	(0.08)
Peace Years			-1.82	
			(0.11)	
Spline(1)[c]			-0.24	
			(0.003)	
Spline(2)[c]			-0.08	
			(0.01)	
Spline(3)[c]			-0.01	
			(0.003)	
Log Likelihood	-3477.6	-2554.7	-2582.9	-2554.1
df	20983	20036	20979	20949
N=20990				

Source: Beck et all (1999)
Standard errors in parentheses
[a] 31 temporal dummy variables in specification not shown
3 dummy variables and 916 observations dropped
due to outcomes being perfectly predicted
[b]34 temporal dummy variables in specification not shown
[c]Coefficients of Peace Years cubic spline segments

Table E.2: Involvement in Militarized Interstate Disputes, All Dyads, 1950-92. SOURCE: Russett and O'Neal.

		All Dyads/IMF Trade Data		All Dyads	
Variable		Yrs Peace Correction	GEE	Yrs Peace Correction	GEE
Lower	β	-40.5	-122	-34.0.0	-182
Dependence	SE_β	22.1	53	21.8	66
	p	.07	.02	.12	.006
Higher		1.40	0.71	1.52	0.810
Dependence		1.69	2.43	1.96	2.72
		.41	.77	.44	.77
Joint Democracy		-0.00363	-0.00281	-0.00332	-0.00239
		0.00077	0.00087	0.00068	0.00075
		$< .001$	$< .001$	$< .001$	$< .001$
Contiguity		2.14	2.80	2.46	2.92
		0.22	0.26	0.21	0.23
		$< .001$	$< .001$	$< .001$	$< .001$
Log Distance		-0.399	-0.489	-0.592	-0.701
		0.079	0.088	0.076	0.081
		$< .001$	$< .001$	$< .001$	$< .001$
Major Power		1.44	1.58	1.91	1.94
		0.22	0.27	0.23	0.25
		$< .001$	$< .001$	$< .001$	$< .001$
Allied		-0.430	-0.666	-0.0532	-0.870
		0.212	0.242	0.168	0.196
		.04	.006	.002	$< .001$
Log Capability		-0.188	-0.169	-0.231	-0.224
Ratio		0.056	0.0778	0.051	0.061
		$< .001$	.034	$< .001$	$< .001$
Constant		-0.351	-1.85	0.517	-0.362
		0.623	0.678	0.626	0.656
		.57	$< .001$	.41	.58
χ^2		1481.4	970.4	2166.8	1691.5
χ^2 d.f.		12	8	12	8
P of χ^2		$< .001$	$< .001$	$< .001$	$< .001$
Pseudo-R^2		.33	—	.36	—
N		118,466	118,382	271,262	269,712

Home Region: Output will include an integer marking the region that the country (if country-year output is selected) or countries (if dyad-year output is selected) is in. Regional memberships are defined as given by the COW Interstate System Members list. Integers correspond to regions as follows: Europe=1; Middle East=2; Africa=3; Asia=4; North and South America=5.

Risk Attitude–EUGene: Output will include the values Ri from the risk attitude calculations performed by EUGene. If the "Country-Year" unit of analysis is selected for output, the output will include risk scores for a given state in all regions. If the "Dyad-Year" unit of analysis is selected, risk scores will be reported for the two states in the dyad for the region that is relevant for their conflict and so to the calculation of expected utility.

Risk Attitude - WTR: Output will include the values from the risk attitude calculations performed by Bueno de Mesquita (1985).

Risk Details: This outputs the values from the intermediate components of the risk attitude calculation from EUGene. Specifically, this outputs actual, maximum hypothetical, and minimum hypothetical security values. If the "Dyad-Year" unit of analysis is selected, these values will be reported for the two states in the dyad for the relevant region of their conflict and expected utility calculations. If the "Country-Year" unit of analysis is selected for output, the output will include detailed intermediate information on the risk scores for the given state in all regions. Values are from -1 to +1, with -1 indicating a highly risk-averse actor, and a +1 indicating a highly risk-acceptant actor.

Regional Uncertainty: Output will include regional uncertainty as defined by Bueno de Mesquita and Lalman (1992).

Tau with System Leader: Output will include the tau between the state in question (country-year unit) or states in question (dyad-year unit) with the system leader, which is Britain up to 1945, and the US from 1946 forward. Options: User may set whether the tau computed with the system leader is based on the alliances of states only involved in the relevant region of the ccode vs. the system leader dyad (regional option), or is based on all states in the international system (global option).

Polity III Data: Output will include selected variables from the Jaggers and Gurr (1995) Polity III data set. These variables are democ, autoc, xrreg, xrcomp, xropen, mono, xconst, parreg, parcomp, cent. In addition, the derived variable "dem" used by Russett and others is available (Dem = Democ - Autoc). In addition, lagged versions of the democ, autoc, and dem variables are available, along with democratization computed as demchg = dem - lag(dem). Select a specific subset of Polity III variables by pressing the "Variable Selection" button.

E.6.2 Available for Dyad-Year and Dispute-Dyad Output Only

Relevant Region: Output will include an integer marking the region that is relevant for the computation of expected utility. Regional memberships are defined as given by the COW Interstate System Members list. Integers correspond to regions as follows: Europe=1; Middle East=2; Africa=3; Asia=4; North and South America=5. Politically Relevant: Output will include a dummy variable marking the cases of politically relevant dyads. Politically relevant dyads are those where at least one state is a major power, or the states are contiguous. You may change the degree of contiguity required with the "Change Contiguity" button under the variable option. Note that setting contiguity for variable output will also affect contiguity for outputting politically relevant dyads only, if you select politically relevant dyads for your output population. A '1' marks a politically relevant dyad, while a '0' marks a non-politically relevant dyad.

Contiguity: Output will include a dummy variable marking dyads where the members are contiguous on land. Dyad Duration: Output will include an integer representing the number of years that both members of the dyad have been states continuously since 1816. For example, Britain and France receive a "0" in 1816, a "1" in 1817, etc. Canada becomes a state in 1920, and so the US-Canada dyad would have missing values to 1920, a 0 in 1920, a 1 in 1921, etc. This counter resets when a state drops out of the state system by COW criteria. So the France-West Germany duration variable starts at 0 in 1955. [Note: this was modified in v1.19. Before v1.19, the duration counter counted the number of years from the first time the two states became states, and did not reset even when states left the system.

Distance Between States: This outputs the distance between states in the dyad as calculated by the program. Note: EUGene will compute distance based on the method in its memory, which by default (unless the user changed options under recalculation options) is the distance between capitals, adjusted for contiguity and allowing multiple cities for each country. Tau-b Scores: Output will include the tau-b between the two states. Both global tau-values (calculated using every state in the system) and regional tau-values (calculated using only states in the relevant region for the dyad) will be reported. S Scores: (See Signorino and Ritter 1997) Not yet implemented.

Expected Utility - War Trap: Output will include the expected utility of country 1 vs. Country 2, based on the operational rules specified in The War Trap as discussed in this documentation. Utility - War and Reason: Output will include various values related to the expected utility of Country 1 vs. Country 2, based upon the measures developed in War and Reason. A number of values are output: From Bueno de Mesquita and Lalman (1992:293-294): UA((A), UA((B), UA(SQ), UB((A), UB((B), UB(SQ), StakesA, StakesB. From Bueno de Mesquita and Lalman (1992:297): PA, PB. From Bueno de Mesquita and Lalman (1992:47): Ui(SQ), Uj(SQ), Ui(Acqi), Ui(Acqj), Uj(Acqi), Uj(Acqj), Ui(Nego), Uj(Nego), Ui(Capi), Ui(Capj), Uj(Capi), Uj(Capj), Ui(Wari), Ui(Warj), Uj(Wari), and Uj(Warj) State i corresponds to state A which is the first state in the dyad A vs. B, while state j corresponds to state B.

Equilibria - War and Reason: Output will include dummy variables marking what equilibrium in the international interaction game is expected given the various utility values in the dyad. Options: User may set whether to generate the equilibrium for a dyad-year by either 1) using the logical conditions given in War and Reason, or by using backwards induction using the computed utility values for each dyad-year. For more details about this choice, see the discussion under section "Equilibria (War and Reason)" on page 17

MID Data: Output will include COW MID dispute data, converted into a dyadic form, marking four items (additional details of converting the COW MID data into dyadic form is given in the next section, beginning on page 31): 1) whether state A initiated a MID vs. state B in this year. Whether or not A is considered to have initiated a dispute depends on user settings for a) marking subsequent years as initiations, b) marking either side A or revisionists as initiators, and c) marking initiators as only originators or including joiners. Note that initiation marks specifically whether A initiated vs. state B. This variable is directed; if the user wants to explore non-directed dispute onset within this directed dyad, use the highest hostility levels instead (if the highest hostility level is ¿ 1 for both sides in a year, there is a dispute onset). 2 and 3) the relevant highest hostility level reached by state A vs. state B. in this year, and the highest hostility level reached by B vs. A. in this year. "Relevant" is specified to determine the hostility level when there are multiple disputes involving A and B in a given year. Rules for selecting the proper dispute from which to determine hostility levels are as follows. First, if there's a new initiation in this year (with initiator defined either as side A or revisionist, as specified by the user), the hostility level for the year is taken from the first initiated MID. Second, if there is not a new initiation, then the value of the first new event in this year is taken, such as joining an ongoing dispute. Third, if there is no new event/dispute but there is an ongoing dispute, then hostility values are taken from the ongoing MID. Finally, if no dispute is occurring, then a 0 is coded for hostility. For years with a MID, coding follows the COW MID data set codings: 1=no militarized response to a MID, 2=threat of force, 3=show of force, 4=use of force, 5=war. 4) the COW MID number of the relevant dispute (if any) between A and B in this year. Selecting MID data for output will enable the options on the "Exclusions" and "Dispute Initiator" output tabs. Identify MID

Joiners: Outputs two variables marking whether state A was a joiner on the initiating side against B in a dispute that had already-started before A became involved, or if A joined on the target side against B. That is, this will mark dyads where ccode1 was on the initiating side against ccode2, but was not an originator (was not involved on day 1), or where A was on the target side against B but was not an originator. Rules for coding initiators as Side A or Revisionist apply; a Joiner will be coded only if it is a state on Side A (if Side A is marked to be the initiator) or the revisionist side (if revisionists are specified as the initiator). Note that if states are to be marked as joiners, then they are marked as such in subsequent years of the same dispute whether or not the user has specified wanting subsequent years coded as an initiation. That is, the variable marking joining is unaffected by the setting on coding subsequent initiation. Peace

Years: Outputs a variable counting the number of years since the last dispute in the dyad to use in creating Beck, Katz, and Tucker spline variables to account for serial autocorrelation. This variable takes a value of 0 for all dyads in 1816, and for the first year a country is considered a state. It increments by one for each year a dyad goes without a dyadic dispute. In the first year that a MID occurs, Peace Years takes the regular incremented value. But in the next year, either the year after the MID if the MID lasts one year, or in the 2nd year of the MID if the MID lasts longer than a year, it resets to 0. As long as the MID continues, the variable takes the value 0, as it does in the first year after the MID ends. In the 2nd year after a MID ends, it again increments starting at 1. So, if a MID lasts from 1820 to 1825, the value of Peace Years would be 0 from 1821 through 1826, and a

"1" in 1827 (a full year passed without a MID).

E.7 Factor categories

E.7.1 Previous or general relations between sides

- Both sides appear to have limited objectives
- One or both sides do not appear to have limited objectives
- One or both sides try to penetrate deeply into the other's territory
- Neither side attempts to penetrate deeply into the other's territory
- Leader of one side desires to avoid wider war with other side
- "Non-status quo" side succeeds in establishing a rival government to "status quo" side
- "Non-status quo" side fails to establish a rival government to "status quo" side
- "Non-status quo" side achieves its primary goals
- "Non-status quo" side has not achieved its primary goals
- "Non-status quo" side is willing to discuss ending hostilities
- Both sides are willing to negotiate a settlement
- Sides appear unwilling to negotiate a settlement
- The sides are negotiating
- Sides share much common heritage
- Partial agreements have been reached
- Leader of one side announces a cease fire
- Leader of one side calls for a summit meeting
- "Status quo" side makes some concessions
- Leader of one side heeds warning that harsh measures will result in action by the other side
- "Status quo" side removes leader of other side who could have restraining influence

- "Status quo" side rallies support among members of the other side
- One side's leaders assure other side that regular forces will not be used to overthrow them

E.7.2 Great power and allied involvement

- Strong superpower diplomatic support to "status quo" side
- Strong superpower diplomatic support to "non-status quo" side
- A superpower appears to be neutral
- A superpower fears growth of conflict into a wider war
- The prestige of a superpower is committed to settlement
- Major powers supply arms and equipment to "status quo" side to redress imbalance
- Major powers supply arms and equipment to "non-status quo" side to redress imbalance
- Arrival of arms in one side raises the prospect of a wider war
- Great power interest in the area increases
- A great power's diplomatic support encourages one side to continue the struggle
- Great power gives "status quo" side substantial economic aid
- Great power gives "non-status quo" side substantial economic aid
- Great power supporter of "status quo" side contemplates intervention
- Great power supporter of "non-status quo" side contemplates intervention
- A great power urges one side to avoid provoking a full-scale war
- A great power indicates interest in terminating hostilities and negotiating a settlement
- Great powers urge cease fire and avoidance of intensification
- Great powers/superpowers are losing interest in the conflict
- One great power/superpower becomes active mediator

- A previously supportive great power/superpower drops out
- Great powers/superpowers collaborate to resolve conflict
- Regional powers want a peaceful settlement
- Great power supporter of "non-status quo" side refuses to assure aid
- Great power supporter of "status quo" side refuses to assure aid

E.7.3 External relations generally

- One side receives aid from a supporter
- Supporter of one side threatens unilateral intervention
- Use of force by one side alienates potential allies
- States in the region give material support to "non-status quo" side
- States in the region give material support to "status quo" side
- After major ally of one side withdrew, another ally was acquired
- After major ally of one side withdrew, another ally was not available
- "Non-status quo" side fearful of potential ally's intentions
- Countries on which "non-status quo" side is dependent are unable or unwilling to give further support
- Countries on which "status quo" side is dependent are unable or unwilling to give further support
- Both sides dependent for aid on outside party pressing for an end to hostilities
- Despite one side's military success, its ally is unwilling to impose solution
- Actions of country supporting one side confirm its threat to intervene

E.7.4 Military-strategic

- "Status quo" side has the military advantage
- "Non-status quo" side has the military advantage
- Military balance remains heavily in favor of one side
- Military technology of one side significantly superior
- Militarily the hostilities are inconclusive
- One side has overwhelming military and logistical preponderance for the terrain
- One side has very weak military forces
- The military strength of one side increases
- "Status quo" side uses superior military power to win military victory
- One side is ill-informed on size of forces needed to execute its avowed policy
- Initial hostilities fail to delay change in status quo
- "Status quo" side's forces are ineffective and unable to stop or deflect attack of "non-status quo" side
- Only a small proportion of each side's forces are engaged
- A large proportion of each side's forces are engaged
- "Non-status quo" side uses superior military power to win military victory
- The manner in which fighting breaks out suggests that hostilities are largely accidental
- Hostilities having broken out almost accidentally, neither side can follow up
- One side resorts to guerrilla warfare
- The terrain is unsuitable for guerrilla warfare
- "Non-status quo" side has few, primitive arms, and weak training and organization
- Armed forces of supporters of both sides become involved
- Armed forces of supporters of both sides do not become involved

- Major ally of one side responds to attacks with large reinforcements
- Military action by one side leads to combat with allies of the other side
- Larger strategic concerns constrain pressure from states influential with "status quo" side
- One side's ally's military mission has expanded from logistic support to advice on military operations
- Prolonged or intensified hostilities may trigger a mutual security agreement to which one side belongs
- Outside parties halt military aid to both sides
- One side unable to cut off arms supply to other side
- External pressures for termination develop
- One side's move against the other side's supply lines risks retaliation
- Foreign officers in one side's armed forces threaten withdrawal if that side continues military activities
- Neither side can obtain a decisive military victory at an acceptable level of commitment and risk
- Rapid growth of one side's armed forces sacrifices quality of training
- Force is not used for military victory, but to strengthen diplomacy by threatening a wider war
- Commander of one side's army advises against more military activity
- One side failed to move beyond terrorism and isolated guerrilla activity
- Military and para-military units act on their own initiative
- New military effectiveness on one side discourages other side from belief in military victory
- Raids by one side into other's territory inflict no military damage
- One side must commit more troops than anticipated
- One side feels that a cease fire in place would leave the other's troops too close
- Reasons for initial intervention by one side remain, but forces committed are inadequate

- The strategic interests of the side that intervened in hostilities retain importance
- Military tactics of "status quo" side restrict scale and scope of hostilities
- One side possesses significant nuclear technology
- Both sides possess significant nuclear technology
- One side believed to be developing nuclear weapons capability
- Both sides believed to be developing nuclear weapons capability

E.7.5 International organization (UN, legal, public opinion)

- The UN begins to assert itself strongly
- The UN does not assert itself strongly
- The UN presses for an immediate cease fire
- The adversaries agree to a UN cease fire resolution
- "Non-status quo" side is anxious to see the conflict in the UN
- "Status quo" side is anxious to see the conflict in the UN
- The UN actively seeks a political formula to end the hostilities
- The UN Secretary General urges restraint on both sides
- UN Security Council membership favors end to hostilities and negotiated settlement
- UN Security Council adopts a resolution calling for a ceasefire
- A great power vetoes UN Security Council resolution
- UN General Assembly is convened under the Uniting for Peace resolution
- UN General Assembly declines to place question on its agenda
- UN General Assembly favors end to hostilities
- UN General Assembly discusses the conflict but takes no action
- The UN creates a body to make an on-the-spot report and facilitate negotiations

- A UN body condemns those aiding "non-status quo"
- A UN body condemns those aiding "status quo"
- UN agrees to send a peacekeeping force to the area
- Plans for a UN force are elaborated and contingents committed
- UN force's mandate supports objectives of "non-status quo" side
- UN force's mandate supports objectives of "status quo" side
- Regional organization to which sides belong offers to mediate
- One side withdraws from international organization

E.7.6 Ethnic (refugees, minorities)

- Ethnic rivalries exist in "status quo" side
- Ethnic rivalries exist in "non-status quo" side
- Refugees from one side return to carry out raids
- Violence occurs between ethnic groups within country supporting one side
- Strife between ethnic groups in one side raise doubt about that side's viability

E.7.7 Economic/resources

- "Status quo" side faces economic problems as a result of hostilities
- "Non-status quo" side faces economic problems as a result of hostilities
- The costs of hostilities for both sides are becoming burdensome in terms of other goals
- Great power ally of "status quo" side threatens to cut economic aid
- Great power ally of "non-status quo" side threatens to cut economic aid
- Great powers threaten to withdraw economic aid from both sides unless cease fire established
- One side's advances threaten important resource on other side

E.7.8 Internal politics of the sides

- "Non-status quo" side's military and territorial objectives appear limited
- "Non-status quo" side's military and territorial objectives do not appear to be limited
- Opposition in one side increases as violence escalates
- Domestic public opinion comes to favor "status quo" side
- Domestic public opinion comes to favor "non-status quo" side
- Public opinion in one side rallies against the other side
- Heavy domestic pressure is generated in one side to cease hostilities
- Public opinion in "status quo" side favors a negotiated settlement
- Internal unrest likely to be triggered by continued hostilities
- Internal unrest unlikely to be triggered by continued hostilities
- "Status quo" side counters violence by arrests and strong reprisals
- "Non-status quo" side wins political concessions
- At times extremist political groups in both sides gain control over policy
- "Non-status quo" side denies any connection with the hostilities
- "Non-status quo" side creates a rival government
- One side dramatizes its position by an international incident
- One side feels responsible for the fate of its proxy forces
- Some military officers on one side believe a military solution is possible
- Opposition within "status quo" side reaches high into the military
- One side's prime role in the military action is widely assumed despite attempts to keep it covert
- Splits occur in the leadership of one side
- Splits within one side become open rifts
- Unity of one side is reinforced as members take action against suspected traitors

- A coalition is emerging within one side in opposition to party in power
- One side is ideologically committed to prolonging the fight
- Ideological split exists among "non-status quo" side's supporters
- Ideological split exists among "status quo" side's supporters
- One side seeks to isolate the other side from its base of support
- Prolongation of hostilities erodes resources of one side
- One side is over-confident in seeking military solution
- "Non-status quo" side resorts to guerrilla war
- Initial objective of one side conceived as being limited and for humanitarian reasons
- "Non-status quo" side's hope for political gains is not fulfilled
- "Non-status quo" side not prepared to act independently

E.7.9 Communication and information

- Sides have open and accessible means of communication
- Sides do not have open and accessible means of communication
- Some military officers of both sides, as citizens of a third party, provide communication channel between opposing forces
- Communications are inadequate between one side's political leader and its military forces in the field

E.7.10 Actions in disputed area

- "Status quo" side lacks anticipated support in disputed area
- "Non-status quo" side lacks anticipated support in disputed area
- "Status quo" side misinterprets nature of events in disputed area
- Supporters of "status quo" side in disputed area begin to turn against it
- Both sides worry about effects of military activity on hostile ethnic groups in disputed area
- UN focuses world attention on developments in disputed area

- Both sides agree to let the UN ascertain the wishes of population in disputed area
- International organization body is slow in reaching disputed area
- Moderating third-party forces in disputed area begin to phase out
- Troops of interested parties present in disputed area form a peacekeeping force
- One side warns that any offensive action in disputed area will result in wider war
- Dangers of all-out war increase when hostilities spill out of disputed area
- Geography of disputed area makes it difficult to prevent one side from reinforcing
- Practical geographical limits in disputed area make extended operations difficult
- One side has a sanctuary in terrain difficult for other to reach
- Geographic isolation of disputed area keeps "non-status quo" side activity at relatively low level
- Nature/location of "non-status quo" side's attack weakens its claim to be supporting groups in disputed area
- Each side labels other the aggressor and sees its own actions as defensive
- Important interests of one side in disputed area threatened by other side's actions
- There is united opposition to "non-status quo" side's action
- One side's administration near disputed area is unable to prevent military activity
- One side claims proof of the other's complicity in military activity in disputed area
- Raids by one side into disputed area do not cause significant damage or provoke local unrest
- Forces of one side sufficient to control raiding groups in disputed area

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