

Fee versus royalty licensing in a Cournot duopoly with increasing marginal costs

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Abstract

We consider a symmetric homogeneous Cournot duopoly operating under increasing marginal costs. One of the firms owns a patented superior technology that reduces the intercept of the marginal cost function. We compare the incentives of the insider patentee to license the technology to the rival firm either through a fixed fee or through a royalty. We obtain that royalty licensing does not necessarily dominates in our setting: when decreasing returns are important, a royalty is superior only for small enough innovations, whereas a fixed fee is chosen for large innovations. Additionally, we show that our model is able to replicate the results in Wang (2002), which analyzes the same question in a differentiated duopoly with constant marginal costs.

KEYWORDS

fixed fee, increasing marginal costs, patent licensing, royalty

JEL CLASSIFICATION

L13, L41, L42

1 | INTRODUCTION

The literature on patent licensing is huge and most of the papers share the assumption that firms operate at constant marginal costs. Lately, some papers have relaxed this assumption by introducing non-constant returns in licensing models (Sen and Stamatopoulos (2009, 2016 and 2019), Mukherjee (2014) and Karakitsiou and Mavrommati (2013)). Mukherjee (2014) considers that firms produce with increasing linear marginal costs and deals with the case of a drastic process innovation.

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Considering “auction plus royalty” licensing, he obtains that the technology is licensed to all firms both for the cases of an outsider and an insider innovator. Licensing to more firms has the advantage of reducing costs and possibly the disadvantage of increasing (profit reducing) competition. However, this problem can be avoided by raising the royalty rate, which explains the result.

Sen and Stamatopoulos (2009) consider two firms that operate with linear marginal costs, but they allow for both the case of increasing and decreasing marginal costs. An external patentee has an innovation that reduces the intercept of the marginal cost and uses two-part tariff licensing contracts. The distinction between increasing and decreasing marginal costs is important because it leads to very different results in terms of diffusion of the technology and the form of the licensing contracts. With increasing marginal cost, the technology is licensed to both firms and the licensing contract includes a royalty. With decreasing marginal costs, however, the technology is licensed to only one firm with no royalty in the contract.

In this paper, we focus on the case of increasing linear marginal costs and the innovation also reduces its intercept. The difference is that we consider an internal patentee and restrict attention to two simple licensing policies, either a fixed fee or a royalty. We show that a royalty contract is chosen if the marginal cost is not very steep. This generalizes the result obtained by Wang (1998) for the case of constant marginal costs. However, if the marginal cost is steep enough, the patentee uses a royalty contract in equilibrium only for small enough innovations. Compared to fixed fee licensing, royalty licensing has the advantage of protecting the patentee's market profits by limiting competition. However, it has the disadvantage of introducing productive inefficiencies. Cost minimization would require that total output is split equally between the two firms but, under a royalty contract, the patentee produces more than the licensee in equilibrium. Under a fixed fee contract, costs are minimized, as both firms produce the same amount. It turns out that the positive effect on profits dominates the negative effect on costs only when the innovation is small.

Vishwasrao (2007) uses a data set including the foreign licensing agreements implemented by firms in India between 1989 and 1993. With the aim to explain the structure of licensing contracts (whether they include royalties, fixed fees, or a combination of both) in terms of industry, licensee, or licensor characteristics. It is very related to the objective of the present paper, studying whether firms use royalties or fixed fees in their licensing contracts. And indeed, in our theoretical model we obtain one of Vishwasrao (2007)'s predictions, namely, that “licensing contracts are more likely to use royalties when sales are relatively high” (abstract). As we have explained, the licensee in our model uses royalties instead of fees when the innovation is lower than a certain threshold. Vishwasrao (2007)'s prediction holds because this threshold increases with demand. Therefore, when demand increases (and sales are higher) the likelihood that the licensees use royalties also increases.

An interesting aspect in our framework is that many of the results of these new brand models can be traced back to the abundant literature of licensing models where firms sell differentiated goods. Both increasing marginal costs and product differentiation increase the incentives of a patent holder to license the new technology to a competitor. As stated in Katz and Shapiro (1985, p. 508): “In practice, there are many motivations for patent licensing. If the patent holder faces capacity constraints, or in general produces subject to increasing marginal costs, he may license his innovation to rivals to expand the scale of use of the new technology. Similarly, the innovator may have limited expertise in some markets where his discovery can reduce costs; licensing to the producer of a differentiated product is an example”. What we claim is that, in the linear case, we can import some of the results obtained in the licensing literature dealing with constant marginal costs and product differentiation, to the licensing literature with increasing marginal costs and homogeneous goods. For example, San Martín and Saracho (2021) show that they can replicate the results in Colombo and Filippini (2016), in a setting with homogeneous goods and increasing marginal costs, using their results in San Martín

and Saracho (2015), where differentiated goods with constant marginal costs are considered. In doing so, they identify some flaws in the proofs of Colombo and Filippini (2016).

To understand this result, we have to introduce some notation. Following Vives (2002), we can transform a duopoly model with linear demand and quadratic cost (linear marginal cost) into a model of product differentiation with linear demand and constant marginal cost. The former model leads to a firm's profit function:

$$(a - q_i - q_j) q_i - (c q_i + f q_i^2).$$

Manipulating the expression we have:

$$(a - (1 + f)q_i - q_j) q_i - c q_i = (1 + f) \left(\frac{a}{1 + f} - q_i - \frac{q_j}{1 + f} \right) q_i - c q_i,$$

which represents a firm's profit function in a model of product differentiation with constant marginal cost and a substitution coefficient

$$d = \frac{1}{1 + f}. \quad (1)$$

Notice that parameter c stands for the intercept of the (increasing) marginal cost in the former model and for a constant marginal cost in the latter.

Wang (2002) also compares the performance of fixed fee licensing and royalty licensing in a Cournot duopoly where one of the firms owns an innovation that reduces the intercept of the marginal cost. The distinctive assumptions between our model and Wang (2002) is that he considers that: i) firms sell differentiated goods with inverse demands $P_i = a - q_i - d q_j$ and ii) firms produce with constant marginal cost. Interestingly, we show that we can replicate the results in Wang (2002) with our model, just by replacing f with $\frac{1}{d} - 1$.¹ However, in doing the comparison, we realized that there is a flaw in the proof of Proposition 3 in Wang (2002) and that, once corrected, the region where fixed fee licensing is superior to royalty licensing is enlarged.

The rest of the paper is organized as follows. In the next Section, we present the model. Section 3 studies the situation without licensing. The optimal licensing contracts with fixed fee licensing and royalty licensing are studied in Sections 4 and 5 respectively. In Section 6 we compare, from the licensor's point of view, both licensing methods and derive the main results of the paper. Section 7 points out an empirical regularity that can be derived from the results of the model. In Section 8, we present two more papers with homogenous goods and increasing marginal costs whose results can be replicated using the ones obtained in Fauli-Oller et al. (2013), where differentiated goods and constant marginal costs are considered. Concluding remarks put the paper to an end. All the formal proofs have been relegated to an [Appendix](#).

2 | THE MODEL

We have two Cournot duopolists (firm 1 and firm 2) that compete in a homogenous good market with inverse demand $P = a - Q$. Firm 1 owns a patented superior technology that allows to produce the good with quadratic costs $C_1(q_1) = (c - \varepsilon)q_1 + f q_1^2$ whereas firm 2 produces with costs

¹ We obtain that expression by solving for f in condition (1).

$C_2(q_2) = cq_2 + fq_2^2$ with $a > c \geq \varepsilon > 0$ and $f \geq 0$. Firm 1 has to decide whether to license its superior technology to Firm 2. The licensing mechanisms available are fixed fee licensing and royalty licensing.

We analyze the following three stage game. In the first stage, firm 1 sets a fixed-fee or a royalty rate. In stage 2, firm 2 decides whether to accept the licensing contract. In the third stage, both firms compete à la Cournot with the cost functions inherited from the licensing stage. We will obtain the subgame perfect Nash equilibrium, solving the proposed game by backward induction.

3 | THE BENCHMARK CASE: NO LICENSING

Let us separate the cases of drastic and non-drastic innovations. If the innovation is non-drastic (which occurs when $\varepsilon < \varepsilon_D = (a - c)(1 + 2f)$),² simple computations show that the firms' equilibrium outputs and profits are given by:

$$q_1^{NL} = \frac{(a - c + \varepsilon)(1 + 2f) + \varepsilon}{3 + 4f(2 + f)}, \quad q_2^{NL} = \frac{(a - c)(1 + 2f) - \varepsilon}{3 + 4f(2 + f)};$$

$$\Pi_1^{NL} = (1 + f) \left(\frac{(a - c + \varepsilon)(1 + 2f) + \varepsilon}{3 + 4f(2 + f)} \right)^2, \quad \Pi_2^{NL} = (1 + f) \left(\frac{(a - c)(1 + 2f) - \varepsilon}{3 + 4f(2 + f)} \right)^2.$$

If the innovation is drastic (which occurs when $\varepsilon \geq \varepsilon_D = (a - c)(1 + 2f)$),³ firms' equilibrium outputs and profits are given by:

$$q_1^{NL} = \frac{a - c + \varepsilon}{2(1 + f)}, \quad q_2^{NL} = 0,$$

$$\Pi_1^{NL} = \frac{(a - c + \varepsilon)^2}{4(1 + f)}, \quad \Pi_2^{NL} = 0.$$

Notice that if $f = \frac{1}{d} - 1$, the above condition for an innovation to be drastic turns out to be $\varepsilon \geq \frac{(a-c)(2-d)}{d}$, which is exactly the condition for a drastic innovation in Wang (2002). From now on, we will remark the correspondence between our results and the ones in Wang (2002).

4 | FIXED FEE LICENSING

Under fixed fee licensing, the patent holder, firm 1, offers to the potential licensee, firm 2, a fixed fee contract F . If firm 2 does not accept the licensing contract, it obtains equilibrium profits Π_2^{NL} in the third stage. If firm 2 accepts the licensing contract, both firms face the same cost function $C_i(q_i) = (c - \varepsilon)q_i + fq_i^2$. In this case, equilibrium outputs and profits are given respectively by:

$$q_1^F = q_2^F = \frac{a - c + \varepsilon}{3 + 2f};$$

$$\Pi_1^F = \Pi_2^F = (1 + f) \left(\frac{a - c + \varepsilon}{3 + 2f} \right)^2.$$

² Following the patent licensing literature, we define an innovation as drastic in our setting if the monopoly price with the new technology is below (or equal to) the lowest possible marginal cost under the old technology (which would be c) so that the firm using the old technology would be driven out of the market.

³ To allow for the possibility of drastic innovations, given that $0 < \varepsilon \leq c$, we assume that $a - c$ is small enough.

In the second stage, firm 2 will accept any contract F such that:

$$\Pi_2^F - F \geq \Pi_2^{NL} \tag{2}$$

In the first stage, if firm 1 decides to license the technology, it will set the highest F that satisfies (2) that is, $\bar{F} = \Pi_2^F - \Pi_2^{NL}$. Therefore, fixed fee licensing is profitable for firm 1 if:

$$\Pi_1^F + \bar{F} = \Pi_1^F + \Pi_2^F - \Pi_2^{NL} \geq \Pi_1^{NL}. \tag{3}$$

Notice that (3) holds as long as technology licensing increases industry profits, because the previous condition can be rewritten as:

$$\Pi_1^F + \Pi_2^F - (\Pi_1^{NL} + \Pi_2^{NL}) \geq 0. \tag{4}$$

Looking for the conditions such that 4 holds, we can write the following proposition:

Proposition 1. *Under fixed fee licensing, firm 1 will license both non-drastic and drastic innovations if $f \geq 0.2071$. If $f < 0.2071$, firm 1 will only license non-drastic innovations satisfying $\epsilon \leq \frac{2(a-c)(1+2f)^2}{3-4f^2}$.*

The result that drastic innovations are licensed when decreasing returns are large enough may look surprising at first sight because, when the innovation is drastic, no licensing implies that firm 1 becomes a monopolist and one could think that industry profits are maximized under monopoly. This holds true under constant marginal costs⁴ but it is not necessarily true under decreasing returns. It can be seen that for convex enough cost functions, industry profits in a duopolistic industry may be higher than industry profits under monopoly.⁵ The intuition for the result in Proposition 1 is the following: when decreasing returns are important enough ($f \geq 0.2071$), the cost saving incentive associated to fixed fee licensing is so important that it suffices to explain its profitability, regardless of the size of the innovation. When decreasing returns are not so important however ($f < 0.2071$), the cost savings produced by fixed fee licensing have to be compared with its negative competition effect, which increases with the size of the innovation. As a result of the comparison, we get that only for small enough innovations the efficiency effect dominates, making fixed fee licensing profitable.

Notice that if $f = \frac{1}{d} - 1$, condition $f \geq 0.2071$ turns out to be $d \leq 0.8284$, which is the cut-off value for the substitution parameter in Proposition 1 in Wang (2002). And $\epsilon < \frac{2(a-c)(1+2f)^2}{3-4f^2}$ becomes $\epsilon < \frac{2(2-d)^2(a-c)}{8d-4-d^2}$, which coincides with condition (12) in Proposition 1 in Wang (2002). Therefore, Proposition 1 in our setting (with homogenous product and increasing marginal costs) replicates the result in Proposition 1 in Wang (2002) (in a setting with product differentiation and constant marginal costs).

⁴ Wang (1998) shows that under constant marginal costs, an insider innovator would never license a drastic innovation. It would prefer, instead, to become a monopolist in the market.

⁵ For example, the equilibrium industry profits in a market where n symmetric firms compete à la Cournot with inverse demand $P = a - Q$ and costs $C(q) = (c - \epsilon)q + fq^2$, are given by $\Pi(n) = \frac{n(a-c+\epsilon)^2(1+f)}{(1+2f+n)^2}$. In this setting, $\Pi(2) - \Pi(1) \geq 0$ if $f \geq 0.2071$. (see Amir (2003)).

5 | ROYALTY LICENSING

If firm 2 accepts a royalty contract r , its cost function will be $C_2(q_2) = (c - \varepsilon + r)q_2 + fq_2^2$. Notice that firm 2 will accept any royalty contract such that $r \leq \varepsilon$. In this case, the equilibrium outputs and profits will be given respectively by:

$$q_1^R = \frac{(a - c + \varepsilon)(1 + 2f) + r}{3 + 4f(2 + f)}, \quad q_2^R = \frac{(a - c + \varepsilon)(1 + 2f) - 2(1 + f)r}{3 + 4f(2 + f)} \text{ and}$$

$$\Pi_1^R = (1 + f) \left(\frac{(a - c + \varepsilon)(1 + 2f) + r}{3 + 4f(2 + f)} \right)^2, \quad \Pi_2^R = (1 + f) \left(\frac{(a - c + \varepsilon)(1 + 2f) - 2(1 + f)r}{3 + 4f(2 + f)} \right)^2.$$

In the first stage, firm 1 will maximize its total income $\Pi_1^R + rq_2^R$ subject to $r \leq \varepsilon$. The objective function $\Pi_1^R + rq_2^R$ is strictly concave in r , with a global maximum in:

$$r^* = \frac{(a - c + \varepsilon)(1 + 2f)(5 + 2f(5 + 2f))}{2(1 + f)(5 + 8f(2 + f))}. \quad (5)$$

Therefore, the optimal royalty will be r^* , if $r^* \leq \varepsilon$. This condition is satisfied whenever:

$$\varepsilon \geq \hat{\varepsilon}, \text{ where } \hat{\varepsilon} = (a - c) \left(1 - \frac{2f}{5 + 2f(11 + 4f(3 + f))} \right). \quad (6)$$

When (6) is not satisfied, the optimal royalty is ε . It is direct to check that (6) is satisfied for drastic innovations, which implies that the optimal royalty in this case is r^* . Again, if $f = \frac{1}{d} - 1$, (5) turns out to be the optimal royalty in Equation (20) in Wang (2002) and (6) coincides with condition (19) in Wang (2002).

It is very intuitive that royalty licensing is always strictly profitable for firm 1 in our setting (it always prefers royalty licensing to no licensing). If it does not license its superior technology, firm 1 obtains Π_1^{NL} . By setting $r = \varepsilon$, it obtains Π_1^{NL} plus the royalty revenues, which are positive if the innovation is non-drastic. If the innovation is drastic, one can check that $\Pi_1^R + rq_2^R$ evaluated at $r = r^*$ is higher than Π_1^{NL} , except for the case $f = 0$, in which case both coincide. Notice that if $f = 0$ (constant marginal costs) we are back to Wang (1998), where drastic innovations are not licensed.

6 | COMPARISON: FIXED FEE VERSUS ROYALTY LICENSING

We have to compare firm 1's total income under fixed fee licensing and under royalty licensing. This comparison is very easy when firm 1 does not license the technology under a fixed fee policy. Proposition 1 shows that this is the case when $f < 0.2071$ and $\varepsilon > \frac{2(a - c)(1 + 2f)^2}{3 - 4f^2}$. In this case, firm 1's total income is higher under royalty licensing than under fixed fee licensing for the same reason that, in the previous section, we showed that royalty licensing is always better than no licensing.⁶

For the rest of the cases, we have to distinguish three different regions in the domain of the difference between firm 1's total income under fee licensing and under royalty licensing, because it has different functional forms in each region:

- If condition (6) does not hold, the optimal royalty is $r = \varepsilon$.
- If condition (6) holds and the innovation is non-drastic, the optimal royalty is r^* .
- If the innovation is drastic, the optimal royalty is r^* .

⁶ The only exception being when $f = 0$ and the innovation is drastic.

We devote Proposition 2 to study the case of drastic innovations and Proposition 3 to the case of non-drastic innovations both when (6) holds and when (6) does not hold.

Proposition 2. *With a drastic innovation ($\varepsilon \geq \varepsilon_D$), fixed fee licensing is superior to royalty licensing for firm 1 if $f > 0.2693$, whereas royalty licensing dominates if $0 \leq f \leq 0.2693$.*

Again, if $f = \frac{1}{d} - 1$, condition $f > 0.2693$ turns out to be $d \leq 0.7878$ in Proposition 4 in Wang (2002). Nevertheless, discrepancies between our results and those in Wang (2002) appear in the comparison between fixed fee licensing and royalty licensing for the case of non-drastic innovations. The reason is that there is a mistake in Wang (2002) for the case in which the optimal royalty is r^* (condition (19) in Wang (2002) and condition (6) in our case). Wang (2002) claims that, in this case, royalty licensing is always superior to fixed fee licensing and we find that this is not always the case. Therefore, comparing our results with those in Wang (2002), we enlarge the region of parameters where fixed fee licensing dominates.

Proposition 3. *With a non-drastic innovation, if licensing does not occur under fixed fee licensing then royalty licensing is superior to fixed fee licensing. If licensing occurs under fixed fee licensing, two threshold values for the size of the innovation ε_1 and ε_2 exist such that:*

- i) *if (6) does not hold, then*
 - i1) *if $f \leq 0.3735$, royalty licensing is superior to fixed fee licensing for firm 1.*
 - i2) *if $f > 0.3735$, royalty licensing is superior to fixed fee licensing for firm 1 if $\varepsilon < \varepsilon_1$ and fixed fee licensing dominates otherwise.*
- ii) *if (6) holds, then*
 - ii1) *if $f > 0.3735$, fixed fee licensing is superior to royalty licensing for firm 1.*
 - ii2) *if $0.2693 < f \leq 0.3735$, royalty licensing is superior to fixed fee licensing for firm 1 if $\varepsilon < \varepsilon_2$ and fixed fee licensing dominates otherwise.*
 - ii3) *if $f \leq 0.2693$, royalty licensing is superior to fixed fee licensing for firm 1.*

It is interesting to remark that the results in the two previous propositions are consistent, in the sense that we could summarize both results stating that there exists a cut-off value for the size of the innovation such that below this cut-off, royalty licensing is superior to fixed fee licensing and above the cut-off, fixed fee licensing is superior to royalty licensing, whenever decreasing returns are important enough. Notice that, stated in this way, the previous sentence is true for the particular cases of drastic and non-drastic innovations. The following proposition formalizes the previous intuition, summarizing Propositions 2 and 3.

Proposition 4. *Two threshold values for the size of the innovation exist such that:*

- if $f > 0.3735$, royalty licensing is superior to fixed fee licensing for firm 1 if $\varepsilon < \varepsilon_1$ and fixed fee licensing dominates otherwise.*
- if $0.2693 < f \leq 0.3735$, royalty licensing is superior to fixed fee licensing for firm 1 if $\varepsilon < \varepsilon_2$ and fixed fee licensing dominates otherwise.*
- if $f \leq 0.2693$ royalty licensing is superior to fixed fee licensing for firm 1.*

Notice that when f is low, we are back to the result in Wang (1998) showing that under constant marginal costs, royalty licensing is superior to fixed fee licensing. However, if decreasing returns are important enough (when f is high enough), we obtain the clear-cut result that royalty licensing is superior to fixed fee licensing only for small enough innovations. Compared to fixed fee licensing, royalty licensing has the advantage of protecting firm 1's profits by limiting competition. However, it has the disadvantage of introducing productive inefficiencies. Cost minimization requires that total output is split equally between the two firms but, with royalty licensing, firm 1 ends up producing more than firm 2. With fixed fee licensing, however, costs are minimized because both firms produce the same amount. It turns out that the positive effect on profits dominates the negative effect on costs only when the innovation is small.

7 | SOME EMPIRICAL EVIDENCE

There are very few empirical studies that explain the structure of licensing contracts as a function of industry, licensee and licensor characteristics. In this case, we want to focus in the work of Vishwasrao (2007), where the determinants of the use of royalties, fixed fees or a combination of both in licensing contracts are studied. Reality is very complex and many of the determinants considered in Vishwasrao (2007) have no correspondence with the ingredients of our simple model as, for example, the assets of the firms, their level of R&D or their exports, among others. However, there is a particular causality that indeed can be checked in our model that refers to the fact that higher sales of the licensee makes more likely the use of contracts with only a royalty. In the author own words: "Higher existing sales would reduce the risk associated with royalty payments and would be more likely to result in contracts using royalties" (p. 752).

In our model, pre-licensing sales of the licensee are given by:

$$q_2^{NL} = \frac{(a - c)(1 + 2f) - \varepsilon}{3 + 4f(2 + f)},$$

and if $f > 0.3735$, royalties are used if

$$\varepsilon < \frac{(a - c)(1 + 2f)^2}{f(5 + 4f(2 + f))}.$$

So increases in $(a - c)$ increase both the existing sales of the licensee and the region where royalty contracts are used. Therefore, we would observe a positive correlation between the sales of the licensee and the likelihood that a royalty contract is used, as predicted in Vishwasrao (2007).

8 | REPLICATING RESULTS IN LICENSING MODELS WITH INCREASING MARGINAL COSTS USING RESULTS OBTAINED IN LICENSING MODELS WITH DIFFERENTIATED GOODS

In the present paper, we have considered a duopoly model where the firms, operating under increasing marginal costs, sell a homogenous good. One of the firms owns a patented process innovation that reduces the intercept of the marginal cost function. We have shown that our results replicate the ones obtained in Wang (2002), where the same question is analyzed in a differentiated duopoly with

constant marginal costs. With this aim, we use a formula that relates the slope of the marginal cost in our model with the substitution coefficient of the demand with differentiated goods in Wang's model.

In this section, we present two more examples where this correspondence between licensing models with differentiated goods and constant marginal costs and models with homogeneous goods and increasing marginal costs holds. The reference paper in this section will be Fauli-Oller et al. (2013), which analyzes the optimal two-part tariff contract for the licensing of a cost reducing innovation to an *n*-firm oligopolistic industry producing differentiated goods, whose demands are given by:

$$p_i = a - q_i - \gamma \sum_{j \neq i} q_j.$$

where $\gamma \in [0, 1]$. The innovation allows to reduce the (constant) marginal production cost from c to $c - \varepsilon$. Both the cases of an external and an internal patentee are considered. We are going to show that the results obtained in Fauli-Oller (2013) can be used to replicate the results in models of licensing with homogenous goods and increasing marginal costs. Let us present the two examples.

8.1 | Comparison with Sen and Stamatopoulos (JITE 2019)

Sen and Stamatopoulos (2019) study a duopoly market, formed by firms 1 and 2, serving a linear demand. The pre-licensing cost functions of firms 1 and 2 are given by $C(q_1) = (c - \varepsilon)q_1 + \left(\frac{b}{2}\right)q_1^2$ and $C(q_2) = cq_2 + \left(\frac{b}{2}\right)q_2^2$ respectively. They analyze the optimal two-part tariff licensing contract that firm 1 can use to license the cost-reducing innovation to its rival firm.

It is direct to see that, in this case, the equivalence between the substitution coefficient γ in Fauli-Oller et al. (2013) and the slope of the marginal cost b in Sen and Stamatopoulos (2019) is given by:

$$\gamma = \frac{1}{1 + \frac{b}{2}} = \frac{2}{2 + b}.$$

The optimal royalties in Fauli-Oller et al. (2013) for the case of a duopoly with one of the firms being an internal patentee (the result is included in Proposition 2, taking into account that, in the duopoly case, $r_1^I = r_2^I$) are given by:

- a) if $\varepsilon \leq e(\gamma)$, the optimal royalty is ε , where $e(\gamma) = \frac{(a-c)(-2+\gamma)^2\gamma}{8-4\gamma-2\gamma^2-\gamma^3}$.
- b) if $\varepsilon > e(\gamma)$, the optimal royalty is $r(\gamma) = \frac{(a-c-\varepsilon)(-2+\gamma)^2\gamma}{8-6\gamma^2}$.

It is straightforward to show that in the optimal two-part tariff licensing contract that can be found in point (III) of Proposition 1 in Sen and Stamatopoulos (2019), the optimal royalty can be written as follows:

- (a) if $\varepsilon \leq e\left(\frac{2}{2+b}\right) = \frac{(b+1)^2(a-c)}{b^3+5b^2+7b+1}$, the optimal royalty is ε .
- (b) if $\varepsilon > e\left(\frac{2}{2+b}\right)$, the optimal royalty is $r\left(\frac{2}{2+b}\right) = \frac{(b+1)^2(a-c+\varepsilon)}{(b+2)(b^2+4b+1)}$.

8.2 | Comparison with Mukherjee (Journal of Economics 2014)

In this paper, an innovator I owns a patent on a new product and there are n symmetric potential licensees, which can produce the product if they obtain a license from I . The innovation is licensed by means of an *auction plus royalty* policy, where the licensor determines the number of licenses to auction (possibly with a minimum bid) and also announces the royalty, so that the up-front fixed-fee that a licensee pays is its winning bid (in this sense, this licensing policy is equivalent to the two-part tariff policy in Fauli-Oller et al. (2013)). To capture zero opportunity costs of the licensees, he assumes that the potential licensees have no existing technologies to produce the new good invented by the innovator. In terms of Fauli-Oller et al. (2013), this means that the innovation is so important that non-licensees do not produce in any circumstance and, therefore, their outside option is zero. Both the cases of an outsider and an insider innovator are considered also in Mukherjee (2014). Firms produce homogeneous goods with a linear demand $P = a - q$. If granted a license, the i th licensee can produce the new product with total (convex) costs $C_i(q_i) = \frac{cq_i^2}{2}$, $i = 1, 2, \dots, n$. As in Fauli-Oller et al. (2013), the author proves that for both the cases of an external and an internal patentee and regardless of the number of firms, the innovator always licenses the innovation to all firms in the industry.

We next show that we can replicate the results in Mukherjee (2014), departing from the differentiated goods model in Fauli-Oller et al. (2013) by assuming in the latter paper i) that the innovation is so important that non-licensees do not produce and ii) that the licensees produce with zero marginal cost (which requires that $c - \varepsilon = 0$).⁷ In the transformation between the two models, the equivalence between the substitution coefficient γ in Fauli-Oller et al. (2013) and the slope of the marginal cost c in Mukherjee (2014) is given by:

$$\gamma = \frac{1}{1 + \frac{c}{2}} = \frac{2}{2 + c}.$$

This transformation will allow us to replicate the results in Mukherjee (2014) departing from the results in Fauli-Oller et al. (2013). In particular, for the case of an external patentee, the optimal royalty in Fauli-Oller et al. (2013) for the particular case where the innovation is so important that non-licensees do not produce ($\varepsilon \geq \varepsilon_1$) and where $c = \varepsilon$ (see their Proposition 1), is given by:

$$r^E(\gamma) = \frac{a(n-1)\gamma}{2 + 2\gamma(n-1)}.$$

It is straightforward to show that the optimal royalty in the optimal “auction plus royalty” licensing contract that can be found in Proposition 1 in Mukherjee (2014), can be obtained as $r^E\left(\frac{2}{2+c}\right) = \frac{a(n-1)}{c+2n}$.

For the case of an internal patentee, one should take into account that Fauli-Oller et al. (2013) considers the case of one internal patentee and $n - 1$ potential licensees, whereas Mukherjee (2014) considers one internal patentee and n potential licensees. In particular, the optimal royalty in Fauli-

⁷ Recall that this second assumption comes from the fact that if we depart from a model of homogeneous goods with licensees' convex costs $hq_i + \frac{cq_i^2}{2}$, and transform it into a model of product differentiation, parameter h stands for the intercept of the (increasing) marginal cost in the former model and for a constant marginal cost in the latter (see the Introduction section above). Given that in Mukherjee (2014) $h = 0$, this implies that we must assume $c = \varepsilon$ in Fauli-Oller et al. (2013), so that licensees produce with marginal cost equal to zero.

Oller et al. (2013) for the case of an internal patentee, a large innovation ($\varepsilon \geq \varepsilon_2^I$) and $c = \varepsilon$ (see their Proposition 2), is given by:

$$r^I(n, \gamma) = \frac{a(-2 + \gamma)^2 \gamma(n - 1)}{8 - 2\gamma(8 + 3\gamma(n - 1) - 4n)}.$$

To get the equivalence between the royalties in both models we have to compute the optimal royalty in the case where there are, instead of n , $n + 1$ potential licensees. It is then straightforward to show that the optimal royalty in the optimal “auction plus royalty” licensing contract that can be found in Proposition 2 in Mukherjee (2014), can be obtained as:

$$r^I\left(n + 1, \frac{2}{2 + c}\right) = \frac{an(1 + c)^2}{(2 + c)(n + c(2 + c + 2n))}.$$

9 | CONCLUSION

In this paper, we have taken up the classical question in the patent licensing literature about the relative performance of royalties and fixed fees, in a new setting where the firms' marginal costs are increasing. When decreasing returns are not very important, we obtain the same result as under constant marginal costs (Wang (1998)), namely, that royalty licensing is superior to fixed fee licensing. However, when decreasing returns become important, we get that royalties are superior to fixed fee licensing only for small innovations, whereas for large innovations, fixed fee licensing dominates.

Additionally, we show that our model is able to replicate the results in Wang (2002), which analyzes the same question in a differentiated duopoly with constant marginal costs and also replicate, as an extension of the model, the results in the papers by Sen and Stamatopoulos (2019) and Mukherjee (2014), which analyze models with homogeneous goods and increasing marginal costs, departing from the model with differentiated goods and constant marginal costs used in Fauli-Oller et al. (2013).

We have considered in the present paper a case in which the innovation reduces the intercept of the marginal cost. A possible avenue for future research could be to extend the analysis to the case where the innovation, instead of the intercept, reduces the slope of the marginal cost and check whether the result that royalties are superior to fixed fee licensing only for small innovations still holds in this new setting. This is part of our future research agenda.

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APPENDIX

Proof of Proposition 1

Let us find conditions such that (4) holds. Let us start with the case of a non-drastic innovation. In this case, (4) results in:

$$\frac{\varepsilon(1+f)(2(a-c)(1+2f)^2 - \varepsilon(3-4f^2))}{(3+4f(2+f))^2} \geq 0. \quad (\text{A1})$$

Simple inspection shows that (A1) holds if $3-4f^2 \leq 0$ that is, if $f \geq 0.8660$. Now, if $f < 0.8660$, it holds if $\varepsilon < \frac{2(a-c)(1+2f)^2}{3-4f^2}$. It is direct to see that if $f \geq 0.2071$, then $\frac{2(a-c)(1+2f)^2}{3-4f^2} \geq (a-c)(1+2f) > \varepsilon$ holds (notice that the second inequality is just the definition of the non-drastic case), which implies

that all non-drastic innovations are licensed. Finally, if $f < 0.2071$, only small non-drastic innovations satisfying $\varepsilon < \frac{2(a-c)(1+2f)^2}{3-4f^2}$ are licensed.

For the case of a drastic innovation, (4) results in:

$$(a - c + \varepsilon)^2 \left(\frac{2(1 + f)}{(3 + 2f)^2} - \frac{1}{4(1 + f)} \right) \geq 0.$$

It can be checked numerically that the previous condition holds if $f \geq 0.2071$.

Proof of Proposition 2

Let $T(\varepsilon)$ be the difference between firm 1's total income under fixed fee licensing and under royalty licensing when the innovation is drastic, namely, when $\varepsilon \geq (a - c)(1 + 2f)$. It is direct to check that $\text{sign}\{T(\varepsilon)\} = \text{sign}\{-5 + 4f + 40f^2 + 48f^3 + 16f^4\}$. Numerical computations lead to the result.

Proof of Proposition 3

First, we prove point (i), where $\varepsilon < \hat{\varepsilon}$. We define $P(\varepsilon) = \frac{\varepsilon(- (a-c)(1+2f)^2 + \varepsilon f(5+4f(2+f)))}{(3+4f(2+f))^2}$ as the difference between firm 1's total income under fee licensing and under royalty licensing for $\varepsilon < \hat{\varepsilon}$ (so that the optimal royalty rate is $r = \varepsilon$). Expression $P(\varepsilon)$ is positive iff $\varepsilon > \varepsilon_1$, with $\varepsilon_1 = \frac{(a-c)(1+2f)^2}{f(5+4f(2+f))}$. It can be verified numerically that if $f \leq 0.3735$, then $\varepsilon_1 \geq \hat{\varepsilon}$ and, therefore, given that we are in the region $\varepsilon < \hat{\varepsilon}$, we have that $\varepsilon < \varepsilon_1$ holds, so $P(\varepsilon)$ is negative and royalty licensing is superior to fixed fee licensing. This proves point (i1). Now, if $f > 0.3735$, $P(\varepsilon)$ is negative if $\varepsilon < \varepsilon_1$ and positive if $\varepsilon_1 \leq \varepsilon < \hat{\varepsilon}$. This proves point (i2).

Next, we prove point (ii), where $\varepsilon \geq \hat{\varepsilon}$. We define $B(\varepsilon) = \frac{2(a-c+\varepsilon)^2(1+f)}{3+2f^2} - \frac{(a-c+\varepsilon)^2(1+2f)^2(5+4f)^2}{4(1+f)(5+8f(2+f))^2} - \frac{(1+f)(c+\varepsilon+2cf-a(1+2cf))^2}{(3+4f(2+f))^2} - \frac{(a-c+\varepsilon)^2 f(1+2f)(5+2f(5+2f))}{(1+f)(5+8f(2+f))^2}$ as the difference between firm 1's total income under fixed fee licensing and under royalty licensing if $\varepsilon \geq \hat{\varepsilon}$ (so that the optimal royalty rate is $r = r^*$). In a Lemma in the Appendix, we prove that $B'(\varepsilon) > 0$ if $f > 0.2693$. Point (ii1) follows from the Lemma (see below) and from the fact that (i2) implies $P(\hat{\varepsilon}) > 0$ and, given that by definition $P(\hat{\varepsilon}) = B(\hat{\varepsilon})$, it follows that $B(\hat{\varepsilon}) > 0$. Point (ii2) follows from the Lemma and the fact that (i1) implies $P(\hat{\varepsilon}) < 0$, and then, $B(\hat{\varepsilon}) < 0$ holds, and that Proposition 2 implies $T(\varepsilon_D) > 0$, and then, $B(\varepsilon_D) > 0$, because by definition $T(\varepsilon_D) = B(\varepsilon_D)$. Therefore ε_2 exists, such that $\hat{\varepsilon} < \varepsilon_2 < \varepsilon_D$ and $B(\varepsilon_2) = 0$. Finally, proving point (ii3) requires a different approach, because the Lemma does not apply. For $f \leq 0.2693$, we check numerically that $B''(\varepsilon) < 0$ (so function $B(\varepsilon)$ is concave in that interval) and that $B(\varepsilon_3) < 0$, where ε_3 is such that $B'(\varepsilon_3) = 0$ (that is, ε_3 is the value where function $B(\varepsilon)$ reaches its maximum).

Lemma 1

$B'(\varepsilon) > 0$ if $f > 0.2693$ (and the innovation is non-drastic).

Proof of Lemma 1

$B'(\varepsilon) = (a - c)g(f) + e h(f)$, where

$$g(f) = \frac{4(1+f)}{(3+2f)^2} + \frac{2(1+f)(1+2f)}{(3+4f(2+f))^2} - \frac{(1+2f)^2(5+4f)^2}{2(1+f)(5+8f(2+f))^2} - \frac{2f(1+2f)(5+2f(5+2f))}{(1+f)(5+8f(2+f))^2}$$

$$h(f) = \frac{4(1+f)}{(3+2f)^2} - \frac{2(1+f)}{(3+4f(2+f))^2} - \frac{(1+2f)^2(5+4f)^2}{2(1+f)(5+8f(2+f))^2} - \frac{2f(1+2f)(5+2f(5+2f))}{(1+f)(5+8f(2+f))^2}$$

One can check numerically that $g(f) > 0$ for all non-negative f and that $h(f) > 0$ if $f > 0.7219$. Then $B'(\varepsilon) > 0$ if $f > 0.7219$. For $f < 0.7219$, $B'(\varepsilon) > B'((a - c)(1 + 2f))$ because we are considering non-drastic innovations. One can check numerically that $B'((a - c)(1 + 2f)) > 0$ if $f > 0.2693$.

Cut-off values

$$\varepsilon_1 = \frac{(a - c)(1 + 2f)^2}{f(5 + 4f(2 + f))}$$

$$\varepsilon_2 = \frac{(a - c)(1 + 2f)(4(1 + f)^2(5 + 8f(2 + f))\sqrt{\frac{(-5 + 4f(1 + 2f(5 + 2f(3 + f))))}{5 + 8f(2 + f)}} - (15 + 2f(49 + 2f(57 + 4f(16 + f(9 + 2f))))))}{-25 + 8f(-15 + 2f(-9 + f(6 + f(21 + 4f(4 + f)))))}$$

$$\varepsilon_3 = \frac{(a - c)(1 + 2f)(15 + 2f(49 + 2f(57 + 4f(16 + f(9 + 2f)))))}{25 - 8f(-15 + 2f(-9 + f(6 + f(21 + 4f(4 + f)))))}$$