Self-consistence of the Standard Model via the renormalization group analysis

Fred Jegerlehner[1,2]**, Mikhail Kalmykov**[3,4] **and Bernd A. Kniehl**[3]

[1] Humboldt-Universität zu Berlin, Institut für Physik, Newtonstraße 15, 12489 Berlin, Germany
[2] Deutsches Elektronen-Synchrotron (DESY), Platanenallee 6, 15738 Zeuthen, Germany
[3] II. Institut für Theoretische Physik, Universität Hamburg, Luruper Chaussee 149, 22761 Hamburg, Germany
[4] Joint Institute for Nuclear Research, 141980 Dubna (Moscow Region), Russia

E-mail: `mikhail.kalmykov@desy.de`

Abstract. A short review of recent renormalization group analyses of the self-consistence of the Standard Model is presented.

1. Introduction
The recent discovery of the Higgs boson [1,2] at the LHC with mass

$$M_H = 125.03^{+0.26}_{-0.27}(\text{stat})^{+0.13}_{-0.15}(\text{syst}) \text{ GeV} , \quad [3]$$

$$M_H = 125.36 \pm 0.37(\text{stat}) \pm 0.18(\text{syst}) \text{ GeV} , \quad [4] ,$$

and the fact that so far no signal for new physics has been found, gives rise to the necessity to analyze in detail the self-consistency of the Standard Model. The Higgs boson is a necessary ingredient of the Standard Model (SM), required by its perturbative renormalizability [5, 6]. However, the renormalizability does not give rise to any constraints on the values of parameters of the Lagrangian, but only on the type of interactions. Some additional restrictions on the values of masses and coupling constants are coming from *unitarity*, *triviality* and *vacuum stability*. These three ideas have been used within last decades for an analysis of the self-consistency of the SM.

Unitarity bound: bounds on masses of fermions and Higgs bosons of any renormalizable model can be derived from considerations of the radiative corrections to decays and/or scattering processes [7, 8]. The breakdown of unitary can be avoided by adding a new particles (which implies the existence of new physics) or by the requirement that the perturbative approach must be meaningful (this implies a bound on the particles mass or its coupling constants) [9]. Unitary bounds depend on the type of process under consideration and the precise definition of the breakdown of the perturbative approach. For the Standard Model, the unitary bound for the Higgs boson mass is $M_H \lesssim 1$ TeV .

Triviality and Landau pole: The *triviality* constraint is related with the high energy behavior of the running couplings. Is it well known, that a running coupling $h(\Lambda)$ may suffers from a Landau pole when the corresponding β-function is positive:

$$\frac{d}{d\Lambda}h(\Lambda) = \beta_0 h^3(\Lambda) \longrightarrow h^2(\Lambda) = \frac{h_R^2}{1 - \beta_0 h_R^2 \ln\frac{\Lambda}{M}} , \quad (1)$$

where Λ is the ultra-violet cutoff, M is a characteristics scale of the process under consideration and h_R is the renormalized (running) charge at scale M. As follows from Eq. (1), the running coupling $h^2(\Lambda)$ diverges at scale Λ_{Landau} defined as $\Lambda_{\text{Landau}} = M \exp\left[\frac{1}{\beta_0 h_R^2}\right]$, when $\beta_0 > 0$. In the SM, Landau poles may exists for the $U(1)$-gauge coupling, g_1, Yukawa couplings, y_f and the Higgs self-coupling, λ. For the most important couplings the SM renormalization group (RG) equations read

$$\frac{dh}{dt} = \frac{1}{16\pi^2}\beta_h , \quad h \in \{g_1, g_2, g_3, y_f, \lambda\} ,$$

where in the one-loop approximation the corresponding β-functions have the following form:

$$\beta_1 = \frac{41}{6}g_1^3 , \quad \beta_2 = -\frac{19}{6}g_2^3 , \quad \beta_3 = -7g_3^3 , \quad \beta_{y_t} = y_t\left[\frac{9}{2}y_t^2 - 8g_3^2 - \frac{9}{4}g_2^2 - \frac{17}{12}g_1^2\right] ,$$

$$\beta_\lambda = \lambda\left[24\lambda + 12y_t^2 - 9g_2^2 - 3g_1^2\right] - \left[6y_t^4 - \frac{9}{8}g_2^4 - \frac{3}{8}g_1^4 - \frac{3}{4}g_1^2 g_2^2\right] . \tag{2}$$

It has been shown in [10] that the Landau pole in the Higgs self-coupling λ would be below Planck scale if the Higgs is heavier than 180 GeV. The results of recent analyses [11,12] have confirmed that all coupling constants are free from Landau singularities and have smooth behavior in interval between $M_Z \sim 90$ GeV and $M_{Planck} = 2.435 \times 10^{18}$ GeV (see Fig. 1):
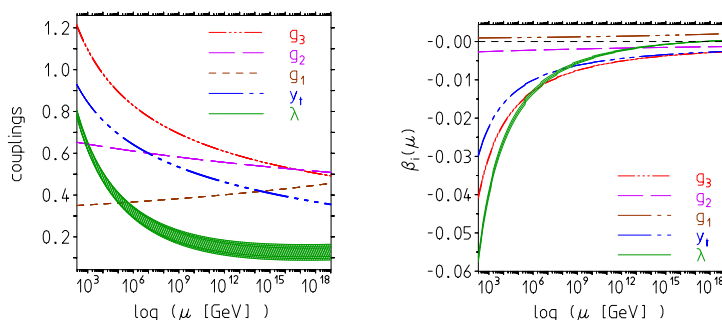

Figure 1. The running coupling constants(left) and corresponding β-functions (right). The plots are taken from [11].

1.1. Vacuum stability and effective potential

For the analysis of the *vacuum stability* one needs to recall some basic definitions related with the scalar potential and its generalization in Quantum Field Theory. Let us remind, that at the classical level, the Higgs potential in the SM

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{1}{4!}\lambda\phi^4 , \tag{3}$$

is bounded from below for $\lambda > 0$ and has a trivial minimum for $m^2 > 0$ at $\phi = 0$, and non-trivial minima at $\phi^2 = \frac{-m^2}{\lambda}$ for $m^2 < 0$. At the quantum level, instead of the classical potential defined by Eq. (3), the *effective potential* should be analysed [13]. It can be written in the Landau gauge and the $\overline{\text{MS}}$ scheme as [14]

$$V(\phi) \quad \rightarrow \quad V_{eff}(\phi(t)) = -\frac{1}{2}m^2(t)\phi^2(t) + \frac{1}{4!}\lambda(t)\phi^4(t) + \frac{1}{16\pi^2}V_1 + \frac{1}{(16\pi^2)^2}V_{rest} , \tag{4}$$

where (we use the notations of [15]):

$$
\begin{aligned}
V_1 &= \sum_i \frac{k_i}{4} M_i^4(\phi(t)) \left[\ln \frac{M_i^2(\phi(t))}{\mu^2} - C_i \right] , \\
\phi(t) &= \exp\{- \int_0^t \gamma(\tau) d\tau \} \phi_{clas} , \quad M_i^2(\phi(t)) = a_i h_i(t) \phi(t)^2 + b_i ,
\end{aligned}
\tag{5}
$$

$h_i(t)$ are the running couplings, $\gamma(t)$ is the anomalous dimension of the Higgs field, and k_i, C_i, a_i, b_i are numerical constants. By V_{rest} we denote the higher-order contributions, which, in particular, also include the higher dimension operators (it begins at four loop) [14, 16]:

$$
V_{rest} \sim \lambda \phi^4 \sum_{L>4} \left(\frac{\lambda^2 \phi^2}{m^2 + \frac{1}{2} \lambda \phi^2} \right)^{L-3} ,
\tag{6}
$$

where L is the number of loops (for more details see Section 4 of [14] or [17]).

The quantum corrections modify the shape of the effective potential such that a second minimum at large (Planck) scale may be generated. This second minimum (see Fig. (2)) is: (left plot) stable $V_{eff}(v) < V_{eff}(\phi_{\min})$, (middle plot) critical (two minima are degenerate in energy) $V_{eff}(v) = V_{eff}(\phi_{\min})$, (right plot) unstable/metastable $V_{eff}(v) > V_{eff}(\phi_{\min})$, electroweak vacuum and v is the location of the EW minimum and $\phi_{\min}$ is the value of a new minimum.
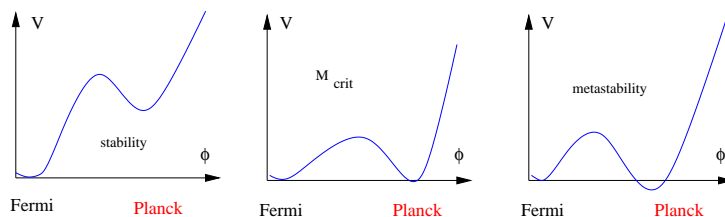

Figure 2. The form of the effective potential for the Higgs field ϕ which corresponds to the stable (left), critical (middle) and metastable (right) electroweak vacuum. The plot is taken from [18].

Depending on the values of the Higgs boson and top quark masses the lifetime of the EW vacuum can be larger or smaller then the age of the Universe [19–21]. The first case corresponds to the metastability scenario.

To get *practical* criteria of stability, the following step-by-step approximations are used [22]: (i) the potential at very high values of the field is dominated by the quartic term $V_{eff} \sim \lambda(\phi)\phi^4$ and $\lambda(\phi)$ depends on ϕ as the running coupling $\lambda(\mu)$ depends on the running scale μ; (ii) it's looking the large value of the field $\phi = \phi_{crit}$, where $\lambda(\phi_{crit}) = 0$; (iii) combine together the previous two approximations, we get at very high values of the Higgs field $V_{eff}|_{\phi \sim \phi_{crit}} \simeq \lambda(\phi_{crit})\phi_{crit}^4$. Then, the effective potential becomes negative (unbounded from below) when $\lambda(\mu) < 0$ and the vacuum at the EW scale is not the absolute minimum. As follows from Eqs. (2), the Higgs self-coupling λ is the only SM dimensionless coupling that can change sign with the scale variation since its beta-function, β_λ contains a part which is not proportional to λ. In the considered approximation,

the requirement that the electroweak vacuum $<\phi> = (\sqrt{2}G_F)^{-\frac{1}{2}} = 246.22 \ GeV$ is the absolute minimum of the potential up to scale Λ, implies

$$
\lambda(\mu) > 0 , \quad for \ any \quad \mu < \Lambda .
\tag{7}
$$

The role of quadratic term or higher dimensional terms (see Eq.(6)) is discussed in Section 3.

2. Analysis

2.1. Renormalization group equations and Matching Conditions

Both the questions, the one concerning vacuum stability and the one concerning triviality, in the Standard Model are reduced to the renormalization group analysis of the Higgs self-coupling λ. The evolution of λ includes the evolution of all coupling constants, see Eq. (2).

The starting point for solving the evolution equations is provided by the matching conditions: the relations between the running coupling constants $h_i(\mu)$ and the relevant (pseudo-) physical observables, σ_i. The simplest form for the matching conditions follows when physical masses are taken as the referring point: $\sigma_i \equiv M_i$. In this case the matching conditions have the following form [23]:

$$h_i(\mu^2) = c_i \frac{G_F}{\sqrt{2}} M_i^2 \left(1 + \delta_{\alpha_s} + \delta_{\alpha} + \delta_{\alpha\alpha_s} + \delta_{\alpha^2} + \delta_{\alpha_s^2} + \cdots \right) , \qquad (8)$$

where c_i are normalization constants and $\delta_{\alpha_j^k}$ are including only propagator type diagrams of order $O(\alpha_j^k)$. The evaluation of the one-loop EW matching conditions of order $O(\alpha)$ was partially done in [23–25] and has been completed in [26,27]. The two-loop matching conditions of order $O(\alpha^2)$ for the Higgs self-coupling λ in the limit of heavy Higgs boson have been evaluated in [28,29], and full two-loop results were completed in [30–35]. The $O(\alpha\alpha_s)$ corrections for the top-quark Yukawa coupling within the SM have been evaluated in [36,37] and $O(\alpha^2)$ corrections are available only in the gaugeless limit [38,39]. A state of the art evaluation attempting matching at the two-loop level has been reported in [12] (results see below).

The evaluation of the 3-loop RG equations for the SM gauge coupling was started in [40,41], and full 3-loop results for the Standard Model have been published in [42–48].

2.2. Results of RG analysis

After the top-quark discovery the results of RG analyses of self-consistence of the SM are typically fitted by three parameters and can be written as follows:

$$M_{\min} > \left[M_{crit} + c_1 \times \left(M_t - M_{\exp} \right) + c_2 \times \left(\alpha_s - \alpha_{s,\exp} \right) \right] \text{ GeV} , \qquad (9)$$

where M_{crit} is the critical value of the Higgs boson mass, c_1, c_2 are some numerical coefficients and $M_{\exp}$ and $\alpha_{s,\exp}$ are the latest experimental values of the pole mass of the top-quark and of the strong coupling constant, respectively.

The detailed analysis of stability/metastability bounds based on the two-loop RG equations for all SM couplings and one-loop EW matching conditions (as well as two-loop QCD corrections to the top-quark Yukawa coupling y_t) have been presented in [49]:

$$M_{\min} \geq \left[130 + 1.8 \times \left(\frac{M_t^{\text{pole}} - 173.2 \text{ GeV}}{0.9 \text{ GeV}} \right) - 0.5 \times \left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) \pm 3 \right] \text{ GeV} , \qquad (10)$$

with $M_t = (173.2 \pm 0.9)$ GeV and $\alpha_s(M_Z) = 0.1184 \pm 0.0007$ as input parameters and where the error of 3 GeV is an estimation of unknown higher-order corrections.

The analysis of vacuum stability performed with inclusion of 3-loop RG equations and 2-loop matching conditions was presented in [50] and [51] with the following results:

$$M_{\min} \geq \left[128.95 + 2.2 \times \left(\frac{M_t^{\text{pole}} - 172.9 \text{ GeV}}{1.1 \text{ GeV}} \right) - 0.56 \times \left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) \pm 1 \right] \text{ GeV} , \quad [50]$$

(11)

$$M_{\min} \geq \left[129.4 + 1.4 \times \left(\frac{M_t^{\text{pole}} - 173.1 \text{ GeV}}{0.7 \text{ GeV}} \right) - 0.5 \times \left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) \pm 0.7 \right] \text{ GeV} , \quad [51]$$

(12)

The latest published analysis have been presented in [12]:

$$M_{\min} > \left[129.6 + 2.0 \times \left(M_t^{\text{pole}} - 173.35 \text{ GeV} \right) - 0.5 \times \left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) \pm 0.3 \right] \text{ GeV} . \quad (13)$$

2.3. The analysis of uncertainties

As follows from Eqs. (10)-(13) adding two-loop matching conditions and 3-loop RG equations does not changed dramatically the central value of critical mass: $130 \rightarrow \sim 129.0 \div 129.6$, but essentially reduces the theoretical uncertainties: $(3 \rightarrow 0.3)$ GeV. Adding the leading 3-loop corrections to the matching conditions for the Higgs self-coupling [52, 53] does not modified the final result, Eq. (13). The 3σ variations of α_s or the small variation of the value of the matching scale [54] also do not produce the significant corrections to the fitted expression.

The main error in Eqs. (10)-(13) is coming from the input value of top-quark mass: $\Delta M_{\min} \sim 2.0 \times \Delta M_t^{\text{pole}}$. Currently the most precise measurement of the top-quark mass has been reported as the world combination of the ATLAS, CDF, CMS and D0 [56] (for completeness we present also the result based only on CDF/D0 data [55]):

$$M_t = 173.18 \pm 0.94 \text{ GeV} , \quad [55] \tag{14}$$

$$M_t = 173.76 \pm 0.76 \text{ GeV} , \quad [56] \tag{15}$$

It has been pointed out in [57] that the values of the top quark mass quoted by the experimental collaborations correspond to parameters in Monte Carlo event generators in which, apart from parton showering, the partonic subprocesses are calculated at the tree level, so that a rigorous theoretical definition of the top quark mass is lacking (see more detailed discussion of this issue in [58]). In particular, the following issues in precision top mass determination at hadron colliders are relevant [59]: MC modeling, reconstruction of the top pair, unstable top and finite top width effects, bound-state effects in top pair production at hadron colliders, renormalon ambiguity in top mass definition, alternative top mass definitions, higher-order corrections, non-perturbative corrections, contributions from physics beyond the Standard Model.

To reduce the uncertainties related with undetermined differences between Monte-Carlo and pole masses (it was estimated in [57, 58] as 1 GeV) the mass of top-quark can be extracted directly from a measurement of the total top-pair production cross section $\sigma_{\exp}(p\bar{p} \rightarrow t\bar{t} + X)$. Such analysis performed in [60] with NNLO accuracy with inclusion of the full theoretical uncertainties (the scale variation as well as the (combined) PDF and α_s uncertainties) gives rise to the following result, $M_t^{\text{pole}} = 173.3 \pm 2.8 \text{ GeV}$. The central value is very close to the one in Eq. (15), but the theoretical uncertainty is much larger. Similar analyses (direct extraction of the pole mass of top-quark from measured total cross section) have been performed by a few

other groups [58, 61–64] with the following results:

$$M_t^{\text{pole}} = 171.4^{+5.4}_{-5.7} \text{ GeV} \quad [61] \,, \tag{16}$$

$$M_t^{\text{pole}} = 174.3^{+4.9}_{-4.4} \text{ GeV} \quad [62] \,, \tag{17}$$

$$M_t^{\text{pole}} = 174.2^{+3.6}_{-3.9} \text{ GeV} \quad [58] \,, \tag{18}$$

$$M_t^{\text{pole}} = 171.2 \pm 2.4 \pm 0.7 \text{ GeV} \quad [64] \,, \tag{19}$$

where the full NNLO QCD corrections evaluated in [65, 66] have been combined with the soft-gluon resummation at NNLL accuracy [63] and Coulomb-gluon NNLL resummation [61]. All results in Eqs. (16)-(19) have large theoretical uncertainties. To improve the current precision of the top-mass determination from the total cross section the higher order corrections as well as a reduction of PDF and α_s uncertainties are required [67, 68].

3. Some additional sources of uncertainties

3.1. EW contribution to the running mass of the top-quark

In order to achieve percent level precision theoretical predictions for cross section $\sigma_{pp \to t\bar{t}}$ not only QCD NNLO radiative corrections should be applied. The EW part as well as mixed EW $\times$ QCD corrections have to be included in a systematic way. For example, the QCD interaction is not responsible for the non-zero width of the top-quark, which can be understood precisely only by inclusion of the EW interactions. In any case, the EW effect [69, 70] as well as the non-zero width should be included in addition to the QCD corrections [71].

In contrast to QCD, where the mass of a quark is the parameter of the Lagrangian, the notion of $\overline{\text{MS}}$-mass in EW theory is not determined entirely by the prescriptions of minimal subtraction. It depends on the value of vacuum expectation value $v(\mu^2)$ chosen as a parameter of the calculations so that the running mass is $m_t(\mu^2) = 1/\sqrt{2} y_f(\mu^2) v(\mu^2)$. It has been shown in a series of papers [72–74] that in the scheme with explicit inclusion of tadpoles [25] the RGE for the running vacuum expectation value $v^2_{MS}(\mu^2)$ which is defined as (see Section 4 in [36])

$$v^2_{MS}(\mu^2) = \frac{1}{\sqrt{2} G_F} \frac{1}{1 - \overline{\Delta} r} \left[\frac{m_W^2(\mu^2)}{M_W^2} \right] \left[\frac{\alpha(M_Z)}{\alpha_{MS}(\mu^2)} \right] \left[\frac{\sin^2 \theta_W^{MS}(\mu^2)}{\sin^2 \theta_W^{OS}} \right] \,, \tag{20}$$

coincides with RGE for the classical definition of tree-level scalar potential vacuum

$$\mu^2 \frac{d}{d\mu^2} v^2_{MS}(\mu^2) = \mu^2 \frac{d}{d\mu^2} \left[\frac{m^2(\mu^2)}{\lambda(\mu^2)} \right] \,, \tag{21}$$

where m^2 and λ are the parameters of the scalar potential, see Eq. (3). The asymptotic behavior and properties of this vacuum at low and high energies have been analysed in [75, 76]. In particular, for a current values of Higgs and top-quark masses an IR point μ_{IR} close to the value of the Z-boson mass exist such that $v^2_{MS}(\mu^2)\big|_{\mu \sim M_Z} = (\sqrt{2} G_F)^{-1/2} = 246.22$ GeV (see details in [76]). In the framework of the effective potential approach, a similar condition for the minimum of effective potential is imposed by hand (see details in [15]).

In the SM the decoupling theorem [77] is not valid in the week sector, the "decoupling by hand" prescription does not work and we have to take full SM parameter relations as they are. In this approximation we got [75] that the EW contribution is large and has opposite sign relative to the QCD contributions, so that the total SM correction is small and approximately equal to $M_t - m_t(m_t) \sim 1 \div 2$ GeV (see left plot in Fig. (3)). The complete $O(\alpha^2)$ correction to the relation between pole and $\overline{\text{MS}}$ masses of the top-quark are not yet available, but our numerical estimation [75] is in agreement with result of [38, 39] and it is of the order of the $O(\alpha_s^4)$ corrections [78, 79]. For the evaluation of the NNLO corrections three different schemes have been used: with explicit

inclusion of tadpole [72–74], the so called β-renormalization scheme [33–35], and the method of minimization of the effective potential [12, 51, 80]. The equivalence of these three methods of evaluation of matching conditions have never been analyzed. In particular, the structure of 2-loop UV-counterterms in [72–74] and [33,34] are different; the gauge dependence of the effective potential method versus gauge independence of the scheme including the tadpoles [72–74] (the gauge (in)dependence of prediction of the critical value of the Higgs boson mass via effective potential scheme have been analysed in [81,82]), etc. The difference between these methods is of order $O(G_F^2 M_W^4)$ and may reach the value $\sim 10^{-3} \div 10^{-4}$. Another source of numerical difference is related with the choice between complex mass [35] or pole mass [72, 74] in Eq. (8). It is equivalent to systematic additional uncertainty of order $O(\Gamma^2/(4M^2))$ (see discussion in [37,83]), where Γ is the width and M is the mass of particle. This effect is the most significant for the top-quark Yukawa-coupling: $\delta y_f \sim 1/173^2 = 3 \times 10^{-5}$. The unknown QCD corrections of order $O(\alpha_s^4)$ to the top-quark Yukawa-coupling is $\delta_{\alpha_s^4} y_f \sim 1 \times 10^{-3}$ [78, 79]. The two-loop EW correction $\delta_{\alpha^2} y_t(\mu = M_t) \approx \frac{5}{(4\pi)^2} = 6 \times 10^{-4}$ (see Eq.(2.49) in [12]) and three-loop corrections can be estimated as $\delta_{\alpha_s^2\alpha} y_t(\mu = M_t) \sim [\delta_{\alpha_s^2} y_t(\mu = M_t)] \times [\delta_\alpha y_t(\mu = M_t)] = 0.011 \times 0.00135 = 1.5 \times 10^{-5}$. All these effects do not change the central value of the critical mass of the Higgs boson, Eqs. (10)-(13), but affect the value of theoretical uncertainties.
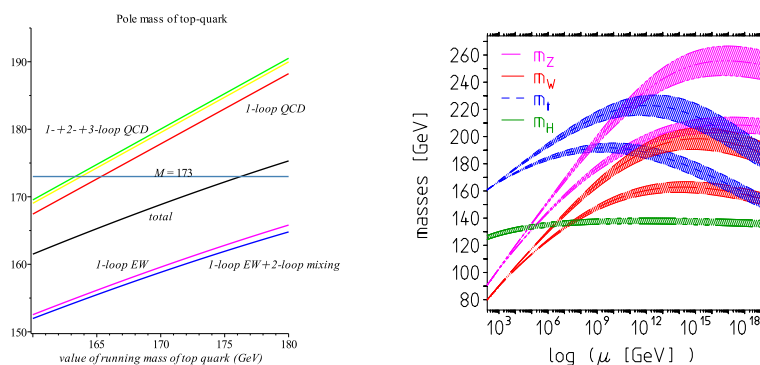

Figure 3. The running top-quark mass at low value of μ (left) and behavior of running masses at large values of μ (right). At the right the bands corresponds to Higgs mass in interval 124 GeV $< M_H <$ 127 GeV. The right plot is taken from [11].

3.2. The first order phase transition and quadratic divergences

It was pointed out in [11] that in the region where the Higgs self-coupling is positive and close to zero, $\lambda(\Lambda) \sim 0$, the quadratic term of effective potential may start play the essential role since $\lambda(\Lambda)\phi^2(\Lambda) \ll m^2(\Lambda)$. The massive parameter m^2 suffers from quadratic divergences: $\Delta m^2(\Lambda) = \frac{\Lambda^2}{16\pi^2}Q_1$, where the coefficients Q_1 are expressible in terms of coupling constants [84], $Q_1 = \lambda + \frac{1}{8}g_1^2 + \frac{3}{8}g_2^2 - y_t^2$. The coefficient Q_1 may vanish at some high scale Λ^* [85]. If $\Lambda \sim \Lambda^*$ then m^2-term may change the sign and that leads to a phase transition of the first order restoring the EW symmetry [11]. Realization of this scenario (the value of scale where $Q_1 = 0$) strongly depends on the value of top-quark mass (see discussion in [86, 87]). The Higgs inflation and hierarchy problem within this scenario have been discussed in [88,89].

3.3. Inclusion of higher dimension operators

As well known [14] the effective potential contains also the higher dimensional operators (see Eq. (6)). The impact of new physics interaction at Planck scale was analysed in [90] by adding two higher dimensional operators ϕ^6 and ϕ^8 suppressed by inverse powers of Planck scale M_{Planck} to the Higgs potential. It has been shown that higher dimension operators may change the

lifetime of the metastable vacuum, τ, from $\tau = 1.49 \times 10^{714} T_U$ to $\tau = 5.45 \times 10^{-212} T_U$, where T_U is the age of the Universe.

4. Conclusion

The main results of the LHC is the discovery of the Higgs boson. The second important result is the absence of a signal of new physics. After the Higgs boson discovery the Standard Model is completed. Since all parameters of the Standard Model are defined now experimentally, one can analyze the extrapolation of the SM up to the Planck scale. The results of the recent analysis can be summarized as follows:

- The Standard Model is a self-consistent QFT that can be extrapolated from M_W to M_{Planck} since all SM couplings remain perturbative (no Landau pole) in that range, see Fig. 1.

- With the current precision in $M_H (\sim 126$ GeV$)$, the value of top-quark mass in accordance with CDF/DO/CMS/ATLAS $M_t (\sim 173$ GeV$)$ and 3-loop RG functions and 2-loop matching conditions, one concludes that the EW vacuum would most likely be *metastable* [12]: the *stability condition* is $M_H > (129.6 \pm 1.5)$ GeV for a given value of top-quark mass. In terms of top quark mass, the stability bound is [22] $M_t < (171.36 \pm 0.46)$ GeV.

- It was shown in [49] that the lifetime of the electroweak vacuum is longer than the age of the Universe for $M_H > 111$ GeV so that
  Metastability of vacuum with very long lifetime cannot be used as motivation for a New Physics
  A lot of effort has been made recently to analyzed EW vacuum during inflation [91, 92]. The condition of vacuum (meta)stability imposes the constraint on the rate of inflationary expansion that are in tension with BICEP2 result [93]. However, the recent result from the Plank collaboration [94] does not confirm the BICEP2 result [93] so that the further experimental verification is necessary.

- The result of the recent analysis [60] reveals a large theoretical uncertainties in the value of top-quark mass (see Eqs. (16)-(19)). This leads to a large uncertainty ~ 5 GeV in the critical value of the Higgs boson mass.

- The mass of the Higgs boson is very close to the values of "critical Higgs mass", M_{crit}: the "multiple point principle" [95], Higgs inflation [96–98], asymptotic safety scenario [99] The explicit realization $M_H = M_{crit}$ would be a strong indication for the absence of a new physics scale between the Fermi and Planck scales.

- To clarify the situation, more precise measurements of the coupling constants α_s, λ, y_t are needed. Unfortunately, there are no ways of directly measuring the Higgs self-coupling and top-quark Yukawa coupling with high precision in the immediate future [100–103].

- The higher order calculations are also desirable.

Therefore,
precision determinations of parameters are more important than ever and a real challenge for experiments at the LHC and at a future ILC.

5. Acknowledgments

References

[1] Aad G *et al.* [ATLAS Collaboration] 2012 *Phys. Lett.* B **716** 1 (Preprint 1207.7214)
[2] Chatrchyan S *et al.* [CMS Collaboration] 2012 *Phys. Lett.* B **716** 30 (Preprint 1207.7235)
[3] [CMS Collaboration] 2014 Precise determination of the mass of the Higgs boson and studies of the compatibility of its couplings with the standard model (Preprint CMS-PAS-HIG-14-009)
[4] G. Aad *et al.* [ATLAS Collaboration], *Phys. Rev.* D **90** 052004 (Preprint 1406.3827)
[5] 't Hooft G 1971 *Nucl. Phys.* B **35** 167
[6] 't Hooft G and Veltman M J G 1972 *Nucl. Phys.* B **44** 189
[7] Dicus D A and Mathur V S 1973 *Phys. Rev.* D **7** 3111
[8] Lee B W, Quigg C and Thacker H B 1977 *Phys. Rev.* D **16** 1519
[9] Vetlman M J G 1977 *Acta Phys. Polon.* B **8** 475
[10] Hambye T and Riesselmann K 1997 *Phys. Rev.* D **55** 7255 (Preprint 9610272)
[11] Jegerlehner F 2014 *Acta Phys. Polon.* B **45** 1167 (Preprint 1304.7813)
[12] Buttazzo D et al. 2013 *JHEP* **1312** 089 (Ppreprint 1307.3536)
[13] Coleman S R and Weinberg E J 1973 *Phys. Rev.* D **7** 1888
[14] Ford C et al. 1993 *Nucl. Phys.* B **395** 17 (Preprint 9210033)
[15] Casas J A et al. 1995 *Nucl. Phys.* B **436** 3 [Erratum-ibid. B **439** (1995) 466] (Preprint 9407389)
[16] Nakano H and Yoshida Y 1994 *Phys. Rev.* D **49** 5393 (Preprint 9309215)
[17] Einhorn M B and Jones D R T 2007 *JHEP* **0704** 051 (Preprint 0702295)
[18] Shaposhnikov M 2013 *PoS EPS* **-HEP2013** 155 (Preprint 1311.4979)
[19] Coleman S R 1977 *Phys. Rev.* D **15** 2929 [Erratum-ibid. D **16** (1977) 1248].
[20] Callan C G and Coleman S R 1977 *Phys. Rev.* D **16** 1762
[21] Arnold P B 1989 *Phys. Rev.* D **40** 613
[22] Espinosa J R 2013 Vacuum Stability and the Higgs Boson (Preprint 1311.1970)
[23] Sirlin A and Zucchini R 1986 *Nucl. Phys.* B **266** 389
[24] Sirlin A 1980 *Phys. Rev.* D **22** 971
[25] Fleischer J and Jegerlehner F 1981 *Phys. Rev.* D **23** 2001
[26] Böhm M, Spiesberger H and Hollik W 1986 *Fortsch. Phys.* **34** 687
[27] Hempfling R and Kniehl B A 1995 *Phys. Rev.* D **51** 1386
[28] van der Bij J and Veltman M J G 1984 *Nucl. Phys.* B **231** 205
[29] Ghinculov A and van der Bij J J 1995 *Nucl. Phys.* B **436** 30
[30] Awramik M et al. 2004 *Phys. Rev.* D **69** 053006 (Preprint 0311148)
[31] Aglietti U et al. 2004 *Phys. Lett.* B **595** 432 (Preprint 0404071)
[32] Degrassi G and Maltoni F 2004 *Phys. Lett.* B **600** 255 (Preprint 0407249)
[33] Actis S and Passarino G 2007 *Nucl. Phys.* B **777** 35 (Preprint 0612123)
[34] Actis S and Passarino G 2007 *Nucl. Phys.* B **777** 100 (Preprint 0612124)
[35] Actis S et al. 2008 *Phys. Lett.* B **670** 12 (Preprint 0809.3667)
[36] Jegerlehner F and Kalmykov M 2003 *Acta Phys. Polon.* B **34** 5335 (Preprint 0310361)
[37] Jegerlehner F and Kalmykov M 2004 *Nucl. Phys.* B **676** 365 (Preprint 0308216)
[38] Martin S P 2005 *Phys. Rev.* D **72** 096008
[39] Kniehl B A and Veretin O V 2014 *Nucl. Phys.* B **885** 459 (Preprin 1401.1844)
[40] Kazakov D I, Tarasov O V and Vladimirov A A 1979 *Sov. Phys. JETP* **50** 521
[41] Tarasov O V, Vladimirov A A and Zharkov A Y 1980 *Phys. Lett.* B **93** 429
[42] Mihaila L N, Salomon J and Steinhauser M 2012 *Phys. Rev. Lett.* **108** 151602 (Preprin 1201.5868)
[43] Mihaila L N, Salomon J and Steinhauser M 2012 *Phys. Rev.* D **86** 096008 (Preprint 1208.3357)
[44] Chetyrkin K G and Zoller M F 2012 *JHEP* **1206** 033
[45] Bednyakov A V, Pikelner A F and Velizhanin V N 2013 *JHEP* **1301** 017 (Preprint 1210.6873)
[46] Bednyakov A V, Pikelner A F and Velizhanin V N 2013 *Phys. Lett.* B **722** 336 (Preprin 1212.6829)
[47] Bednyakov A V, Pikelner A F and Velizhanin V N 2013 *Nucl. Phys.* B **875** 552 (Preprint 1303.4364)
[48] Chetyrkin K G and Zoller M F 2013 *JHEP* **1304** 091 [Erratum-ibid. **1309** 155] (Preprint 1303.2890)
[49] Elias-Miro J et al. 2012 *Phys. Lett.* B **709** 222 (Preprint1112.3022)
[50] Bezrukov F et al. 2012 *JHEP* **1210** 140 (Preprint 1205.2893)
[51] Degrassi G et al. 2012 *JHEP* **1208** 098 (Preprint 1205.6497)
[52] Martin S P 2014 *Phys. Rev.* D **89** 013003 (Preprint 1310.7553)
[53] Martin S P and Robertson D G 2014 (Preprint 1407.4336)

[54] Masina I 2012 *Phys. Rev.* D **87** 053001 (Preprint 1209.0393)
[55] T. Aaltonen *et al.* [CDF and D0 Collaborations] 2012 Phys. Rev. D **86** 092003 (Preprint 1207.1069)
[56] [ATLAS and CDF and CMS and D0 Collaborations] 2014 First combination of Tevatron and LHC measurements of the top-quark mass (Preprint 1403.4427)
[57] Hoang A H and Stewart I W 2008 *Nucl. Phys. Proc. Suppl.* **185** 220 (Preprint 0808.0222)
[58] Moch et al. 2014 High precision fundamental constants at the TeV scale (Preprint 1405.4781)
[59] Juste A et al. 2013 Determination of the top quark mass circa 2013: methods, subtleties, perspectives (Preprint 1310.0799)
[60] Alekhin S, Djouadi A and Moch S 2012 *Phys. Lett.* B **716** 214
[61] Beneke et al. 2012 *JHEP* **1207** 194 (Preprint 1206.2454)
[62] Beneke et al. 2013 *Nucl. Phys. Proc. Suppl.* **234** 101 (Preprint 1208.5578)
[63] Czakon M and Mitov M 2014 *Comput. Phys. Commun.* **185** 2930 (Preprint 1112.5675)
[64] Alekhin S, Blümlein J and Moch S 2014 *Phys. Rev.* D **89** 054028 (Preprint 1310.3059)
[65] Bärnreuther P, Czakon M and Mitov A 2012 *Phys. Rev. Lett.* **109** 132001 (Preprint 1204.5201)
[66] Czakon M, Fiedler P and Mitov A 2013 *Phys. Rev. Lett.* **110** 252004 (Preprint 1303.6254)
[67] Czakon M et al. 2013 *JHEP* **1307** 167 (Preprint 1303.7215)
[68] Ball R D *et al.* [The NNPDF Collaboration] 2014 Parton distributions for the LHC Run II (Preprint 1410.8849)
[69] Beenakker W et al. 1994 *Nucl. Phys.* **B411** 343
[70] Kühn J H, Scharf A and Uwer P 2007 *Eur. Phys. J.* **C51** 37 (Preprint 0610335)
[71] Falgari P, Papanastasiou A S and Signer A 2013 *JHEP* **1305** 156 (Preprint 1303.5299)
[72] Jegerlehner F, Kalmykov M and Veretin O 2002 *Nucl. Phys.* B **641** 285 (Preprint 0105304)
[73] Jegerlehner F, Kalmykov M and Veretin O 2003 *Nucl. Phys. Proc. Suppl.* **116** 382 (Preprint 0212003)
[74] Jegerlehner F, Kalmykov M and Veretin O 2003 *Nucl. Phys.* B **658** 49 (Preprint 0212319)
[75] Jegerlehner F, Kalmykov M and Kniehl B A 2013 *Phys. Lett.* B **722** 123 (Preprint 1212.4319)
[76] Jegerlehner F, Kalmykov M and Kniehl B A 2013 *PoS DIS* **2013** 190 (Preprint 1307.4226)
[77] Appelquist A and Carazzone J 1975 *Phys. Rev.* D **11** 2856
[78] Kataev A L and Kim V T 2010 *Phys. Part. Nucl.* **41** 946 (Preprint 1001.4207)
[79] Sumino Y 2014 *Phys. Lett.* B **728** 73 (Preprint 1309.5436)
[80] Martin S P 2004 *Phys. Rev.* D **70** 016005 (Preprint 0312092)
[81] Loinaz W and Willey R S 1997 *Phys. Rev.* D **56** 7416 (Preprint 9702321)
[82] Di Luzio L and Mihaila L 2014 *JHEP* **1406** 079 (Preprint 1404.7450)
[83] Smith M C and Willenbrock S S 1997 *Phys. Rev. Lett.* **79** 3825 (Preprint 9612329)
[84] Veltman M J G 1981 *Acta Phys. Polon.* B **12** 437
[85] Hamada Y, Kawai H and Oda K 2013 *Phys. Rev.* D **87** 053009 (Preprint 1210.2538)
[86] Masina I and Quiros M 2013 *Phys. Rev.* D **88** 093003 (Preprint 1308.1242)
[87] Jones D R T 2013 *Phys. Rev.* D **88** 098301 (Preprint 1309.7335)
[88] Jegerlehner F 2014 *Acta Phys. Polon.* B **45** 1215 (Preprint 1402.3738)
[89] Jegerlehner F 2013 The hierarchy problem of the electroweak Standard Model revisited, (Preprint 1305.6652)
[90] Branchina V and Messina E 2013 *Phys. Rev. Lett.* **111** 241801 (Preprint 1307.5193)
[91] Kobakhidze A and Spencer-Smith A 2013 *Phys. Lett.* B **722** 130 (Preprint 1301.2846)
[92] Enqvist E, Meriniemi T and Nurmi S 2014 JCAP **1407** (2014) 025 (Preprint 1404.3699)
[93] Ade P A R *et al.* [BICEP2 Collaboration] 2014 *Phys. Rev. Lett.* **112** 241101 (Preprint 1403.3985)
[94] Adam R *et al.* [Planck Collaboration] 2014 Planck intermediate results. XXX. The angular power spectrum of polarized dust emission at intermediate and high Galactic latitudes (Preprint 1409.5738)
[95] Froggatt C D and and Nielsen H B 1996 *Phys. Lett.* B **368** 96 (Preprint 9511371)
[96] Bezrukov F L and Shaposhnikov M 2008 *Phys. Lett.* B **659** 703 (Preprint 0710.3755)
[97] Hamada Y et al. 2014 *Phys. Rev. Lett.* **112** 241301 (Preprint 1403.5043)
[98] Bezrukov F and Shaposhnikov M 2014 Higgs inflation at the critical point (Preprint 1403.6078)
[99] Shaposhnikov M and Wetterich C 2010 *Phys. Lett.* B **683** 196 (Preprint 0912.0208)
[100] Dawson S et al 2013 Working Group Report: Higgs Boson (Preprint 1310.8361)
[101] Englert C et al. 2014 *J. Phys.* G **41** 113001 (Preprint 1403.7191)
[102] Bechtle P et al. 2014 Probing the Standard Model with Higgs signal rates from the Tevatron, the LHC and a future ILC (Preprint 1403.1582)
[103] Baak M *et al.* [Gfitter Group Collaboration] 2014 *Eur. Phys. J.* C **74** 3046 (Preprint 1407.3792)