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UNCERTAINTY QUANTIFICATION AND FILM COOLING

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ABSTRACT

In gas turbine cooling, hundreds of ducts are fed by common plenums connected to small channels. The inlet stagnation pressure, temperature and turbulence levels are unknown in the ducts and subjected to a strong variability, due to the uncertainty associated with operating conditions and/or manufacturing defects. Despite the uncertainty level in boundary values, it is a common practice to use deterministic values.

In this work, a Monte Carlo Method Lattice Sampling (MCMLS) and a Probabilistic Collocation Method (PCM) are used to assess the uncertainty quantification problem in film cooling. By assuming Gaussian distributions for the inlet total pressures, 202 CFD simulations have been performed for MCMLS and the probabilistic distribution of the adiabatic effectiveness is obtained. It provides the average value for the stochastic output and the level of confidence related to that value. The results show that 40% variation in the stochastic inputs provides a variation of the adiabatic effectiveness of about 80%, and reduces the blade life by 5 times.

The MCMLS is two orders of magnitude less computational expensive than a standard MCM, robust and accurate but still computationally expensive for everyday design. Therefore, using the MCMLS as baseline, a new technique has been proposed: the Probabilistic Collocation Method (PCM), in order to both reduce the number of simulations and obtain accurate results. The developed PCM methodology is 30 times faster than the MCMLS with negligible differences in the results and three orders of magnitude faster than standard MCM. This work shows that in nowadays design, computational fluid dynamics must use stochastic methods and it is possible to integrate probabilistic analysis in the design phase to investigate the robustness by using PCM and MCMLS.

INTRODUCTION

Uncertainty on manufacturing and/or operating conditions for gas turbines, leads to a reduced knowledge of their behaviour along with high risk of unexpected failures. Fadlun et al. [1] showed that the operating point, fuel composition, and other macro-parameters are subjected to variations which have strong impact on gas turbine performance and reliability. Spieler et al. [2] improved the agreement between real and predicted performance of a complete engine, by taking into account the probability distribution of operating point values. For this reason, there is a growing interest in the analysis of influence of stochastic parameters in gas turbines. New design tools are required to assess probabilistic analyses in order to consider the strong variability due to manufacturing tolerances, assembly process and in service operations. The question rising is how to take into account these features with their associated aleatory variability.

Walters and Hyuse [3] reviewed various uncertainty quantification methods applied to fundamental problems in fluid mechanics. The methods discussed and implemented are: Interval Analysis, Propagation of Errors using sensitive derivatives, Monte Carlo Method (MCM), Polynomial Chaos and Moment methods. The analysis shows that MCM converges to the exact solution by increasing the number of samples, and despite the high computational cost, it may be considered as baseline. They concluded that the methods based on Polynomial Chaos Expansion may give results with the same accuracy of MCM being less demanding from the computational point of view. Chantrasmi et al. [4] studied heat conduction and Burgers' equation under uncertainty conditions by using MCM and Polynomial Chaos Expansion. Their analysis was able to predict both the temperature stochastic distribution and the probability to overcome the critical value, which has a strong impact on the life estimation of high temperature components.

The life of hot gas components in gas turbines is strictly related to the coolant performance. A local increment of 20 K of the stator surface temperature can reduce the blade life by 50%, and further increase in turbine entry temperature will be required in the near future. Most of the uncertainty quantification studies are applied to simple geometries with simplified equations, and only few studies are based on realistic conditions. Montomoli et al. [5] investigated the impact on performance of uncertainty fillet radius at the rotor tip in a transonic stage. The authors pointed out that some discrepancies, between experiments and CFD, may be due to the presence of small fillets in the real geometry. This effect is particularly important in transonic machines where, the non linearities introduced by the high Mach number, increase the impact of uncertainty on the solution. Montomoli et al. [6] applied a similar methodology to study the impact of geometrical uncertainties in a realistic three dimensional film cooling configuration. The authors analysed the impact of geometrical modification, and found that small variations in the internal fillet radius can modify the discharge coefficient by more than 10%.

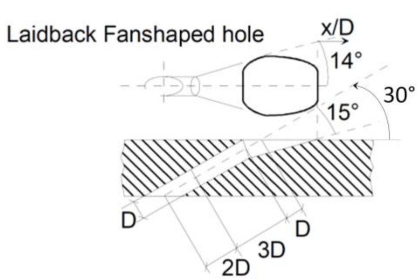
To the authors' knowledge, there are no uncertainty quantification studies on film cooling under stochastic operating conditions. Hence, the overall goal of this work is to show the impact of stochastic operating conditions on film cooling performance. The first part of the paper describes a standard laidback shaped hole with the same geometry as in Saumweber et al. [7]. Consequently, are introduced the two Gaussian inputs: the inlet stagnation pressures of main and coolant, both with a variation of 20%. The second part of the paper describes the two methods used to evaluate the influence of uncertainty on the performance. The MCM technique with Lattice Sampling (MCMLS), reduces the number of simulations in comparison to the standard MCM, while retaining the same accuracy. The PCM is 30 times less computational expensive than the MCMLS. The last part of the paper presents the average and the standard deviation of the adiabatic effectiveness, and a comparison between the two methods. The 20% variation in input gives a variation of the adiabatic effectiveness of about 80%, which corresponds to a blade life reduction of about 5 times. The results of the two probabilistic methods are in good agreement, hence it is possible to have a reduction in computational cost and integrate stochastic analyses in the design process.

NOMENCLATURE

D	Film cooling diameter [mm]	P_0	Total pressure [Pa]
L	Length of the coolant channel [mm]	T_0	Total temperature [K]
P	Hole picth [mm]	η	Adiabatic film cooling effectiveness
θ	Duct inclination [deg]	σ	Standard deviation
CFD	Computational Fluid Dynamics	μ	Mean value
MCM	Monte Carlo Method	**SUBSCRIPT**	
MCMLS	Monte Carlo Method Lattice Sampling	c	Coolant
PCM	Probabilistic Collocation Method	m	Main

TEST CASE AND PROBABILISTIC INPUT DESCRIPTION (A-F)

To assess the probabilistic analysis on film cooling, it has been chosen to use a classical test case, details about this configuration can be found in Saumweber [7]. The film cooling geometry is shown in Figure 1 and the geometrical characteristics are summarised in Table 1. The inlet diameter is 5 mm and the duct is inclined at 30 ° and an additional forward expansion of 15 °, the fan exit angle is 14 °, the length to diameter, L/D, is 6. The pitch to diameter, P/D, has been doubled in comparison with the value in Saumweber [7] to represent a more modern design philosophy, in agreement with Montomoli et al. [6].



Coolant duct		
Diameter, D	[mm]	5
Lateral expansion angle	[deg]	14
Laidback expansion angle	[deg]	15
Duct inclination, θ	[deg]	30
Length/D, L/D	[-]	6
Pitch/D, P/D	[-]	8

Figure 1 Cooling hole [7] **Table 1 Geometrical Parameters [7]**

The inlet stagnation pressure for main ($P_{0,m}$) and coolant ($P_{0,c}$) are known only within a certain degree of accuracy, hence they are chosen as stochastic input variables. Then, it is assumed a variation of 20% relative to the average pressure value and a Gaussian distribution. The mean and the standard deviation along with the probabilistic distribution functions are summarized in Table 2. Figure 2 shows a sketch of the stochastic input distribution for the coolant stagnation pressure; on the x-axis there is the pressure and on the y-axis the probability. The Gaussian is considered in the interval +/- 3 σ.

Variable	Mean [Pa] μ	Deviation [Pa] σ	Probability distribution function
$P_{0,m}$	84303.75	5620.25	$\text{Prob} = \frac{1}{\sigma_m\sqrt{2\pi}} exp\left[-\frac{\left(P_{0,m}-\mu_m\right)^2}{2\sigma_m^2}\right]\ P_{0,m} \in\ [\mu_m \pm 3\sigma_m]$
$P_{0,c}$	101550.2	6770.00	$\text{Prob} = \frac{1}{\sigma_c\sqrt{2\pi}} exp\left[-\frac{\left(P_{0,c}-\mu_c\right)^2}{2\sigma_c^2}\right]\ P_{0,c} \in\ [\mu_c \pm 3\sigma_c\ ]$

Table 2 Statistics of Aleatory Input

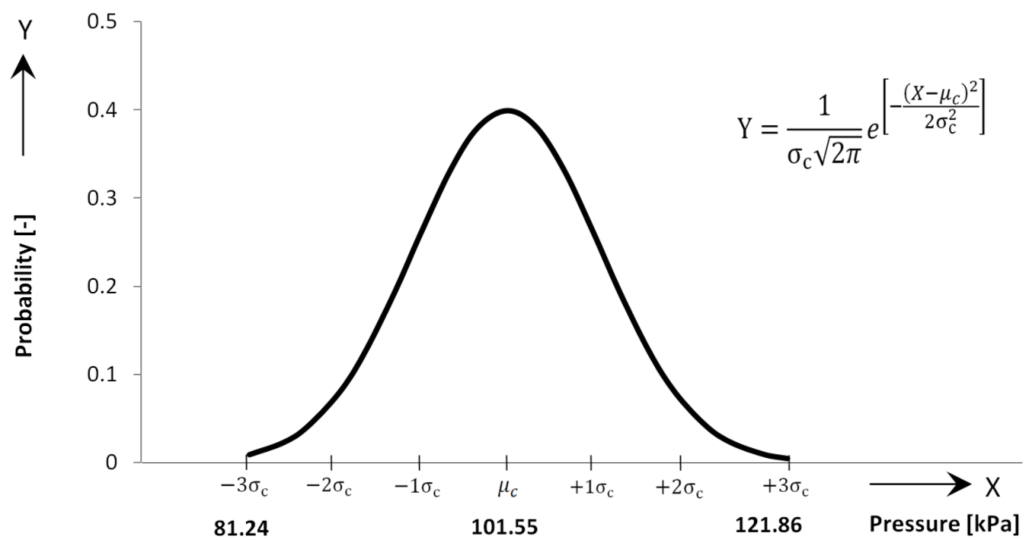


Figure 2 Stochastic input distribution inlet coolant

COMPUTATIONAL APPROACH

CFD solver

The analysis of film cooling subjected to stochastic input has been carried out using the commercial code ANSYS CFX, which has been largely validated. The software solves the Reynolds-averaged Navier Stokes equations and uses the k-ω SST turbulence model to compute the averaged turbulent stresses, more details can be found in Wilcox [8]. The average Reynolds number, based on the coolant diameter and the main flow values, is 3000; hence the Langtry-Menter transition model is used.

Mesh

The computational mesh is generated by using a commercial semiautomatic mesh generator: Centaur™. The zone in the core of the flow is tessellated using tetrahedrons, while regular prismatic layers cover solid walls for accurate discretisation of the viscous boundary layer. A grid sensitivity study has been carried out by using four meshes with 0.3, 0.6, 1 and 2.6 million of elements with 15 prismatic layers. The Richardson extrapolation criterion applied to this configuration gives a grid independency at about 600,000 elements. The denser mesh with 2.6 million of elements has been used in this work and it is represented along with its boundary details in Figure 3. The top of the main stream is considered inviscid and all the viscous surfaces are adiabatic with no slip condition. A periodic boundary is applied to lateral surfaces to model the row of coolant holes. The boundary conditions applied are the stagnation temperature and pressure at inlet surfaces and the outlet static pressure. A check on the y+ has been performed and its value was always below one.

For all cases, convergence was determined based on three criteria:

- value of normalized residuals (≈ 10 E-6)
- mass balance error (≈ 10 E-6)
- variation of local adiabatic effectiveness (≈ 10 E-2)

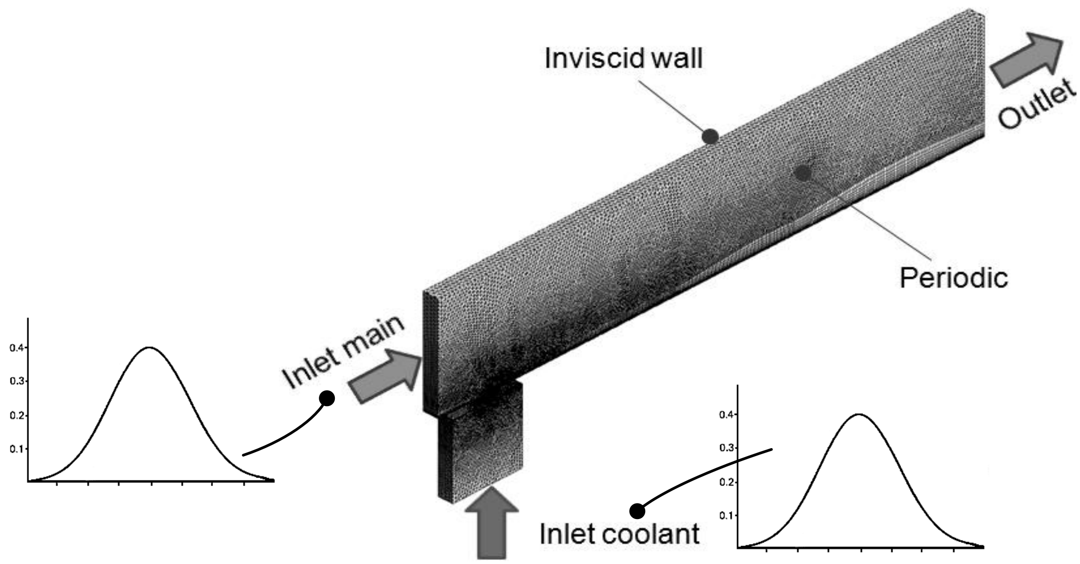


Figure 3 Computational mesh and boundary conditions

Stochastic output

The objective of this research is to accurately predict the coolant coverage at the wall, taking into account the variability of the input. The adiabatic effectiveness, η, is the parameter chosen for this purpose:

$$\eta = \frac{T - T_m}{T_c - T_m} \quad \textbf{Eq. 1}$$

It represents the ratio between the actual cooled surface temperature and the lowest temperature achievable when the wall surface is perfectly covered by the coolant flow. This formulation differs from the original one since the coolant is actually modelled as hot flow and the main as cold.

NUMERICAL APPROACH FOR THE PROBABILISTIC ANALYSIS

Monte Carlo Simulations

Walters and Hyuse [3] stated that MCM is the baseline for uncertainty analysis. In order to perform a MCM simulation it is required to sample the probability space of the input parameters, and perform thousands or millions of simulations which cover the whole design space. Iaccarino [9] described the MCM as a non-intrusive method: since it uses existing deterministic solver as a black box, but computationally expensive. MCM is usually used as validation against other probabilistic approach since it is general and converges to the exact stochastic solution for a number of samples which goes to infinity. Sandor and Andras [10] presented variations for the MCM in order to reduce the computational cost and obtain equally accurate results. Among them the lattice based approach has been chosen in this work MCMLS. The design space is discretised using regularly spaced points where the solution is evaluated. The discretisation choice depends on the input parameters and the accuracy required. In this work, the design space for the two input parameters was divided in 11 points for each variable, giving 121 points. By applying a refinement step, the overall number of cases doubles to 242. However, some of the cases where not physical, since the inlet pressure for the coolant was smaller than the main, hence the sampling space has been bounded. Considering the design space bounded, as in real life, 202 points were chosen. The black points in Figure 4 are the simulations performed and the graph shows the joint probability distribution for the two stochastic inputs. The contours are used to highlight the highest probability region. Despite the reduced computational cost using the lattice based MCM, a new model is suggested to reduce this cost even further.

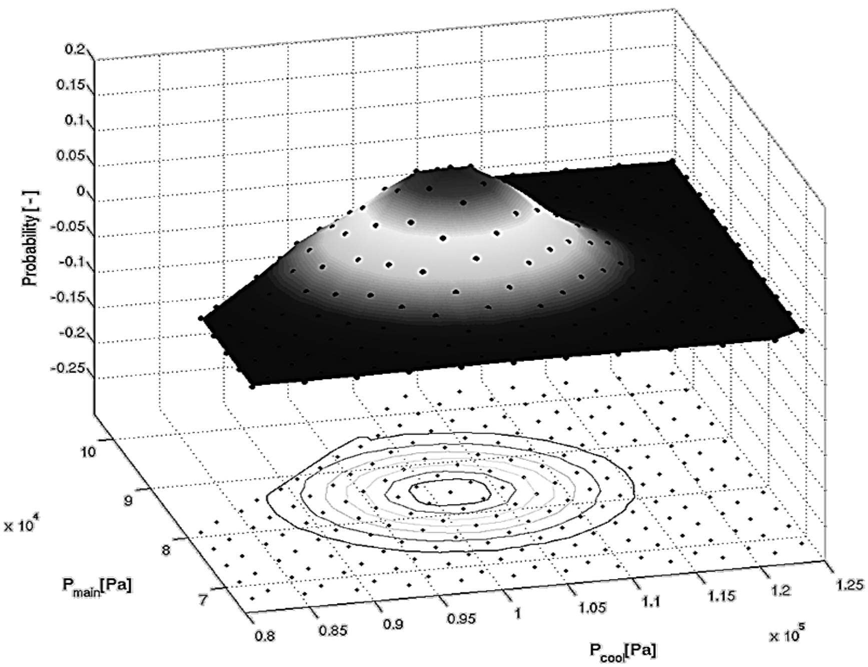


Figure 4 Design Space and cases evaluated for MCM

Probabilistic Collocation Method

The Probabilistic Collocation Method (PCM), Tatang [11], is a non-intrusive technique to perform uncertainty analysis. It is based on the idea of a chaos expansion introduced by Wiener [12] combined with the collocation method, Hairer et al. [13]. It is beyond the scope of this paper to provide a deep explanation of the PCM but few elements will be presented in order to understand this method and its capabilities.

The PCM uses polynomial expansion to shape the output of random processes $y(\boldsymbol{x},\boldsymbol{\xi})$:

$$y(\boldsymbol{x},\boldsymbol{\xi}) = \sum_{j=0}^{P} a_j(\boldsymbol{x})\Psi_j(\boldsymbol{\xi}) \qquad \textbf{Eq. 2}$$

Where the coefficients $a_0(\boldsymbol{x}), a_1(\boldsymbol{x}),\ldots, a_{\mathrm{P}}(\boldsymbol{x})$ are deterministic functions of $\mathbf{x}$ to be estimated. $\Psi_0, \Psi_1(\boldsymbol{\xi}), \ldots, \Psi_P(\boldsymbol{\xi})$ are multi-dimensional orthogonal polynomials with regard to the random variables $\boldsymbol{\xi} = \left(\xi_{i_1}, \xi_{i_2}, \ldots, \xi_{i_N}\right)$. Xiu and Karniadakis [14] presented a criterion to choose the orthogonal polynomials, depending on the input parameters distribution. For example, let $\left(\xi_{i_1}, \ldots, \xi_{i_N}\right)$ be a set of independent standard Gaussian random variables with zero mean and unit variance, the Hermite polynomials form the best orthogonal basis to represent Ψ_{j}, as reported in Ghanem and Spanos [15]. Let truncate the polynomial expansion Eq. 2 to the 2nd order and assume we have only two variables, though: $\boldsymbol{\xi} = (\xi_1, \xi_2)$. Using the Hermite orthogonal polynomial, Eq. 2 can be rewritten as:

$$y(\boldsymbol{x}, \xi_1, \xi_2) = a_0(\boldsymbol{x}) + a_1(\boldsymbol{x})\xi_1 + a_2(\boldsymbol{x})\xi_2 + a_3(\boldsymbol{x})\xi_1\xi_2 + a_4(\boldsymbol{x})(\xi_1^2 - 1) + a_5(\boldsymbol{x})(\xi_2^2 - 1) \qquad \textbf{Eq. 3}$$

Eq. 3 represents the chaos expansion of the stochastic response, and the unknowns are the deterministic functions $a_j(x)$. In order to calculate these coefficients we need to evaluate the random output $y(\boldsymbol{x}, \boldsymbol{\xi})$ for *P=6* times. *P* is a function of the random dimensionality i.e. the number of random variables *N*, which is finite, and the degree of the polynomial expansion *d*:

$$P = \frac{(N + d)!}{N!\,d!} \qquad \textbf{Eq. 4}$$

Usually *d* is chosen even, since the PCM performs better in this case. In fact Li and Zhang [16] showed that even orders are more accurate in presence of large variability. Hence, the simulations have been conducted using the 2nd order.

The choice of the collocation points has been made using the roots of the $(d + 1)^{th}$ order orthogonal polynomials. However, since the number of collocation points available $(d + 1)^N$ is greater than the necessary (Eq. 4), it is required an optimum choice. Hence, the roots have been ranked in decreasing probability order and the first *P* terms were used to perform the simulations. The six points chosen for the simulations at 2nd order are shown as black points in Figure 5. In the same figure, the coloured graph shows the joint probability distribution for the two stochastic inputs and contours are used to highlight the highest probability region.

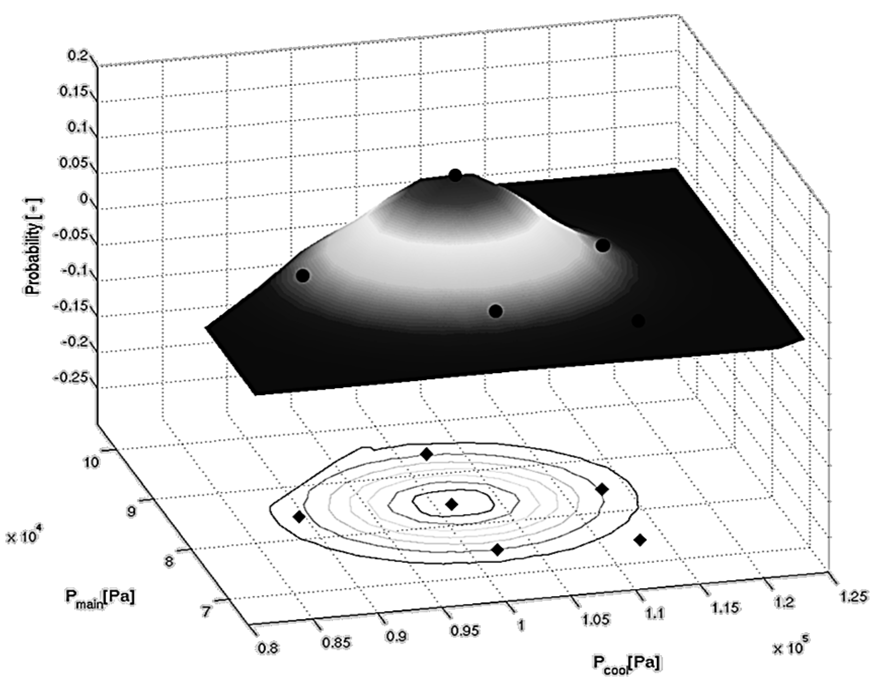


Figure 5 Design Space and cases evaluated for PCM

Once the simulations are performed and the outputs have been collected, it is possible to calculate the deterministic coefficients: $a_0(\boldsymbol{x}), a_1(\boldsymbol{x}),\ldots, a_P(\boldsymbol{x})$. They are used to estimate the statistics of the output $y(\boldsymbol{x}, \boldsymbol{\xi})$ like average and variance:

$$\mu_y = a_0(\boldsymbol{x}) \qquad \textbf{Eq. 5}$$

$$\sigma_y^2 = \sum_{i=1}^{P} a_i^2 \langle \psi_i^2 \rangle$$

RESULTS

The following section shows the impact of the stochastic input variability on the output, using the MCMLS. Hence, the comparison between the two probabilistic methods aforementioned is presented and further analyses are carried on. This work considers normal distribution for the two random variables: inlet total pressure for the main flow and the coolant channels, whose statistics are summarised in Table 2 and illustrated in Figure 2. The random output monitored is the adiabatic effectiveness at the mean line of the coolant for a maximum downstream distance of *x/D=20*.

This work considers a traditional film cooling test case with normal distribution for two random variables, and analyse the impact of stochastic input on the output. Hence, the MCMLS solutions with lower and higher adiabatic effectiveness distribution are shown with different scale in Figure 6; the inlet values are summarised in Table 3. The effectiveness distribution at the wall is different: the spreading of the coolant for the minimum effectiveness case is thicker on the central line, whereas the maximum effectiveness has wider spreading. The 20% variation in the stochastic input gives a variation in the effectiveness at the exit of the hole of 80%.

.	Low eta	High eta
$P_{0,c}$ [Pa]	121860	81240
$P_{0,main}$ [Pa]	74187	80932

Table 3 Important design

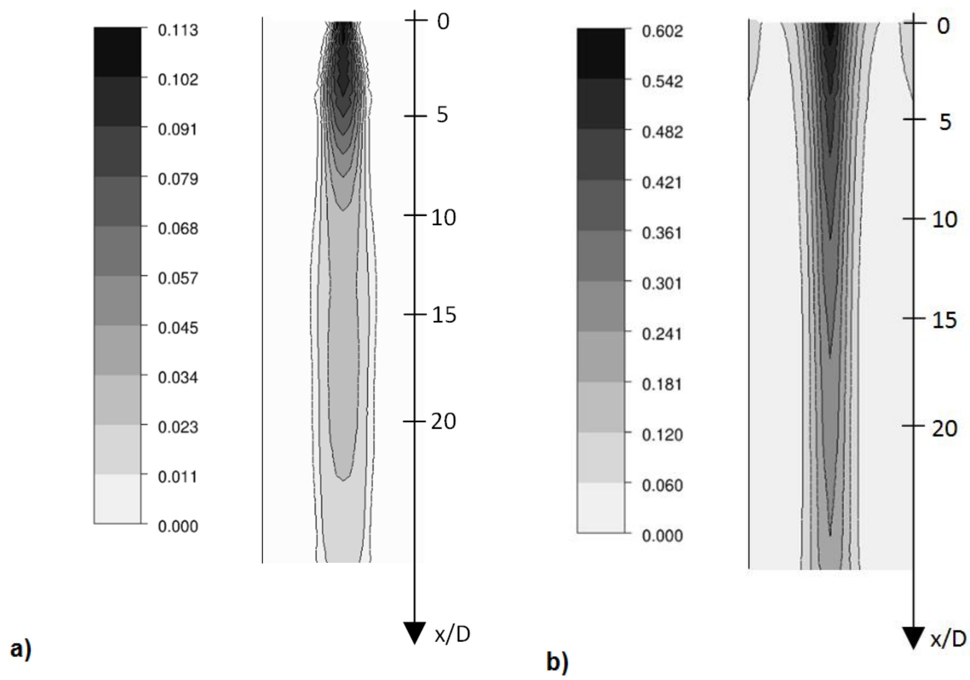
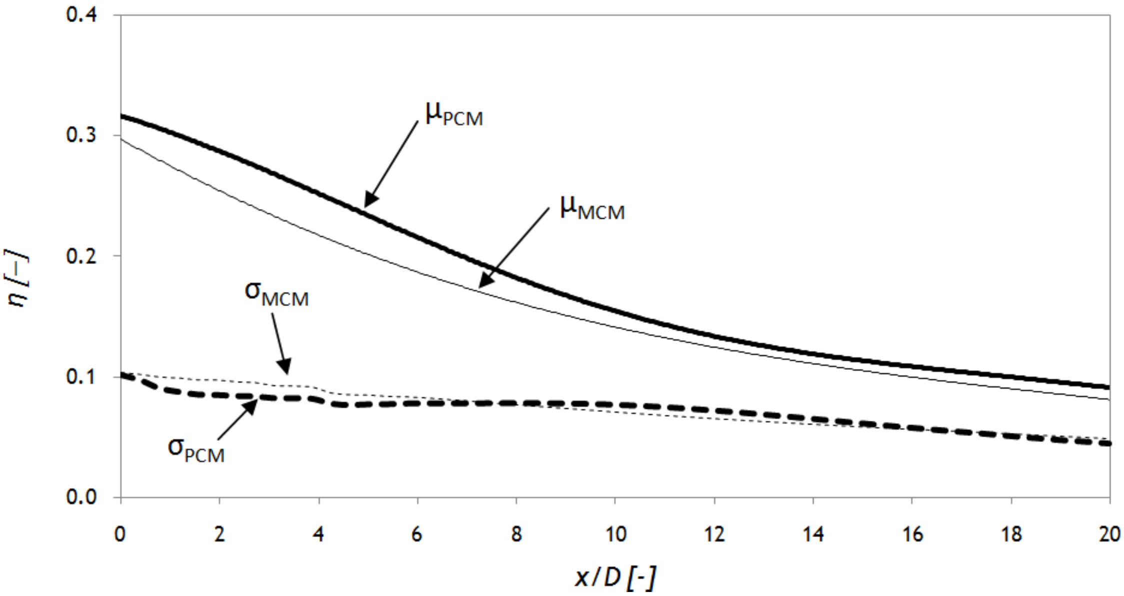


Figure 6 a) Minimum, b) Maximum Adiabatic Effectiveness

The stochastic input greatly affect the performance, is it possible to assess uncertainty analyses more quickly than with MCMLS? There are different methods which reduce the computational cost, the PCM has been used in this work. Figure 7 represents the average and the standard deviation of the adiabatic effectiveness for the PCM 2nd order polynomial and the MCMLS. It is evident a good agreement between the two techniques, proof of the good



implementation of the method and its validity, hence it was not considered necessary to further increase the order of the expansion. For *x/D>10* the two lines are almost the same and the higher difference near the downstream edge in the area near the hole exit is due to the greater sensitivity of this zone to the change in pressure, which is reflected on the higher standard deviation for the output in the same zone. PCM has a computational cost 30 times smaller than MCMLS and can be integrated in the current design process.

Figure 7 Statistics comparison for the PCM and MCM

How can designers develop more performing and robust designs? Figure 8 represents three different effectiveness distributions at the wall, respectively minimum, maximum and average. Taking into account that a 20 K reduction in surface temperature will roughly double the life of the blade, we can conclude that in the worse scenario the blade will last 5 times less than the average value and in the most favourable scenario it will last 8 times more than the average value. The same picture contains other important information represented by the red lines; these give the probability to obtain a certain value of effectiveness at different wall locations. As example, we can consider a case with *x/D=5*, there is a 10% probability to obtain an effectiveness of *0.35*. Taking into account the stochastic variation of the operating conditions the engineers will make more robust designs, since the more probable conditions will be known.

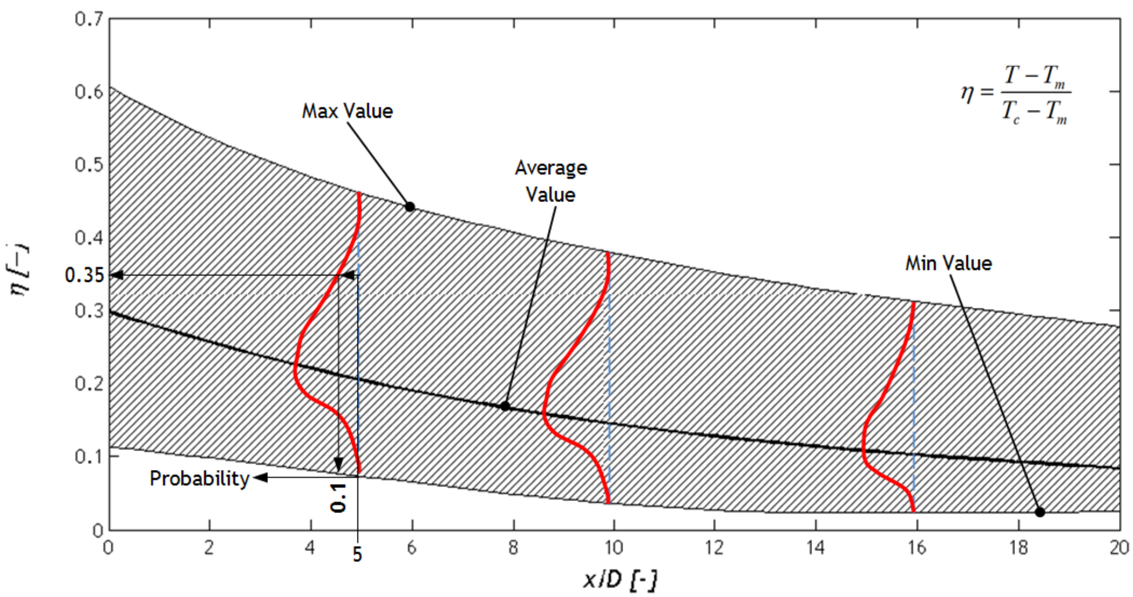


Figure 8 Stochastic distribution of coolant effectiveness: 0.8 <$P_{0,c}$ / $\overline{P}_{0,c}$ <1.2, 0.8 <$P_{0,m}$ / $\overline{P}_{0,m}$ <1.2

CONCLUSIONS

This work presents a standard film cooling configuration and two different uncertainty quantification methods. Afterwards, it analyses how much the change in total inlet pressure for main and coolant channels impacts on the central line adiabatic effectiveness. The variation of the latter can be up to 80% due to variation on the inlet total pressure for main and coolant of 20%. The results show that the PCM has comparable results with the MCMLS and the former requires 30 times less computational effort. Hence, the tool is not only robust but efficient as well. Both methods underlined that the standard deviation is higher near the hole exit, which suggests to designers to have greater carefulness in this area. Further analysis on other variables which are affected by manufacturing uncertainty or variable operating conditions will help in designing more reliable film cooling designs as well as gas turbine components.

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