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Application of Constrained Nonlinear Minimization to Estimate Parameters of Transmission Line in Power System

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ABSTRACT

One of the essential components of power systems is the transmission line. It is used in the power system to transfer electricity energy from generation units to load units. Modern transmission line is a complex interconnected network. The mathematical model of transmission line is in nonlinear equations. This paper applied the constrained nonlinear minimization to estimate parameters of transmission line in power system. The great advantage in this proposed method is that the data of the Phase Measurement Unit (PMU) is not required. The ready data at power substations voltage magnitude(V), active (P) and reactive power (Q) is incorporated into mathematical model to estimate the parameters. The various methods of nonlinear iterative available in MATLAB toolbox such as *fsolve*, *fmin*, and *fmincon* are also compared in this paper.

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1. INTRODUCTION

The accurate parameters of the transmission line consisting of the series impedance ($\boldsymbol{Z}$) and the shunt admittance($\boldsymbol{Y}$) play are very important data in analysis, designing, planing, operation, and protection of powersystem [1]. The $\boldsymbol{Z}$ and $\boldsymbol{Y}$ are due to the effect of magnetic field and electric fields, respectively. There are two possible ways to access the parameters of the transmission line. The first method uses the commissioning data such as, transmission line type, and arrangement, etc., to calculate the parameters [2]. However, commissioning data may be changed by many factors such as conductor degradation, temperature, sag, length, etc. Then it affects on the accuracy of the parameters. The second approach is the so called estimation method. This method uses the measurable data such as voltage, power, etc., with some iterative numerical methods to estimate parameters as can be seen in many previous researches [3, 4]. Research [3] applied the state estimation to eliminate inaccuracies and errors of calculation. The effects of temperature and sag of transmission were also considered in algorithm as given in [4]. Most of previous researches required the data from PMU [5–7]. However, the PMU is generally implemented at some important power substations because it is an expensive instrument.

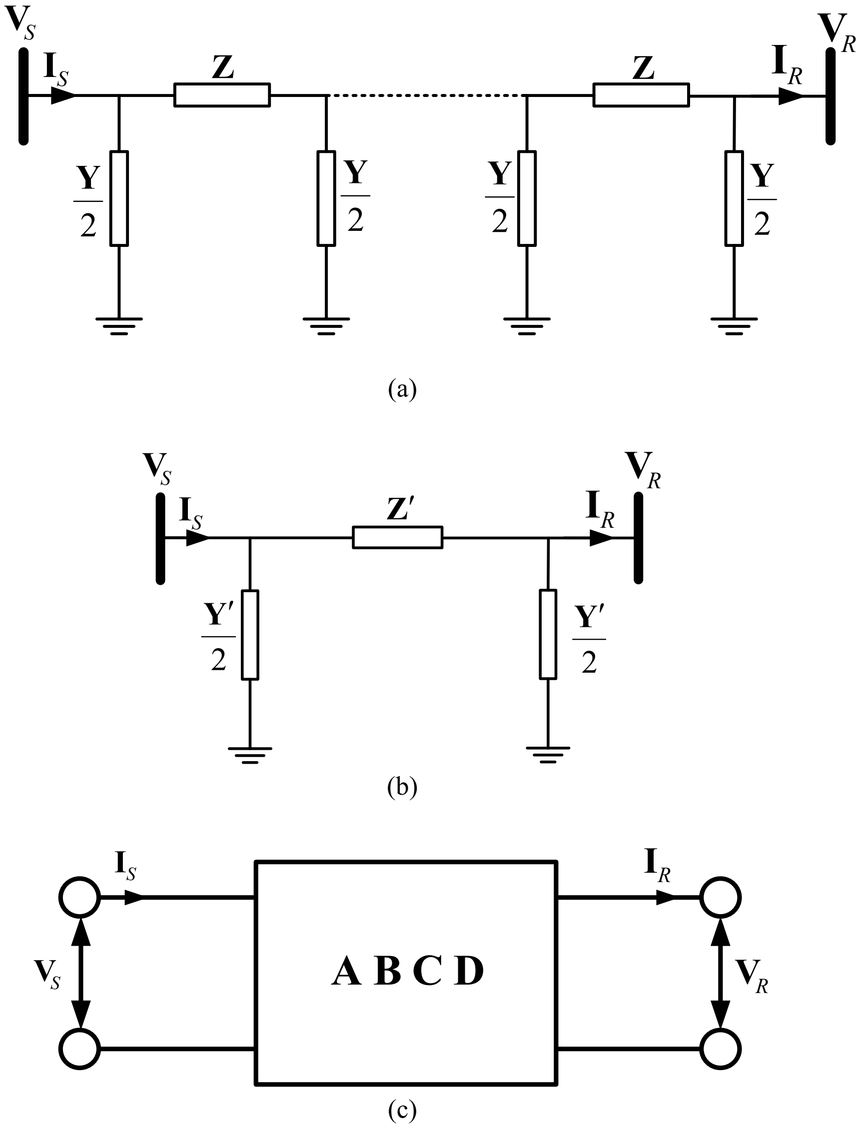
This paper proposes the method of estimating parameters of transmission line without the use of the PMU data. The ready data at power substations voltage magnitude (V), active (P) and reactive power (Q) is incorporated into mathematical model to estimate the parameters. The constrained nonlinear minimization available in MATLAB optimization toolbox is applied in this paper.

2. MATHEMATICAL MODEL

This section will provide the mathematical model of transmission line in power system. Figure 1 (a) shows the lumped π model of transmission line equivalent. Each π model consists of the series impedance ***Z*** and the shunt admittance ***Y***. After some mathematical manipulation of nonlinear differential equation, the transmissionline equivalent in one π model is represented in Figure 1(b). The **Z′** and **Y′** are given by [8]

$$\mathbf{Z}' = R' + jX' = \sqrt{\frac{\mathbf{Z}}{\mathbf{Y}}}\sinh\sqrt{\mathbf{ZY}} \tag{1}$$

$$\mathbf{Y}' = jB' = \frac{\mathbf{Y}\tanh\sqrt{\mathbf{ZY}}}{\sqrt{\mathbf{ZY}}} \tag{2}$$



(a)

(b)

(c)

Figure 1. Transmission line equivalent (a) distributed π model (b) with a π model (c) two-port representation

The transmission line equivalent in Figure 1(b) can be represented in two-port network as shown in Figure 1(c). The sending end voltage ***V**S* and current ***I**S* are given by

$$\mathbf{V_S} = \mathbf{AV}_R + \mathbf{BI}_R \tag{3}$$

$$\mathbf{I_S} = \mathbf{CV}_R + \mathbf{DI}_R \tag{4}$$

Here

$$\mathbf{A} = \mathbf{D} = A\angle\theta_A = \left[1 + \frac{\mathbf{Z'Y'}}{2}\right] \tag{5}$$

$$\mathbf{B} = B\angle\theta_B = \mathbf{Z}' \tag{6}$$

$$\mathbf{C} = C\angle\theta_C = \mathbf{Y}'\left[1 + \frac{\mathbf{Z'Y'}}{4}\right] \tag{7}$$

Expressing the sending end voltage $\mathbf{V_S}$ in polar form as $\mathbf{V}_S = V_S\angle\delta$ and the receiving end voltage as reference $\mathbf{V}_R = V_R\angle 0$, the complex power at sending end ***S**S* is written by

$$\mathbf{S}_S = P_S + jQ_S = \mathbf{V}_S\mathbf{I}_S^* \tag{8}$$

After some mathematical manipulations of (4) and (8), the real (P_S) and reactive power (Q_S) at sending end are given by

$$P_S = \mathrm{Re}[\mathbf{S}_S] = \frac{AV_S^2\cos(\theta_B - \theta_A)}{B} - \frac{V_S V_R}{B}\cos(\theta_B + \delta) \tag{9}$$

$$Q_S = \mathrm{Im}[\mathbf{S}_S] = \frac{AV_S^2\sin(\theta_B - \theta_A)}{B} - \frac{V_S V_R}{B}\sin(\theta_B + \delta) \tag{10}$$

The complex power at recieving end ***S**R* is written by

$$\mathbf{S}_R = P_R + jQ_R = \mathbf{V}_R\mathbf{I}_R^* \tag{11}$$

Similarly, some mathematical manipulations of (4) and (11), the real (P_R) and reactive power (Q_R) at recieving end are given by

$$P_R = \mathrm{Re}[\mathbf{S}_R] = \frac{V_S V_R\cos(\theta_B - \delta)}{B} - \frac{AV_R^2}{B}\cos(\theta_B - \theta_A) \tag{12}$$

$$Q_R = \mathrm{Im}[\mathbf{S}_R] = \frac{V_S V_R\sin(\theta_B - \delta)}{B} - \frac{AV_R^2}{B}\sin(\theta_B - \theta_A) \tag{13}$$

It can be seen in (9), (10), (12), and (13) that they are in the nonlinear equations of the parameters of the transmission line (R, X, and B) and measurable data at power substation (V_S, δ, V_R, P_S, Q$_S$, P_R, and Q_R). The V_S, V_R, P_S, Q_S, P_R, and Q_R are typically available at power substation. However, the δ must utilize the PMU which is usually implemented at some important power substation because it is expensive instrument. Thus, this paper proposes the method of access the δ. The general form of four equations in four variables of the above equations written by

$$\mathbf{F_x}(\mathbf{x}) = 0 \tag{14}$$

Here

$$\mathbf{F_x} = [f_1,\ f_2, f_3, f_4] \tag{15}$$

$$\mathbf{x} = [R,\ X, X, \delta] \tag{16}$$

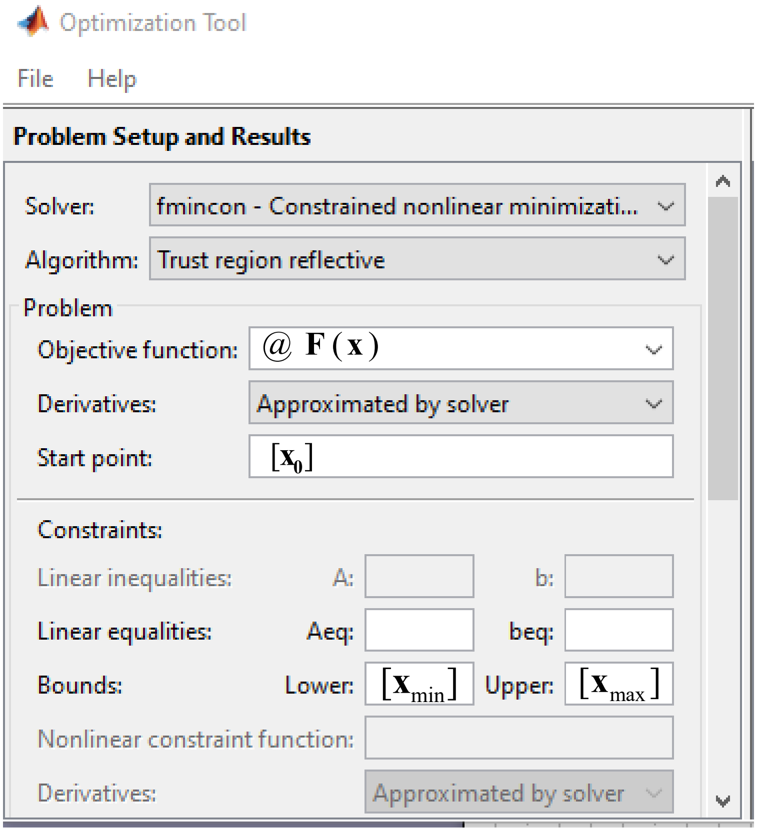
3. Application of MATLAB Optimization Toolbox and Verification

Nonlinear function optimization is important method to solving parameter in the objective function (***F***(***x***)). The necessary condition to minimize the objective equations written by [9, 10]

$$Min\ \mathbf{F}_{\mathbf{x}}(\mathbf{x}) = \sum_{i=1}^{4} \left| f_i(x_i) \right| \qquad (17)$$

Subject to the inequality constrains

$$\mathbf{x}_{\min} \le \mathbf{x} \le \mathbf{x}_{\max} \qquad (18)$$



Problem Setup and Results

Solver: fmincon - Constrained nonlinear minimizati...

Algorithm: Trust region reflective

Problem

Objective function: $@\ \mathbf{F}(\mathbf{x})$

Derivatives: Approximated by solver

Start point: $[\mathbf{x}_0]$

Constraints:

Linear inequalities: A: b:

Linear equalities: Aeq: beq:

Bounds: Lower: $[\mathbf{x}_{\min}]$ Upper: $[\mathbf{x}_{\max}]$

Nonlinear constraint function:

Derivatives: Approximated by solver

Figure 2. Window of MATLAB optimization toolbox

This paper applies the MATLAB optimization toolbox to solve the above objective equations. Figure 2 shows the window of the MATLAB optimization toolbox [11]. It may be mentioned here that normalization techniques called per unit power system will reduce the effects of the variation in the scale of the solution. In the practical power system industry, the lower x_{min} and upper x_{max} bounds in the range between 0 and 1. The initial point x_0 may come from the historical data of transmission line implementation. The proposed method is tested on an example to validate the concept involved. A three-phase, 50 *Hz*, 220 *kV* transmission line 340 *km* long. The transmission line parameters considered as the actual parameters are

$$\boldsymbol{Z} = 0.04 + j37.7\ \Omega \qquad \boldsymbol{Y} = j3.761\times10-4\text{siemens}$$

The measurable data at power substation is

P_S=147.03 MW Q_S=111.68 MVAR V_S=241.14 kV
P_R=144 MW Q_R=108 MVAR V_R=220 kV

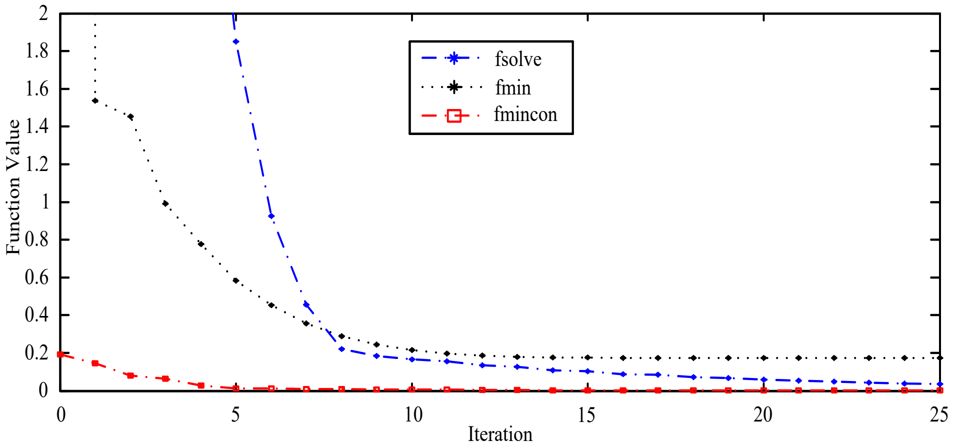


Figure 3. Comparisons of various numerical methods

There are many solver types in Optimization Toolbox such as *fsolve*, *fmin*, *fmincon*, *ga*, etc. In this paper, the *fslove*, *fmin* and *fmincon* are compared. It can be seen from Figure 3 that the parameter estimation using the *fmincon* with constrained nonlinear minimization provides the best results. With 25 iterations, the value of the objective function is less than 1×10^{-5} and it is considered as global minimum. Then the estimated parameters are approximately equal to the actual parameters.

4. CONCLUSION

The parameters of the transmission line may be changed by many factors such as the physical properties of a transmission line, temperature, etc. This paper applied constrained nonlinear minimization available in the MATLAB optimization toolbox for estimating parameters of transmission line in power system. It was found that the *fmincon* with nonlinear constrained minimization provided the best results. This method allows us to utilize the historical data to faster achieve the desired accuracy. The normalization techniques called per unit power system also help to reduce the effects of the variation in the scale of the solution. The simulation results indicated that the proposed method provides the perfect agreement of estimating parameters of transmission line.

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