INVESTIGATING CONVERGENCE OF A CAPACITY PLANNING MODEL USING GENERALIZED BENDERS'S DECOMPOSITION

by

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ABSTRACT

This paper presents two major methods used to speed up the rate of convergence of a long-range capacity expansion planning model for electric utilities which utilizes Generalized Benders's Decomposition. The first method was to add initial capacity constraints to the master problem. The second required the disaggregation of the reliability constraint from one per iteration to one for every infeasible time period.

Both methods tried yielded significant improvements in the algorithm's convergence rate. A major factor which needed to be considered during these runs, however, was the computational error introduced into the calculation of the Lagrange multipliers. This paper shows that these numerical inaccuracies result from the piece-wise linear representation of the equivalent load duration curves used in the operating subproblem. These errors create small nonconvexities within the linear program, causing such inconsistencies as the same data yielding different solutions when run on different versions of the algorithm.

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CHAPTER ONE

Long-range planning for electric utilities has changed dramatically within the past decade. Uncertainties in supply and demand, new technologies, and environmental, financial, and regulatory considerations all conspire to make the planning environment more complicated than the one in which straightforward cost-minimization techniques had been previously applied quite successfully. One approach recently proposed to help the utility planner with long-range investment decisions is Generalized Benders' Decomposition, hereinafter referred to as GBD. This mathematical programming technique was developed by Benders [3] in 1962 and applied to the electric utility planning problem by Bloom [4] in 1978. It is being incorporated as one of five analysis options into a modular software package currently under development by MIT and Stone & Webster Engineering Corporation through Electric Power Research Institute (EPRI) funding.

The GBD approach holds great promise for the utility planner as one of its central propositions includes the satisfaction of a probabalistic reliability constraint. Since the benefit of every plant is measured by the amount it contributes (relative to its cost) toward meeting this constraint, all plants, including time-dependent generation sources (such as solar and wind) are treated equally within this formulation. Other advanced features which may be treated include load modification strategies, preventive maintenance, and storage plants. Financial and environmental constraints may also be easily incorporated into the GBD model.

The original algorithm at MIT which implemented GBD made use of two existing software packages, SYSGEN [11] and GEM [32]. SYSGEN is a production costing model which utilizes Booth-Baleriaux probabilistic simulation (to be discussed in greater detail in Chapter Two). GEM is a linear program which can explicitly handle environmental constraints. It was originally designed to be run iteratively with SYSGEN, with plant capacity factors being passed back and forth; however, it was found not to converge to the true optimum. GBD is also an iterative procedure; however, after a run of SYSGEN, Lagrange multipliers on operating cost and unserved energy are passed back to its linear program (called the master problem), not capacity factors. GBD starts with a few or no constraints in its master, constructing others as needed throughout the procedure. Convergence at every iteration can be measured, as the algorithm generates both a lower and an upper bound. These may be used to terminate the procedure short of optimality, with known error bounds.

It was empirically found that the GBD algorithm originally implemented at MIT: a) did not converge very quickly to the optimal solution, and b) converged to different (although close) solutions. This depended on factors such as initial constraints used, and representation of the feasibility constraint (to be discussed in Chapter Five), which requires different numbers of iterations necessary to reach optimality. Work was obviously needed investigating ways of speeding up convergence, and understanding why these numerical differences were occurring. This thesis presents the author's work in these areas.

Chapter Two reviews the basic formulas necessary for an understanding of GBD. (For greater in-depth discussions, the reader is invited to consult the references by Bloom [4,5,6,7] on GBD and by Finger [10,11] on SYSGEN.) Chapter Three talks about the upper and lower bounds used in this algorithm. Chapter Four discusses numerical discrepancies encountered, while Chapter Five presents computational results of different approaches tried to speed up convergence. Chapter Six contains conclusions and recommendations for further areas of investigation. The technical details of one of these recommendations, subgradient optimization, are handled in Appendix A. Appendix B presents results of a simple problem worked by hand to verify results from the SYSGEN code, while Appendix C does the same for an example concerning the upper bound calculation. Lastly, Appendix D contains information on computer times and storage requirements for the runs presented in Chapter Five.

CHAPTER TWO

The problem faced by electric utilities of planning long-range capacity expansion may be broken down into two parts. The first part determines the optimal plant capacities to be installed in every year of the planning horizon, while the second part calculates the expected operating costs and reliability levels associated with any given set of plant capacities. This natural decomposition structure is efficiently exploited by the GBD algorithm, which proceeds in an analogous manner. The master problem performs the first task, while the operating subproblems handle the second. Mathematically, this planning problem can be written as a two-stage optimization problem.

$$\min_{\substack{\underline{X} \geq 0 \\ \underline{X} \in \Omega}} \underline{C}'\underline{X} + \sum_{t=1}^{T} \quad \min_{\underline{Y}_t \geq 0} EF_t(\underline{Y}_t)$$

$$\text{s.t.} \quad EG_t(\underline{Y}_t) \leq \varepsilon_t$$

$$\underline{Y}_t \leq \delta_t \underline{X}$$

where:

$\underline{X}$ = vextor of plant installed capacities (MW)

$\underline{C}$ = vector of present-value capital costs ($/MW)

$\underline{Y}_t$ = vector of plant operating capacities in the time period t (MW)

ε_t = desired reliability level in time period t (MWH)

$EF_t(.)$ = present-value expected operating cost function in time period t ($)

$EG_t(.)$ = expected unserved energy function in time period t (MWH)

T = number of years in the planning horizon (which may include an extension period)

δ_t = matrix which sorts plants into economic loading order

Ω = set of all $\underline{X}$ for which all subproblems are feasible

The functions $EF_t(.)$ and $EG_t(.)$ are highly nonlinear, making this a difficult problem to solve by ordinary methods. However, once the master problem selects a trial set of plant capacities, $\underline{X}$ becomes fixed, and the inner minimization becomes a function of $\underline{Y}_t$ only. This inner minimization is the operating subproblem, which calculates operating costs and reliabilities. Since customer demand is a random variable, as well as plant outages, probabilistic simulation must be employed within this subproblem. A load duration curve (LDC) is constructed which gives the probability that customer demand is greater than or equal to any given x (the LDC is thus a reverse cumulative probability distribution). When a plant is loaded onto the system, its failures create additional load which must be made up by later plants. Convolving plant outages with customer demand results in an equivalent load duration curve (ELDC), which gives the probability that the load on the system, from both customers and plant outages, is greater or equal to x. The ELDC is computed recursively as follows:

$$G_{i+1}(x) = p_i G_i(x) + q_i G_i(x - Y^i) \qquad i = 1,\ldots,I$$

and

$$G_1(x) = G(x) = \text{original system LDC}$$

where

p_i = availability of unit i

q_i = forced outage rate of unit i

$\quad = 1 - p_i$

Y_i = operating capacity of unit i

I = number of units in the economic loading order.

This recursive formula is the heart of Booth-Baleriaux probabilistic simulation [8,9]. Using the ELDC, one determines the amount of energy generated by the i^{th} unit as the amount of area which that plant cuts out from the ELDC times its availability:

$$E^i = p_i \cdot \int_{U^{i-1}}^{U^i} G_i(x)\, dx$$

where

$$U^{i-1} = \text{loading point for unit } i = \sum_{i=1}^{i-1} Y^i$$

To find a unit's operating cost, one merely multiplies the energy generated by the i^{th} unit by its operating cost F^i (in $/MWH). It is obvious that to minimize operating costs, plants should be loaded in order of increasing operating cost:

$$F^1 \leq F^2 \leq \ldots \leq F^I$$

This is known as the <u>economic loading order</u>, or sometimes, <u>merit order</u>.

SYSGEN [10,11] is a code which performs the above probabilistic simulation. To fit into the GBD context, a reliability constraint had to be imposed. While loss-of-load probability (the value of the ELDC after all units have been loaded) was traditionally the measure of reliability used by utility planners, it has been argued [38] that expected unserved energy is a better indicator of the loss suffered, as it reflects the magnitude of the loss as well as its frequency:

$$\text{expected unserved energy} = \text{initial customer demand, } E - \text{energy generated by I plants, } \sum_{i=1}^{I} E^i \Rightarrow EG(\underline{Y}) = \int_{U^I}^{\infty} G_{I+1}(x)dx$$

The inner minimization of the two-stage optimization problem may now be written as:

$$\min_{\underline{Y}} \quad EF(\underline{Y}) = \sum_{i=1}^{I} F^i p_i \int_{U^{i-1}}^{U_i} G_i(q)dq$$

$$\text{s.t.}$$

$$EG(\underline{Y}) = \int_{U^I}^{\infty} G_{I+1}(q)dq \leq \varepsilon$$

$$0 \leq Y^i \leq X^i$$

where the index on time has been omitted for clarity. The optimal solution to this subproblem requires units to be loaded in economic loading order, and operated to their maximum capacity up to that unit in the loading order which just meets the unserved energy constraint. This unit is known as the <u>marginal plant</u>, and its operating capacity, Y^n, is calculated such that:

$$\int_{U^{n-1}+Y^n}^{\infty} G_{n+1}(x)dx = \varepsilon.$$

Any plants above the marginal plant are not operated at all.

Once the optimal solution for each time period is found, Lagrange (dual) multipliers also have to be computed for every plant. These multipliers measure the value associated with a small change in capacity X^i. Define the Lagrangian function for the subproblem to be:

$$L(\underline{Y},\underline{\lambda},\pi) = EF(\underline{Y}) + \pi[EG(\underline{Y})-\varepsilon] + \underline{\lambda}[\underline{Y}-\underline{X}]$$

where π is the dual multiplier associated with the reliability constraint and $\underline{\lambda}$ is the vector of dual multipliers associated with the capacity constraints. Since all the constraints are inequalities, π and $\underline{\lambda}$ must be nonnegative. The multiplier π represents the marginal cost of decreasing the unserved energy level ε , and is equal to the operating cost of the marginal plant. The multipliers λ^i represent the marginal operating cost reduction from increasing the capacity X^i. The values for these multipliers are obtained by referring to the Kuhn-Tucker conditions:

(a) $$\frac{\partial}{\partial Y^i} L(\underline{Y},\underline{\lambda},\pi) \geq 0 \qquad i=1,\ldots,I$$

with equality if $Y^i > 0$

(b) $$\pi[EG(\underline{Y})-\varepsilon] = 0$$

$$\underline{\lambda}[\underline{Y}-\underline{X}] = 0$$

Therefore it follows that:

$$\hat{\lambda}^i = \begin{cases} -\frac{\partial}{\partial Y^i} EF(\hat{\underline{Y}}) - \hat{\pi}\frac{\partial}{\partial Y^i} EG(\hat{\underline{Y}}) & i < n \\ 0 & i \geq n \end{cases}$$

$$\hat{\pi} = F^n$$

when $\underline{Y} = \hat{\underline{Y}}$

The above has assumed that the solution to the operating subproblem is feasible. If, however, it is infeasible (meaning that after all the plants have been loaded, the unserved energy constraint is still not met), the inner optimization is replaced by its dual:

$$\max_{\underline{\lambda},\pi \geq 0} \; \min_{\underline{Y} > 0} \{EF(\underline{Y}) + \pi[EG(Y)-\varepsilon] + \underline{\lambda}\,[\underline{Y}-\underline{X}]\}$$

Infeasibility in the primal subproblem is equivalent to unboundedness in this dual subproblem. This means that for an infeasible solution, the maximum value is driven to infinity. To constrain the set to be comprised of only feasible solutions is equivalent to the following condition:

$$\max_{\underline{\mu},\nu \geq 0} \; \min_{\underline{Y} > 0} \{\nu[EG(\underline{Y})-\varepsilon] + \underline{\mu}\,[\underline{Y}-\underline{X}]\} \leq 0 \quad .$$

where ν and $\underline{\mu}$ are dual multipliers generated only when the subproblem is feasible. The multiplier ν can be taken equal to 1 with no loss of generality, while the multipliers $\underline{\mu}$ measure the change in unserved energy (reliability) due to small changes in capacities:

$$\mu^i = \frac{\partial EG}{\partial Y^i} \qquad i = 1,\ldots,I$$

Once the subproblem has finished computing operating costs, reliabilities, and shadow prices, this information is sent to the master problem to create additional constraints, referred to as Benders' cuts. The master problem is a linear program of the form:

$$\min_{Z,\underline{X}} Z$$

s.t.

$$Z \geq \underline{C}'\underline{X} + \sum_{t=1}^{T} \{EF_t(\hat{\underline{Y}}_t) + \hat{\pi}_t(EG_t(\hat{\underline{Y}}_t) - \varepsilon_t) + \hat{\underline{\lambda}}_t(\hat{\underline{Y}}_t - \delta_t\underline{X})\}$$

$$\forall \hat{\pi}_t, \hat{\underline{\lambda}}_t \geq 0$$

$$0 \geq \sum_{t=1}^{T} \min_{\hat{\underline{Y}}_t \geq 0} \{\hat{\nu}_t(EG_t(\underline{Y}_t) - \varepsilon_t) + \hat{\underline{\mu}}_t(\hat{\underline{Y}}_t - \delta_t\underline{X})\}$$

$$\underline{X} \geq 0 \qquad \forall \hat{\nu}_t, \hat{\underline{\mu}}_t \geq 0$$

Using the following complementary slackness conditions:

$$\hat{\pi}_t[EG_t(\hat{\underline{Y}}_t) - \varepsilon_t] = 0$$

$$\hat{\underline{\lambda}}_t(\hat{\underline{Y}}_t - \delta_t\hat{\underline{X}}) = 0$$

$$\hat{\underline{\mu}}_t(\hat{Y}_t - \delta_t\hat{\underline{X}}) = 0$$

and $\hat{\nu}_t = 1$, the master problem may be re-written as

$$\min_{Z,\underline{X}} Z$$

s.t.

$$Z \geq \underline{C}'\underline{X} + \sum_{t=1}^{T} [EF_t(\underline{Y}_t^k) + \underline{\lambda}_t^k\delta_t(\underline{X}^k - \underline{X})] \qquad k=1,\ldots,K$$

$$\sum_{t\varepsilon T_k'} [EG_t(\underline{Y}_t^k) + \underline{\mu}_t^k\delta_t(\underline{X}^k - \underline{X})] \leq \sum_{t\varepsilon T_k'} \varepsilon_t \qquad k=1,\ldots,K$$

$$\underline{X} \geq 0$$

where:

k = iteration number

T_k' = set of time periods for which the subproblem was infeasible during iteration k.

The above master problem is a relaxed version of the original planning problem (i.e., it does not contain all of the constraints present in the original problem). Therefore, any set of capacities $\underline{X}$ which solves the master problem must also solve the original problem. If, however, the trial Z,$\underline{X}$ generated by the master problem violate some original constraints not yet included in the master, the most violated of these are generated and added to the master problem. The cost constraints in the master may be regarded as linear approximations to the original cost function. As this function is convex, the approximations are tangent to it from below. Similarly, the reliability constraints in the master may be regarded as linear approximations to the original feasible region. Since GBD is an outer linearization - relaxation procedure [13], these constraints form an outer approximation to the feasible region. Therefore, trial solutions will always be infeasible in the subproblem until optimality is reached, as feasibility is approached from the outside.

In summary, the benefits of formulating the long-range capacity planningproblem into the GBD context are that the GBD algorithm needs to solve only a smaller mathematical program than this original problem, and the smaller program is also linear. Also, all of the nonlinearities are confined to the operating subproblems, which may be readily solved without recourse to any complicated nonlinear optimization technique. The algorithm iterates between the master problem sending trial capacities to the subproblem, and the subproblem sending back costs, reliabilities, and Lagrange multipliers to the master. Optimality is reached when a given set of $\underline{X}$ satisfies all the constraints. However, the procedure may be terminated when the user deems that the upper and lower bounds on the cost of the optimal solution are sufficiently close.

CHAPTER THREE

This chapter is devoted to the discussion of bounds used in the GBD algorithm. Section 3.1 deals with the derivation of the lower bound, while Section 3.2 discusses issues involved with the upper bound computation.

Section 3.1

The lower bound is readily extracted from the problem formulation. Since the master problem is a relaxed version of the original capacity planning problem, it is less constrained than the original problem; therefore, the value of the objective function (hereafter referred to as Z) in the master should be less than the corresponding Z in the original problem. This value of Z is also lower than the total cost (TC) of the trial solution for that iteration, as convexity implies that the linearized costs are always less than the actual ones:

$$Z \leq TC = CX + \Sigma EF$$

Therefore, Z is the lower bound (LB) generated by this algorithm. As more Benders' cuts are constructed, this value of the objective function must increase (or stay the same) as the problem becomes increasingly constrained. When Z equals the total cost of the trial solution (implying that the newly-generated constraint satisfies all of the remaining constraints in the original problem) and the reliability constraint is satisfied, then the trial solution for that iteration is optimal, and Z equals minimum total cost for that problem. In the results presented in Chapter Five, optimality is considered reached when both Z and TC remain unchanged for two successive iterations. This is done as these two figures do not always equal each other exactly, for numerical reasons which are explained in Chapter Four.

Section 3.2

Now upper bounds must be considered. It is clear that the cost of any feasible solution is an upper bound on the optimal cost. The problem becomes one of generating a feasible solution from an infeasible one, as the master problem generally selects infeasible trial capacity plans. The original upper bound calculation implemented at MIT added "fictitious" capacity, after all real plants had been loaded, until the reliability constraint was met. This fictitious plant was to have characteristics which rendered it less desirable than any other alternative. Determining precisely what these characteristics should be, however, was a difficult task. At first, the fictitious plant had the same capital cost as the GTB alternative, a slightly higher operating cost, and a much higher forced outage rate. Upper bounds generated by this method were observed to be smaller than the lower bound of later iterations.

After much re-thinking, it was decided that the proper way of achieving a feasible solution for upper bcund calculations was not to add this new, fictitious capacity at the end, but rather to augment the capacity of an existing alternative. To see the difference each type of capacity would have on reliability, a simple numerical example was worked out (see Appendix C). It shows that 1 MW of fictitious capacity can generate as much energy as, perhaps, 5 MW of augmented capacity at the margin. This ratio starts at 1 and increases as the amount of existing capacity increases and approaches the reserve margin. As so much less new capacity is needed to generate the same amount of energy as an extra increment of capacity added to an existing unit, this explains why the lower bounds generated using fictitious capacity were too low. In order to utilize the current code before this correction was implemented, the capital cost of the fictitious plans was multiplied by a number (3) which seemed like a good ratio of augmented to fictitious capacity, considering the ratios derived in the Appendix example. While this yielded valid results for most of the runs performed, some upper bounds were still too low, which required computing the percent range of optimality after the optimal solution had been achieved, instead of at each iteration.

A second correction to the algorithm is that trial capacity plans from the master problem should be represented in terms of discrete unit sizes. The code currently models, say, 2.3 units of nuclear as one 2300-MW nuclear unit, instead of as two 1000-MW units and one 300-MW unit. Without discrete-unit representation, the limit on an achievable reliability level is the unserved energy under the equivalent LDC after all the committed and existing units are loaded, times the product of the forced outage rates of the available alternatives. The ε of 0.1% used in original test runs was found to be unachievable according to this limit; it was therefore increased to 0.9% for all the runs presented in Chapter Five.

Once the multiple plant representation has been implemented the correct algorithm for determining a feasible solution will search backwards in the merit order until it finds the last alternative loaded. It will then deconvolve out the fractional part of this plant, and add more capacity of this type, in discrete plant units, until the reliability level has been satisfied, truncating the last increment if necessary to meet the reliability constraint exactly.

While the lower bound is monotonically nondecreasing, the upper bound, although exhibiting a downward trend, oscillates in an irregular fashion from iteration to iteration. Therefore, the smallest of all (correct) upper bounds generated up to any point should be used as the upper bound for that iteration. A major advantage of the GBD algorithm is that it need not proceed until optimality is reached, but may be terminated when the lower and upper bounds at any iteration are sufficiently "close." If a user is satisfied with a trial solution that is within x% of optimality, s/he should allow the algorithm to continue until:

$$100 \cdot \frac{UB-LB}{LB} \leq x.$$

If, at any point, the lower bound ever exceeds the total cost of a feasible solution, then also stop. (Lower bound-trial solution cost crossovers are explained in Chapter Four.)

CHAPTER FOUR

This chapter will discuss accuracy issues, the first section explaining how lower bounds may occasionally exceed the trial solution costs, and why different versions of the same problem converge to different optimal solutions. The second section goes into a new way of computing the multipliers on unserved energy, which reduces the amount of error included.

Section 4.1

After a simple test problem solved by hand (see Appendix B) verified the absence of obvious coding errors with SYSGEN, it became necessary to resolve why λ of the marginal plant for that simple problem (hereafter referred to as λ_{IMRG}) equalled 3.48637×10^{-4}, instead of 0.0, implied by the formula for this Lagrange multiplier (see Appendix B for a proof of this result). It was decided to investigate the effects of the spacing of points representing the load duration curve and linear interpolation between points. Errors would be especially prominent in those regions where the LDC displayed a small bump, or "knee." Therefore, three points from the final equivalent LDC were selected (1500±75), and equations of the line segments between these points were computed. Table 4.1.1 displays the selected values of curve, and also their corresponding equations. The 75-MW spacings were then divided up into 15-MW spacings, and hand calculations computed these intermediate values using the convolution formula:

$$G_4(x) = p_3 G_3(x) + q_3 G_3(x-Y^3).$$

The values of the final equivalent LDC calculated in this fashion were then compared to values obtained by substitution into the derived equations (i.e., by linear interpolation). Table 4.1.2 presents these results.

It was seen that the largest discrepancies were of the order of approximately 4×10^{-4}, a very significant error. The test problem was then run on the computer halving the number of points of the ELDC, and then doubling the number of points. As hoped, λ_{IMRG} increased in the first case, and decreased in the second. The 20-point LDC originally used in this test problem was now abandoned in favor of the 40-point case, which yielded $\lambda_{IMRG} = 1.74291 \times 10^{-4}$. It was decided to run a case in which the peak load and the plant capacities would be exact multiples of the LDC spacing, in order to eliminate this source of error and compute the percentage differences between the Lagrancge multipliers calcualted both ways. Table 4.1.3 presents these results. λ_{IMRG} now equalled -2.4993×10^{-9}, which is approximately zero, given that the computer is good to eight significant digits in single-precision mode. To discover how this error would affect the value of the objective function and the trial solutions generated, one iteration of SYSGEN and the master problem was run for the unexact (40-pt.) and exact (30-pt.) test cases. These results are shown in Table 4.1.4. This procedure could not be carried past the first iteration, as the LP generates trial solutions which are made up of plant capacities that are not exact multiples of the LDC spacing, thus destroying the validity of that case for comparison. The differences in the values of the objective function were of the order of the lower bound "crossovers" observed in the results of the next chapter (a lower bound "crossover" is said to occur whenever the lower bound exceeds the total cost of the trial solution during that iteration.) The % errors shown in Table 4.1.4 are actually valid for comparison only with one time period studies. With multiple time periods, one would expect somewhat larger % errors, although probably still of the same order of magnitude.

Table 4.1.1

Region of $G_4(x)$ under investigation

$$G_4(1425) = 0.48137774$$
$$G_4(1500) = 0.41199996$$
$$G_4(1575) = 0.39239996$$

Between 1425 - 1500 MW:

$$G_4(x) = -.000925037x + 1.7995559$$

Between 1500 - 1575 MW:

$$G_4(x) = -.000261333x + 0.80399996$$

Table 4.1.2

Comparison of $G_4(x)$ results

Load (MW)	$G_4(x)$ (15-MW spacing)	$G_4(x)$ (linear interpolation)
1425	.48137774	.4813781
1440	.46756440	.4675026
1455	.45375100	.4536270
1470	.43993770	.4397515
1485	.42599990	.4258759
1500	.41199996	.4120004
1515	.40807980	.4080804
1530	.40415990	.4041604
1545	.40023990	.4002404
1560	.39631990	.3963204
1575	.39239996	.3924004

Table 4.1.3

Comparison of Lagrange multipliers from exact and non-exact cases

	Exact	Non-exact	% error
λ^1	.0401507	.0403250	.43
λ^2	.0420373	.0422117	.42
λ^3	$-2.4993 \times 10^{-9} \approx 0$	1.74291×10^{-4}	-
μ^1	.0597333	.0603555	1.04
μ^2	.2400000	.2406220	.26
μ^3	.3204000	.3210220	.19

Table 4.1.4

Comparison of Z and capacity plans from exact and non-exact cases

	Exact	Non-exact	% error
Z	1078.870	1084.44	.52
X^1	1386.995	1393.98	.54
X^2	0.0	0.0	-

The reason for the lower bound crossovers is thus explained, and this revealing of numerical errors in the computation of the Lagrange multipliers, due to discrete-point LDC calculations, can be used to explain other discrepancies, such as convergence to different (although close) solutions when starting with different initial constraints, and when using an aggregated reliability constraint as opposed to disaggregated reliability constraints. The errors in the Lagrange multipliers are responsible for introducing small nonconvenxities into the problems, thus causing results which appear inconsistent with theory.

Section 4.2

The formulas used for computing the Lagrange multipliers in all the results presented in this thesis (except the special set presented in this section) are:

$$\lambda^j = \begin{cases} -\frac{\partial}{\partial Y^j} EF(\underline{Y}) - \pi \frac{\partial}{\partial Y^j} EG(\underline{Y}) & j < I \\ 0 & j \leq I \end{cases}$$

$$\pi = F^I$$

$$\mu^j = \begin{cases} -\frac{\partial}{\partial Y^j} EG(\underline{Y}) & \text{if infeasible} \\ 0 & \text{if feasible} \end{cases}$$

$$\frac{\partial}{\partial Y^j} EG(\underline{Y}) = F^j p_j G_j(U^j) + \sum_{i=j+1}^{I} F^i p_i [H_{ij}(U^i) - H_{ij}(U^{i-1})]$$

$$\frac{\partial}{\partial Y^j} EG(\underline{Y}) = -H_{I+1,j}(U^I)$$

$$H_{ij}(U) = \begin{cases} \frac{\partial}{\partial Y^j} \int_0^U G_i(Q;Y^1, \ldots, Y^{I-1})dQ & \text{for } i > j \\ 0 & \text{for } i < j \\ G_j(U^i) & \text{for } i = j \end{cases}$$

where H is defined by:

$$H_{i+1,j}(U) = \begin{cases} p_i H_{ij}(U) + q_i H_{ij}(U-Y^i) & \text{for } i=j+1,\ldots,I \\ p_j G_j(U) & \text{for } i=j+1 \end{cases}$$

$$H_{ij}(U) = \begin{cases} \sum_{\ell=0}^{k} \left(\frac{-q_i}{p_j}\right)^{\ell} G_i(U-\ell \cdot Y^j) + \left(\frac{-q_i}{p_j}\right)^{k+1} p_j & \text{for } Y^j > 0 \\ p_j G_i(U) & \text{for } Y^j = 0 \end{cases}$$

where k is defined such that:

$$k \cdot Y^i < U \leq (k+1) \cdot Y^j$$

and

I = merit order index of the marginal plant.

A proposed change in the μ calculations is to compute them <u>before</u> the marginal plant is convolved into the equivalent load duration curve, not after, which is the current procedure, thus avoiding errors introduced by the convolution of the marginal unit. While this procedure will not make much of a difference in a study with many units, it becomes significant in a system with only a few units, as in this example. The new algorithm is described below.

For all plants i in the merit order from 1 to I, compute the following:

$$\mu^1 = p_I H_{I,1}(U^I) + q_I H_{I,1}(U^{I-1})$$

$$\mu^2 = p_I H_{I,2}(U^I) + q_I H_{I,2}(U^{I-1})$$

$$\vdots$$

$$\mu^{I-1} = p_I H_{I,I-1}(U^I) + q_I H_{I,I-1}(U^{I-1})$$

$$\mu^I = p_I G_I(U^I)$$

where:

$$U^{I-1} = U^I - Y^I.$$

This procedure was tried by hand on the 40-pt. test case. Table 4.2.1 shows the extremely good agreement between values obtained by this method and values from the exact case. (In fact, when run on the computer, the new results corresponded precisely with the exact ones).

Table 4.2.1

Comparison of μ_{old}, μ_{new}, and μ_{exact}

	Old	New	Exact
μ^1	.0609777	.0597332	.0597333
μ^2	.2412444	.2399999	.2400000
μ^3	.3216443	.3203999	.3204000

CHAPTER FIVE

Various approaches were tried to speed up the rate of convergence of the GBD algorithm. Two basic approaches are discussed below, and their results are presented. The data used will be described in Section 5.1. Section 5.2 deals with the base case, while Sections 5.3-5.4 handle the variants.

Section 5.1

Development work on the GBD algorithm was originally done using the following 9 time period dataset. When investigation into convergence started, it was decided to use mostly 1 and 2 time period cases, in order to save money and facilitate hand-verification of the results. Table 5.1.1 shows the peak load and customer energy demand for the full 9 time periods (when $i<9$ time periods are used, only the first i rows are relevant). Table 5.1.2 gives the values of the initial load duration curve, while Table 5.1.3 presents plant data, including unit sizes, availabilities, and capital and operating costs. There are 5 committed and existing units: one 1000-MW base-loaded nuclear (NUC) unit, two 800-MW intermediate combined-cycle oil (CCO) units, and two 150-MW peaking gas turbines (GTB), for a combined available capacity of 2900 MW.

Table 5.1.1

Year	Peak Demand (MW)	Customer Energy Demand (MWH)
1	2100.0	11275147.0
2	2268.0	12177154.0
3	2449.4	13151113.0
4	2645.4	14203460.0
5	2857.0	15339569.0
6	3085.6	16566947.0
7	3332.4	17892023.0
8	3599.0	19323440.0
9	3887.0	20869744.0

Table 5.1.2

Load Duration Curve

(42 MW spacing)

1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000
0.99818	0.99497	0.96335	0.95248	0.92464
0.89211	0.85250	0.79034	0.78151	0.70502
0.69512	0.64347	0.61328	0.54936	0.54846
0.49885	0.47049	0.41260	0.41112	0.32364
0.31990	0.24839	0.17687	0.16822	0.10930
0.07381	0.04906	0.03111	0.01317	0.01222
0.00446	0.00159	0.00045	0.00001	0.00000

Table 5.1.3

Name	Capacity (MW)	Availability	Install. Cost ($/MW)	Operating Cost ($/MWH)
LWR BASE	1000	0.70	500000.0	6.28
CCO INTR	800	0.75	300000.0	12.02
GTB PEAK	150	0.80	130000.0	32.07
NUC BASE	1000	0.70	500000.0	6.18
FOS INTR	800	0.75	300000.0	11.93
GTB PEAK	150	0.80	130000.0	31.93

Alternatives fall into these three basic plant types, and, as a GTB installed in year 1 is different from a GTB installed in year 4, the number of alternatives in any of the following cases equals three times the number of time periods in that particular case. Load growth is set at 8%/yr. Escalation factors for fuel, operation and maintenance (O & M), and capital costs are held constant at 6%, while the discount rate equals 10.6% (no adjustment for the rate of inflation is made here). All capacities and loads in the last year of the study are assumed to remain constant (via replacement in kind) throughout the extension period. The desired reliability level ε equals 0.009* (customer energy demand), and most cases, except those which explicitly state otherwise, start with an initial trial solution of 0.0 MW for all alternatives, and no initial constraints.

Section 5.2

The results (including lower bound, total cost of trial solution, and upper bound) of the 1 time period base case are displayed in Table 5.2.1. It is seen that for this test case, only 15 iterations are needed to reach optimality (where optimality is defined as that point when values of the lower bound and the trial solution stabilize, i.e., when $LB_{k-1} = LB_k$ and $TC_{k-1} = TC_k$). Table 5.2.2 shows the amount of "fictitious" capacity added in every iteration, and the % unserved energy after the last real plant has been loaded. An interesting observation is that, although the % unserved energy does not decrease monotonically, it does decrease rapidly until the 10th iteration, after which it levels off, and slowly approaches feasibility. Table 5.2.3 shows the trial solutions generated at every iteration, in terms of MW's of capacity. Again, by the 10th iteration, the total trial solution was just 1.8% away from the optimal solution.

Table 5.2.1

Iter. #	Lower Bound	Solution	Upper Bound	% Range of Optimality
		1050.35	2730.68	
1	1126.09	1301.23	1790.52	59.0
2	1198.86	1281.41	1567.50	30.749
3	1294.75	1363.71	1635.89	21.107
4	1310.93	1358.49	1439.53	9.810
5	1350.12	1371.38	1458.65	6.622
6	1353.98	1372.90	1481.38	6.318
7	1363.50	1375.02	1408.09	3.270
8	1381.00	1386.55	1414.06	1.962
9	1384.26	1391.28	1404.70	1.477
10	1385.26	1389.22	1392.30	0.508
11	1389.34	1389.89	1393.63	0.213
12	1389.86	1390.23	1391.68	0.131
13	1390.17	1390.58	1391.71	0.109
14	1390.44	1390.59	1390.91	0.0003
15	1390.65	1390.68	1390.68	0.000
	1390.68			

Table 5.2.2

Iter. #	Fict. Cap. Added (MW)	% Unserved Energy
0	2541.3	4.683
1	729.9	2.161
2	425.5	1.658
3	404.3	1.630
4	120.6	1.114
5	129.5	1.137
6	163.2	1.157
7	49.6	0.982
8	40.9	0.973
9	20.1	0.933
10	4.6	0.908
11	5.6	0.909
12	2.2	0.903
13	1.7	0.903
14	0.5	0.901
15	0.0	0.900

Table 5.2.3

Proposed Trial Solutions (MW)

Iter. #	1	2	3	4	5	6	7	8
NUC	528.0		72.0	287.0			102.0	212.0
CCO			578.4		391.2	11.2	152.0	243.2
GTB		580.5		522.6	377.4	928.8	621.3	377.3
Total	528.0	580.5	650.4	809.6	768.6	940.0	875.3	832.5

Iter. #	9	10	11	12	13	14	15
NUC	258.0	167.0	190.0	143.0	192.0	162.0	166.0
CCO	12.0	176.0	107.2	167.2	140.8	170.4	156.8
GTB	677.4	554.1	629.0	605.1	579.0	575.3	591.8
Total	947.4	897.1	926.2	915.3	911.8	907.7	914.6

Section 5.3

The first major approach used was to add some initial capacity in the master problem. Various options were tried: using the trial solution of an intermediate solution as the alternatives' initial values, constraining the sum of alternative capacities to fall within a certain range, constraining individual alternatives to be less than certain values, and constraining individual alternatives to be greater than certain values. Not surprisingly, the last option yielded the best results, requiring 10 iterations for complete convergence. Tables 5.3.1 - 5.3.3 present the results for this 1 time period case, in which the lower values chosen were 100 MW for NUC, 150 MW for CCO, and 500 MW for GTB. It was believed that this method would help convergence as it does not allow the master problem to install all of its capacity in one or two alternatives. In the absence of such constraints, the master problem does just this in the beginning iterations, as LP solutions exist at the corners of the feasible set, but the capacity constraints force it to generate a mix of all the alternatives from the start. Inspecting the results, one sees that by the 5th iteration the algorithm has essentially converged, its total proposed solution coming within 1% of the total optimal solution.

Table 5.3.1

Iter. #	Lower Bound	Solution	Upper Bound	% Range of Optimality
		1050.35	2730.68	
1	1196.54	1342.87	1440.46	20.385
2	1373.32	1380.97	1404.06	2.238
3	1382.19	1386.91	1399.23	1.233
4	1386.43	1389.00	1393.71	0.525
5	1388.80	1389.45	1391.84	0.219
6	1390.22	1390.68	1391.66	0.104
7	1390.25	1390.53	1391.32	0.077
8	1390.50	1390.67	1390.92	0.030
9	1390.62	1390.82	1391.63	0.0002
10	1390.66	1390.60	1390.60	0.000
	1390.66			

Table 5.3.2

Iter. #	Fict. Cap. Added (MW)	% Unserved Energy
0	2541.3	4.683
1	145.2	1.159
2	34.7	0.956
3	18.4	0.932
4	7.0	0.912
5	3.6	0.906
6	1.5	0.902
7	1.2	0.902
8	0.4	0.901
9	1.2	0.902
10	0.0	0.900

Table 5.3.3

Proposed Trial Solutions (MW)

Iter. #	1	2	3	4	5	6
NUC	100.0	100.0	226.0	133.5	170.0	130.0
CCO	150.0	150.0	150.0	210.4	150.0	191.2
GTB	500.0	651.2	500.0	547.2	590.9	588.0
Total	750.0	901.2	876.0	891.1	910.9	909.2

Iter. #	7	8	9	10
NUC	167.0	151.0	191.0	166.0
CCO	180.0	170.8	152.0	164.0
GTB	553.4	590.1	562.5	580.2
Total	900.4	911.9	905.5	910.2

Section 5.4

In this second approach, a break from the theory presented in Chapter Two was taken. Using data for two time periods, the code was modified to write one feasibility constraint for each infeasible time period, instead of one which sums multiplier data for all time periods. The justification for this approach is that several individual constraints are more restrictive than one aggregated constraint. Tables 5.4.1 - 5.4.3 display results for the 2 time period, aggregated reliability constraint case, while Tables 5.4.4 - 5.4.6 do so for the 2 time period, disaggregated reliability constraints case.

Table 5.4.1

Iter. #	Lower Bound	Solution	Upper Bound	% Range of Optimality*
		1209.35	3551.19	
1	1225.49	1457.65	2101.82	23.47
2	1379.75	1484.54	1872.73	13.83
3	1446.66	1551.44	1966.33	9.65
4	1479.36	1549.59	1693.67	7.6
5	1556.36	1583.58	1739.59	2.8
6	1561.35	1584.54	1720.76	2.5
7	1566.10	1578.56	1686.84	2.2
8	1576.96	1593.25	1637.54	1.5
9	1583.54	1602.45	1676.44	1.1
10	1586.27	1597.96	1674.32	0.94
11	1589.31	1596.55	1620.87	0.75
12	1593.32	1598.04	1618.00	0.50
13	1597.57	1598.04	1605.44	0.23
14	1597.93	1599.10	1603.46	0.21
15	1599.11	1599.02	1606.97	0.13
16	1599.49	1600.21	1602.59	0.11
17	1600.41	1601.58	1602.42	0.052
18	1600.54	1601.05	1601.44	0.044
19	1600.77	1600.85	1600.85	0.030
20	1600.80	1600.86	1600.86	0.028
21	1600.82	1600.96	1600.96	0.027
22	1600.91	1601.08	1601.25	0.021
23	1601.07	1601.14	1601.14	0.011
24	1601.08	1601.15	1601.15	0.0106
25	1601.11	1601.47	1601.50	0.0087
26	1601.23	1601.31	1601.37	0.0001
27	1601.25	1601.24	1601.24	0.0
28	1601.25	1601.23	1601.23	0.0

*% range of optimality computed by: $\frac{L.B._{28} - L.B._{i}}{L.B._{28}}$

Table 5.4.2

Iter. #	Fict. Cap. Added (MW) in Year 1	Fict. Cap. Added (MW) in Year 2	% Unserved Energy in Year 1	% Unserved Energy in Year 2
0	2665	3582	4.683	5.535
1	790	953	2.091	2.620
2	408	575	1.564	1.989
3	451	616	1.636	2.030
4	51.7	220	0.985	1.285
5	68.9	237	1.016	1.321
6	56.5	210	0.984	1.218
7	0.0	168	0.901	1.178
8	69.9	27.2	1.009	0.941
9	115	55.8	1.101	0.995
10	125	0.0	1.117	0.900
11	38.8	9.6	0.965	0.916
12	32.7	0.0	0.953	0.900
13	6.7	11.2	0.911	0.917
14	1.2	6.8	0.902	0.910
15	0.0	12.4	0.901	0.919
16	3.9	0.0	0.906	0.900
17	0.62	1.3	0.901	0.902
18	0.64	0.0	0.901	0.900
19	0.0	0.0	0.900	0.900
20	0.0	0.0	0.900	0.900
21	0.0	0.0	0.900	0.900
22	0.28	0.0	0.900	0.900
23	0.0	0.0	0.900	0.900
24	0.0	0.0	0.900	0.900
25	0.17	0.0	0.900	0.900
26	0.0	0.23	0.900	0.900
27	0.0	0.0	0.900	0.900
28	0.0	0.0	0.900	0.900

Table 5.4.3

Proposed Trial Solutions (MW)

Iter. #	1	2	3	4	5	6	7
NUC 1	556.0			279.5			112.7
NUC 2							
CCO 1			696.4	68.3	497.5	132.7	254.3
CCO 2							
GTB 1		622.5		515.9	358.4	850.8	573.8
GTB 2							
Total	556.0	622.5	696.4	763.7	855.9	983.5	940.8

Iter. #	8	9	10	11	12	13	14
NUC 1	198.0	355.4	125.7	297.1	162.9	275.1	217.0
NUC 2			73.9				
CCO 1	35.0	235.1	237.0	110.8	159.7	79.2	99.4
CCO 2							
GTB 1	651.4	207.9	393.1	454.1	546.3	561.4	611.4
GTB 2	154.2	172.4	165.9	149.1	157.3	119.0	116.6
Total	1038.6	970.8	995.6	1011.1	1026.2	1034.7	1044.4

Iter. #	15	16	17	18	19	20	21
NUC 1	248.8	178.2	158.5	205.0	174.3	179.8	184.9
NUC 2							
CCO 1	95.9	136.5	138.2	110.7	137.7	131.6	125.9
CCO 2							
GTB 1	583.3	601.3	632.3	610.0	610.5	612.5	614.4
GTB 2	112.6	126.0	119.7	124.7	122.0	122.4	122.8
Total	1040.6	1042.0	1048.7	1050.4	1044.5	1046.3	1048.0

(Table 5.4.3, cont'd.)

Iter. #	22	23	24	25	26	27	28
NUC 1	186.7	168.6	180.8	196.7	165.5	166.8	166.8
NUC 2							
CCO 1	122.2	137.9	125.7	109.4	138.0	137.9	137.9
CCO 2							
GTB 1	617.7	618.5	621.0	624.8	622.8	620.9	620.9
GTB 2	122.8	121.2	121.9	122.8	120.6	120.9	120.9
Total	1049.4	1046.2	1049.4	1053.7	1046.9	1046.5	1046.5

Table 5.4.4

Iter. #	Lower Bound	Solution	Upper Bound	% Range of Optimality
		1209.35	3551.19	
1	1226.17	1469.32	2078.45	69.5
2	1381.38	1497.00	1850.79	34.0
3	1461.18	1575.10	1908.49	26.7
4	1493.44	1560.23	1676.33	12.2
5	1543.54	1576.94	1680.65	8.60
6	1558.50	1576.17	1693.73	7.56
7	1565.89	1582.15	1619.14	3.40
8	1590.37	1599.80	1620.59	1.81
9	1592.56	1601.86	1642.61	1.67
10	1592.84	1598.58	1605.10	0.77
11	1593.62	1597.40	1613.97	0.72
12	1595.06	1598.53	1602.67	0.48
13	1596.55	1597.66	1600.94	0.27
14	1597.70	1598.37	1600.18	0.155
15	1598.36	1598.59	1599.53	0.073
16	1598.53	1598.92	1599.85	0.0626
17	1598.62	1598.76	1599.25	0.0394
18	1598.79	1598.89	1598.89	0.0063

Table 5.4.5

Iter. #	Fict. Cap. Added (MW) in Year 1	Fict. Cap. Added (MW) in Year 2	% Unserved Energy in Year 1	% Unserved Energy in Year 2
0	2665	3582	4.683	5.535
1	736	900	2.039	2.549
2	475	523	1.682	1.800
3	496	487	1.712	1.676
4	156	172	1.170	1.197
5	153	152	1.166	1.160
6	173	176	1.164	1.160
7	54.9	54.2	0.991	0.988
8	30.2	31.0	0.949	0.949
9	60.8	53.7	1.006	0.992
10	9.78	8.56	0.916	0.914
11	23.5	24.9	0.936	0.937
12	5.59	6.20	0.909	0.909
13	4.93	4.35	0.908	0.907
14	2.67	2.67	0.904	0.904
15	1.17	1.40	0.902	0.902
16	1.40	1.40	0.902	0.902
17	0.75	0.54	0.901	0.901
18	0.0	0.0	0.900	0.900

Table 5.4.6

Proposed Trial Solutions (MW)

Iter. #	1	2	3	4	5	6	7
NUC 1	579.5			360.4		67.1	174.8
NUC 2			57.8			112.7	
CCO 1			648.7		363.6		154.5
CCO 2							
GTB 1		571.0		424.2	387.2	795.9	511.1
GTB 2		86.2	74.4	123.3	125.8	17.0	124.4
Total	579.5	657.2	780.9	896.9	876.6	992.7	964.8

Iter. #	8	9	10	11	12	13	14
NUC 1	357.0	309.7	270.1	203.4	157.1	235.2	199.5
NUC 2							
CCO 1		273.4	168.3	55.3	170.7	128.1	186.3
CCO 2							
GTB 1	563.4	250.2	440.8	675.0	575.1	537.8	502.7
GTB 2	122.5	132.2	126.2	118.8	121.9	123.6	124.0
Total	1042.9	965.5	1005.4	1051.5	1024.8	1024.7	1022.5

Iter. #	15	16	17	18	19
NUC 1	197.3	239.6	222.5	210.6	211.9
NUC 2					
CCO 1	161.8	156.9	153.4	147.1	144.7
CCO 2					
GTB 1	540.7	497.2	522.9	546.5	548.4
GTB 2	123.0	124.2	123.6	123.0	123.0
Total	1022.8	1017.9	1022.4	1027.2	1028.0

The disaggregated case is seen to be faster, converging in 10 fewer iterations than the aggregated case (18 vs. 28 iterations). Another point of interest is that the aggregated case converges to a higher total solution case than the disaggregated one. This result is due to the errors revealed in the Lagrange multipliers in the previous chapter. Since fewer iterations are required for complete convergence in the disaggregated case, errors have less time over which to compound; therefore the error in the disaggregated case is smaller, which results in a lower value of the objective function. The lower bound crossovers are also explained by the numerical inaccuracies in the multipliers. Since the temporarily fixed-up upper bounds in the aggregated case fall below lower bounds of later iterations, the % range of optimality is measured in terms of the optimal solution. Notice in this case, however, that termination should occur after the 21st iteration, as $LB_{22} > TC_{19}$, which is a feasible solution. Had this termination been allowed to occur, the disaggregated case would converge in only three fewer iterations than the aggregated.

It was wondered whether a more dramatic difference could be obtained with more time periods. To this end, a 4 time period case was constructed and run. Tables 5.4.7 - 5.4.9 present results for the aggregated case, while Tables 5.4.10 - 5.4.12 do so for the disaggregated case. The aggregated case converged in 39 iterations, while the disaggregated case now took longer, converging in 44 iterations. However, a nearly feasible solution (less than 1 MW of fictitious capacity added during the planning period) occurred on the 25th iteration, and again on the 38th. The percent difference between Z_{25} in the disaggregated case and $Z_{optimal}$ in the aggregated case (0.85%) is of the same order of magnitude as the discrepancies caused by the inaccuracies in the Lagrange multiplier calculation (see Table 4.1.4). Therefore, the final iterations in the disaggregated case really gain no new information, as the algorithm has now passed within its accuracy resolution. This problem stresses the need for a correct upper bound implementation, since once the lower bound has exceeded a true upper bound, the procedure may be safely terminated. Any further iterations would be meaningless, as the errors introduced through the Lagrange multipliers have now accumulated to the point where they are of the same order of magnitude as the algorithm's resolution, and thus can introduce noticeable nonconvexities. In addition to saving on the cost of running these later iterations, such a termination rule would also produce a lower cost solution, as Z's upward creeping during these extra iterations would now be eliminated.

Table 5.4.7

Iter. #	Lower Bound	Solution	Upper Bound	% Range of Optimality*
0	---	1546.74	6965.1	---
1	0.0	6758.62	7381.812	---
2	1451.56	1819.77	3572.004	30.46
3	1782.36	1883.74	2466.69	14.61
4	1859.31	1977.37	2638.426	10.92
5	1873.86	2093.02	3706.48	10.23
6	1874.97	1980.29	2455.952	10.17
7	1940.84	1989.90	2405.114	7.02
8	1955.32	1974.71	2231.49	6.32
9	1986.04	2032.03	2210.49	4.73
10	1988.62	2024.48	2199.89	4.73
11	2003.65	2023.29	2353.25	4.01
12	2006.05	2027.73	2158.99	3.89
13	2011.40	2013.64	2242.60	3.64
14	2013.28	2017.69	2259.51	3.55
15	2036.46	2039.16	2279.14	2.44
16	2041.21	2056.70	2181.36	2.21
17	2045.86	2056.05	2182.85	1.99
18	2050.20	2076.66	2231.05	1.78
19	2053.35	2075.71	2192.91	1.63
20	2057.15	2085.22	2122.92	1.45
21	2065.98	2081.22	2083.96	1.02
22	2072.14	2078.42	2181.90	0.728
23	2075.97	2090.00	2091.15	0.544
24	2083.37	2090.60	2159.69	0.190
25	2084.30	2089.15	2134.41	0.145
26	2085.87	2082.63	2102.40	0.0700
27	2086.22	2082.28	2094.00	0.0532
28	2087.17	2080.20	2083.96	0.00767
29	2087.23	2081.43	2083.03	0.00479
30	2087.24	2080.70	2082.40	0.00431
31	2087.26	2082.17	2083.03	0.00335

(Table 5.4.7, cont'd.)

Iter. #	Lower Bound	Solution	Upper Bound	% Range of Optimality*
32	2087.27	2081.87	2082.43	0.00287
33	2087.30	2083.31	2083.89	0.00144
34	2087.31	2082.50	2083.04	0.000958
35	2087.31	2082.59	2083.08	0.0000958
36	2087.32	2083.27	2083.88	0.0000479
37	2087.32	2082.24	2082.64	0.0000479
38	2087.32	2082.80	2082.95	0.0000479
39	2087.33	2082.14	2082.14	0.0

* % range of optimality calculated with respect to the optimal solution.

Table 5.4.8

Iter. #	Fict. Cap. Added in Year 1 (MW) [% unserved energy]	Fict. Cap. Added in Year 2 (MW) [% unserved energy]	Fict. Cap. Added in Year 3 (MW) [% unserved energy]	Fict. Cap. Added in Year 4 (MW) [% unserved energy]
0	2665 [4.683]	3582 [5.535]	6082 [6.632]	8957 [8.098]
1	341.5 [1.405]	491.8 [1.662]	695.9 [1.992]	963.9 [2.432]
2	2371 [3.514]	772.3 [2.062]	2447 [2.647]	2729 [3.401]
3	189.7 [1.228]	358.5 [1.585]	584.5 [2.061]	887.5 [2.684]
4	351.4 [1.462]	480.6 [1.710]	699.2 [2.139]	1007 [2.758]
5	2280 [3.129]	2581 [3.582]	384 [1.471]	678 [1.848]
6	730 [2.046]	247.6 [1.343]	132 [1.127]	390.6 [1.554]
7	629 [1.929]	0.0 [0.900]	186 [1.202]	449 [1.607]

(Table 5.4.8, cont'd.)

Iter. #	Fict. Cap. Added in Year 1 (MW) [% unserved energy]	Fict. Cap. Added in Year 2 (MW) [% unserved energy]	Fict. Cap. Added in Year 3 (MW) [% unserved energy]	Fict. Cap. Added in Year 4 (MW) [% unserved energy]
8	920 [1.058]	0.0 [0.901]	181 [1.220]	402 [1.659]
9	97.8 [1.070]	270.5 [1.414]	22.3 [0.936]	238.7 [1.292]
10	101.3 [1.070]	269.7 [1.391]	0.0 [0.900]	208.2 [1.234]
11	0.0 [0.900]	168.5 [1.181]	288.6 [1.413]	517.3 [1.867]
12	115.3 [1.104]	31.4 [0.953]	0.0 [0.900]	200.8 [1.257]
13	0.0 [0.900]	112.5 [1.090]	147.0 [1.152]	362.3 [1.571]
14	250.2 [1.344]	0.0 [0.900]	144.5 [1.146]	361.9 [1.576]
15	0.0 [0.900]	0.0 [0.901]	182.9 [1.185]	387.0 [1.579]
16	44.95 [0.975]	55.67 [0.991]	0.0 [0.900]	199.5 [1.234]
17	0.0 [0.900]	0.0 [0.900]	0.0 [0.900]	210.6 [1.248]
18	237.2 [1.267]	0.0 [0.900]	0.0 [0.900]	155 [1.128]
19	185.2 [1.218]	0.0 [0.900]	0.0 [0.900]	61.2 [0.997]
20	59.1 [0.997]	11.4 [0.918]	0.0 [0.900]	23.7 [0.937]
21	4.5 [0.907]	0.0 [0.900]	0.0 [0.900]	0.0 [0.900]
22	0.0 [0.901]	0.0 [0.901]	0.0 [0.900]	173.7 [1.161]
23	1.9 [0.903]	0.0 [0.900]	0.0 [0.900]	0.0 [0.900]

(Table 5.4.8, cont'd.)

Iter. #	Fict. Cap. Added in Year 1 (MW) [% unserved energy]	Fict. Cap. Added in Year 2 (MW) [% unserved energy]	Fict. Cap. Added in Year 3 (MW) [% unserved energy]	Fict. Cap. Added in Year 4 (MW) [% unserved energy]
24	109.9 [1.089]	6.6 [0.911]	0.0 [0.900]	28.4 [0.945]
25	74.1 [1.025]	0.0 [0.900]	0.0 [0.900]	0.0 [0.900]
26	19.6 [0.933]	32.7 [0.953]	0.0 [0.900]	0.0 [0.900]
27	19.2 [0.932]	0.0 [0.900]	0.0 [0.900]	0.0 [0.900]
28	4.7 [0.908]	6.2 [0.910]	0.0 [0.900]	0.0 [0.900]
29	1.5 [0.902]	2.6 [0.904]	0.0 [0.900]	0.0 [0.900]
30	2.8 [0.905]	0.0 [0.900]	0.0 [0.900]	0.0 [0.900]
31	1.4 [0.902]	1.1 [0.902]	0.0 [0.900]	0.0 [0.900]
32	0.923 [0.902]	0.0 [0.900]	0.0 [0.900]	0.0 [0.900]
33	0.937 [0.902]	0.790 [0.901]	0.0 [0.900]	0.0 [0.900]
34	0.0 [0.900]	0.938 [0.902]	0.0 [0.900]	0.0 [0.900]
35	0.796 [0.901]	0.0 [0.900]	0.0 [0.900]	0.0 [0.900]
36	0.280 [0.900]	1.01 [0.902]	0.0 [0.900]	0.0 [0.900]
37	0.0 [0.900]	0.679 [0.901]	0.0 [0.900]	0.0 [0.900]
38	0.0 [0.900]	0.254 [0.900]	0.0 [0.900]	0.0 [0.900]
39	0.0 [0.900]	0.0 [0.900]	0.0 [0.900]	0.0 [0.900]

Table 5.4.9

Proposed Trial Solutions (MW)

Iter. #	1	2	3	4	5	6	7
NUC 1	7904.0		478.6				438.0
NUC 2		495.4		23.9		239.7	
NUC 3					1035.8	266.5	
NUC 4							
CCO 1		173.9		849.8	133.6	201.7	
CCO 2							
CCO 3							
CCO 4							
GTB 1			314.8		95.9	242.4	109.2
GTB 2					38.0	112.7	550.3
GTB 3							
GTB 4							
Total	7904.0	669.3	793.4	873.7	1303.3	1063.0	1097.5

Iter. #	8	9	10	11	12	13	14
NUC 1	299.8	133.6	270.4	461.1	213.6	333.4	229.1
NUC 2	266.3				222.9		105.1
NUC 3			287.6				
NUC 4							
CCO 1	97.7	374.8	40.6	514.4	263.0	151.5	68.1
CCO 2							
CCO 3							
CCO 4							
GTB 1	412.3	283.2	508.4	132.8	289.5	549.1	374.9
GTB 2					6.9	39.6	419.0
GTB 3		346.9	129.1	45.4	234.9	111.4	32.2
GTB 4							
Total	1076.1	1138.5	1236.1	1153.7	1230.8	1185.0	1228.4

(Table 5.4.9, cont'd.)

Iter. #	15	16	17	18	19	20	21
NUC 1	278.8	341.4	362.0		269.9	130.3	341.5
NUC 2	152.3	134.6		285.0	258.9	5.0	149.3
NUC 3			122.2	191.4	171.5	94.5	
NUC 4						155.5	
CCO 1	18.3	26.7	212.4	32.4	11.9	34.9	85.6
CCO 2							
CCO 3							
CCO 4							
GTB 1	894.6	520.4	404.5	759.8	463.6	519.2	484.9
GTB 2			126.9	134.4	197.1	157.6	56.7
GTB 3		375.2	133.7	29.8	102.3	189.2	177.6
GTB 4							168.0
Total	1344.03	1398.3	1361.7	1432.8	1475.2	1286.2	1463.6

Iter. #	22	23	24	25	26	27	28
NUC 1	262.2	348.5	249.8	38.9	186.3	190.3	261.6
NUC 2	159.2		72.1	173.6	109.5	127.6	146.7
NUC 3				195.5	211.7	178.1	87.4
NUC 4							
CCO 1		21.5	384.3	308.4	206.1	211.0	199.6
CCO 2							
CCO 3							
CCO 4							
GTB 1	737.8	444.1	408.6	471.5	472.1	461.7	417.7
GTB 2	41.9	135.2		58.3	19.6	43.3	
GTB 3	243.5	218.5	161.1	31.6	59.9	60.6	152.2
GTB 4	17.7	190.2	129.5	154.6	195.2	185.4	192.4
Total	1462.3	1358.0	1405.4	1432.4	1460.4	1458.0	1457.6

(Talbe 5.4.9, cont'd.)

Iter. #	29	30	31	32	33	34	35
NUC 1	213.0	238.4	225.8	215.0	181.6	202.7	194.1
NUC 2	146.4	152.5	116.9	130.8	132.1	127.0	138.8
NUC 3	115.3	89.5	166.9	145.3	161.5	159.4	140.9
NUC 4							
CCO 1	230.0	218.1	198.4	216.2	238.5	221.5	236.1
CCO 2							
CCO 3							
CCO 4							
GTB 1	435.4	421.5	459.2	450.0	460.5	457.8	449.8
GTB 2			23.8	13.7	11.3	15.0	6.7
GTB 3	114.1	140.6	68.0	84.9	62.6	69.1	83.8
GTB 4	199.3	193.7	204.5	202.5	207.1	205.9	204.0
Total	1453.5	1454.3	1463.5	1458.4	1455.2	1458.4	1454.2

Iter. #	36	37	38	39	40
NUC 1	203.2	210.3	206.4	211.3	211.3
NUC 2	111.0	124.4	117.3	126.4	126.4
NUC 3	191.1	160.3	176.8	155.7	155.8
NUC 4					
CCO 1	208.3	214.7	211.3	215.6	215.6
CCO 2					
CCO 3					
CCO 4					
GTB 1	473.0	457.8	466.0	455.6	455.7
GTB 2	28.1	17.7	23.7	16.6	16.6
GTB 3	39.2	69.2	52.7	73.2	73.2
GTB 4	209.4	205.5	207.5	204.9	204.9
Total	1463.3	1459.9	1461.7	1459.3	1459.5

Table 5.4.10

Iter. #	Lower Bound	Solution	Upper Bound	% Range of Optimality*
0	---	1546.74	6965.1	---
1	0	6758.62	7381.81	---
2	1467.36	1896.19	2590.82	31.60
3	1843.74	1959.97	2347.91	14.05
4	1891.69	2069.10	2349.66	11.82
5	1937.49	2030.69	2149.88	9.68
6	1977.07	2025.80	2278.35	7.83
7	2008.23	2068.05	2154.10	6.38
8	2031.04	2070.45	2096.28	5.32
9	2033.84	2060.68	2087.62	5.19
10	2046.62	2065.19	2091.34	4.59
11	2048.92	2057.00	2082.07	4.49
12	2054.81	2061.82	2088.48	4.21
13	2055.38	2059.37	2083.97	4.19
14	2057.32	2059.47	2086.83	4.09
15	2063.43	2069.52	2071.59	3.81
16	2063.57	2069.78	2071.25	3.80
17	2066.04	2069.14	2073.13	3.69
18	2067.01	2068.78	2070.82	3.64
19	2067.26	2071.18	2072.29	3.63
20	2068.11	2071.42	2074.67	3.59
21	2068.17	2069.42	2070.74	3.58
22	2068.28	2072.55	2074.01	3.58
23	2069.10	2068.45	2078.57	3.55
24	2069.70	2071.05	2073.21	3.52
25	2069.75	2071.09	2071.52	3.52
26	2069.80	2071.89	2072.63	3.51
27	2070.17	2073.32	2095.02	3.50
28	2070.94	2061.11	2159.35	3.46
29	2086.18	2074.05	2199.30	2.75
30	2103.53	2106.24	2156.12	1.94

(Table 5.4.10, cont'd.)

Iter. #	Lower Bound	Solution	Upper Bound	% Range of Optimality*
31	2104.81	2096.65	2194.53	1.88
32	2114.30	2102.29	2227.71	1.44
33	2131.53	2125.72	2226.70	0.635
34	2136.12	2137.07	2143.62	0.421
35	2137.26	2138.80	2139.40	0.368
36	2138.49	2136.49	2169.74	0.311
37	2143.18	2144.86	2146.63	0.0923
38	2143.65	2144.60	2144.72	0.0739
39	2143.97	2145.76	2145.96	0.0555
40	2145.14	2139.90	2141.14	0.000932
41	2145.16	2140.22	2140.69	0.0
42	2145.16	2139.62	2139.85	0.0
43	2145.16	2140.10	2140.1	0.0

* % range of optimality calculated with respect to the optimal solution.

Table 5.4.11

Iter. #	Fict. Cap. Added in Year 1 (MW) [% unserved energy]	Fict. Cap. Added in Year 2 (MW) [% unserved energy]	Fict. Cap. Added in Year 3 (MW) [% unserved energy]	Fict. Cap. Added in Year 4 (MW) [% unserved energy]
0	2665 [4.683]	3582 [5.535]	6082 [6.632]	8957 [8.098]
1	341.5 [1.405]	491.8 [1.662]	695.9 [1.992]	963.9 [2.432]
2	773.8 [2.081]	444.2 [1.638]	802.1 [2.129]	1027.6 [2.775]
3	260.9 [1.343]	315.8 [1.446]	367.7 [1.519]	588.2 [1.992]
4	397.4 [1.545]	362.2 [1.476]	354.6 [1.456]	409.6 [1.542]

(Table 5.4.11, cont'd.)

Iter. #	Fict. Cap. Added in Year 1 (MW) [% unserved energy]	Fict. Cap. Added in Year 2 (MW) [% unserved energy]	Fict. Cap. Added in Year 3 (MW) [% unserved energy]	Fict. Cap. Added in Year 4 (MW) [% unserved energy]
5	172.9 [1.209]	147.6 [1.159]	152.5 [1.163]	161.7 [1.173]
6	98.6 [1.067]	0.0 [0.900]	179.4 [1.209]	396.6 [1.626]
7	11.9 [0.919]	111.1 [1.084]	0.0 [0.901]	132.9 [1.113]
8	9.88 [0.916]	18.6 [0.930]	0.0 [0.900]	41.1 [0.963]
9	39.4 [0.967]	32.0 [0.953]	0.0 [0.900]	39.4 [0.963]
10	27.5 [0.943]	39.4 [0.960]	0.0 [0.900]	35.9 [0.954]
11	18.7 [0.932]	17.8 [0.930]	0.0 [0.900]	39.1 [0.963]
12	7.37 [0.912]	8.32 [0.913]	0.0 [0.900]	43.6 [0.966]
13	11.16 [0.918]	11.97 [0.919]	0.0 [0.900]	39.5 [0.962]
14	5.11 [0.908]	11.47 [0.919]	0.0 [0.900]	44.4 [0.971]
15	1.68 [0.903]	3.44 [0.906]	0.0 [0.900]	0.0 [0.900]
16	1.44 [0.902]	1.80 [0.903]	0.0 [0.900]	2.23 [0.903]
17	5.95 [0.910]	6.51 [0.910]	0.0 [0.900]	0.0 [0.900]
18	2.84 [0.905]	3.35 [0.905]	0.0 [0.900]	0.0 [0.900]
19	1.82 [0.903]	1.66 [0.903]	0.0 [0.900]	0.0 [0.900]
20	1.42 [0.902]	5.47 [0.909]	0.0 [0.900]	0.0 [0.900]

(Table 5.4.11, cont'd.)

Iter. #	Fict. Cap. Added in Year 1 (MW) [% unserved energy]	Fict. Cap. Added in Year 2 (MW) [% unserved energy]	Fict. Cap. Added in Year 3 (MW) [% unserved energy]	Fict. Cap. Added in Year 4 (MW) [% unserved energy]
21	2.17 [0.904]	1.15 [0.902]	0.0 [0.900]	0.0 [0.900]
22	1.03 [0.902]	2.45 [0.904]	0.0 [0.900]	0.0 [0.900]
23	0.0 [0.900]	0.0 [0.900]	0.0 [0.900]	16.9 [0.926]
24	1.56 [0.903]	3.6 [0.906]	0.0 [0.900]	0.0 [0.900]
25	0.68 [0.901]	0.28 [0.900]	0.0 [0.900]	0.0 [0.900]
26	0.0 [0.901]	1.26 [0.902]	0.0 [0.900]	0.0 [0.900]
27	3.52 [0.906]	0.0 [0.900]	3.71 [0.958]	10.1 [0.916]
28	0.0 [0.900]	0.0 [0.900]	155.9 [1.158]	155.6 [1.152]
29	0.60 [0.901]	4.39 [0.907]	0.0 [0.900]	208.2 [1.235]
30	0.0 [0.900]	0.0 [0.900]	88.4 [1.043]	4.02 [0.906]
31	0.0 [0.900]	0.0 [0.900]	158.4 [1.160]	131.8 [1.111]
32	0.0 [0.900]	0.31 [0.900]	0.0 [0.900]	209 [1.235]
33	0.0 [0.901]	0.0 [0.900]	168.4 [1.178]	94.4 [1.051]
34	0.0 [0.900]	1.2 [0.902]	0.0 [0.900]	10.9 [0.917]
35	0.0 [0.900]	1.03 [0.902]	0.0 [0.900]	0.0 [0.900]

(Table 5.4.11, cont'd.)

Iter. #	Fict. Cap. Added in Year 1 (MW) [% unserved energy]	Fict. Cap. Added in Year 2 (MW) [% unserved energy]	Fict. Cap. Added in Year 3 (MW) [% unserved energy]	Fict. Cap. Added in Year 4 (MW) [% unserved energy]
36	1.1 [0.902]	0.58 [0.901]	0.0 [0.900]	55.4 [0.987]
37	1.76 [0.903]	2.92 [0.905]	0.0 [0.900]	0.0 [0.900]
38	0.0 [0.900]	0.21 [0.900]	0.0 [0.900]	0.0 [0.900]
39	0.32 [0.900]	0.28 [0.900]	0.0 [0.900]	0.0 [0.900]
40	0.295 [0.900]	2.11 [0.903]	0.0 [0.900]	0.0 [0.900]
41	0.0 [0.900]	0.80 [0.901]	0.0 [0.900]	0.0 [0.900]
42	0.362 [0.901]	0.0 [0.900]	0.0 [0.900]	0.0 [0.900]
43	0.0 [0.900]	0.0 [0.900]	0.0 [0.900]	0.0 [0.900]

Table 5.4.12

Proposed Trial Solutions (MW)

Iter. #	1	2	3	4	5	6	7
NUC 1	7904.0		616.1		155.7	365.4	324.0
NUC 2		292.0			4.2	442.4	
NUC 3			121.0				420.1
NUC 4							
CCO 1		492.5			60.3		57.1
CCO 2							
CCO 3							
CCO 4							
GTB 1			197.2	632.1	245.0	476.4	528.2
GTB 2			79.3	141.4	118.9		47.0
GTB 3				139.9	137.7		
GTB 4				117.1	148.2		
Total	7904.0	784.5	1013.6	1030.5	870.0	1284.2	1376.4

Iter. #	8	9	10	11	12	13	14
NUC 1	447.4	152.2	147.0	300.8	252.1	179.3	300.7
NUC 2		29.6	50.9		67.0	100.2	109.4
NUC 3		139.5	97.7	97.2	133.2	71.4	98.2
NUC 4		142.3			134.4	70.7	
CCO 1	53.2	269.9	125.8	191.7	70.5	194.0	140.1
CCO 2							
CCO 3							
CCO 4							
GTB 1	504.8	415.0	630.0	376.3	602.6	506.1	451.6
GTB 2	117.0	104.4	65.5	127.1	61.6	35.5	24.9
GTB 3	176.8	58.7	81.1	101.7	53.1	125.2	126.6
GTB 4	58.7		120.3	122.5		57.9	117.2
Total	1357.9	1311.6	1318.3	1317.3	1374.5	1340.3	1368.7

(Table 5.4.12, cont'd.)

Iter. #	15	16	17	18	19	20	21
NUC 1	318.6	301.5	229.0	243.8	194.0	272.5	270.6
NUC 2	53.0	97.3	67.7	83.2	85.5		81.7
NUC 3			95.0	98.3	53.9	142.5	27.1
NUC 4	110.9	64.9					
CCO 1	163.2	110.0	224.7	173.8	226.2	145.3	185.8
CCO 2							
CCO 3							
CCO 4							
GTB 1	411.3	494.6	420.0	468.6	460.0	477.4	428.3
GTB 2	76.0	38.5	64.7	50.9	49.0	120.9	53.4
GTB 3	194.0	197.5	105.6	105.4	134.2	45.2	171.5
GTB 4	56.1	92.1	168.3	164.6	167.2	184.9	158.3
Total	1383.1	1396.4	1375.0	1388.6	1370.0	1388.7	1376.7

Iter. #	22	23	24	25	26	27	28
NUC 1	228.8	297.7	343.5	322.7	300.3	225.7	293.3
NUC 2	23.7	148.5	45.5	39.1	27.0	53.2	73.9
NUC 3	62.6		85.6	83.1	77.9	62.6	
NUC 4							
CCO 1	203.6	132.4	98.6	106.3	118.6	133.1	140.4
CCO 2							
CCO 3							
CCO 4							
GTB 1	450.5	469.9	469.3	479.8	485.7	543.8	464.0
GTB 2	102.1		82.2	88.9	99.3	139.5	231.3
GTB 3	116.0	208.9	113.9	109.9	108.6	51.9	22.1
GTB 4	179.3	139.1	164.2	168.0	173.2	174.0	153.3
Total	1366.6	1396.5	1402.5	1397.8	1390.6	1383.8	1378.3

(Table 5.4.12, cont'd.)

Iter. #	29	30	31	32	33	34	35
NUC 1	236.3	336.6	289.7	233.3	296.5	255.8	285.7
NUC 2	140.5		71.5	144.1	71.4	117.9	65.1
NUC 3		79.6	22.9		15.5		37.7
NUC 4							
CCO 1	138.1	78.6	121.9	137.9	125.8	138.8	116.6
CCO 2							
CCO 3							
CCO 4							
GTB 1	528.1	503.8	492.0	532.7	481.2	505.2	503.3
GTB 2		342.8	390.9		503.4	21.1	65.3
GTB 3	430.5			578.0		563.4	515.5
GTB 4		221.7	172.2		208.1	143.4	162.2
Total	1473.5	1563.1	1561.1	1626.0	1701.9	1745.6	1751.4

Iter. #	36	37	38	39	40	41	42
NUC 1	273.8	280.3	263.3	303.8	258.9	253.7	272.0
NUC 2	13.7	142.5	88.4	86.0	145.1	146.7	147.6
NUC 3	86.2						
NUC 4							
CCO 1	115.0	77.5	140.2	94.6	173.7	170.5	166.3
CCO 2							
CCO 3							
CCO 4							
GTB 1	517.5	564.5	495.5	515.0	456.0	465.7	452.1
GTB 2	111.4		45.6	47.7			
GTB 3	519.9	540.4	548.1	546.5	547.8	545.8	546.3
GTB 4	106.6	191.5	192.7	192.9	187.6	187.9	187.0
Total	1744.1	1796.7	1773.8	1786.5	1769.1	1770.3	1771.3

(Table 5.4.12, cont'd.)

Iter. #	43	44
NUC 1	256.2	256.1
NUC 2	147.5	147.5
NUC 3		
NUC 4		
CCO 1	169.9	169.9
CCO 2		
CCO 3		
CCO 4		
GTB 1	463.9	463.9
GTB 2		
GTB 3	545.3	545.3
GTB 4	187.7	187.7
Total	1770.5	1770.4

A final interesting development that arose at the beginning of these runs concerned negative λ's which occasionally emerged. This is understandable for the marginal unit, because of the discrepancies which arise from the piece-wise representation of the LDC within SYSGEN. After normalization, these λ's were on the order of 10^{-3} or (usually) less. More disturbing, however, were relatively large negative λ's associated with earlier units, which sometimes occurred during the beginning iterations, as a result of the master problem installing all its capacity in a single alternative, and SYSGEN modelling that capacity as a single huge unit. When discrete-plant representation has been implemented, this problem should disappear. Adding constraints which limit the amount of capacity that can be installed in any one alternative should also help. These negative λ's are seen to greatly affect the convexity of the problem. In one 4 time period, aggregated case in which these large negative λ's appear in the early iterations, the algorithm seemingly "converged," with Z and TC stabilizing after 29 iterations; however, almost 250 MW of fictitious capacity were still being built in this last iteration. When a condition was implemented which set any negative λ's equal to 0, this strange behavior disappeared. As this correction only affected a few early iterations, when the LP is still fluctuating between extremes, the slight change caused in these few constraints was not believed to significantly disturb the final solution, and it was consequently used in all the multi-period test cases (repeating the 1 time period case showed negligible difference between the results).

CHAPTER SIX

This chapter shall present conclusions of the work presented in previous chapters, and also mention some areas of further research which might be fruitful.

The most promising approach tried was that of adding minimum capacity constraints for every alternative. These lower bounds may be the result of some approximate technique, such as screening curves, to determine that within some range the alternative is indeed competitive, or they may be from a prior optimization run, or from the planner's own intuitive judgment. If concern is felt about over-constraining the problem, these capacity constraints may be removed after several iterations, thus insuring that enough constraints have already been generated to avoid the great fluctuations in capacity that generally occur in the beginning iterations.

Disaggregating the reliability constraint into one constraint for every infeasible time period is also recommended, as it leads to convergence in fewer iterations, which means less accumulated error, and therefore an optimal solution which costs less than that of the aggregated case. It was also seen that (although the results were not presented here) that creating a reliability constraint for <u>all</u> time periods, instead of only infeasible ones, is not a method to be pursued, as the additional constraints introduce more errors and thus lead to the slow convergence due to passing the algorithm's resolution limit observed in the previous chapter.

The strongest recommendation to emerge from this study is the implementation of the correct way of computing upper bounds as soon as possible. Once this is accomplished, then convergence may be easily determined: once the lower bound exceeds the correct upper bound, then terminate the procedure. Another stopping rule might be to look at the trial capacity plans proposed, and terminate when these stop fluctuating wildly and have settled around some values.

Some areas which may be investigated further for speeding convergence shall now be mentioned. First, more theoretical work needs to be done on incorporating the Lagrange multipliers calculated after the marginal plant has been augmented to achieve feasibility. While it was speculated that this might help quicken convergence by calculating the capacity plans on the edge of the feasible region instead of far outside it, preliminary work done here shows that this is not the case. The problem lies in using the μ's and unserved energy after; the unserved energy is always ε (to within computational tolerance), and the μ's are extremely small, as well as non-varying, so that the master problem is not given much information on the best way to proceed. A second area for investigation might be to try adding different kinds of initial constraints to the master, and see what effect they have on convergence. A third area would be to use another mathematical programming technique, namely, subgradient optimization, in the beginning iterations. As this is only a local optimization technique, control would have to be returned to the GBD algorithm, as this insures global optimality. A technical discussion of subgradient optimization is presented in Appendix A. A fourth area of investigation might be to run identical test cases with the GBD option on, using first the piece-wise load duration curve representation utilized in this work, and then a load duration curve represented by the method of cumulants [32]. The latter case should yield consistent values for λ's and μ's, as its continuous representation of the LDC's should eliminate errors introduced by SYSGEN's piecewise linear representation.

In summary, although numerical errors were discovered to have crept into the Lagrange multiplier calculation through the linear interpolation used for the load duration curves, the GBD algorithm has proved to be fairly robust and quite effective as a tool to aid in utility planning. Iterating between GBD and some sort of perturbation/sensitivity analysis might be useful, keeping as initial constraints for each GBD run the results of previous iterations. To give the utility planner a general idea of the computer time and storage requirements for the GBD algorithm, Appendix D was prepared, which includes tables of computational times and other pertinent information.

APPENDIX A

Consider the capacity expansion planning problem in the following form (suppressing the index on time for clarity, assuming that the capacities are already sorted into merit order, and viewing Y as a function of X):

$$\min \underline{C}'\underline{X} + \min EF(\underline{Y})$$

$$\text{s.t.} \quad -\underline{X} + \underline{Y} \leq 0$$

$$EG(\underline{Y}) \leq \varepsilon$$

$$\underline{X} \geq 0, \ \underline{Y} \geq 0$$

The Lagrangean function associated with this problem is:

$$L(\underline{X},\underline{\lambda},\pi) = \underline{C}'\underline{X} + \min \{EF(\underline{Y}) + \underline{\lambda}(\underline{Y}-\underline{X}) + \pi(EG(\underline{Y}) - \varepsilon)\}$$

The Lagrangean dual becomes:

$$L^* = \max_{\underline{\lambda},\pi\geq 0} L(\underline{\lambda},\pi)$$

where

$$L(\underline{\lambda},\pi) = \min_{\underline{X}\geq 0} L(\underline{X},\underline{\lambda},\pi)$$

$$\Rightarrow \quad L^* = \max_{\underline{\lambda},\pi\geq 0} \ \min_{\underline{X}\geq 0} [\underline{C}'\underline{X} + \min_{\underline{Y}\geq 0}\{ EF(\underline{Y}) + \underline{\lambda}(\underline{Y}-\underline{X}) + \pi(EG(\underline{Y}) -)\}]$$

Vectors $(\overline{\gamma},\overline{\delta})$ are called <u>subgradients</u> of $L(\lambda,\pi)$ at $(\overline{\lambda},\overline{\pi})$ if:

$$L(\lambda,\pi) \leq L(\overline{\lambda},\overline{\pi}) + (\lambda-\overline{\lambda})\overline{\gamma} + (\pi-\overline{\pi})\overline{\delta} \qquad \forall \lambda,\pi$$

Subgradients point in the direction of steepest ascent of $L(\lambda,\pi)$ at $(\overline{\lambda},\overline{\pi})$. <u>Subgradient optimization</u> uses these subgradients to generate a sequence of nonnegative solutions $\{(\lambda^{\ell},\pi^{\ell})\}_{\ell=1}^{\infty}$ to the Lagrangean dual by the rule:

$$\lambda_i^{\ell+1} = \max\ \{0, \lambda_i^\ell + \theta^\ell \gamma_i^\ell\} \qquad \forall_i$$

$$\pi_j^{\ell+1} = \max\ \{0, \pi_j^\ell + \theta^\ell \delta_j^\ell\} \qquad \forall_j$$

where θ^ℓ is the step length which satisfies $\sum_{\ell=1}^{\infty} \theta^\ell = +\infty$, but $\theta^\ell \to 0^+$. Polyak [30] shows that the (λ^ℓ, π^ℓ) given by this rule converge to an optimal solution of L^*. Since this method converges very slowly, Polyak [31] proposes the following rule, which converges in a finite number of steps to any target value $\overline{L} < L^*$:

$$\theta^\ell = \rho^\ell \frac{(\overline{L} - L(\lambda^\ell, \pi^\ell))}{||(\gamma^\ell, \delta^\ell)||^2}$$

where $0 < \varepsilon_1 < \rho^\ell < 2 - \varepsilon_2 < 2$. Choosing θ^ℓ in this fashion is very tricky, as the target value selected $\overline{L}$ must be less than L^* (which is unknown) for convergence (if a target value greater than L^* is chosen, oscillation occurs).

Although there is no theoretical guarantee that using this last rule for determining θ^ℓ yields increasing lower bounds, experience in the literature has shown that this will occur "using the correct combination of artistic expertise and luck." [36, p.124]

APPENDIX B

Section B.1

The following pages display the results of a complete SYSGEN iteration done manually. Only one time period was considered, and the size of the original load duration curve was reduced to 20 points. Its peak was set at 1500 MW, the first half of which remained constant at the value 1.0, the second half of which it decreased linearly to zero. The initial customer energy demand was 9828000 MWH. Only one committed and existing plant (NUC-1000 MW) was considered, and the number of alternatives was cut to two (CCO-500 MW; GTB-200 MW). The peak demand and plant capacities were deliberately chosen not to be exact multiples of the LDC-spacing (75 MW in this example) so as not to avoid generating errors due to interpolating between two points. The availability of the nuclear unit was 0.7, the combined cycle oil unit was 0.8, and the gas turbine was 0.9. All other data remained the same as that described in Chapter Five.

It was seen that very good agreement was obtained with the values of the equivalent load duration curves, the most serious defect being that the first point at which the curves go to zero had to occur on a 75-MW increment, which in actuality was not always where it should land. This resulted in discrepancies between the unserved energy figures. However, the shadow prices agreed very closely, providing positive proof that SYSGEN was correctly computing the dual multipliers as specified by the equations in Chapter Two.

Table B.1.1

Original Load Duration Curve

Load, x	$G_1(x)$
0	1.0
75	1.0
150	1.0
225	1.0
300	1.0
375	1.0
450	1.0
525	1.0
600	1.0
675	1.0
750	1.0
825	0.9
900	0.8
975	0.7
1050	0.6
1125	0.5
1200	0.4
1275	0.3
1350	0.2
1425	0.1
1500	0.0

Table B.1.2

Second Equivalent Load Duration Curve

Load, x	$G_2(x)$	Load, x	$G_2(x)$
0	1.0	1875	0.2499999
75	1.0	1950	0.2199999
150	1.0	2025	0.1899999
225	1.0	2100	0.1599999
300	1.0	2175	0.1299999
375	1.0	2250	0.0999999
450	1.0	2325	0.0699999
525	1.0	2400	0.0399999
600	1.0	2475	0.00999999
675	1.0	2550	0.0
750	1.0		
825	0.93		
900	0.86		
975	0.79		
1050	0.72		
1125	0.65		
1200	0.58		
1275	0.51		
1350	0.44		
1425	0.37		
1500	0.30		
1575	0.30		
1650	0.30		
1725	0.30		
1800	0.2799999		

actual terminal pt.: 2500

Table B.1.3

Third Equivalent Load Duration Curve

Load, x	$G_3(x)$	Load, x	$G_3(x)$
0	1.0	1875	0.2833332
75	1.0	1950	0.2453332
150	1.0	2025	0.2119999
225	1.0	2100	0.1879999
300	1.0	2175	0.1639999
375	1.0	2250	0.1386665
450	1.0	2325	0.1099998
525	1.0	2400	0.0799998
600	1.0	2475	0.0499998
675	1.0	2550	0.0359999
750	1.0	2625	0.0299999
825	0.944	2700	0.0239999
900	0.888	2775	0.0179999
975	0.832	2850	0.0119999
1050	0.776	2925	0.00599998
1125	0.720	3000	0.0
1200	0.664		
1275	0.6033333		
1350	0.5333333		
1425	0.4633333		
1500	0.3933333		
1575	0.3793333		
1650	0.3653333		
1725	0.3513333		
1800	0.3213332		

actual terminal pt.: 3000

Table B.1.4

Final Equivalent Load Duration Curve

Load, x	$G_4(x)$	Load, x	$G_4(x)$
0	1.0	1875	0.29106654
75	1.0	1950	0.25493312
150	1.0	2025	0.22166655
225	1.0	2100	0.19626655
300	1.0	2175	0.17102211
375	1.0	2250	0.14519979
450	1.0	2325	0.11699979
525	1.0	2400	0.08755534
600	1.0	2475	0.0579109
675	1.0	2550	0.04239998
750	1.0	2625	0.0339998
825	0.9496	2700	0.02613332
900	0.8992	2775	0.0195999
975	0.84693333	2850	0.0135999
1050	0.79093333	2925	0.00759997
1125	0.73493333	3000	0.00159999
1200	0.67893333	3075	0.000999992
1275	0.6187333	3150	0.00039998
1350	0.5501333	3225	0.0
1425	0.48137774		
1500	0.41199996		
1575	0.39239996		
1650	0.37279996		
1725	0.3550666		
1800	0.32666654		

actual terminal pt.: 3200

Table B.1.5

Computational Results

Second Equivalent Load Duration Curve

1.0000000000	1.0000000000	1.0000000000	1.0000000000	1.0000000000
1.0000000000	1.0000000000	1.0000000000	1.0000000000	1.0000000000
0.9299999845	0.8600000107	0.7899999952	0.7200000215	0.6500000060
0.5799999905	0.5100000167	0.4400000012	0.3700000274	0.3000000119
0.3000000119	0.3000000119	0.3000000119	0.2800000064	0.2500000099
0.2200000075	0.1900000111	0.1600000087	0.1300000004	0.1000000040
0.0700000016	0.0400000052	0.0100000028	0.0	

Third Equivalent Load Duration Curve

1.0000000000	1.0000000000	1.0000000000	1.0000000000	1.0000000000
1.0000000000	1.0000000000	1.0000000000	1.0000000000	1.0000000000
0.9439999868	0.8880000069	0.8319999937	0.7760000138	0.7200000006
0.6639999874	0.6033333401	0.5333333274	0.4633333508	0.3933333381
0.3793333414	0.3653333391	0.3513333397	0.3213333383	0.2833333413
0.2453333423	0.2120000100	0.1880000077	0.1640000007	0.1386666696
0.1100000004	0.0800000036	0.0500000016	0.0359999999	0.0299999994
0.0239999989	0.0179999996	0.0119999998	0.0060000005	0.0

Final Equivalent Load Duration Curve

1.0000000000	1.0000000000	1.0000000000	1.0000000000	1.0000000000
1.0000000000	1.0000000000	1.0000000000	1.0000000000	1.0000000000
0.9495999894	0.8992000089	0.8469333308	0.7909333487	0.7349333377
0.6789333256	0.6187333440	0.5501333316	0.4813777972	0.4120000090
0.3924000106	0.3728000083	0.3550666738	0.3266666731	0.2910666763
0.2549333443	0.2216666785	0.1962666764	0.1710222254	0.1452000051
0.1170000026	0.0875555607	0.0579111146	0.0424000016	0.0340000007
0.0261333329	0.0196000000	0.0136000002	0.0076000008	0.0016000003
0.0010000002	0.0004000001	0.0		

Table B.1.6

Comparison of Relevant Results

		hand	computer
Energy generated by first plant (MWH)		5860400	5860397
First iteration: λ^1		-2.8839113	-2.88391
Unserved energy after first plant (MWH)		3965414.2	3966137.0
Energy generated by Second plant (MWH)		1863679.9	1863676.0
Second iteration: λ^2		-2.884416	-2.88442
λ^1		1.6029574	1.60297
Unserved energy after second plant (MWH)		2324429.8	2104059.0
Energy generated by third plant (MWH)		589155.74	589153.50
Third iteration: λ^3		-10.27468	-10.2747
λ^1		2.6804542	2.68046
λ^2		- 2.8844131	- 2.88442
Unserved energy after third plant (MWH)		1515102.26	1515130.0
Final computations (before normalizing)	λ^1	4.6359063	4.63592
	λ^2	4.8518847	4.8518800
	λ^3	0.0399081	0.0*
(after normalizing**)	λ^1	0.0404992***	0.0404993***
	λ^2	0.042386	0.042386
	λ^3	0.000348637	0.0*
	μ^1	0.0609777***	0.0609778***
	μ^2	0.2412444	0.2412440
	μ^3	0.3216443	0.3216440

*code contained statement which set λ of the marginal plant equal to zero

**normalization factor = $8736/10^6$

***shadow prices for CEX plants are not used in master problem

Section B.2

The discrepancy in the λ of the marginal plant, λ_{IMRG}, which Bloom [4] maintains should equal zero, but which computationally turns out to be non-zero, was extremely worrisome. Having eliminated the possibility of a coding error through the hand verification of the previous section, the formulas which lead to the equations for $H_{ij}(U)$ were re-derived carefully. With no errors turning up here, it was decided to go back to the original equation for λ, and verify that it does indeed yield $\lambda_{IMRG} = 0.0$ theoretically. The following equations step through the derivation ($I_{mrg} \equiv I$ below):

$$\lambda^i = - \frac{\partial}{\partial Y^i} EF(\underline{Y}) - \Pi \frac{\partial}{\partial Y^i} EG(\underline{Y}) \qquad \text{for } i < I$$

$$\lambda^i \stackrel{?}{=} 0 \text{ for } i \geq I \text{ implies } \pi \frac{\partial}{\partial Y^I} EG(\underline{Y}) \stackrel{?}{=} - \frac{\partial}{\partial Y^I} EF(\underline{Y})$$

$$\pi \frac{\partial}{\partial Y^I} \int_{U^I}^{\infty} G_{I+1}(Q)dQ \stackrel{?}{=} - F^I p_I \frac{\partial}{\partial Y^I} \int_{U^{I-1}}^{U^I} G_I(Q)dQ \,]$$

$$\pi \frac{\partial}{\partial Y^I} [\int_0^{\infty} G_{I+1}(Q)dQ - \int_0^{U^I} G_{I+1}(Q)dQ]$$

$$\stackrel{?}{=} - F^I p_I \frac{\partial}{\partial Y^I} [\int_0^{U^I} G_I(Q)dQ - \int_0^{U^{I-1}} G_I(Q)dQ$$

$$\pi[H_{I+1,I}(\infty) - H_{I+1,I}(U^I)] \stackrel{?}{=} - F^I p_I [H_{I,I}(U^I) - H_{I,I}(U^{I-1})]$$

Since:

$$H_{I+1,I}(\infty) = 0, \qquad \text{by definition}$$

$$H_{I,I}(U^{I-1}) = \frac{\partial}{\partial Y^I} \int_0^{U^{I-1}} G_I(Q;Y', \ldots, Y^{I-1})dQ$$

$$= - \underset{=1}{G_I(0)} \underset{=0}{\frac{\partial 0}{\partial Y^I}} \quad \text{since the derivative of a constant equals zero}$$

$$+ G_I(U^{I-1}) \underset{=0}{\frac{\partial U^{I-1}}{\partial Y^I}} \quad \text{since } U^{I-1} = \sum_{i=1}^{I-1} Y^i \text{ is not a function of } Y^I$$

$$+ \int_0^{U^{I-1}} \underset{= 0}{\frac{\partial}{\partial Y^I} G_I(Q;Y^1, \ldots, Y^{I-1})}dQ \quad \text{since } G_I \text{ is not a function of } Y^I$$

$$= 0$$

$$= - \ H_{I+1,I}(U^I) \overset{?}{=} -F^I p_I H_{I,I}(U^I)$$

$$- \ [p_I H_{I,I}(U^I) + q_I H_{I,I}(U^I - Y^I)] \overset{?}{=} - F^I p_I H_{I,I}(U^I)$$

Again, since:

$$H_{I,I}(U^I - Y^I) = H_{I,I}(U^{I-1}) = 0$$

$$- \pi p_I H_{I,I}(U^I) \overset{?}{=} F^I p_I H_{I,I}(U^I).$$

If $\pi = F^I$, which is the value currently used, then the above equality holds, confirming the result that $\lambda_{Imrg} = 0.0$.

APPENDIX C

The following example was constructed by Michael Caramanis of the Massachusetts Institute of Technology Energy Laboratory.

Assume a triangular LDC with U = system peak. Only two units are considered in this example, unit 1 being the first loaded, 2 being the second, with capacities of X_1 and X_2, and availabilities of p_1 and p_2, respectively. The energy generated by each unit shall be calculated below (invoking the rule of similar triangles to determine points on the LDC):

$$E_1 = [\ \frac{1}{2} \cdot U \cdot 1 - \frac{1}{2} \cdot (U-X_1) \cdot \frac{(U-X_1)}{U}\ ]p_1 = \frac{p_1 U}{2} - \frac{p_1(U-X_1)^2}{2}$$

$$E_2 = [\ \frac{1}{2} \cdot U \cdot 1 - \frac{1}{2} \cdot (U-X_2) \cdot \frac{(U-X_2)}{U}](1-p_1)p_2$$

$$+ [\ \frac{1}{2} \cdot U \cdot 1 - \frac{1}{2} \cdot (U-X_1-X_2) \cdot \frac{(U-X_1-X_2)}{U}$$

$$- \frac{1}{2} \cdot U \cdot 1 + \frac{1}{2} \cdot (U-X_1) \frac{(U-X_1)}{U}]p_1 p_2$$

$$= \frac{(1-p_1)p_2 U}{2} - \frac{(1-p_1)p_2(U-X_2)^2}{2U} - \frac{p_1 p_2(U-X_1-X_2)^2}{2U} + \frac{p_1 p_2(U-X_1)^2}{2U}$$

Therefore the sum of the energies generated is given by:

$$E_1 + E_2 = \frac{U}{2} p_1 + \frac{U}{2}(1-p_1)p_2 + \frac{1}{2U}[(U-X_1)^2(p_1 p_2 - p_1)$$

$$- (U-X_2)^2(1-p_1)p_2 - (U-X_1-X_2)^2 p_1 p_2] = \frac{U}{2}[1-q_1 q_2]$$

$$- \frac{1}{2U}[(U-X_1)^2(p_1 - p_1 p_2) + (U-X_2)^2 q_1 p_2 + (U-X_1-X_2)^2 p_1 p_2]$$

Now assume that $q_1 = q_2 = q$ and $p_1 = p_2 = p$:

$$E_1 + E_2 = \frac{U}{2}[1-q^2] - \frac{1}{2U}[(U-X_1)^2 pq + (U-X_2)^2 pq + (U-X_1-X_2)^2 p^2]$$

$$= \frac{U}{2}[1-q^2] - \frac{p}{2U}[(U-X_1)^2 q + (U-X_2)^2 q + (U-X_1-X_2)^2 p] \quad (*)$$

Let $p = 0.9$, $U = 2000$ MW, $X_1 = 1500$ MW.

It is clear that:

$$E_1 + E_2 = \frac{1}{2} \cdot U \cdot 1 \cdot p = 900 \text{ MW-years.}$$

Now equating both sides of equation (*), using the above values, and solving for X_2, yields:

$$900 = 1000\ (1-.01) - \frac{.9}{4000}[25000 + (U^2 + X_2^2 - 2X_2U)*.1$$

$$+ (250000 + X_2^2 - 1000X_2)p]$$

$$900 = 990 - \frac{9}{40000}[25000 + 400000 + \frac{X_2^2}{10}$$

$$- 400X_2 + 225000 + X_2^2*.9 - 900X_2]$$

$$900 = 990 - \frac{9}{40000}[\ 650000 - 1300X_2 + X_2^2\ ]$$

$$90 - 146.25 + .2925X_2 - .000225X_2^2 = 0; - .000225X_2^2 + .2925X_2 - 56.25 = 0$$

$$X_2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-.2925 \pm \sqrt{(.2925)^2 - .050625}}{-.000450}$$

$$= \frac{-.2925 \pm .18689904}{-.000450} = 234.668, 1065.33 = 234.668$$

as $X_1 + X_2 \leq U$.

Therefore, starting with X_1 = 1500 MW, it is seen that X_2 = 234.668 MW generates as much energy as an extra 500 MW of the first unit. Carrying out the same calculation in reverse, it is seen that, with a starting value of X_1 = 234.668 MW, X_2 = 1500 MW of fictitious capacity would generate as much energy as adding an extra 1765.33 MW of the first unit to X_1. These results are summarized in Table C.1, which clearly shows that the ratio of augmented capacity to fictitious capacity increases as the margin is approached.

Table C.1

Comparison of Augmented vs. Fictitious Capacity Required

(all capacities in MW)

alternate capacity	amount by which one needs to augment some alternative	necessary fictitious capacity	ratio
1500	500	234.668	2.13
234.668	1765.33	1500.00	1.17688

APPENDIX D

The runs in this thesis were done on an IBM 370/168. The default on memory size is 440K. The version of SYSGEN-4 used with GBD required 700K to be loaded into core. Most of the jobs were run in 800K, to leave a margin for the LP. The ordinary LP was dimensioned to handle up to 4 alternatives, 15 time periods, and 100 constraints. This was good for up to 50 iterations in the aggregated case. The disaggregated case required a larger matrix, so one was created which extended the 100-constraint capacity to 200 constraints, SEXOP's limit. This required 1024K to load into core, but no account was taken of the fact that the number of nonzero entries would be about 60-70% of the (# rows)*(# columns). Setting this variable down would conserve on storage requirements.

The maximum number of constraints in the aggregated case is 2 • K, where K = # iterations. The maximum number constraints in the disaggregated case is (T+1)*K, where T = # time periods. Therefore, with a 20 time period study, at worst only 9 iterations could be run keeping all the constraints; then a procedure would have to be arranged to discard old, unused constraints.

The following tables give a listing of the actual CPU times (in seconds) required to run 1, 2, and 4 time period cases, aggregated and disaggregated. It can be seen that SYSGEN requires about 2.5 seconds for 1 and 2 time periods, and slightly over 3 seconds for 4 time periods.

Within each run, the times are fairly constant. The master problem, naturally, increases its time requirements as the number of iterations increase. It takes roughly 1.5 seconds in the 1 and 2 time period case. With 4 time periods (aggregated), it increases in the later iterations to about 2.25 seconds, while the 4 time period (disaggregated) case increases to just slightly under 3 seconds. A run of SYSGEN with 9 time periods was done, and found that it requires approximately 5 seconds of CPU time.

Table D.1

CPU times (sec.) - 1 time period

SYSGEN		MASTER	
0)	2.51	1)	1.42
1)	2.81	2)	1.49
2)	2.74	3)	1.62
3)	2.62	4)	1.55
4)	2.59	5)	1.58
5)	2.45	6)	1.49
6)	2.71	7)	1.57
7)	2.42	8)	1.52
8)	2.62	9)	1.56
9)	2.49	10)	1.54
10)	2.48	11)	1.56
11)	2.42	12)	1.60
12)	2.43	13)	1.59
13)	2.36	14)	1.63
14)	2.58	15)	1.68
15)	2.33	16)	1.61
16)	2.41	17)	1.64

Table D.2

CPU times (sec.) - 2 time periods (aggregated)

	SYSGEN		MASTER
0)	2.37		
1)	2.38	1)	1.36
2)	2.47	2)	1.37
3)	2.46	3)	1.42
4)	2.51	4)	1.44
5)	2.43	5)	1.40
6)	2.59	6)	1.46
7)	2.46	7)	1.52
8)	2.65	8)	1.49
9)	2.51	9)	1.54
10)	2.46	10)	1.53
11)	2.59	11)	1.52
12)	2.54	12)	1.52
13)	2.62	13)	1.53
14)	2.54	14)	1.61
15)	2.38	15)	1.63
16)	2.48	16)	1.51
17)	2.49	17)	1.60
18)	2.59	18)	1.64
19)	2.51	19)	1.69
20)	2.43	20)	1.55
21)	2.41	21)	1.63
22)	2.74	22)	1.58
23)	2.53	23)	1.72
24)	2.58	24)	1.80
25)	2.63	25)	1.81
26)	2.76	26)	1.77
27)	2.55	27)	1.81
28)	2.51	28)	1.79
29)	2.61	29)	1.72

Table D.3

CPU times (sec.) - 2 time periods (disaggregated)

SYSGEN		MASTER	
0)	2.40		
1)	2.56	1)	1.43
2)	2.53	2)	1.46
3)	2.56	3)	1.46
4)	2.50	4)	1.45
5)	2.56	5)	1.53
6)	2.64	6)	1.50
7)	2.58	7)	1.58
8)	2.51	8)	1.54
9)	2.54	9)	1.51
10)	2.59	10)	1.64
11)	2.66	11)	1.63
12)	2.62	12)	1.64
13)	2.58	13)	1.65
14)	2.54	14)	1.73
15)	2.67	15)	1.81
16)	2.82	16)	1.75
17)	2.68	17)	1.80
18)	2.61	18)	1.85
19)	2.54	19)	1.87
20)	2.71	20)	1.82

Table D.4

CPU times (sec.) - 4 time periods (aggregated)

Run	SYSGEN	MASTER
0)	2.92	
1)	3.07	1.56
2)	3.13	1.51
3)	2.92	1.56
4)	3.26	1.57
5)	3.18	1.64
6)	3.16	1.73
7)	3.06	1.65
8)	3.12	1.69
9)	3.05	1.61
10)	3.14	1.65
11)	2.97	1.64
12)	3.27	1.75
13)	3.08	1.77
14)	3.36	1.78
15)	3.04	1.82
16)	3.25	1.67
17)	3.00	1.79
18)	3.08	1.74
19)	3.16	1.82
20)	3.42	1.87
21)	3.01	1.82
22)	2.97	1.75
23)	2.94	1.84
24)	3.12	1.96
25)	3.24	2.13
26)	3.28	1.96
27)	3.21	2.08
28)	3.15	2.13
29)	3.13	2.09
30)	3.19	2.05
31)	3.23	2.20
32)	3.18	2.18
33)	3.14	2.14
34)	3.19	2.17
35)	3.21	2.25
36)	3.21	2.22
37)	3.14	2.24
38)	3.24	2.24
39)	3.11	2.33
40)	3.01	2.24

Table D.5

CPU times (sec.) - 4 time periods (disaggregated)

0)	2.76						
1)	2.93	31)	3.22	1)	1.51	31)	2.64
2)	2.87	32)	3.11	2)	1.43	32)	2.42
3)	2.88	33)	3.23	3)	1.49	33)	2.60
4)	2.72	34)	3.10	4)	1.46	34)	2.82
5)	3.07	35)	3.12	5)	1.56	35)	2.84
6)	2.88	36)	3.27	6)	1.57	36)	2.81
7)	2.94	37)	3.03	7)	1.65	37)	2.70
8)	2.93	38)	3.03	8)	1.70	38)	2.93
9)	3.16	39)	3.03	9)	1.77	39)	2.78
10)	3.23	40)	2.95	10)	1.79	40)	2.68
11)	3.01	41)	2.90	11)	1.81	41)	2.92
12)	3.27	42)	2.93	12)	1.90	42)	2.80
13)	3.24	43)	2.94	13)	2.07	43)	2.78
14)	3.23	44)	3.01	14)	2.01	44)	2.91
15)	3.12			15)	2.07		
16)	3.16			16)	2.09		
17)	3.11			17)	2.17		
18)	3.12			18)	2.17		
19)	3.14			19)	2.19		
20)	3.05			20)	2.20		
21)	3.21			21)	2.36		
22)	3.18			22)	2.40		
23)	3.00			23)	3.00		
24)	3.26			24)	2.39		
25)	3.23			25)	2.49		
26)	3.28			26)	2.55		
27)	3.31			27)	2.49		
28)	3.13			28)	2.57		
29)	3.01			29)	2.56		
30)	3.11			30)	2.47		

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