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Towards 't Hooft parameter corrections to charge transport in strongly-coupled plasma

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Abstract

We study R-charge transport in a wide class of strongly-coupled supersymmetric plasmas at finite temperature with 't Hooft coupling corrections. To achieve this, we use the gauge/string duality and include the full set of $\mathcal{O}(\alpha'^3)$ corrections to the supergravity backgrounds given at zeroth order by the direct product of the AdS$_5$-Schwarzschild black hole with a five-dimensional compact Einstein manifold. On general grounds, the reduction leads to a large number of higher derivative operators, which we reduce using the symmetries of the solution. We are left with a universal set of operators whose coefficients can in principle be fixed by carrying out an explicit compactification. We apply our results to the computation of the R-charge conductivity of the supersymmetric plasma at finite yet strong coupling.

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1 Introduction

The past few years have seen increasing interest in the properties of the deconfined quark-gluon plasma (QGP), obtained as a result of the collision of heavy nuclei. Observations at the Relativistic Heavy Ion Collider (RHIC) imply that once this plasma is produced at temperatures of order a hundred MeV, it behaves like an ideal fluid. We refer the reader to several review articles discussing the phenomenology of the QGP [1, 2, 3, 4, 5, 6, 7]. Such experimental motivation necessitates an understanding of the hydrodynamic properties of the QGP from the theoretical side, and an excellent tool to use for this investigation is the AdS/CFT correspondence [9, 10, 11, 12]. The latter conjectures a duality between a class of highly symmetric strongly-coupled quantum field theories and strings propagating in certain gravity backgrounds. The reason why the AdS/CFT correspondence is a good candidate to approach the QGP is that the latter is strongly-coupled in the relevant regime of temperature. An immediate focus would be to use the AdS/CFT correspondence to compute the transport coefficients of highly-symmetric plasmas in the hydrodynamic regime, defined as the regime in which the perturbations of the plasma have a momentum much smaller than the temperature. The important observables, which hopefully can be used to compare with experiment, are the usual transport coefficients of fluid dynamics entering the Navier-Stokes equation. The recipe is to calculate the retarded two-point functions of conserved currents of the theory at thermal equilibrium, following the rules established in [13, 14]. Using these rules, and working in the gravitational holographic dual model, one is able to obtain the transport coefficients of both mass (energy-momentum) and charge, extracting quantities such as viscosity [15, 16, 17] and the R-charge conductivity [14, 16, 18].

Now we must bear in mind that the results obtained in these references, and in fact in all of the literature utilizing the holographic duality, apply to theories with a large number of degrees of freedom (large-N), and with a certain degree of supersymmetry and/or conformal invariance. These are of course the limitations of the AdS/CFT correspondence, and so a direct comparison to experimental observations is difficult. It is nonetheless important to understand the strong-coupling regime of these highly-symmetric theories fully, as they share many features with QCD. In fact, the shear viscosity results for $\mathcal{N} = 4$ SYM theory [15, 16, 17] obtained via the AdS/CFT correspondence are very close to those measured for the QGP [1]. Moreover, the AdS/CFT correspondence allows us to identify universal properties of the hydrodynamic coefficients [19], which would be very useful if QCD were to be shown to be within the class of theories in which this universality is operative. In addition, one may improve the approach to QCD by incorporating some of its essential aspects in the gravity dual. This includes adding flavour fields in the fundamental representation into the gravity dual [20], for instance using D3-D7 brane systems as in the Karch and Katz model [21]. Another important direction to pursue is to go to finite coupling, by including corrections to the infinite 't Hooft parameter results. Within the context of the AdS/CFT correspondence, this is achieved by adding string-theoretic higher-curvature corrections to the gravitational background. This is the premise of the present article.

There is by now a large volume of literature on finite-coupling corrections to the transport properties of plasma in the hydrodynamic regime [19, 22, 23, 24, 25, 26, 27, 28]. Such transport properties include the shear viscosity and the mass-density diffusion constants, both of which can be obtained by studying tensor fluctuations of the supergravity metric with higher-curvature corrections. On the other hand, the vector fluctuations of the metric yield quantities such as the R-charge diffusion and conductivity. The finite coupling corrections to the latter quantities have been considered so far only for the cases where the additional curvature terms have been of mass-dimension four and six. In type IIB string theory, the stringy corrections made up of the metric and the Ramond-Ramond five-form field strength are known explicitly, and are found to yield dimension eight operators. In this paper, we analyze the effect of these dimension-eight operators on the vector fluctuations of the supergravity metric. We should mention that in a recent paper we have studied the effect of the full $\mathcal{O}(\alpha'^3)$ corrections from type IIB string theory, including those derived from the Ramond-Ramond five-form field strength, on the retarded current-current correlators at the high energy regime, where the plasma is probed at distances smaller than the inverse of the temperature [29].

Let us describe the general idea of the computation we carry out firstly and summarize our results. The type IIB string theory action with leading order $\mathcal{O}(\alpha'^3)$ corrections, is given by the usual two-derivative minimal Lagrangian with certain eight-derivative corrections added. The schematic form of the $\mathcal{O}(\alpha'^3)$ corrections is $Weyl^4 + Weyl^3\mathcal{T} + Weyl^2\mathcal{T}^2$, where $Weyl$ is the ten-dimensional Weyl tensor and $\mathcal{T}$ is based on the five-form field strength F_5 of type IIB string theory. A general solution to this complicated Lagrangian is a *warped* product of a deformed AdS-Schwarzschild black hole with a five-dimensional Einstein manifold M_5, which for instance can be a Sasaki-Einstein manifold [30]. This is dual to a supersymmetric conformal field theory (SCFT), with an R-symmetry group that contains at least one $U(1)$ factor.

We want to examine the R-charge correlators in this dual theory, focussing on the diagonal $U(1)$ which is dual to the graviphoton. Note that a weak gauging of this $U(1)$ on the field theory side, with a small gauge coupling in the manner of [18], allows us to interpret our results as pertaining to an embedding of $U(1)_{em}$ in the R-symmetry group of the field theory. This obviously gives our computations below added significance, because we may view them as an investigation into the finite 't Hooft coupling corrections to electromagnetic charge transport in a wide class of theories. The fields dual to the field-theory $U(1)$ are vectorial perturbations of the metric and the five-form field strength. We thus must perturb the background supergravity solution in the vectorial (mixed) directions $A_\mu = G_{\mu a}$, plug the perturbed supergravity solution into the corrected Lagrangian, and integrate out the M_5 directions a to obtain a five-dimensional Lagrangian for the gauge fields A_μ. At infinite 't

Hooft coupling, one simply uses the minimal type IIB supergravity action, and obtains the Einstein-Maxwell Lagrangian in AdS-Schwarzschild black hole, *i.e.* five-dimensional gravity (with a cosmological constant) coupled to a $U(1)$ graviphoton. The computation at finite coupling is much more complicated, as one must take into account all of the dimension eight string-theory operators when performing the compactification. To achieve this, we focus our attention on the construction of the full (complete) set of five-dimensional operators that can be induced by the ten-dimensional operators $Weyl^4 + Weyl^3\mathcal{T} + Weyl^2\mathcal{T}^2$. We find 26 such operators, and they have the schematic form C^2F^2, $C^2(\nabla F)^2$ and $(\nabla F)^2$, where F is the field-strength of the $U(1)$ boson. Observe that these operators will be present in any type IIB string theory dual model, giving our computation an added incentive. The numerical values of the coefficients of the operators *may* be dependent on the explicit type of internal manifold, but the hope is that some of them are universal (*i.e.* independent of the internal manifold M_5).

Having obtained the complete five-dimensional Lagrangian with arbitrary coefficients, we apply our results to obtain the equations of motion of the transverse gauge field A_x, whose solution we require to compute the R-charge conductivity. This field decouples from the other perturbations, as we show explicitly. We solve the equations of motion of this field in the hydrodynamic regime as a series in the momentum, requiring ingoing boundary conditions at the horizon. We find that the frequency of the waves at the horizon is unchanged with respect to the infinite 't Hooft coupling results, for any gauge-invariant set of operators: our results therefore strongly suggest that any gauge invariant Lagrangian for the vector perturbations yields an equation of motion with the same singularity structure and indices at the horizon. We then obtain a general expression for the leading 't Hooft coupling corrections to the R-charge conductivity.

We view our results as a step towards a better understanding of charge-transport in strongly-coupled gauge theory plasmas for a range of theories. We emphasize that the set of operators enumerated in this work are present in any type IIB string theory holographic dual model, because the order $\mathcal{O}(\alpha'^3)$ ten-dimensional terms are present regardless of the details of the dual theory (*e.g.* whether it has flavour branes or not). Thus, obtaining this complete set of operators in this setting is of intrinsic value even if the exact coefficients of the operators in five dimensions are unknown. The hope is that some (or many) of these coefficients will be universal, as we speculate in later sections of the paper.

2 The corrected background

Let us define the premise of the paper more carefully: we are interested in the retarded correlators of the vector currents associated with a gauged $U(1)$ subgroup of the global R-symmetry group of a SCFT plasma. For example, the conductivity is extracted from the retarded current-current commutator

$$R_{\mu\nu}(q) = i \int d^4x \, e^{-iq\cdot x} \, \Theta(x_0) \, < [J_\mu(x), J_\nu(0)] > , \tag{1}$$

where $\Theta(x_0)$ is the Heaviside function, while $J_\mu(x)$ is the conserved current associated with the gauged $U(1)$ subgroup mentioned above. The expectation value is understood as a thermal average over the statistical ensemble of the SYM plasma at temperature T. Let us first consider the string theory holographic dual description of this theory at infinite 't Hooft coupling. This is a solution of type IIB supergravity with only the leading curvature terms, namely the Einstein-Hilbert action coupled to the dilaton and the Ramond-Ramond five-form field strength:

$$S_{10} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left[R_{10} - \frac{1}{2} (\partial\phi)^2 - \frac{1}{4.5!} (F_5)^2 \right] . \tag{2}$$

The solution to the equations of motion of this Lagrangian sourced by N D3-branes at finite temperature is given by the direct product of an AdS-Schwarzschild black hole with a compact Einstein manifold M_5. The five-form field strength is given as the sum of the volume forms on the two manifolds,

$$F_5^{(0)} = -\frac{4}{R}(1+*)\epsilon_5 \, , \tag{3}$$

where the superscript $^{(0)}$ indicates that this is a pure supergravity solution, *i.e.* with no stringy corrections. Its total flux through the compact manifold gives N units. The current operator $J_\mu(x)$ is dual to the s-wave mode of the vectorial fluctuation about this background. In order to obtain the correct AdS-Schwarzschild Lagrangian for the vectorial perturbation, one must construct a consistent perturbed *ansatz* for both the metric and the five-form field strength, such that a proper $U(1)$ subgroup of the R-symmetry group is obtained (see [31, 32, 33, 34] for the S^5 solution, and [30] for the five-dimensional Sasaki-Einstein solution). The result of inserting the consistent perturbation *ansatz* into the minimal type IIB supergravity action is the minimal $U(1)$ gauge field kinetic term in the AdS-Schwarzschild black hole. Therefore, by studying the bulk solutions of the Maxwell equations in the AdS-Schwarzschild black hole with certain boundary conditions, we can obtain the retarded correlation functions [13, 14, 18] of the operator $J_\mu(x)$. Our aim in this section is to describe this procedure for the α'^3-*corrected* type IIB supergravity action, which contains dimension-eight higher curvature operators. These higher-curvature corrections on the supergravity side correspond to finite-coupling corrections in the field theory, hence our interest in their effect. Essentially, for any given field-theoretic observable $\mathcal{O}$, we can write a series $\mathcal{O}_0 + \mathcal{O}_1/\lambda^{n_1} + \cdots$, where λ is the 't Hooft coupling, and n_1 is a positive number which indicates that the lowest order correction to the result at infinite coupling $\mathcal{O}_0$ need not begin at order one. We are interested in the case where $\mathcal{O}_0$ is the two-point current correlator, and we must thus determine the effect of the string-theory corrections on the vectorial perturbations of the metric.

The inclusion of higher-derivative corrections to the supergravity must take place at the level of the ten-dimensional action, through the evaluation of stringy corrections to Eq.(2). The leading corrections were found to begin at $\mathcal{O}(\alpha'^3)$. These corrections were found to have no effect on the metric at zero temperature [35], verifying certain non-renormalization theorems of CFT correlators. At finite temperature [36, 37], much of the work focussed on the corrections to the thermodynamics of the black hole. The corrections were then revisited in references [38, 39, 40], where the computation of the α'-corrected metric was improved and attempts were made to address the issue of the completeness of the corrections at leading order in α'. Much of the interest of the community has focussed on the effect of higher curvature corrections on the spin-2 sector of the fluctuations [22, 24, 41, 42], as these determine the viscosity and mass-diffusion constants of the plasma. In [43, 25] the higher curvature corrections to the dual of $\mathcal{N} = 4$ SYM were parsed thoroughly to determine how they affect the metric. Crucially, the corrections to the metric were found to be universal, *i.e.* independent of the internal manifold for a wide class of internal manifolds, of which Sasaki-Einstein is a member.

For the case of the vectorial fluctuations of the background, there are two distinct parts to the calculation: the first part consists of obtaining the minimal gauge-field kinetic term using new perturbed *and* corrected metric and five-form field strength *ansätze*. The second part of the computation consists of obtaining the corrections to the gauge field Lagrangian coming directly from the higher-derivative operators. The reason why these two steps are distinct is that the first step will require insertion of the corrected perturbation *ansätze* into the minimal ten-dimensional supergravity two-derivative part Eq.(2). The second step requires insertion of the *uncorrected* perturbation *ansätze* into the higher-curvature terms in ten dimensions, for consistency in the α' expansion.

Below we choose to use a specific manifold, the S^5 manifold, as an illustration of the methodology, but we emphasize to the reader that the discussion below applies directly without modification to any five-dimensional compact Einstein manifold. The only restriction comes from the requirement that in the zeroth-order background supergravity solution the only non-trivial fields are the metric and the five-form field strength. Our discussion thus applies to all supergravity backgrounds with an internal component satisfying these requirements, and these internal manifolds happen to be Einstein. Thus, we could have used the *ansätze* espoused in [30] to arrive at the same conclusions. We only use the five-sphere in what follows for simplicity and familiarity.

We begin by examining the corrected metric and F_5 solutions, then proposing *ansätze* for the perturbations that may be inserted into Eq.(2) to obtain the minimal gauge kinetic term. The corrections to the ten-dimensional type IIB action are given by [25]

$$S_{10}^{\alpha'} = \frac{R^6}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left[\gamma e^{-\frac{3}{2}\phi} W_4 + \cdots \right] , \tag{4}$$

where γ encodes the dependence on the 't Hooft coupling λ through the definition $\gamma \equiv$

$\frac{1}{8}\,\xi(3)\,(\alpha'/R^2)^3$, with $R^4 = 4\pi g_s N \alpha'^2$. Setting $\lambda = g_{YM}^2 N \equiv 4\pi g_s N$, we get $\gamma = \frac{1}{8}\,\xi(3)\,\frac{1}{\lambda^{3/2}}$.

The W_4 term is a dimension-eight operator, and is given by

$$W_4 = C^{hmnk}\, C_{pmnq}\, C_h^{\ \ rsp}\, C^q_{\ \ rsk} + \frac{1}{2}\, C^{hkmn}\, C_{pqmn}\, C_h^{\ \ rsp}\, C^q_{\ \ rsk}\,, \tag{5}$$

where $C^q_{\ \ rsk}$ is the Weyl tensor. The dots in Eq.(4) denote extra corrections containing contractions of the five-form field strength F_5, which we can schematically write as $\gamma(C^3\mathcal{T} + C^2\mathcal{T}^2 + C\mathcal{T}^3 + \mathcal{T}^4)$, where C is the Weyl tensor and $\mathcal{T}$ is a tensor found in [25] and composed of certain combinations of F_5. The authors of [25] showed that the metric itself is only corrected by W_4, essentially due to the vanishing of the tensor $\mathcal{T}$ on the uncorrected supergravity solution. This conclusion was found to be independent of the internal manifold, as long as the only non-trivial fields in the zeroth-order supergravity background are the metric and the five-form field strength. As we mentioned this holds for any compact Einstein manifold. Hence, all Sasaki-Einstein manifolds L_{pqr}, as well as $Y^{p,q}$ and $T^{1,1}$ which are special cases of them, are covered by what follows.

After taking into account the contribution of W_4 to the Einstein equations, one finds the corrected metric [36, 37, 39]

$$ds^2 = \left(\frac{r_0}{R}\right)^2 \frac{1}{u}\, \left(-f(u)\, K^2(u)\, dt^2 + d\vec{x}^2\right) + \frac{R^2}{4u^2 f(u)}\, P^2(u)\, du^2 + R^2 L^2(u)\, d\Omega_5^2\,, \tag{6}$$

where $f(u) = 1 - u^2$ and R is the radius of the AdS$_5$. In these coordinates the AdS-boundary is at $u = 0$ while the black hole horizon is at $u = 1$. We denote the AdS$_5$ coordinates by the indices m, where $m = \{(\mu = 0, 1, 2, 3), 5\}$, where

$$K(u) = \exp\left[\gamma\,(a(u) + 4b(u))\right]\,, \quad P(u) = \exp\left[\gamma\, b(u)\right]\,, \quad L(u) = \exp\left[\gamma\, c(u)\right]\,, \tag{7}$$

and

$$\begin{aligned} a(u) &= -\frac{1625}{8}\, u^2 - 175\, u^4 + \frac{10005}{16}\, u^6\,, \quad b(u) = \frac{325}{8}\, u^2 + \frac{1075}{32}\, u^4 - \frac{4835}{32}\, u^6\,, \\ c(u) &= \frac{15}{32}\,(1 + u^2)\, u^4\,, \qquad \text{with} \qquad r_0 = \frac{\pi T R^2}{(1 + \frac{265}{16}\gamma)}\,, \end{aligned} \tag{8}$$

where T is the physical temperature of the plasma. Having obtained the corrected metric, the next step is to deduce the appropriate perturbation *ansätze*. This is in fact where the complications of the problem appear: the vector perturbation enters into both the perturbed metric and the perturbed F_5 solution. This is distinct from the case where one considers tensor perturbations of the background, which are relevant for viscosity computations, because they only enter into the metric *ansatz*, not into the F_5 *ansatz*, making the computations far simpler.

Let us first consider how we would obtain the minimal (two-derivative) kinetic term for the gauge fields. We must insert our corrected *ansatz* into the two-derivative supergravity action Eq.(2). The metric *ansatz* we use is as follows, with obvious substitutions, where we have imposed that the internal metric is the five-sphere (for ease of demonstration)

$$ds^2 = g_{mn}\, dx^m\, dx^n + R^2 L(u)^2 \sum_{i=1}^{3} \left[d\mu_i^2 + \mu_i^2 (d\phi_i + \frac{2}{\sqrt{3}} A_\mu dx^\mu)^2 \right] , \tag{9}$$

where the μ_i are the direction cosines for the sphere, as usual.

As for the *ansatz* for $F_5 = G_5 + *G_5$ we use the fact that we are only interested in the terms which are quadratic in the gauge-field perturbations. Thus we use the following *ansatz*

$$G_5 = -\frac{4}{R}\overline{\epsilon}_5 + \frac{R^3 L(u)^3}{\sqrt{3}} \left(\sum_{i=1}^{3} d\mu_i^2 \wedge d\phi_i \right) \wedge \overline{*} F_2 \,, \tag{10}$$

where $F_2 = dA$ is the Abelian field strength and $\overline{\epsilon}_5$ is the deformation of the volume form of the five-dimensional metric of the AdS-Schwarzschild black hole [3]. The Hodge dual $*$ is taken with respect to the ten-dimensional metric, while $\overline{*}$ denotes the Hodge dual with respect to the five-dimensional metric piece of the black hole. Note that we have not dwelled on the details of the *ansatz* because the main point of the paper is that the operators derived below are in fact independent of the internal manifold in form. The only dependence comes in through their coefficients. Inserting these *ansätze* into Eq.(2), and discarding all the higher (massive) Kaluza-Klein harmonics of the five-sphere, we get the following action for the zero-mode Abelian gauge field A_m:

$$S = -\frac{\tilde{N}^2}{64\pi^2 R} \int d^4x\, du\, \sqrt{-g}\, L^7(u)\, g^{mp}\, g^{nq}\, F_{mn}\, F_{pq} \,, \tag{11}$$

where the Abelian field strength is $F_{mn} = \partial_m A_n - \partial_n A_m$, the partial derivatives are $\partial_m = \partial/\partial x^m$, while $x^m = (t, \vec{x}, u)$, with t and $\vec{x} = (x_1, x_2, x_3)$ being the Minkowski coordinates, and $g \equiv \det(g_{mn})$, where the latter is the metric of AdS-Schwarzschild black hole piece of the corrected metric. The dependence on the dimensionless factor $L(u)$ is acquired by the proper reduction from ten dimensions [16], and ultimately arises as a consequence of the non-factorisability of the corrected metric [37]. This factor is independent of the internal metric M_5. Note also that the volume of the internal manifold has been absorbed into the definition of the factor $\tilde{N}$.

We have thus completed the first step in our programme, that of obtaining the minimal gauge kinetic term from the two-derivative supergravity action. The next step is to obtain the effect of the eight-derivative corrections of Eq.(4). Concretely, we must determine the five-dimensional operators that arise once the perturbed metric and five-form field strength *ansätze* of equations (10) and (9) are inserted into Eq.(4). Crucially, we are able to use

[3] Note that we are not interested in the part of G_5 which does not contain the vector perturbations. This part only contributes to the potential of the metric, and is thus accounted for by the use of the corrected metric in the computation.

the uncorrected *ansätze* in this step, because using the corrected ones results in terms of even higher order in γ. The uncorrected *ansätze* are derived from those of equations (10) and (9) by taking $L(u), K(u), P(u) \to 1$ and $\bar{\epsilon}_5 \to \epsilon_5$. Our philosophy will be to consider the structure of the $C^3\mathcal{T}$ and $C^2\mathcal{T}^2$ terms, and use the symmetries of the various tensors to deduce the most general set of five-dimensional operators that can be obtained via the compactification. We describe how this is achieved in the next section.

3 Operator enumeration

The ten-dimensional corrections in totality are schematically given by

$$C^4 + C^3\mathcal{T} + C^2\mathcal{T}^2 + C\mathcal{T}^3 + \mathcal{T}^4 \,, \tag{12}$$

where $\mathcal{T}$ is given by

$$\mathcal{T}_{abcdef} = i\nabla_a F^+_{bcdef} + \frac{1}{16}\left(F^+_{abcmn}F^{+\ \ mn}_{def} - F^+_{abfmn}F^{+\ \ mn}_{dec}\right) \,, \tag{13}$$

where the RHS must be antisymmetrized in $[a,b,c]$ and $[d,e,f]$ and symmetrized with respect to interchange of $abc \leftrightarrow def$ [43] and we have defined the tensor

$$F^+ = \frac{1}{2}(1+*)F_5 \,. \tag{14}$$

The perturbed *ansatz* for the five-form field strength contains only one power of the gauge field strength. Therefore, we can write $\mathcal{T} = \mathcal{T}_0 + \mathcal{T}_1 + \mathcal{T}_2$, with each subscript denoting the number of powers of the gauge field contained in the tensor. The tensor $\mathcal{T}_0$ is zero for all supergravity backgrounds given by a direct product of an AdS$_5$-Schwarzschild black hole with a five-dimensional compact Einstein manifold, provided that the five-form field strength $F_5^{(0)}$ can be written as in Eq.(3) [25]. Therefore, splitting the ten-dimensional Weyl tensor in a similar fashion, the corrections can be schematically written as

$$C_0^2C_1^2 + C_0^3C_2 + C_0^2C_1\mathcal{T}_1 + C_0^3\mathcal{T}_2 + C_0^2\mathcal{T}_1^2 \,. \tag{15}$$

Now, let us study the term $C_0^3\mathcal{T}_2$. As we are considering a direct product space, the Weyl tensor factorizes into its AdS-Schwarzschild black hole and M_5 factors. Hence, C_0^3 must reside entirely in one of the two factors. If it resides in the AdS-Schwarzschild black hole factor, then the only five-dimensional operators that can result from this are of the form

$$\tilde{C}^3F^2 \,, \tag{16}$$

where $\tilde{C}$ denotes the Weyl tensor of five-dimensional AdS-Schwarzschild black hole and F is the $U(1)$ field strength tensor. We have checked explicitly that, for any given M_5, the non-zero entries of $\mathcal{T}_2$ correspond to zero entries of $\tilde{C}^3$, and so this operator vanishes generally, and operators of the form $\tilde{C}^3F^2$ are therefore not induced. The same argument can be constructed for the contribution of $C_0^3C_2$. However, the tensor C_0^3 may reside in the M_5 factor, in which case the induced operators in five dimensions are of the form F^2. Of course, there is only one such operator, proportional to the kinetic term.

Let us now focus on the terms given by $C_0^2\mathcal{T}_1^2$ and $C_0^2C_1^2$. Again, C_0^2 factorizes; if it resides in the AdS-Schwarzschild black hole factor, then we must obtain operators of the form

$$\tilde{C}^2F^2 \quad \text{and} \quad \tilde{C}^2(\nabla F)^2 \,. \tag{17}$$

If C_0^2 resides in the M_5 factor, then we must obtain all operators of the form

$$F^2 \quad \text{and} \quad (\nabla F)^2 \,. \tag{18}$$

A bit of thought should then convince the reader that the same considerations apply to $C_0^2C_1\mathcal{T}_1$ because in this way of thinking C_1 is entirely equivalent to $\mathcal{T}_1$ in that it also contains only one power of the gauge field. Therefore, the problem reduces to finding the set of independent monomials comprising all contractions of two Weyl tensors and two ∇F, as well as two Weyl tensors and two F. In addition, we require a set of monomials to represent all contractions of two ∇F, and all contractions of two powers of F.

This can all be very quickly computed by Cadabra, the program developed by Kasper Peeters. We find 26 such operators in total, and we list them here. One of them may be eliminated on shell (it does not contribute to the on-shell action), and so the final set contains 25 operators in total. Note that these comprise a full linearly-independent set up to the use of dimension dependent identities that are similar in nature to the Schouten identities of [19]. We expect these identities to reduce the set by four, but we have not undertaken the reduction in what follows. We would like to stress that this result is indeed a massive reduction compared with the very large number of general five-dimensional operators which are possibly induced by the five-dimensional reduction of the ten-dimensional operators of Eq.(12) upon a general compact Einstein manifold. Just to give an idea of this consider for instance that an operator like $C^2(\nabla F)^2$ leads to 720 distinct operators induced from the permutations of the operator $(\nabla F\nabla F)_{abcdef}$ before any symmetry operations are taken into account.

The full set of 26 five-dimensional scalar operators is given by

$$\begin{aligned}
&(C^4 + C^3\mathcal{T} + C^2\mathcal{T}^2 + C\mathcal{T}^3)|_{5d} = \\
&a_1 C_{abcd}C^{abcd}\nabla^e F_{ef}\nabla^g F^f{}_g + a_2 C_{abcd}C^{acbd}\nabla_e F_{fg}\nabla^f F^{eg} + \\
&C_{abc}{}^{d}C^{acbe}\left[b_1\nabla_f F_d{}^f\nabla_g F_e{}^g +_2 \nabla_d F_{ef}\nabla_g F^{fg} + b_3\nabla_d F_{fg}\nabla_e F^{fg}\right] + \\
&b_4 C_{abc}{}^{d}C^{abce}\nabla_f F_{dg}\nabla^f F_e{}^g + \\
&C_a{}^b{}_c{}^d C^{aecf}\left[c_1\nabla_b F_{eg}\nabla^g F_{df} + c_2\nabla_b F_{de}\nabla^g F_{fg} + c_3\nabla_b F_{dg}\nabla_f F_e{}^g + c_4\nabla_b F_{eg}\nabla_d F_f{}^g\right] + \\
&C_a{}^b{}_c{}^d C^{acef}\left[c_5\nabla_b F_{dg}\nabla_e F_f{}^g + c_6\nabla_b F_{ef}\nabla_g F_d{}^g + c_7\nabla_b F_{eg}\nabla_f F_d{}^g\right] +
\end{aligned}$$

$$
\begin{aligned}
&c_8 C_{ab}{}^{cd} C^{abef} \nabla_c F_{eg} \nabla^g F_{df} + \\
&C_a{}^{bcd} C^{aefg} \left[d_1 \nabla_b F_{ce} \nabla_f F_{dg} + d_2 \nabla_c F_{be} \nabla_f F_{dg} + d_3 \nabla_b F_{cf} \nabla_g F_{de} + d_4 \nabla_c F_{bd} \nabla_f F_{eg} \right] + \\
&e_1 C_{abcd} C^{acbd} F_{ef} F^{ef} + f_1 C_{abc}{}^{d} C^{acbe} F_{df} F_e{}^f + \\
&g_1 C_a{}^b{}_c{}^d C^{aecf} F_{be} F_{df} + C_a{}^b{}_c{}^d C^{acef} \left[g_2 F_{bd} F_{ef} + g_3 F_{be} F_{df} \right] + \\
&h_1 F_{ab} F^{ab} + h_2 \nabla_a F_{bc} \nabla^b F^{ac} + h_3 \nabla^b F_{bc} \nabla^a F_a{}^c \,. \qquad (19)
\end{aligned}
$$

We remind the reader that this does not mean that all of these will be induced by the compactification: in all probability only a handful of them will be induced, but the statement we can definitively make is that the operators listed here comprise the most complete allowed set. Note that the final operator, with coefficient h_3, can be eliminated on-shell, and we do this in what follows. Note also that for the sphere the coefficients $h_i = 0$, because the sphere is Weyl flat. This is not necessarily the case for other manifolds, unless there is a miraculous cancellation at work. Perhaps this points the way to a non-universal behaviour in this particular sector of the dual field theory, and we shall have more to say on this later.

4 The Lagrangian for the transverse modes

As an application of the above, we will consider the two-point correlators of the R-symmetry current J_x. The dual field on the supergravity side is the gauge field in the x-direction A_x. Two-point functions of J_x are useful for a range of physical phenomena, including the conductivity and diffusion constant of the EM charge, and the computation of photoemission spectra by gauge-theory plasma [18]. Specifically, we will compute the leading 't Hooft coupling corrections to the conductivity associated with the $U(1)$ R-charge. The conductivity is given by the following relationship:

$$
\sigma = -i \lim_{\omega \to 0} \frac{1}{2T} R_{xx}(\omega, q = 0) \,, \qquad (20)
$$

where we have used Eq.(1) and the Kubo formula from reference [18]. Because we are interested only in the size of the corrections to this quantity, we will measure our conductivity below in terms of the uncorrected conductivity, so that our result will have the form $1 + \frac{\rho}{\lambda^{3/2}}$, where ρ is the number we shall compute below.

As we saw in the last section, the eight-derivative $\mathcal{O}(\alpha'^3)$ corrections introduce a multitude of higher-derivative operators upon compactification on the M_5, and we must take account of them properly to solve the equation of motion within perturbation theory. The situation is entirely analogous to that studied by Buchel, Liu and Starinets in [22]. In that paper, the authors were concerned with the tensor perturbations of the metric, but the logic is the same. From our effective Lagrangian, we see immediately that A_x decouples from the other perturbations. Computing the effect of the minimal kinetic term of Eq.(11) and the above general set of operators of Eq.(19) yields the following Lagrangian for the transverse mode A_x

$$
\begin{aligned}
S_{\text{total}} &= -\frac{\tilde{N}^2 r_0^2}{16\pi^2 R^4}\int \frac{d^4k}{(2\pi)^4}\int_0^1 du\,\Big[\gamma A_W A_k'' A_{-k} + (B_1+\gamma B_W)A_k' A_{-k}' \\
&\quad +\gamma C_W A_k' A_{-k} + (D_1+\gamma D_W)A_k A_{-k} + \gamma E_W A_k'' A_{-k}'' + \gamma F_W A_k'' A_{-k}'\Big]\,, \qquad (21)
\end{aligned}
$$

where we have introduced the following Fourier transform of the field A_x

$$
A_x(t,\vec{x},u) = \int \frac{d^4k}{(2\pi)^4}\, e^{-i\omega t + iqz}\, A_k(u)\,. \qquad (22)
$$

There are also a number of boundary terms that must be included for this higher-derivative Lagrangian to make sense, and this is discussed in detail in [22, 29]. The coefficients B_1 and D_1 arise directly from the minimal kinetic term F^2. The subscript W indicates that the particular coefficient comes directly from the eight-derivative corrections, and the functions $A_W \to F_W$ are listed in the appendix. Moreover, B_1 and D_1 contain some γ-dependence, but they are non-vanishing in the $\gamma \to 0$ limit, while every other coefficient vanishes in that limit. The equation of motion is given by

$$
A_x'' + p_1 A_x' + p_0 A_x = \gamma \frac{1}{2f(u)} V(A_x)\,, \qquad (23)
$$

where

$$
\begin{aligned}
& A_W\, A_x'' + C_W A_x' + 2\left(\delta D_1 + D_W\right) A_x \\
& -\partial_u \left(2\delta B_1 A_x' + 2B_W A_x' + C_W A_x + F_W A_x''\right) \\
& +\partial_u^2 \left(A_W A_x + 2E_W A_x'' + F_W A_x'\right) = V(A_x)\,, \qquad (24)
\end{aligned}
$$

where $B_1 - B_1|_{\gamma\to 0} = \delta B_1$ and $D_1 - D_1|_{\gamma\to 0} = \delta D_1$. First we have the coefficients with no γ-dependence p_0 and p_1, given by

$$
p_0 = \frac{\varpi_0^2 - f(u)\kappa_0^2}{u f^2(u)} \quad \text{and} \quad p_1 = \frac{f'(u)}{f(u)}\,, \qquad (25)
$$

where $\varpi_0 = \omega/(2\pi T)$ and $\kappa_0 = q/(2\pi T)$. For the coefficients originating from the F^2 term in the action of the gauge field, we obtain

$$
\begin{aligned}
B_1 &= \frac{K(u) f(u) L^7(u)}{P(u)}\,, \\
D_1 &= -K(u)P(u)L^7(u)\left[\frac{\varpi^2 - f(u)K^2(u)\kappa^2}{u f(u) K^2(u)}\right]\,, \qquad (26)
\end{aligned}
$$

where $\varpi = \omega R^2/(2r_0)$ and $\kappa = qR^2/(2r_0)$. The terms originating from the higher curvature terms in the action are listed in the appendix. At this stage it is convenient to reduce the equation to a second-order differential equation using a simple trick [28]. The idea is that $\gamma A_x'' = -\gamma\,(p_1 A_x' + p_0 A_x) + \mathcal{O}(\gamma^2)$. Thus, we may reduce the entire RHS of the equation of motion to terms which are first or zeroth order in derivatives. The resulting equation is given by:

$$A_x'' + \left[p_1 - \frac{\gamma}{2f(u)}\left[\theta_1(u) - p_1\theta_2(u)\right]\right] A_x' + \left[p_0 - \frac{\gamma}{2f(u)}\left[\theta_0(u) - p_0\theta_2(u)\right]\right] A_x \; = \mathcal{O}(\gamma^2)\,, \quad (27)$$

where

$$\begin{aligned}
\theta_0(u) &= 2\,(\delta D_1 + D_W) - C_W' + A_W'' - 4E_W'\,p_0' + 2\,E_W\,(p_1 p_0' - p_0'') \\
\theta_1(u) &= 2\,A_W' - 2\,(\delta B_1 + B_W)' + F_W'' - 4E_W'\,(p_1' + p_0) + 2E_W\left[p_1(p_1' + p_0) - p_1'' - 2p_0'\right] \\
\theta_2(u) &= 2\,A_W - 2\,(\delta B_1 + B_W) + F_W' + 2E_W'' - 4E_W'\,p_1 + 2E_W\left[p_1^2 - 2p_1' - p_0\right]. \qquad (28)
\end{aligned}$$

We are now in a position to solve this equation in the hydrodynamic regime. The first step is to examine the singularity structure of the equation at the horizon $u = 1$. As usual, we change variables to $x = 1 - u$, so that the singularity is at $x = 0$, then insert the functional form $A_x = x^\beta$. We obtain the indicial equation:

$$\beta^2 + \left(\frac{\omega}{4\pi T}\right)^2 = 0\,. \qquad (29)$$

This is of course the same indicial equation that would have been obtained in the infinite 't Hooft coupling limit. Thus, as long as the Lagrangian originates from a gauge-invariant series of operators, then the indicial equation is unchanged. We have made several checks of this statement, using operators of arbitrary dimension, containing up to four derivatives of the gauge field. Hence, the fact that the indicial equation is unchanged is a consequence of the gauge-invariance in five dimensions, which is in turn a consequence of the $U(1)$ isometry of the internal manifold M_5, and has nothing to do with supersymmetry. We are aware that this behaviour is expected [24, 26, 27, 28] and in fact Buchel mentions it in his paper [24], focussing on scalar and tensor fluctuations of the metric. It would be very interesting to find a general proof of this statement.

5 Solving the equations of motion in the hydrodynamic regime

We now turn to the solution of the equations for A_x. In order to compute the conductivity, we must solve the equation for A_x up to linear order in γ and ω, which is of course the hydrodynamic regime. Guided by our observations of the behaviour at the horizon, we propose the following form for the solution:

$$A_x(u) = A_0(u) + \gamma A_1(u) \;=\; [1-u]^{-\sigma}\left(\phi_0(u) + \gamma\phi_1(u)\right)\,, \qquad (30)$$

where $\sigma = i\omega/(4\pi T)$. We now write $\phi_{0,1}(u) = h_{0,1}(u) + \sigma g_{0,1}(u)$. We insert this decomposition into the equation of motion, and realize immediately that the only relevant terms are $B_{1,W}$ (in the limit ω and $q \to 0$), F_W and E_W, because everything else enters with at least quadratic powers in ω and q (see the appendix for B_W, E_W, F_W). This is actually a consequence of gauge-invariance: any gauge-invariant Lagrangian of the form of Eq.(47) will have this property. This is *not* the case for tensor-fluctuations of the metric.

First we focus on $\phi_0(u)$. Comparing powers of σ in the equation of motion, we simply obtain

$$h_0(u) = C \quad \text{and} \quad g_0(u) = C\log(1+u) + D\,, \tag{31}$$

where we will not fix any of the constants C, D until the very end. The only requirement at this stage is regularity of all of the functions at the horizon. We now turn to $A_1 = [1-u]^{-\sigma}\,\phi_1(u)$. This function obeys a rather complicated equation of motion which can be deduced straightforwardly from the parent equation:

$$A_1'' + p_1 A_1' + p_0 A_1 \;= \frac{1}{2f(u)} V(A_0)\,. \tag{32}$$

We obtain the following equation for $h_1(u)$

$$\begin{aligned} f(u)h_1'' + f'(u)h_1' &= E_W'' h_0'' + 2E_W' h_0''' + E_W h_0'''' \\ +\quad & \frac{1}{2}\left\{\left(F_W' - 2\left(\delta B_1 + B_W\right)\right)h_0'' + \left(F_W'' - 2\left(\delta B_1' + B_W'\right)\right)h_0'\right\}\,, \end{aligned} \tag{33}$$

which solves to $h_1(u) = C_\gamma$, also a constant. We must now solve for $g_1(u)$. Using the explicit forms for $h_{0,1}(u)$ and $g_0(u)$, the equation for $g_1(u)$ simplifies to

$$\begin{aligned} &\partial_u\left(f(u)g_1(u)' + \frac{C_\gamma f(u)}{1-u}\right) \\ =\quad & \frac{1}{2}\partial_u\left\{\left(F_W' - 2\left(\delta B_1 + B_W\right)\right)\left[g_0'(u) + \frac{C}{1-u}\right]\right\} + \partial_u^2\left\{E_W\left[g_0''(u) + \frac{C}{(1-u)^2}\right]\right\}. \end{aligned} \tag{34}$$

Note the appearance of the combination $F_W' - 2\left(\delta B_1 + B_W\right)$, which could have been anticipated from the work of [27]. The above equation for $g_1(u)$ solves to

$$g_1(u) = \theta\log(1+u) + (C_\gamma - \theta)\log(1-u) + D_\gamma + \Phi(u)\,, \tag{35}$$

where $\Phi(u)$ is given by the following integrals

$$\Phi(u) = C\int du \frac{1}{f^2(u)}\left(F_W' - 2(\delta B_1 + B_W)\right) + 4C\int du \frac{1}{f(u)}\partial_u\left(E_W\, u/f^2(u)\right)\,. \tag{36}$$

Therefore, the function $g_1(u)$ is given by

$$\begin{aligned} g_1(u) \;&=\; \theta\log(1+u) + (C_\gamma - \theta)\log(1-u) + D_\gamma \\ &+\; C\left(\left(\frac{185}{4} + 2\tilde{\alpha}\right)u + \left(\frac{185}{8} + z\right)\log\left[\frac{1-u}{1+u}\right]\right) + \mathcal{O}(u^2)\,, \end{aligned} \tag{37}$$

where $\tilde{\alpha}$ is composed of the coefficients of the Lagrangian

$$\begin{aligned}\tilde{\alpha} &= 216a_2 + 144b_3 + 192b_4 + 30c_1 + 54c_3 - 12c_4 + 6c_5 - 60c_7 - \\ &\quad 12c_8 - 12d_1 - 18d_2 + 18d_3 - 36e_1 - 8f_1 + 5g_1 - 2g_2 - g_3 + h_2\,, \end{aligned} \tag{38}$$

and z drops out upon requiring regularity at the horizon. This yields the following solution to linear order in u

$$g_1(u) = \left(C_\gamma + C\left[\frac{185}{4} + 2\tilde{\alpha}\right]\right) u + D_\gamma\,. \tag{39}$$

We now have the full solution of the equations of motion to linear order in γ and σ:

$$A_x(u) = [1-u]^{-\sigma}\left(C + \gamma C_\gamma + \sigma\left\{D + \gamma D_\gamma + \left(C + \gamma C_\gamma + \gamma C\left[\frac{185}{4} + 2\tilde{\alpha}\right]\right) u\right\}\right)\,. \tag{40}$$

Letting $\overline{C} = C + \gamma C_\gamma$ and $\overline{D} = D + \gamma D_\gamma$, we then have that, to linear order in γ

$$A_x(u) = [1-u]^{-\sigma}\left(\overline{C} + \sigma\left\{\overline{D} + \overline{C}\left(1 + \gamma\left[\frac{185}{4} + 2\tilde{\alpha}\right]\right) u\right\}\right)\,. \tag{41}$$

If we call the boundary value of the field A_T, we then immediately have that $A_T = \overline{C} + \sigma\overline{D}$. A simple calculation then reveals that

$$A_x'(u=0) = 2\sigma A_T\left[1 + \frac{\gamma}{2}\left[\frac{185}{4} + 2\tilde{\alpha}\right]\right]\,. \tag{42}$$

The on-shell action is given by

$$S_{\text{total}} = -\frac{\tilde{N}^2 r_0^2}{16\pi^2 R^4}\int \frac{d^4k}{(2\pi)^4}\int_0^1 du\,\left[\frac{1}{2}A_{-k}\mathcal{L}\,A_k + \partial_u\Psi\right]\,. \tag{43}$$

where $\mathcal{L}A_k = 0$ is simply the equation of motion, and Ψ is a boundary term. Upon evaluating the on-shell action, the only surviving term is the boundary term, as we expect from holography. This is given by

$$\begin{aligned}\Psi &= (B_1 + \gamma B_W - \gamma A_W)A_k' A_{-k} + \frac{\gamma}{2}(C_W - A_W')A_k A_{-k} - \gamma E_W' A_k'' A_{-k} \\ &\quad + \gamma E_W A_k'' A_{-k}' - \gamma E_W A_k''' A_{-k} + \gamma E_W\left(p_1 A_k' + 2p_0 A_k\right)A_{-k}' - \gamma\frac{F_W'}{2}A_k' A_{-k}\,. \end{aligned} \tag{44}$$

The functions A_W, C_W, D_W start at $\mathcal{O}(\varpi_0^2, \kappa_0^2)$, and so do not contribute to the order of momentum in which we are interested. The function E_W starts at $\mathcal{O}(u^2)$, and the regularity of A_x at the boundary ensures that the contribution from terms containing E_W vanishes at the horizon. Therefore, we only get contributions from B_1, B_W and F_W inside Ψ. Remembering that $r_0 = \pi T R^2(1 - 265/16\gamma)$, we obtain that the conductivity is then corrected by a factor

$$1 + \gamma\left(\alpha - 10\right)\,, \tag{45}$$

where

$$\alpha = \tilde{\alpha} - h_1 - 3h_2\,. \tag{46}$$

6 Conclusions

In this work we have considered a strongly-coupled SCFT plasma at finite temperature with $\mathcal{O}(\alpha'^3)$ corrections from type IIB string theory using the gauge/string duality. The corrections include those derived from the Ramond-Ramond five-form field strength and consist of a set of dimension-eight operators in the ten-dimensional type IIB supergravity action. We focused on the effect of these dimension-eight operators on the behaviour of vector fluctuations of the supergravity background. The $\mathcal{O}(\alpha'^3)$ corrections to type IIB supergravity are dual to $1/\lambda^{3/2}$ corrections to the large N limit of the SCFT. Our aim was to study the vectorial fluctuations of the ten-dimensional supergravity background in order to get an insight into R-charge transport in the SCFT. In a certain limit [18], R-charge transport can be equated to EM transport, essentially because a weak gauging of the R-symmetry $U(1)$ allows us to embed $U(1)_{em}$ into the theory, and then reinterpret our results with this is mind.

We recall that the type IIB string theory action with leading $\mathcal{O}(\alpha'^3)$ corrections is given by the usual two-derivative minimal Lagrangian plus a number of eight-derivative operators. The tensor structure of these operators is given by contractions of four factors which are the Weyl tensor and the $\mathcal{T}$ tensor, where the latter is constructed from the five-form field strength F_5 of type IIB string theory. This complicated Lagrangian leads to a background which is a *warped* product of a deformed AdS-Schwarzschild black hole with a compact five-dimensional Einstein manifold M_5, providing that $\mathcal{T}_0$ vanishes. We emphasize that the corrections in AdS factor of the background are independent of the internal manifold as long as the latter is Einstein [19]. Using this corrected background we consider vector perturbations of the metric and investigate the dimensional reduction of the ten-dimensional type IIB string theory action at $\mathcal{O}(\alpha'^3)$ on the M_5. This leads to a five-dimensional effective action for the $U(1)$ gauge fields, and we study the full set of five-dimensional operators induced by the ten-dimensional operators $Weyl^4 + Weyl^3\mathcal{T} + Weyl^2\mathcal{T}^2$. We find 26 independent five-dimensional operators of the form C^2F^2 and $C^2(\nabla F)^2$, F^2 and $(\nabla F)^2$. One of these operators vanishes on-shell leading to only 25 general five-dimensional scalar operators. In principle this Lagrangian can be used to study the finite-coupling corrections to the two-point functions of R-symmetry currents in a wide range of strongly-coupled SCFTs.

As an application of our general effective Lagrangian, we then study the transverse gauge fields A_x, whose solution we need in order to obtain the conductivity of the QGP. In order to solve the EOM of A_x in the hydrodynamic regime we require ingoing boundary conditions at the horizon. Importantly, we find that the frequency of the waves at the horizon does not change compared with the infinite 't Hooft coupling results. Our results indicate that any gauge invariant Lagrangian for the vector perturbations yields an equation of motion with the same singularity structure and indices at the horizon. Finally, we obtain a general expression for the leading 't Hooft coupling corrections to the conductivity for any Lagrangian quadratic in the gauge field and containing up to four derivatives.

The results of this work constitute a step towards the understanding of charge-transport at finite-coupling in a range of SCFTs. We emphasize that the set of operators enumerated in here are present in any type IIB string theory holographic dual model, because the order $\mathcal{O}(\alpha'^3)$ ten-dimensional terms are present regardless of the details of the dual theory. Thus, computing the conductivity in this setting is of intrinsic value even if the exact coefficients of the operators in five dimension are unknown. For simple internal manifolds the exact contributions of the ten-dimensional terms can be determined exactly. For S^5, where the dual theory is $\mathcal{N} = 4$ SYM, we will consider the full ten-dimensional calculation of the conductivity with $1/\lambda^{3/2}$ corrections in a future work [44].

We end with a word on the universality of the corrections computed here. We recall that Buchel *et al.* found that the corrections to the shear viscosity to entropy density ratio for all theories dual to $AdS_5 \times M_5$, where M_5 is Einstein, was a universal quantity. This was shown by proving that the five-dimensional Lagrangian for the tensorial metric perturbations, which govern both the viscosity and entropy of the theory, is the same regardless of the internal manifold. This was obviously a very exciting result, as it shows us a universal feature shared by the strongly-coupled regime of a wide class of theories which have totally different field contents in the weak-coupling description. Naturally, we must ask if there is a possibility that the five-dimensional effective Lagrangian we compute above also exhibits universal behaviour. The problem we have here is that the operators with coefficients h_i have a direct dependence on the Weyl tensor of the internal manifold, making it seem unlikely that the h_i are manifold-independent. However, the operators with coefficients $a_i \longrightarrow g_i$ have no such dependence on the internal manifold, and it could be that those coefficients are universal. These are, however, very speculative comments, and clearly require a lot of work to settle them.

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Appendix: The full Lagrangian

The Lagrangian for the higher-dimensional terms is given by:

$$
\begin{aligned}
A_W &= -2u^5\left[a_1^w f(u)\kappa_0^2 + a_2^w \varpi_0^2\right] + \tilde{A}_W \,, \\
B_W &= -4u^4\left[(b_1^w - b_2^w u^2 + b_3^w u^4) - b_5^w \varpi_0^2 u - b_4^w u f(u)\kappa_0^2\right] + \tilde{B}_W \,, \\
C_W &= -4\frac{u^4}{f(u)}\left[3f(u)(c_1^w u^2 - c_2^w)\kappa_0^2 + \left(c_4^w - c_3^w u^2\right)\varpi_0^2\right] + \tilde{C}_W \,, \\
D_W &= -\frac{u^3}{f^2(u)}\left[d_6^w u f^2(u)\kappa_0^4 + d_7^w u \varpi_0^4 + d_8^w u f(u)\kappa_0^2\varpi_0^2\right. \\
&\quad \left. +4f^2(u)(d_1^w u^2 - d_2^w)\kappa_0^2 + 4(d_3^w - d_4^w u^2 + d_5^w u^4)\varpi_0^2\right] + \tilde{D}_W \,, \\
E_W &= -e_1^w u^6 f^2(u) + \tilde{E}_W \,, \\
F_W &= 4u^5 f(u)(f_1^w u^2 - f_2^w) + \tilde{F}_W \,,
\end{aligned} \tag{47}
$$

where we denote the contributions coming from F^2 and ∇F^2 by $\tilde{A}_W$, $\tilde{B}_W$, $\tilde{C}_W$, $\tilde{D}_W$, $\tilde{E}_W$, $\tilde{F}_W$, and we write the rest explicitly. From our Lagrangian of Eq.(19) we have the identifications:

$$
\begin{aligned}
a_1^w &= 2\left(72a_1 - 4b_1 + 8b_2 - 2c_2 + 5c_3 - c_4 + 2c_6 - c_7 + d_3 + d_4\right), \\
a_2^w &= 2\left(-72a_1 + 4b_1 - 12b_2 + 6c_2 + 3c_3 + 9c_4 - 2c_6 + 9c_7 + 3d_1 + 3d_2 - 2d_3 + d_4\right), \\
b_1^w &= \frac{1}{4}\ (2[11c_1 + 3(13c_3 + 2c_4 + c_5 - 2(3c_7 + c_8))] + 2(-2d_1 - 5d_2 + 9d_3 + 4d_4) \\
&\quad -36(-6a_2 - e_1) + 8(16b_3 + 22b_4 + f_1) - 5g_1 + 2g_2 + g_3)\ , \\
b_2^w &= \frac{1}{4}(576a_2 - 32b_2 + 352b_3 + 480b_4 + 64c_1 + 32c_2 + 256c_3 + 80c_4 + 16c_5 - 48c_7 - 32c_8 \\
&\quad +8d_1 - 8d_2 + 40d_3 + 32d_4 + 36e_1 + 8f_1 - 5g_1 + 2g_2 + g_3), \\
b_3^w &= \frac{1}{4}(-576a_1 + 504a_2 + 32b_1 - 128b_2 + 320b_3 + 432b_4 + 62c_1 + 80c_2 + 238c_3 + 132c_4 \\
&\quad +14c_5 - 16c_6 + 20c_7 - 28c_8 + 28d_1 + 14d_2 + 18d_3 + 32d_4), \\
b_4^w &= -36a_2 - 16b_3 - 24b_4 - c_1 - c_3 + 6c_4 - c_5 + 2(c_7 + c_8) + d_1\,, \\
b_5^w &= 36a_2 + 24b_3 + 32b_4 + 5c_1 + 9c_3 - 2c_4 + c_5 - 10c_7 - 2c_8 - 2d_1 - 3d_2 + 3d_3\,, \\
c_1^w &= \frac{2}{3}(-72a_1 - 36a_2 + 4b_1 - 10b_2 - 16b_3 - 20b_4 + c_1 + 4c_2 - 10c_3 + 5c_4 - c_5 - 2c_6 + 3c_7 \\
&\quad +2c_8 + d_1 - 2(d_3 + d_4)), \\
c_2^w &= -\frac{2}{3}\left(36a_2 + 2b_2 + 16b_3 + 20b_4 - c_1 - 2c_2 + 5c_3 - 4c_4 + c_5 - 2c_7 - 2c_8 - d_1 + d_3 + d_4\right), \\
c_3^w &= -144a_1 + 36a_2 + 8b_1 - 28b_2 + 24b_3 + 32b_4 + 5c_1 + 16c_2 + 23c_3 + 24c_4 + c_5 - 4c_6 + 16c_7 \\
&\quad -2c_8 + 7d_1 + 6d_2 - 2d_3 + 4d_4\,, \\
c_4^w &= -72a_2 - 4b_2 - 48b_3 - 64b_4 - 10c_1 + 4c_2 - 10c_3 + 12c_4 - 2c_5 + 28c_7 + 4c_8 + 7d_1 + 9d_2 \\
&\quad -7d_3 + 2d_4\,,
\end{aligned}
$$

$$
\begin{aligned}
d_1^w &= \frac{1}{4}\left(-108a_2-56b_3-64b_4+c_1-3c_3+22c_4-3c_5+6c_7+6c_8+4d_1-d_2+d_3\right), \\
d_2^w &= \frac{1}{4}\left(-108a_2-56b_3-64b_4+c_1-3c_3+22c_4-3c_5+6c_7+6c_8+4d_1-d_2+d_3-36e_1\right. \\
&\quad \left. -4f_1-3g_1-2g_2-g_3\right), \\
d_3^w &= \frac{1}{4}\left(-108a_2-64b_3-104b_4-11c_1-19c_3+10c_4-7c_5+26c_7+14c_8+8d_1+7d_2-7d_3\right. \\
&\quad \left. -36e_1-8f_1+5g_1-2g_2-g_3\right), \\
d_4^w &= \frac{1}{4}\left(16b_3-16b_4+8c_1+16c_3+8c_4-8c_5-8c_7+16c_8+4d_1-4d_2+4d_3-36e_1-8f_1\right. \\
&\quad \left. +5g_1-2g_2-g_3\right), \\
d_5^w &= \frac{1}{4}\left(-36a_2-16b_3-40b_4-c_1-c_3+6c_4-5c_5+6c_7+10c_8+4d_1+d_2-d_3\right), \\
d_6^w &= -2\left(72a_1-36a_2-4b_1+4b_2-8b_3-16b_4+3c_1+2c_2-4c_3-7c_4-c_5+2c_6+c_7\right. \\
&\quad \left. +2c_8-d_1-d_3-d_4\right), \\
d_7^w &= 2\left(-72a_1+36a_2+4b_1-12b_2+24b_3+32b_4+5c_1+6c_2+12c_3+7c_4+c_5-2c_6-c_7\right. \\
&\quad \left. -2c_8+d_1+d_3+d_4\right), \\
d_8^w &= 4\left(72a_1-36a_2-4b_1+8b_2-16b_3-24b_4-c_1-2c_2+4c_3+5c_4-c_5+2c_6+c_7+2c_8\right. \\
&\quad \left. +d_1+d_3+d_4\right), \\
e_1^w &= 2\left(-72a_1+36a_2+4b_1-12b_2+24b_3+32b_4+5c_1+6c_2+12c_3+7c_4+c_5-2c_6\right. \\
&\quad \left. -c_7-2c_8+d_1+d_3+d_4\right), \\
f_1^w &= \frac{1}{2}\left(-288a_1+180a_2+16b_1-56b_2+120b_3+160b_4+25c_1+32c_2+73c_3+42c_4\right. \\
&\quad \left. +5c_5-8c_6+2c_7-10c_8+8d_1+3d_2+5d_3+8d_4\right), \\
f_2^w &= \frac{1}{2}\left(108a_2-8b_2+72b_3+96b_4+15c_1+8c_2+43c_3+10c_4+3c_5-14c_7-6c_8-3d_2\right. \\
&\quad \left. +7d_3+4d_4\right).
\end{aligned}
$$

We also have the contributions given by

$$
\begin{aligned}
\tilde{A}_W &= 0\,, \\
\tilde{B}_W &= -h_1 f(u)+2h_2(-3+u(8u-7u^3-2\kappa_0^2 f(u)+2\varpi_0^2))\,, \\
\tilde{C}_W &= -8h_2\kappa_0^2 f(u)+\frac{4h_2(2+u^2)\varpi_0^2}{f(u)}\,, \\
\tilde{D}_W &= \frac{1}{uf(u)^2}\Big(-\kappa_0^2 f(u)^2(h_1+h_2(3f(u)+2\kappa_0^2 u)) \\
&\quad +(h_1 f(u)+h_2(3+u^4+4\kappa_0^2 u f(u)))\varpi_0^2-2h_2 u\varpi_0^4\Big)\,, \\
\tilde{E}_W &= -2h_2u^2 f(u)^2\,, \\
\tilde{F}_W &= -2h_2 u f(u)(3-5u^2)\,.
\end{aligned}
\tag{48}
$$

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