Measurement: Sensors 18 (2021) 100209

Contents lists available at ScienceDirect

Measurement: Sensors

journal homepage: www.sciencedirect.com/journal/measurement-sensors

Theoretical insights on the influence of the experimental plan in the calibration of multicomponent force and moment transducers

ARTICLE INFO

Keywords
Experimental plan
Multicomponent transducers
Force metrology
Calibration

ABSTRACT

In recent years, the increasing demand of multicomponent force and moment transducers led the necessity to develop specific calibration procedures. Sensitivity and exploitation terms of these transducers are usually expressed in matrix form to evaluate cross-talks between the different components. According to the seminal work of Ronald Fisher in 1926, to provide accurate results, calibrations shall be performed with different combinations of forces and moments in order to minimize the correlation between them. In this work, a theoretical investigation, based on an ideal transducer, on the influence of the experimental plan in the evaluation of exploitation matrix terms and the associated uncertainties as function of the number of measurements and the correlation between the applied forces and moments is performed. It is found that at decreasing number of measurements and increasing correlations between the applied forces and moments, uncertainties increase, while exploitation matrix terms are poorly affected by the chosen experimental plan.

1. Introduction

In the recent decade, an increasing demand of multicomponent force and moment transducers (MCFMTs) and multicomponent material testing machines to be exploited in many fields of mechanical engineering, from robotics, aerospace, civil engineering and quality and production engineering to force metrology and material testing, has been observed. Such evidence entailed transducer and machine manufacturers to devise new kinds of multicomponent systems, and calibration bodies to develop specific calibration procedures. Among the national metrology institutes, reference calibration systems for MCFMTs have been developed by PTB (Physikalisch-Technische Bundesanstalt) using a hexapod-structured calibration device [1,2] or a deadweight force and torque machine equipped with additional weights coupled to a metallic band and lever arms [3] and by INRiM (Istituto Nazionale di Ricerca Metrologica) using deadweight machines equipped with tilted plates [4] or a system of pulleys and bell crank levers [5] or using calibration systems equipped with crossed flexure levers [6] or rotating table [7]. However, a traceability chain for multicomponent forces and moments, at international level, is still missing together with a standardized calibration method for such transducers and testing machines. As for single-component force or moment transducers equipped with strain-gauges, MCFMTs are typically composed of different strain-gauge bridges each one dedicated to a single component to be measured. However, the intrinsic influence between the different components cannot be neglected and, in calibration processes, the cross-talk signals for combined axial forces and moments shall be evaluated [8]. For this purpose, it is fundamental to perform calibration measurements with different combinations of applied forces and moments to accurately evaluate sensitivity or exploitation matrix terms, each with its associated expanded uncertainty. The set of the different combinations of applied forces and moments to the MFMT under calibration represents the experimental plan. From the seminal work of Ronald Fisher in 1926 [9], it is known that the experimental plan has an influence on the calibration sensitivities and the associated uncertainties. In particular, given a set of combinations of independent parameters (applied forces and moments, in this case), correlations between these components lead to a badly conditioned matrix that, when inverted, provides poorly defined results and higher uncertainties. Therefore, in calibration processes, it is necessary to establish a suitable experimental plan related to the required level of accuracy and uncertainty. If the lowest level of uncertainty is requested, a full factorial experimental plan, with a large number of applied loads combinations, shall be used with the consequence of increasing timings and costs. On the other hand, if higher uncertainties are tolerated, a lower number of measurements can be performed. In this work, taking into account an ideal MCFMT, the influence of the experimental plan, in terms of accuracy and uncertainty of the exploitation matrix terms as function of the number of calibration measurements and correlation between the independent applied forces and moments is investigated.

2. Theoretical background

2.1. Evaluation of exploitation and sensitivity matrices

In an ideal MCFMT, every output is only dependent on the relevant force or moment component. Actually, this condition is not true since transducer outputs interact with each other and cross-talk sensitivities might play a crucial role. Each force and moment component F_k ($k = 1$, 6) can be expressed, in first analysis, as a linear combination of the MCFMT outputs d_i ($i = 1$, 6), considering the second-order interactions negligible, as shown in (1).

https://doi.org/10.1016/j.measen.2021.100209

$$\begin{cases} F_x = d_1A_{11} + d_2A_{21} + d_3A_{31} + d_4A_{41} + d_5A_{51} + d_6A_{61} \\ F_y = d_1A_{12} + d_2A_{22} + d_3A_{32} + d_4A_{42} + d_5A_{52} + d_6A_{62} \\ F_z = d_1A_{13} + d_2A_{23} + d_3A_{33} + d_4A_{43} + d_5A_{53} + d_6A_{63} \\ M_x = d_1A_{14} + d_2A_{24} + d_3A_{34} + d_4A_{44} + d_5A_{54} + d_6A_{64} \\ M_y = d_1A_{15} + d_2A_{25} + d_3A_{35} + d_4A_{45} + d_5A_{55} + d_6A_{65} \\ M_z = d_1A_{16} + d_2A_{26} + d_3A_{36} + d_4A_{46} + d_5A_{56} + d_6A_{66} \end{cases} \tag{1}$$

$A_{i,k}$ are the coefficients used to calculate the force and moment components F_k from the MCFMT outputs d_i, according to (2), in general matrix form,

$$\mathbf{F} = \mathbf{d}\ \mathbf{A} \tag{2}$$

where **F** is the row 1 × k reference forces and moments matrix, **d** is the 1 × i matrix of the MCFMT outputs, and **A** is the i × k coefficients matrix, also called exploitation matrix, which is the matrix actually used by end-users. In this specific case, matrix **A** is a 6 × 6 squared matrix.

Considering the n linearly independent sets of calibration values deriving from the experimental plan, **F** and **d** in (2) become a $n \times k$ and a $n \times i$ matrix, respectively. Matrix **A** and its $A_{i,j}$ coefficients can be evaluated through a linear regression according to (3) [10].

$$\mathbf{A} = \left[\mathbf{d}^{\mathbf{T}}\mathbf{d}\right]^{-1}\mathbf{d}^{\mathbf{T}}\mathbf{F} \tag{3}$$

In the same way, to evaluate sensitivity matrix, considering n linearly independent sets of values, each MCFMT output d_i (i = 1, 6) can be expressed as a linear combination of the force and moment components F_k (k = 1, 6), according to (4),

$$\mathbf{d} = \mathbf{F}\ \mathbf{A}^{-1} = \mathbf{F}\ \mathbf{S} \tag{4}$$

where **S** is the $k \times i$ (6 × 6) sensitivity matrix, in which the diagonal terms are the main sensitivities, and the out-of-diagonal terms are the cross-talk sensitivities. Sensitivity matrix can be calculated as the inverse of the exploitation matrix, $\mathbf{S} = \mathbf{A}^{-1}$, if i = k.

2.2. Uncertainty assessment

A comprehensive uncertainty evaluation can be performed according to GUM [11]. The $i \times k$ (6 × 6) matrix of the variances referred to the single terms of the exploitation matrix, $\mathbf{u^2(A)}$, is given by the general rule of uncertainty propagation, according to (5),

$$\mathbf{u}^2(\mathbf{A}) = \begin{bmatrix} u^2(A_{11}) & \cdots & u^2(A_{1k}) \\ \vdots & \ddots & \vdots \\ u^2(A_{i1}) & \cdots & u^2(A_{ik}) \end{bmatrix} = \\ = \begin{bmatrix} u^2(A_{11})' & \cdots & u^2(A_{1k})' \\ \vdots & \ddots & \vdots \\ u^2(A_{i1})' & \cdots & u^2(A_{ik})' \end{bmatrix} + \begin{bmatrix} u^2(S_{11})\dfrac{A_{11}^2}{S_{11}^2} & \cdots & u^2(S_{1i})\frac{A_{i1}^2}{S_{1i}^2} \\ \vdots & \ddots & \vdots \\ u^2(S_{k1})\dfrac{A_{1k}^2}{S_{k1}^2} & \cdots & u^2(S_{ki})\frac{A_{ik}^2}{S_{ki}^2} \end{bmatrix}^T \tag{5}$$

where,

$$\begin{bmatrix} u^2(A_{11})' & \cdots & u^2(A_{1k})' \\ \vdots & \ddots & \vdots \\ u^2(A_{i1})' & \cdots & u^2(A_{ik})' \end{bmatrix} = \mathbf{c}\ \mathbf{u}^2(\mathbf{F}) \tag{6}$$

is the $i \times k$ matrix given by the multiplication of **c**, which is a $i \times n$ matrix of the squared terms of $[\mathbf{d^T d}]^{-1}\mathbf{d^T}$ matrix, and $\mathbf{u^2(F)}$ is the $n \times k$ matrix representing the variances of the reference applied forces and moments at each calibration condition, and $u^2(S_{ki})$ are the terms deriving from

$$\begin{bmatrix} u^2(S_{11}) & \cdots & u^2(S_{1i}) \\ \vdots & \ddots & \vdots \\ u^2(S_{k1}) & \cdots & u^2(S_{ki}) \end{bmatrix} = \mathbf{h}\ \mathbf{u}^2(\mathbf{d}) \tag{7}$$

which is the $k \times i$ matrix given by the multiplication of **h**, a $k \times n$ matrix composed of the squared terms of $[\mathbf{F^T F}]^{-1}\mathbf{F^T}$ matrix, and $\mathbf{u^2(d)}$, which is the $n \times i$ matrix representing the variances of the MCFMT outputs containing uncertainty contributions due to repeatability, drift of the zero output and resolution for each calibration condition. These operations with matrices and the assumption that $u^2(A_{ik}) = u^2(A_{ik})' + u^2(S_{ki})A_{ik}^2/S_{ki}^2$, are due to the impossibility to directly propagate $[\mathbf{d^T d}]^{-1}\mathbf{d^T}$ matrix terms of (3).

Table 1
Capacities of the ideal MCFMT.

F_x/kN	F_y/kN	F_z/kN	M_x/kN m	M_y/kN m	M_z/kN m
2	2	10	0.15	0.15	0.15

Table 2
Exploitation matrix terms of the ideal MCFMT in N/(mV/V), columns 1–3, or N m/(mV/V), columns 4–6.

1000	1	5	0.075	0.075	0.075
1	1000	5	0.075	0.075	0.075
1	1	5000	0.075	0.075	0.075
1	1	5	75	0.075	0.075
1	1	5	0.075	75	0.075
1	1	5	0.075	0.075	75

3. The ideal multicomponent force and moment transducer

To evaluate the influence of the experimental plan on the calculation of the exploitation matrix terms and associated uncertainty, an ideal MCFMT, with negligible associated uncertainties $\mathbf{u^2(d)}$, is considered. In this way, the only uncertainties to be propagated are the ones of the reference forces and moments $\mathbf{u^2(F)}$ applied at each calibration condition. Capacities and exploitation matrix of the chosen MCFMT, similar to a real one, are given in Tables 1 and 2. Cross-talk terms are in the order of 0.1% of the relevant diagonal main exploitation term. Given an experimental plan, the outputs of the MCFMT **d** are calculated from the applied forces and moments **F**, according to (4).

4. Theoretical calibration results with different experimental plans

As stated in the introduction, a "complex" design (such as factorial design) is more efficient than studying one factor at a time and allows to determine the effect of several factors and interactions between them with the same number of tests necessary to determine any one of the effects by itself with the same degree of accuracy. A full factorial experiment is an experiment whose design consists of two or more factors, each with discrete possible values or "levels", and whose experimental units take on all possible combinations of these levels across all such factors. Such an experiment allows the investigator to study the effect of each factor and their interaction on the response variable. Given a number of factors k (in this case, the 6 force and moment outputs of the MCFMT) and a number of calibration levels N for each applied force or moment, the total number of calibration conditions n, to have full factorial design, is given by (8)

$$n = N^k = N^6 \tag{8}$$

The number of calibration measurements n increase as a function of the levels for each force or moment component, e.g. from $n = 729$ with $N = 3$ levels (e.g. −100%, 0% and +100% of the MCFMT capacities) up to $n = 531441$ with $N = 9$ (e.g. from −100% to 100% of the capacities with steps of 20%). In the following Sections, by implementing equations (3)–(7), considering only the uncertainty of the reference applied forces and moments $\mathbf{u^2(F)}$, which are assigned a value of 10^{-3} in terms of relative uncertainty [1], and neglecting the uncertainties $\mathbf{u^2(d)}$ of the ideal known MCFMT, different experimental plans are designed to

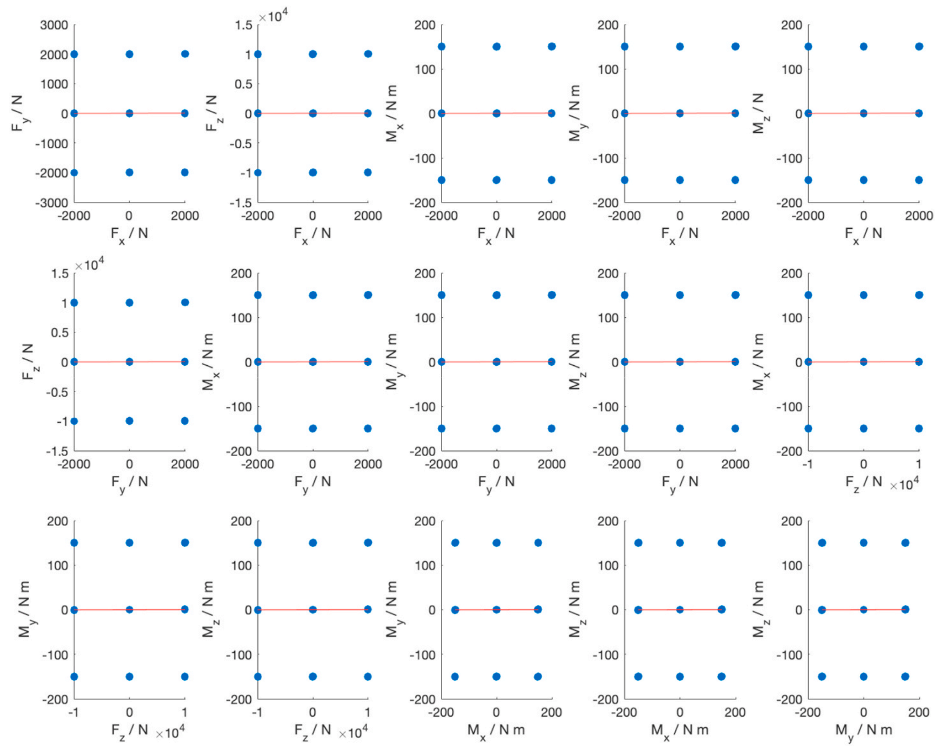

Fig. 1. Full factorial experimental plan with $N = 3$ levels ($n = 726$).

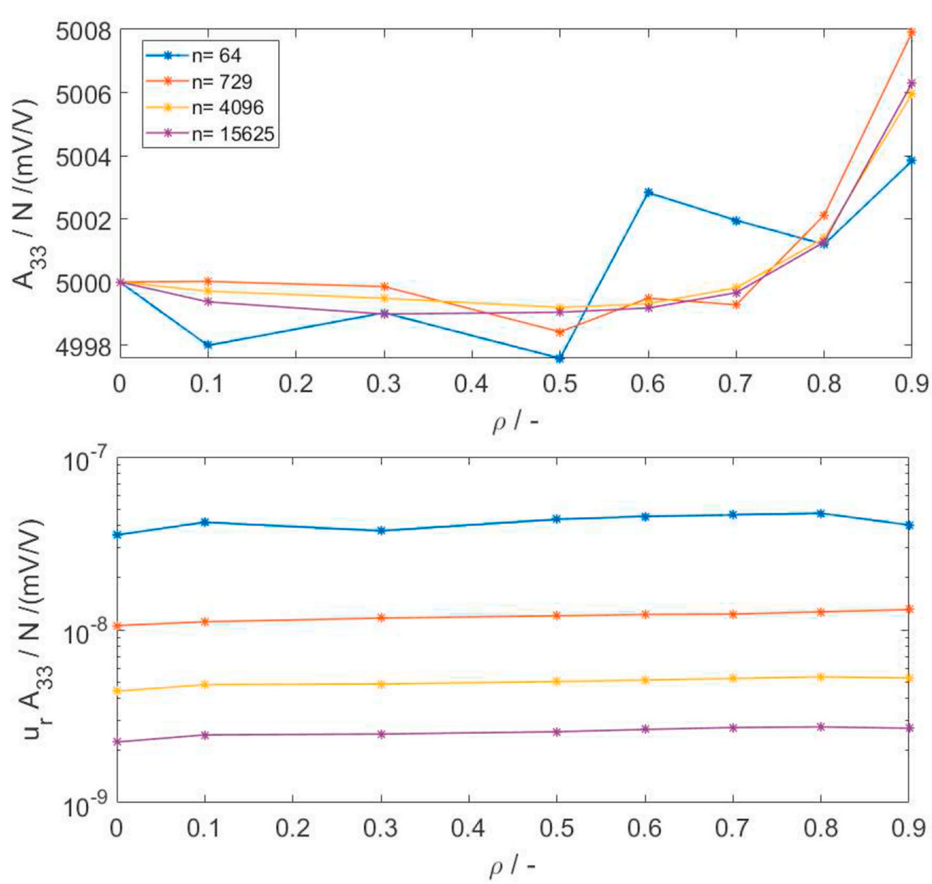

Fig. 2. Exploitation term A_{33} and its associated relative standard uncertainty as a function of the number of calibration conditions n and the correlation ρ between the applied forces and moments.

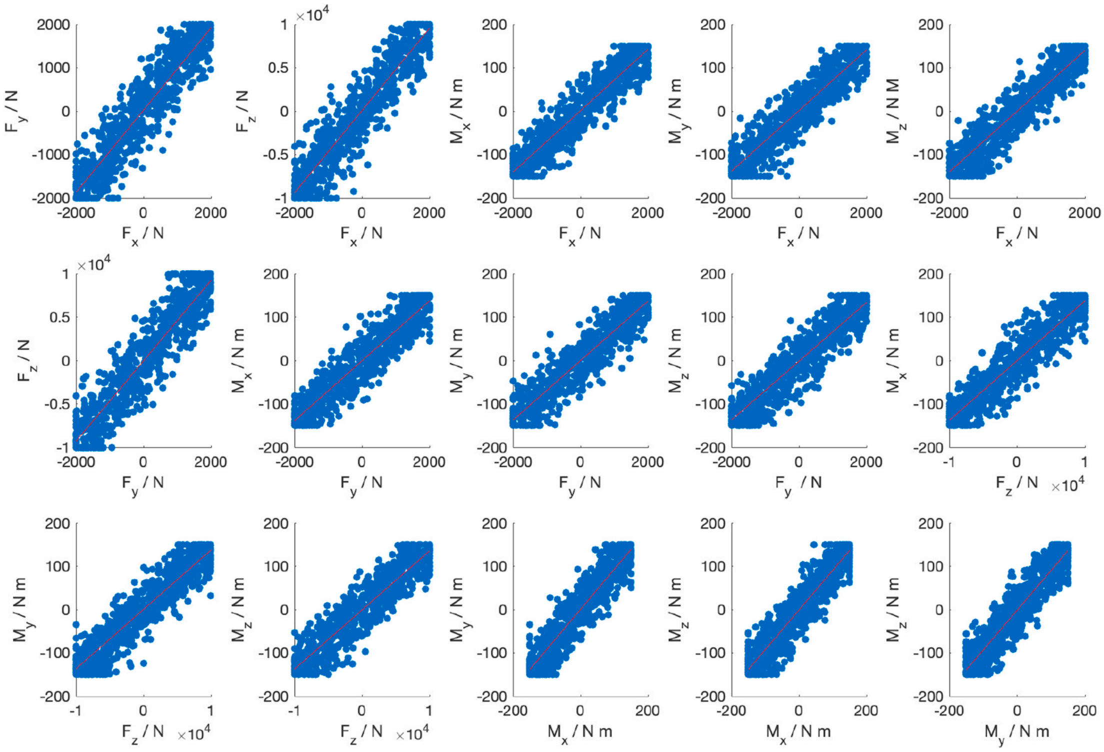

Fig. 3. Experimental plan with $n = 726$.

evaluate the influence of the number of measurement n and the degree of correlation between the reference applied forces and moments $\mathbf{F}$ ($n \times k$ matrix), i.e., the independent variables of the experimental plan in the evaluation of the exploitation matrix terms and the associated uncertainties.

4.1. Full factorial experimental plan as function of the number of levels N

From theory, a full factorial experimental plan is given by a set of levels of two or more factors that are not correlated ($\rho = 0$). In this case, the full factorial experimental plan is represented by all the combinations of the reference forces and moments, at different levels, that are applied to the MCFMT during calibration. Full factorial experimental plans can be designed with different levels N for each component (e.g. ISO 376 [12] requires a minimum number of 8 levels). In this case, the number of levels is changed from $N = 2$ (i.e., −100% and +100% of the capacity of each MCFMT output) to $N = 5$ (i.e., −100%, −50%, 0%, +50% and +100% of the capacity of each MCFMT output), which correspond to $n = 64$ and $n = 15625$ respectively according to (8). By way of example, the experimental plan with $N = 3$ levels ($n = 726$), that is −100%, 0% and +100% of MCFMT capacities, is shown in Fig. 1. No correlation between the applied force and moment components is found (red lines), as expected. In these conditions, the exploitation matrix terms and the associated uncertainties, calculated according to (3)–(7), are constant, whereas uncertainties decrease at increasing levels. In particular, standard uncertainties with $n = 64$ ($N = 2$) are 10 times larger than those with $n = 15625$ ($N = 5$). By way of example, exploitation matrix term A_{33} and its associated relative standard uncertainty are shown in Fig. 2 ($\rho = 0$). Relative standard uncertainties, evaluated according to (5)–(6), of main diagonal terms are much lower than starting one. This is not unusual since starting reference uncertainties spread along the 36 ($i \times k$) terms and during subsequent use are recombined by end-user when using the exploitation matrix.

4.2. Experimental plan as function of the correlation between the independent variables

From theory [9], it is found that correlations between the independent variables of the experimental plan, i.e. the combinations of reference applied forces and moments $\mathbf{F}$, influence the calibration results. For this reason, with the same number of calibration conditions ($n = 64$, 729, 4096 and 15625) previously tested, the experimental plan is modified in order to have correlations between the reference applied forces and moments with values ranging from $\rho = 0$ to $\rho = 0.9$. By way of example, the experimental plan with $n = 726$ and $\rho = 0.9$ is shown in Fig. 3. In these conditions, exploitation terms start diverging up to 0.13% and relative standard uncertainties increase, doubling from $\rho = 0$ to $\rho = 0.9$ (see Fig. 2 for A_{33}).

4.3. Subsets of the full factorial experimental plan

In the end, starting from the full factorial experimental plan with a number of levels per component equal to $N = 5$ (producing a maximum number of combinations equal to $n = 15625$), smaller subsets of calibration conditions are randomly selected in order to evaluate its influence on the calibration results. Since the reduction in the number of calibration conditions entails a deviation from the full factorial plan, correlation values are no longer equal to $\rho = 0$ as seen in Section 4.1, but vary randomly between $\rho = -0.5$ and $\rho = 0.5$. It is found that exploitation matrix terms are constant, although a decreasing number of calibration measurements. This could be due to the fact that, even if correlations of each pair of applied forces and moments appear, they are averagely around 0, contrary to the case of Section 4.2 and Fig. 3. On the other hand, uncertainties increase at decreasing number of calibration conditions, as expected. In particular, relative standard uncertainties with $n = 8$ are 50 times larger than those with $n = 15625$. Furthermore, comparing the uncertainties with $n = 64$ and $n = 15625$ as in Section 4.1, it is found that the ratio is around 15, thus slightly larger than the ratio previously found (i.e., 10). This is due to the fact that, in this case, correlations between the reference applied forces and moments is not 0.

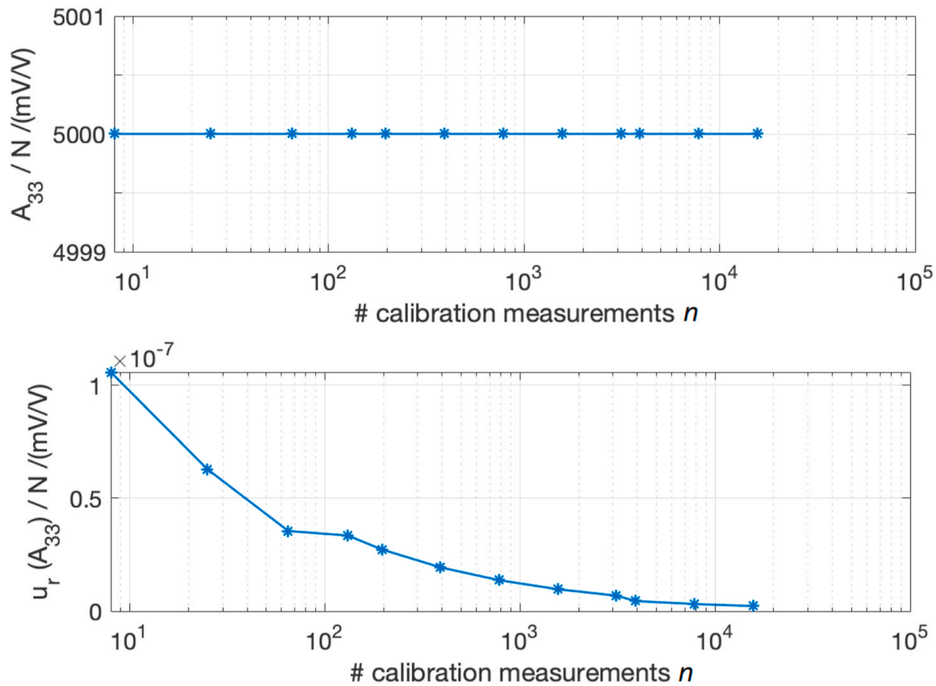

Fig. 4. Exploitation term A_{33} and its associated relative standard uncertainty as a function of the number of calibration measurements n, subsets of the $N = 5$ full factorial plan ($n = 15625$).

By way of example, exploitation term A_{33} and its associated relative standard uncertainty as function of the number of measurements n are shown in Fig. 4.

5. Conclusions

A theoretical investigation on the influence of the experimental plan on the exploitation matrix terms and associated uncertainties is performed assuming an ideal MCFMT. It is found that the evaluation of the exploitation matrix term seems to be affected only by the mean correlation of the reference applied forces and moments, with relative deviations increasing up to 0.13% when $\rho = 0.9$. Associated uncertainties, instead, increase at decreasing number of measurements and at increasing correlations between the applied forces and moments.

Acknowledgments

This work was supported by the EURAMET and the European Union in the framework of EMPIR project ComTraForce [18SIB08, 2019].

References

[1] J. Nitsche, S. Baumgarten, M. Petz, D. Röske, R. Kumme, R. Tutsch, Measurement uncertainty evaluation of a hexapod-structured calibration device for multi-component force and moment sensors, Metrologia 54 (2) (2017) 171.

[2] D. Röske, “Metrological Characterization of a Hexapod for a Multi-Component Calibration Device”, *XVII IMEKO World Congress*, June 2003. Dubrovnik, Croatia.

[3] S. Baumgarten, D. Röske, R. Kumme, Multi-component measuring device - completion, measurement uncertainty budget and signal cross-talk for combined load conditions, ACTA IMEKO 6 (2017) 4.

[4] S. Palumbo, A. Prato, F. Mazzoleni, A. Germak, “Multicomponent Force Transducer Calibration Procedure Using Tilted Plates”, *XXII World Congress*, Sept. 2018. Belfast, UK.

[5] A. Bray, G. Barbato, R. Levi, Theory and Practice of Force Measurement, Academic Press, London, UK, 1990.

[6] C. Ferrero, L.Q. Zhong, C. Marinari, E. Martino, New automatic multicomponent calibration system with crossedflexure levers, The 3rd Int. Symp. on Measurement and Control in Robotics (1993) Cm.I-31–Cm.I-39.

[7] G. Barbato, A. Bray, S. Desogus, F. Franceschini, A. Germak, “Field Calibration Method for Multicomponent Robotic Force/moment Transducers”, *II International Symposium On Measurement And Control In Robotics*, Tsukuba Science City, Japan, Nov. 1992.

[8] S. Baumgarten, H. Kahmann, D. Röske, Metrological characterization of a 2 kN·m torque standard machine for superposition with axial forces up to 1 MN, Metrologia 53 (5) (2016) 1165–1176.

[9] R. Fisher, The arrangement of field experiments, Journal of the Ministry of Agriculture of Great Britain 33 (1926) 503–513.

[10] A. Prato, F. Mazzoleni, A. Schiavi, Traceability of digital 3-axis MEMS accelerometer: simultaneous determination of main and transverse sensitivities in the frequency domain, Metrologia 57 (3) (2020).

[11] Jcgm 100, Evaluation of Measurement Data — Guide to the Expression of Uncertainty in Measurement (GUM), Joint Committee for Guides in Metrology, Sèvres, France, 2008.

[12] Iso 376, Metallic Materials — Calibration of Force-Proving Instruments Used for the Verification of Uniaxial Testing Machines, 2011.

Andrea Prato*
INRIM – National Institute of Metrological Research, Turin, Italy

Davide Borgiattino
Politecnico di Torino, Turin, Italy

Fabrizio Mazzoleni, Alessio Facello, Alessandro Germak
INRIM – National Institute of Metrological Research, Turin, Italy

* Corresponding author.
E-mail address: a.prato@inrim.it (A. Prato).