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Modeling multi-criteria decision-making problems with applications in last mile delivery and school safety assessment

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Modeling multi-criteria decision-making problems with applications in last mile delivery and
school safety assessment

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in Partial Fulfillment of the Requirements
for the Degree of Doctor of Philosophy
in Industrial and Systems Engineering
in the Bagley College of Engineering

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The last-mile delivery option has become a focal point of academic research and industrial development in recent years. Multiple factors such as increased demands on delivery flexibility, customer requirements, delivery urgency, and many others are enforcing to adopt this option. For fulfilling this paradigm shift in delivery and providing additional flexibility, drones can be considered as a viable option to use for last-mile delivery cases. Numerous drones are available in the market with varying capacities and functionalities, posing a significant challenge for decision-makers to select the most appropriate drone type for a specific application. For this purpose, this study proposes a comprehensive list of criteria that can be used to compare a set of available last-mile delivery drones. Additionally, we introduced a systematic multi-criterion, multi-personnel decision-making approach, referred to as the Interval Valued Inferential Fuzzy TOPSIS method. This method is robust and can handle the fuzziness in decision-making, thereby providing quality drone selection decisions under different applications. We then apply this method to a real-life test setting. Results suggest that smaller drones or quadcopters are considered viable to use in urban environments, while long-range drones are preferred for the last mile delivery needs in rural settings.

Keywords: Drone selection, Multi-criteria decision making, Interval Valued Inferential Fuzzy
TOPSIS

DEDICATION

This Dissertation is dedicated to my parents

Ibrahim rahahleh

and

Iman rahahleh

who have given me invaluable educational opportunities and to my spouse, Saed alshawabkeh, who has been my emotional partner through my Ph.D. Program.

To my boys Ayan and Yenal, the sky is your limit.

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CHAPTER I

LAST MILE DELIVERY DRONE SELECTION AND EVALUATION USING THE
INTERVAL VALUED INFERENTIAL FUZZY TOPSIS

1.1 Introduction

Autonomous drone deployment has become a focal point of research in recent years. Market-leading companies, such as Google, Amazon, DHL, have been researching and expanding both the capabilities and feasibilities of using this technology for the last mile delivery needs (Stewart, 2014; Guardian, 2016; DHL, 2018). For instance, Amazon's Prime Air and Google's Project Wing are two such projects designed to examine the applicability of using drones of different specifications into last-mile delivery operations (Stewart, 2014; Guardian, 2016; Rheude, 2018). Most recently, the United States Department of Transportation (U.S. DoT) and the Federal Aviation Administration (FAA) announced ten regions of the nation to participate in the Unmanned Aircraft Systems (UAS) Integration pilot program (U.S. Department of Transportation., 2018). This new initiative has accelerated the attempts for drone integration into the US airspace while widening the opportunities for deploying drones for use within the logistics fields. The main objective of this FAA program is the safe integration of drones into the US airspace beyond the sight of human pilots. With this upgraded regulation, different states have collaborated with private companies (e.g., Flirtey, Zipline) to carry out wide variants of testing such as tests for delivering medical equipment's, package delivery, emergency management, blood delivery, infrastructure inspection, crop inspection, and so on. Several other countries, such as the

UK and Rwanda, also regulated several projects for possible implementation of drones as last-mile delivery needs (Rheude, 2018). While the ideation of and execution with drone delivery systems may be growing in both popularity and scope, selection of the best drone option is constrained by a number of crucial factors such as package size, delivery distance, demographics of the drone deployment area, dominant weather conditions of the area to be served, and many more. Selecting the best-suited drone considering all these solutions' specific and situational attributes is a complicated and time-consuming problem, requiring a thorough exploration to make the selection process more structured.

With the growth of the world economy, the need for faster package delivery is increasing. In the past few years, several studies have been conducted to test the feasibility of using drones in delivery tasks such as pizza delivery (Bamburly, 2015) or medical supplies delivery (Bamburly, 2015; Bryan, 2014; Poudel et al., 2019). Even though drone solutions are a plausible option for last-mile delivery, due to the differences in sizes, shapes, flight distances, loading capacities, and many others, choosing the best drones to deploy for different applications is considered challenging. Some drones are faster but have low load-carrying capacities, while others are slower but have higher load carrying capacities and can cover longer flight distances. Also, not all delivery drone solutions are suitable for use in all environmental conditions, such as in high winds or in different forms of precipitation. Therefore, it is essential to have a standard and structured methodology for last-mile delivery drone selection. However, to date, no research has been conducted that attempts to provide a standard, well-structured methodology to perform drone selection that would be applicable to any delivery-based use cases. To address this gap, this paper proposes a standardized method to select the best drone option among a list of potential drone solutions that could be applicable for last-mile delivery cases.

Till now, the majority of the existing studies have attempted to optimize the drone routing decisions by assuming that a set of drones are available for deployment. The focuses for such mathematical models primarily lie in minimizing the overall delivery cost, energy consumption, travel time, and/or a combination thereof. For instance, Motlagh et al. (2016) propose two integer programming model formulations to minimize the energy consumption and the operation time of a set of known drones. Lee (2017) presents a modular drone delivery method that is beneficial in increasing the fleet readiness and decreasing the overall fleet size. Gatteschi et al. (2015) consider the hardware choice problem for quadcopters delivery drones only. Ponza (2016) optimizes the routing path of the truck-based last-mile delivery drone. The author utilizes the classical traveling salesman problem approach to model the solution. Other notable research in optimizing the drone delivery under diverse application areas can be found in (Oruc and Kara, 2018; Carlsson and Song, 2017; Campbell et al., 2018; Derpich et al., 2018; Li et al., 2018; Kim et al., 2017). For a comprehensive overview of drone routing and optimizing literature, interested readers are referred to review the recent article by Otto et al. (2018). Note that all the prior studies assume that a set of drones are available for possible deployment to different needs. However, to date, no rigorous methodology has been developed that can appropriately characterize drones to find their suitability for a given application and/or environment, more specific to last-mile delivery problem in our case.

To fill this literature gap, this study proposes a rigorous methodology for selecting last-mile delivery drones. First, a comprehensive list of criteria is developed to characterize a drone appropriately. Then, a systematic multi-criterion, multi-personnel decision-making approach, namely, Interval Valued Inferential Fuzzy (IVIF) TOPSIS method, is proposed to select a drone for a given application from a set of available drones. To the end, several realistic last-mile delivery problem scenarios are constructed to derive valuable managerial insights and to demonstrate the

applicability of the proposed methodology. Note that the last mile delivery drone selection problem includes a number of conflicting attributes. Decision-makers or experts prefer to provide linguistic assessments other than the exact numerical judgments, which can later be converted to their corresponding numerical values and used in the MCDM method through the fuzzy set theory. MCDM methods such as fuzzy TOPSIS (Kahraman et al., 2007), fuzzy AHP (Tuysuz and Kahraman, 2006), and fuzzy VIKOR (Kaya and Kahraman, 2011) utilize the ordinary fuzzy sets. Extensions of regular fuzzy sets such as hesitant fuzzy sets (Torra, 2010), intuitionistic fuzzy sets (Bustince and Burillo, 1996), and type-2 fuzzy sets (Zadeh, 1975) are the recent inclusions into multi-criteria decision making which can better define the membership functions compared to ordinary fuzzy sets (Boran, 2011; Boran et al., 2011; Onar et al., 2015). We used Interval Valued Inferential Fuzzy (IVIF) TOPSIS method for our last-mile delivery drone selection problem. This method uses IVIF sets to provide a more robust definition of the membership functions, can better handle the vagueness of user's input, and efficiently handle judgmental biases.

The exposition of this paper is as follows. Section 2 provides the dissection of the last mile delivery drone selection problem and proposes critical factors that have a significant influence on this selection process. Section 3 introduces the IVIF TOPSIS decision-making model by illustrating each step of the model. In section 4, a real-life case study visiting different drone deployment scenarios is presented that demonstrates the suitability of using different drones under varying deployment cases while revealing crucial managerial insights in these scenario-dependent drone deployment situations. Finally, we conclude and present avenues for future research in Section 5.

1.2 Problem Statement And Criterion Selection

From the airspace regulatory perspective, drones are becoming a more realistic option for last-mile delivery among different industries such as logistics, retail, service-sector, military, and anywhere that assets need to be moved and tracked from an origin to a destination. The efficient and enhanced performance (España, 2018), improved flight time (Zeng et al., 2016), and relaxed navigational regulations (Park et al., 2016) are motivating companies (Thomas and Qidwai, 2004) that rely upon product delivery in any capacity to consider upgrading or deploying this technology in the near future. Several variants of delivery drones are available in the market that can successfully handle the last-mile delivery operations. These drones possess distinguishing features that might make one drone more preferred over another depending on particular use cases. Therefore, selecting the appropriate drone for the delivery process is critical for both efficiency and economics. Selection of the proper last-mile delivery drone depends upon a number of device-specific and performance-related factors. This paper proposes a comprehensive list of the key factors that have a significant influence on drone selection. A total of 28 sub-criteria have been identified and grouped under five main criteria, namely, (i) physical specification, (ii) economic, (iii) performance, (iv) environmental, and (v) payload capacity. All these criteria are extracted from online literature and expert opinion. Detailed descriptions for each sub-criterion are provided in this section, while Figure 1 visualizes the hierarchical representation of the sub-criteria under each main criterion. The methodology flow diagram for applying the IVIF TOPSIS method can be seen in Figure 2.

1.2.1 Physical Specification (A)

- **Drone Overall Size (A_1):** There is no particular standard for the classification of drones according to their sizes. Hence, the standard varies among the defense agencies and the

civilians. However, drones can be roughly classified into several subclasses, such as very small (30 cm – 50 cm), small (50 cm – 2 m), medium (5 m – 10 m), and large (above 10 m).

- **Weight (A_2):** The weight of a drone plays a significant role in its performance, agility, and flight duration. Weight varies in drones according to their sizes and shell/casing materials (e.g., plastic, metal, composite, foam). The weight of a drone can vary from 1.2 kg up to as much as 2 tons, but a delivery drone typically falls within 5 kg to 8.6 kg.
- **Drone Type (A_3):** Indicates the genre of the drone according to its rotor or wing formation. On the basis of rotor/wing formation, drones can be divided into several classes, such as fixed-wing, rotary-wing, hybrids that combine fixed and rotary, flapping-wing, and blimps. Fixed, rotary and hybrid types are more conducive for delivery-type operations; thus, in this study, flapping-wing and blimps-type drones are not taken into consideration.
- **Fuel Type (A_4):** This indicates the type of fuel source used to provide power to a UAV. There are six basic ways (e.g., batteries, solar energy, hydro fuel cell, combustion engine, tethered, and laser transmitter) through which drones can be powered. Based on the currently available options, both battery and fuel-powered drones are considered in this study.

1.2.2 Performance (B)

- **Internal Computing Components (B_1):** Internal computing components often include different sensors used in the drone such as accelerometer, stabilizer, 3-axis Gyroscope, Magnetometer, Barometer, and like such.

- **Location and Proximity Accuracy (B_2):** This criterion details the capability of the drone to accurately measure distance and location using different sensors such as distance identification, obstacle detection (ultrasonic, laser, and LIDAR based), occupancy sensors, proximity sensors, motion sensors, and collision detection sensors are included in this sub-criteria.
- **Communication and Data Quality (B_3):** In some cases, communication and data quality influence the performance level of a drone. Internet of Things (IoT) and radio connectivity fall under this section.
- **Traceability (B_4):** The ability of a drone to locate an object as well as the previous route from the drone's history. GPS sensors would typically be grouped in this sub-criterion.
- **Reliability (B_5):** The length of the operational lifetime of the drone without any failure.

1.2.3 Economic (C)

- **Repair Cost (C_1):** After being in operation for some time, drones may damage, or the performance may deteriorate. Repairs are then made to bring a drone back to its initial operating condition or to keep the drone functioning at its current state.
- **Total Unit Cost (C_2):** This cost includes all infrastructure costs (fixed, variable, and overhead costs) associated with each unit of a drone. The overhead cost could consist of items such as storing the equipment or building out charging or refueling infrastructure required to operate each drone over time.
- **Total Lifecycle Cost (C_3):** This cost refers to the total cost of ownership associated with the drone over its entire product lifecycle.

- **Operating Cost (C_4):** The cost for the operation of a drone that could include unmanned pilot expenses, fuel, and any airspace usage charges.
- **Training Cost (C_5):** The costs for training the employees or operators to use the drone for delivery.

1.2.4 Environmental (D)

- **Adaptability (D_1):** This criterion represents the ability of the drone to operate in normal and extreme weather conditions successfully (e.g., icing, wind, rain, hail, and fog).
- **Recharge/Refuel Location (D_2):** Recharge and refuel locations consider the availability and the geographical distribution of the charging facilities for delivery drones. Drones can stop and recharge/refuel in these places to provide continuous service. This criterion significantly impacts the performance and the speed of the delivery process.
- **Environmental Impact (D_3):** This sub-criterion denotes the possible impacts (e.g., noise pollution and carbon footprint) to the environment that might occur due to drone deployment for last-mile delivery.
- **Required Delivery Distance (D_4):** The distance from the inventory or product storage and the actual location of the customer or drop-off point where the product needs to be delivered.

1.2.5 Payload Capacity (E)

- **Max Flight Time (E_1):** This indicates the maximum time that a drone can fly without recharge/refuel both with and without payload.
- **Total Recharge/Refuel Time (E_2):** The total time required for the drone to be recharged or refueled. This impacts the amount the drone will be unavailable for use.

- **Charge/Fuel Usage Rate (E_3):** Charge/Fuel usage rate means the rate at which the drone consumes the charge/fuel per unit time (e.g., hour and second). Also, in consideration for this criterion is the total number of recharges or refuels that can occur and when items such as batteries will need to be replaced.
- **Maximum Load (E_4):** This factor represents the maximum load carrying capacity of the drones and is highly compatible with the size and engine strength of the drone.
- **Maximum Carry Dimensions (E_5):** This indicates the overall dimension of the package to be delivered.
- **Maximum Reachable Altitude (E_6):** The maximum allowable altitude for drones in the U.S. is restricted by the Federal Aviation Administration (FAA) regulations. According to these regulations, drones are not permitted to fly above 400 feet from the ground. On the contrary, the limit is 500 feet for European Union drones. Other than the allowable altitude, the capacity of the drone can be another limiting factor to reach a certain height. This factor considers both cases.
- **Drone Speed (E_7):** This sub-criterion stands for the maximum allowable speed of the drone.
- **Adaptability to Dynamic Assignment (E_8):** Adaptability to dynamic assignment/routing is the ability of a drone to align with the continuously updating routing decisions, which is different from the current routing path the drone is following.
- **Package Handling Flexibility (E_9):** This sub-criterion indicates the safety and damage prevention factors of the drone.

- **Delivery Flexibility (E_{10}):** This defines the ability of the drone to adjust to varying package sizes, shapes, and handling. Some drones will have restrictions and limitations in this case.

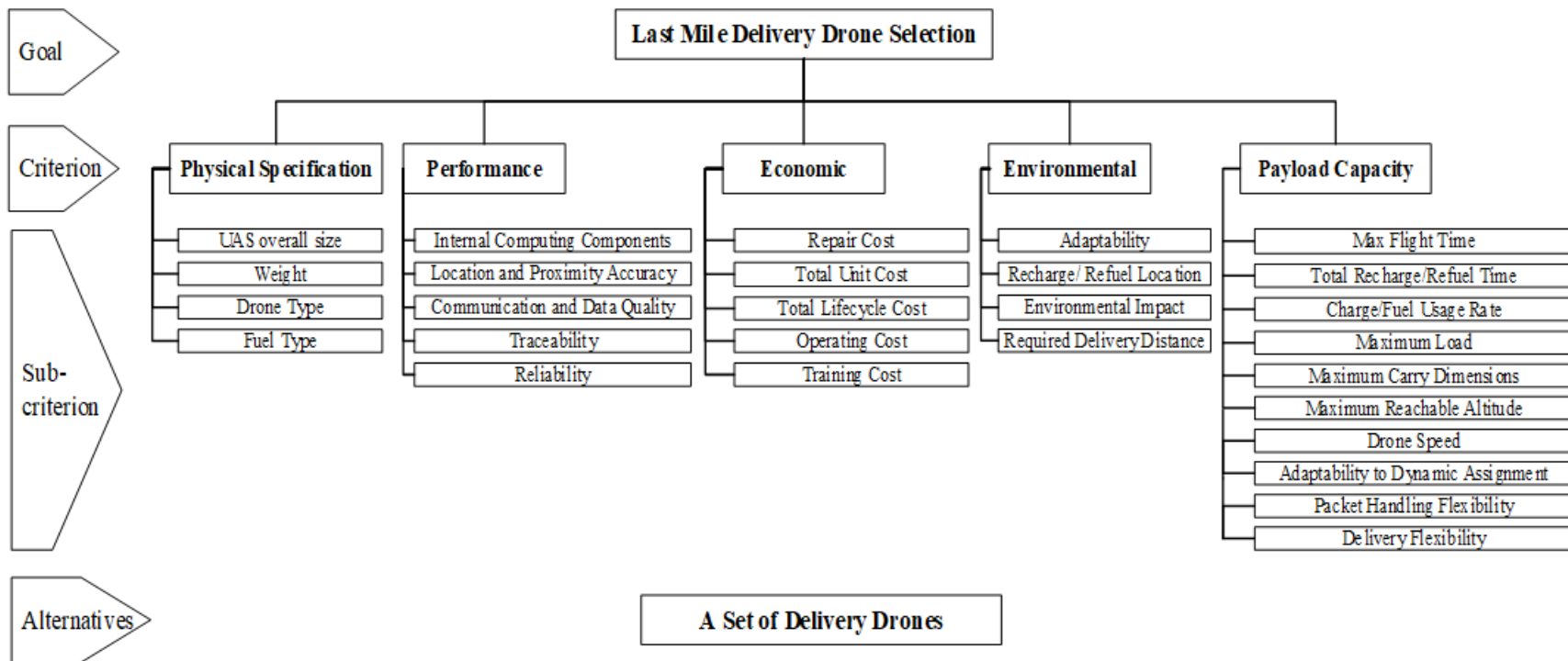


Figure 1.1 Criterion list for last mile delivery drone selection

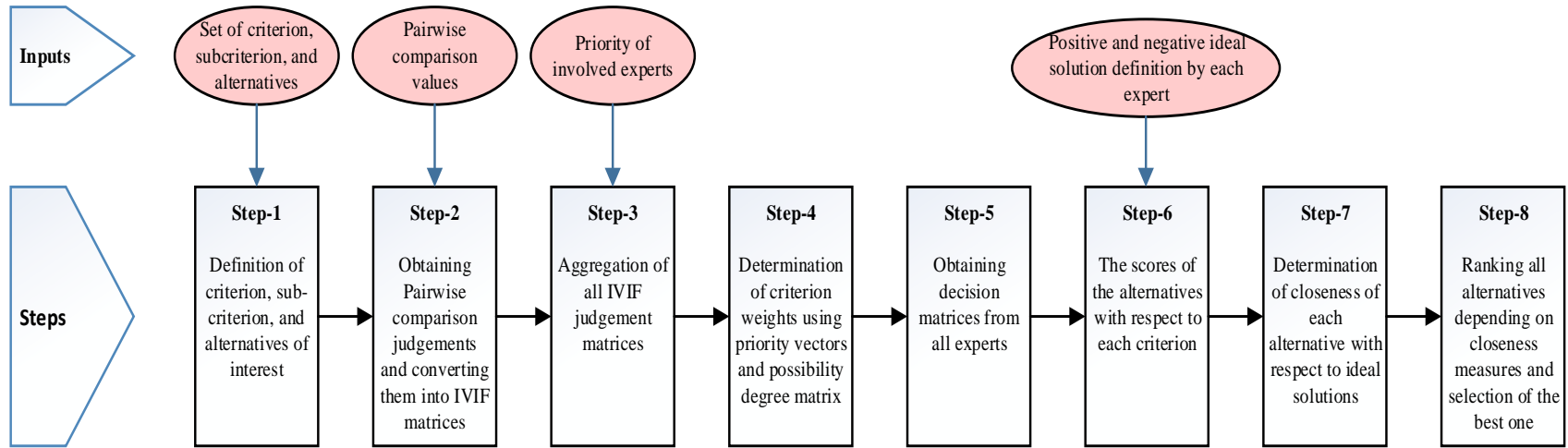


Figure 1.2 Methodology flow diagram with inputs

1.3 Interval Valued Intuitionistic Fuzzy (IVIF) Multi-Criterion Decision Making Model

In this section, we introduce a robust hierarchical group decision-making model that can be applied to any complex multi-criteria decision-making problem. Oztaysi et al. (2017b) first integrate the concept of *interval-valued intuitionistic fuzzy* (IVIF) preference relations along with the IVIF-TOPSIS decision-making method. The methodology begins with pairwise comparisons between different criteria and alternatives, followed by several sequential steps for obtaining the final set of decisions with the overall weight of different alternatives. For creating a better understanding of the method, in this section, we provided some basic definitions of IVIF sets and relevant calculations of IVIFS along with step by step illustration of procedures for applying IVIF sets in the multi-criterion decision-making process. Interested readers are suggested to follow the article Oztaysi et al. (2017b) for a better understanding of IVIF numbers and their fundamental calculations.

1.3.1 Definition 3.1: Interval Valued Intuitionistic Fuzzy Numbers

Interval-valued inferential fuzzy set I over the universe of discourse X can be defined as $I = \{ \langle x, \mu_I(x), \nu_I(x) \rangle \mid x \in X \}$, where $C \subseteq [0, 1]$ is the set of all closed subintervals of intervals, where $C \subseteq [0, 1]$ is the set of all closed intervals of the interval $\mu_I \longrightarrow C \subseteq [0, 1], \nu_I(x) \longrightarrow C \subseteq [0,1]$ with the conditions $0 \leq \sup \mu_i(x) + \sup \nu_i(x) \leq 1, \forall x \in X$.

1.3.2 Definition 3.2: Hesitancy degree of IVIF numbers

The *unknown degree* or *hesitancy degree* of IVIF numbers of $x \in X$ in $I =$

$([\mu_I^-, \mu_I^+], [v_I^-, v_I^+])$ can be represented as follows: $\pi_I(x) = 1 - \mu_I(x) - v_I(x) = (1 - \mu_I^+(x) - v_I^+(x), 1 - \mu_I^-(x) - v_I^-(x)) = (\pi_I^l(x), \pi_I^u(x))$. Here, $\mu_I(x) = [\mu_I^-, \mu_I^+]$ and $v_I(x) = [v_I^-, v_I^+]$.

1.3.3 Definition 3.3: Score function

For any IVIFN I , the *score function* is defined in equation (1).

$$S(I) = \frac{\mu_I^- + \mu_I^+ - v_I^- - v_I^+}{2} \quad (1.1)$$

1.3.4 Definition 3.4: Accuracy function

For any IVIFN I , the *accuracy function* is determined following equation (2).

$$H(I) = \frac{\mu_I^- + \mu_I^+ + v_I^- + v_I^+}{2} \quad (1.2)$$

The step by step procedure for conducting the proposed MCDA approach is described below:

Step 1: In the first step, we should define the MCDA problem of interest and represent it in a hierarchical structure with n criteria, sub-criterion (if any), and m decision alternatives. In addition, the experts' inputs should be collected, and pairwise comparison matrices should be formed following the *linguistic comparison* scale shown in Table 1 (Oztaysi et al., 2017a). Table 3 represents a simple form of data accumulation table having pairwise comparison values in it. This kind of table should be formed for all criteria and sub-criteria levels for every expert providing inputs to the study.

Table 1.1 Linguistic scale with corresponding IVIFS Oztaysi et al. (2017a)

Linguistic terms	IVIFS
Absolutely	$([0.10, 0.25],[0.65, 0.75])$
Low (AL)	
Very Low (VL)	$([0.15, 0.30],[0.60, 0.70])$
Low (L)	$([0.20, 0.35],[0.55, 0.65])$
Medium Low (ML)	$([0.25, 0.40],[0.50, 0.60])$
Exactly Equal (EE)	$([0.50, 0.50],[0.50, 0.50])$
Approximately Equal (AE)	$([0.45, 0.55],[0.30, 0.45])$
Medium High (MH)	$([0.50, 0.60],[0.25, 0.40])$
High (H)	$([0.55, 0.65],[0.20, 0.35])$
Very High (VH)	$([0.60, 0.70],[0.15, 0.30])$
Absolutely High (AH)	$([0.65, 0.75],[0.10, 0.25])$

Table 1.2 Example of a basic pairwise comparison matrix using linguistic scales

k^{th} Decision-maker	1 st criterion	2 nd criterion	3 rd criterion	...	n^{th} criterion
1 st criterion	EE	AL	MH		EE
2 nd criterion		EE	VH		MH
3 rd criterion			EE		ML
....					
n^{th} criterion					EE

Step 2: The linguistic data obtained in the previous step should be converted to their corresponding IVIFS following Table 1. This conversion will yield individual IVIF judgement matrices \tilde{P} for each decision-maker. Since this is a pairwise comparison matrix, the dimension of this matrix is $n \times n$. The definition of this IVIF judgement matrix $\tilde{P} = (\tilde{p}_{ij})_{n \times n}$ can be seen in equation (3) where $i(i = 1, 2, 3, \dots, n)$ and $j(j = 1, 2, 3, \dots, n)$ represents the criterion number.

$$\tilde{P} = \begin{bmatrix} ([\mu_{11}^-, \mu_{11}^+], [v_{11}^-, v_{11}^+]) & \dots & ([\mu_{1n}^-, \mu_{1n}^+], [v_{1n}^-, v_{1n}^+]) \\ \vdots & \ddots & \vdots \\ ([\mu_{n1}^-, \mu_{n1}^+], [v_{n1}^-, v_{n1}^+]) & \dots & ([\mu_{nn}^-, \mu_{nn}^+], [v_{nn}^-, v_{nn}^+]) \end{bmatrix} \quad (1.3)$$

Note that, the reciprocal value of any IVIF number $[\mu_{ij}^-, \mu_{ij}^+], [v_{ij}^-, v_{ij}^+]$ will be simply $[v_{ji}^-, v_{ji}^+], [\mu_{ji}^-, \mu_{ji}^+]$.

Step 3: After constructing all the pairwise matrices and using IVIFS for each expert, all the corresponding matrices are aggregated into a single judgement matrix \tilde{P}_g (equation (4)).

This aggregation is done by following equation (5) where $\tilde{\alpha}_i = \langle [\mu_i^-, \mu_i^+], [v_i^-, v_i^+] \rangle$ ($i = 1, 2, \dots, n$) is a collection of IVIF numbers and w represents the set of experts weights $w = (w_1, w_2, \dots, w_n)$ such that $w_i \in [0, 1], \sum_{i=1}^n w_i = 1$.

$$\tilde{P}_g = \begin{bmatrix} ([\mu_{g_{11}}^-, \mu_{g_{11}}^+], [v_{g_{11}}^-, v_{g_{11}}^+]) & \dots & ([\mu_{g_{1n}}^-, \mu_{g_{1n}}^+], [v_{g_{1n}}^-, v_{g_{1n}}^+]) \\ \vdots & \ddots & \vdots \\ ([\mu_{g_{n1}}^-, \mu_{g_{n1}}^+], [v_{g_{n1}}^-, v_{g_{n1}}^+]) & \dots & ([\mu_{g_{nn}}^-, \mu_{g_{nn}}^+], [v_{g_{nn}}^-, v_{g_{nn}}^+]) \end{bmatrix} \quad (1.4)$$

$$WA_w(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = \left\langle \left[1 - \prod_{i=1}^n (1 - \mu_i^-)^{w_i}, 1 - \prod_{i=1}^n (1 - \mu_i^+)^{w_i} \right], \left[\prod_{i=1}^n (v_i^-)^{w_i}, \prod_{i=1}^n (v_i^+)^{w_i} \right] \right\rangle \quad (1.5)$$

Step 4: In this step, the *score judgement matrix* $\tilde{S} = (\tilde{s}_{ij})_{n \times n}$ and *interval multiplicative matrix*

$\tilde{A} = (\tilde{a}_{ij})_{n \times n}$ should be constructed by using equations (6) and (7), respectively. The

values of this obtained matrix \tilde{A} are between 0 and 1.

$$\tilde{S} = \begin{bmatrix} [\mu_{g_{11}}^- - v_{g_{11}}^+, \mu_{g_{11}}^+ - v_{g_{11}}^-] & \dots & [\mu_{g_{1n}}^- - v_{g_{1n}}^+, \mu_{g_{1n}}^+ - v_{g_{1n}}^-] \\ \vdots & \ddots & \vdots \\ [\mu_{g_{n1}}^- - v_{g_{n1}}^+, \mu_{g_{n1}}^+ - v_{g_{n1}}^-] & \dots & [\mu_{g_{nn}}^- - v_{g_{nn}}^+, \mu_{g_{nn}}^+ - v_{g_{nn}}^-] \end{bmatrix} \quad (1.6)$$

$$\tilde{A} = \begin{bmatrix} [e^{(\mu_{g_{11}}^- - v_{g_{11}}^+)}, e^{(\mu_{g_{11}}^+ - v_{g_{11}}^-)}] & \dots & [e^{(\mu_{g_{1n}}^- - v_{g_{1n}}^+)}, e^{(\mu_{g_{1n}}^+ - v_{g_{1n}}^-)}] \\ \vdots & \ddots & \vdots \\ [e^{(\mu_{g_{n1}}^- - v_{g_{n1}}^+)}, e^{(\mu_{g_{n1}}^+ - v_{g_{n1}}^-)}] & \dots & [e^{(\mu_{g_{nn}}^- - v_{g_{nn}}^+)}, e^{(\mu_{g_{nn}}^+ - v_{g_{nn}}^-)}] \end{bmatrix} = \begin{bmatrix} [\tilde{a}_{11}^-, \tilde{a}_{11}^+] & \dots & [\tilde{a}_{1n}^-, \tilde{a}_{1n}^+] \\ \vdots & \ddots & \vdots \\ [\tilde{a}_{n1}^-, \tilde{a}_{n1}^+] & \dots & [\tilde{a}_{nn}^-, \tilde{a}_{nn}^+] \end{bmatrix} \quad (1.7)$$

Step 5: After computing the interval multiplicative matrix \tilde{A} , the priority vector of this exponential matrix can be calculated as follows.

$$w_i = \left[\frac{\sum_{j=1}^n \tilde{a}_{ij}^-}{\sum_{i=1}^n \sum_{j=1}^n \tilde{a}_{ij}^+}, \frac{\sum_{j=1}^n \tilde{a}_{ij}^+}{\sum_{i=1}^n \sum_{j=1}^n \tilde{a}_{ij}^-} \right] = [w_i^-, w_i^+], i = 1, 2, \dots, n \quad (1.8)$$

Step 6: In this step, the weights \tilde{w}_i obtained in **Step 5** should be compared, and the possibility degree matrix is obtained following equations (9) and (10), respectively.

$$\psi(w_i > w_j) = \psi_{ij} = \frac{\max(0, w_i^+ - w_j^-) - \max(0, w_i^- - w_j^+)}{(w_i^+ - w_i^-) + (w_j^+ - w_j^-)} \quad (1.9)$$

$$\psi(w_j > w_i) = \psi_{ji} = \frac{\max(0, w_j^+ - w_i^-) - \max(0, w_j^- - w_i^+)}{(w_i^+ - w_i^-) + (w_j^- - w_j^+)} \quad (1.10)$$

where $\psi_{ij} \geq 0, \psi_{ij} + \psi_{ji} = 1, \psi_{ii} = 1/2$. The possibility matrix $\psi = (\psi_{ij})_{n \times n}$ prioritization is done according to (11).

$$w_i = \frac{\sum_{j=1}^n \psi_{ij} - 1}{n} + 0.5 \quad (1.11)$$

Step 7: Next, the vector $\{w_i\}; \forall i \in n$ comprising of weights for criteria and sub-criterion are normalized, and this normalized matrix is obtained by the following equation.

$$w_i^T = \frac{w_i}{\sum_{i=1}^n w_i} \quad (1.12)$$

Step 8: The weight calculation should be done for all criteria and sub-criterion of the hierarchy following **Steps 1 to 7**. After performing this step, we should have all the weights for criteria and sub-criteria listed as vectors which will be used in the alternative evaluation.

Step 9: From this step, we should start the incorporation of the IVIF-TOPSIS method into our model. First, we obtain the decision matrix $[\tilde{D}_k]$ from each decision-maker using Table 1 as a basis for comparison. Equation (13) depicts the anatomy of the decision matrix $[\tilde{D}_k]$ where n and m stands for the number of criteria ($i = 1, 2, \dots, n$) and the number of alternatives ($j = 1, 2, \dots, m$), respectively. The overall IVIF-TOPSIS method is elaborated in **Step 10**.

$$\tilde{D}_k = \begin{matrix} & \begin{matrix} A_1 & A_2 & \dots & A_m \end{matrix} \\ \begin{matrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{matrix} & \begin{pmatrix} (\mu_{11k}^-, \mu_{11k}^+) [v_{11k}^-, v_{11k}^+] \\ (\mu_{21k}^-, \mu_{21k}^+) [v_{21k}^-, v_{21k}^+] \\ \vdots \\ (\mu_{n1k}^-, \mu_{n1k}^+) [v_{n1k}^-, v_{n1k}^+] \end{pmatrix} & \begin{pmatrix} (\mu_{12k}^-, \mu_{12k}^+) [v_{12k}^-, v_{12k}^+] \\ (\mu_{22k}^-, \mu_{22k}^+) [v_{22k}^-, v_{22k}^+] \\ \vdots \\ (\mu_{n2k}^-, \mu_{n2k}^+) [v_{n2k}^-, v_{n2k}^+] \end{pmatrix} & \dots & \begin{pmatrix} (\mu_{1mk}^-, \mu_{1mk}^+) [v_{1mk}^-, v_{1mk}^+] \\ (\mu_{2mk}^-, \mu_{2mk}^+) [v_{2mk}^-, v_{2mk}^+] \\ \vdots \\ (\mu_{nmk}^-, \mu_{nmk}^+) [v_{nmk}^-, v_{nmk}^+] \end{pmatrix} \end{matrix} \quad (1.13)$$

Here, C_1, C_2, \dots, C_n represents the *weight vector* of element n .

Step 10: The TOPSIS method requires the definition of positive and negative ideal solutions for each criterion, and these inputs are obtained from experts. The *fuzzy positive ideal solution* $I\tilde{S}_k^+$ and *fuzzy negative ideal solution* $I\tilde{S}_k^-$ for k^{th} decision-maker are obtained from equations (14) and (15), respectively, along with the score function (1) and accuracy functions (2) as appropriate.

$$I\tilde{S}_k^+ = (([\mu_{1^*k}^-, \mu_{1^*k}^+], [v_{1^*k}^-, v_{1^*k}^+]), ([\mu_{2^*k}^-, \mu_{2^*k}^+], [v_{2^*k}^-, v_{2^*k}^+]), \dots, ([\mu_{n^*k}^-, \mu_{n^*k}^+], [v_{n^*k}^-, v_{n^*k}^+])) \quad (1.14)$$

$$I\tilde{S}_k^- = (([\mu_{1-k}^-, \mu_{1-k}^+], [v_{1-k}^-, v_{1-k}^+]), ([\mu_{2-k}^-, \mu_{2-k}^+], [v_{2-k}^-, v_{2-k}^+]), \dots, ([\mu_{n-k}^-, \mu_{n-k}^+], [v_{n-k}^-, v_{n-k}^+])) \quad (1.15)$$

Here $([\mu_{1^*k}^-, \mu_{1^*k}^+], [v_{1^*k}^-, v_{1^*k}^+])$ and $([\mu_{1-k}^-, \mu_{1-k}^+], [v_{1-k}^-, v_{1-k}^+])$ are representatives of maximum and minimum intuitionistic fuzzy sets, respectively, among the alternatives' values for i^{th} criterion. Next, we obtain the separation measures between j^{th} alternative and $I\tilde{S}_k^+$ and $I\tilde{S}_k^-$ for each decision-maker as follows.

$$D_j^{*k} = \sqrt{\frac{1}{2} \sum_{i=1}^n w_i^T \left[(\mu_{ijk}^- - \mu_{1^*k}^-)^2 + (\mu_{ijk}^+ - \mu_{1^*k}^+)^2 + (v_{ijk}^- - v_{1^*k}^-)^2 + (v_{ijk}^+ - v_{1^*k}^+)^2 \right.} \\ \left. + (\pi_{ijk}^l - \pi_{i^*k}^l)^2 + (\pi_{ijk}^u - \pi_{i^*k}^u)^2 \right]} \quad (1.16)$$

$$D_j^{-k} = \sqrt{\frac{1}{2} \sum_{i=1}^n w_i^T \left[(\mu_{ijk}^- - \mu_{1-k}^-)^2 + (\mu_{ijk}^+ - \mu_{1-k}^+)^2 + (v_{ijk}^- - v_{1-k}^-)^2 + (v_{ijk}^+ - v_{1-k}^+)^2 \right.} \\ \left. + (\pi_{ijk}^l - \pi_{i-k}^l)^2 + (\pi_{ijk}^u - \pi_{i-k}^u)^2 \right]} \quad (1.17)$$

Where π_{ijk}^l, π_{ijk}^u and $\pi_{i^*k}^l, \pi_{i^*k}^u$ are calculated following hesitancy degree equation (**Definition 3.2**). Finally, we aggregate the separation measures following (18) and (19). Equation (20) provides the overall *closeness co-efficient* for all alternatives. The values of this closeness coefficient U_j dictate the preference order of alternatives where the larger value means more preference. The final ranking of alternatives is conducted based on this closeness coefficient U_j .

$$D_j^* = \sum_{k=1}^k (\lambda_k D_j^{*k}) \quad (1.18)$$

$$D_j^- = \sum_{k=1}^k (\lambda_k D_j^{-k}) \quad (1.19)$$

$$U_j = \frac{D_j^-}{D_j^- + D_j^+} \quad (1.20)$$

1.4 Case Study

This paper presents a well-structured case study introducing a real-life problem experienced by logistics companies while deploying drones for last-mile delivery. As companies pursue the decision to deploy drones for last-mile product delivery, they need to select one or multiple drone solutions depending upon requirements among a wide variety of available drones with different capability levels and diversified feature sets. After thoroughly reviewing characteristics of drone technologies available for purchase at the time of this study, four device options are designated: *Drone A* (small quadcopter), *Drone B* (larger quadcopter), *Drone C* (tilt-wing), and *Drone D* (fixed wing), which are representative of actual delivery drones having different levels of specified device characteristics. For each of these drones, six major features - dimension, drone type, control panel, payload capacity, delivery distance, and flight time - are considered as distinguishing factors. Each category has different combinations of these characteristics to aid in the proof of concept of the proposed decisions tool selection. The selected drone types are familiar to many regions of the world, and the pictures of these drones (Figure 3) are chosen based on these common characteristic combinations strictly for visualization purposes. First, the input parameters of this case study are discussed, showing the summary of the attributes of these selected drone solutions (Table 3)¹. Next, we apply the IVIF TOPSIS method, a robust MCDA methodology, to obtain the base case results. By analyzing different real-life delivery

¹ Please note that the characteristics of the drones provided in Table 3 are not representative of the actual drones depicted in the Figure 3. Pictures represent the functional intent of the solution while the information in Table 3 are used to provide varying combinations of characteristics of available solutions in order to demonstrate how the identified criterion weight certain solutions differently given the context of real-world use cases

requirements, multiple sensitivity analyses generating different realistic scenarios are conducted. In total, four discrete scenarios are generated: (i) stationary deployment urban canyon drone delivery, (ii) truck-based deployment urban canyon drone delivery, (iii) truck-based deployment rural area drone delivery, and (iv) drone delivery in a disaster affected area. These sensitivity analyses results reveal different key managerial insights that can be used to provide decision-makers with a general baseline to make drone selection depending on different use cases.



(a) Drone A



(b) Drone B



(c) Drone C



(d) Drone D

Figure 1.3 Pictures of studied drones

1.4.1 Base Case Result

In the base case, the robust IVIF TOPSIS methodology is applied to select the best drone option for the last mile delivery. The generalized criterion list for the last mile delivery drone selection, discussed in Section 2, is considered as a baseline for this research. The proposed criterion list contains 5 hierarchical levels with 5 main criteria and 28 sub-criteria. Following the IVIF methodology, we construct the pairwise comparison matrices (PCM's) for each

hierarchical level in (Figure 1) and perform necessary calculations. For this case study, the pairwise comparison matrices are provided by three Subject Matter Experts (SME's) who ranked the criteria separately using their practical judgement and relevant experience on drone research. The PCM's start with comparisons between the main criteria. Later, the PCM's are generated for each sub-criterion. Therefore, for the base case, each decision-maker provided 6 PCM's using the linguistic scale. Table 4 provides an example of the PCM for the main criterion level given by the first expert. Each of the three experts provided PCM's for the 5 main criteria. The linguistic data obtained in the previous table are converted to their corresponding IVIF's using Table 1. This conversion yields individual IVIF judgement matrices, as shown in Table 5.

Table 1.3 Specifications of the selected drones

	Drone A	Drone B	Drone C	Drone D
Type	Quadcopter	Quadcopter	Tilt wing	Tail sitter
Pay Load	Up to 1.2 kg	Up to 1.2 kg	Up to 2 kg	Up to 1.4 kg
Delivery Distance	1 km	12 km	8.3 km	5 km
Flight time	15 min	15 min	8-9 min	9-10 min
Control	Manual	Automatic	Automatic	Automatic
Dimension	1030 mm	1030 mm	2200 mm	1500

Table 1.4 Pairwise Comparison Matrix

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>
Physical Specification (<i>A</i>)	EE	H	AE	MH	ML
Economic (<i>B</i>)		EE	AE	MH	ML
Performance (<i>C</i>)			EE	H	MH
Environmental (<i>D</i>)				EE	L
Payload Capacity (<i>E</i>)					EE

After obtaining the individual judgement matrix for each SME, we aggregate them to get an aggregated judgment matrix following equation (4). Later, we use equations (5)-(7) to obtain an interval multiplicative matrix. Table 6 represents the interval multiplicative matrix. After computing the interval multiplicative matrix, the priority vector is calculated using equation (8). These weights are then compared, and the possibility degree matrix is calculated using equations (9) and (10). Following this, the prioritization is conducted using equation (11). Finally, the weights are normalized using equation (12), and the resultant normalized matrix is presented in Table 7. The same procedure is repeated for obtaining the final rankings for the sub-criteria as well. The overall aggregated rankings for criterion and sub-criterion are presented in Table 8.

Table 1.5 IVIF Judgement Matrix

	Physical Spec (A)				Economic (B)				Performance (C)				Environmental (D)				Payload Capacity (E)			
Physical Specification (A)	0.50	0.50	0.50	0.50	0.55	0.65	0.20	0.35	0.45	0.55	0.30	0.45	0.50	0.60	0.25	0.40	0.25	0.40	0.50	0.60
Economic (B)					0.50	0.50	0.50	0.50	0.45	0.55	0.30	0.45	0.50	0.60	0.25	0.40	0.25	0.40	0.50	0.60
Performance (C)									0.50	0.50	0.50	0.50	0.55	0.65	0.20	0.35	0.50	0.60	0.25	0.40
Environmental (D)													0.50	0.50	0.50	0.50	0.20	0.35	0.55	0.65
Payload Capacity (E)																	0.50	0.50	0.50	0.50

Table 1.6 Interval Exponential Matrix

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>
<i>A</i>	[1, 1]	[1.314, 1.696]	[1, 1.284]	[1.188, 1.531]	[0.619, 0.795]
<i>B</i>	[0.590, 0.761]	[1, 1]	[1, 1.284]	[1.188, 1.531]	[0.661, 0.850]
<i>C</i>	[0.779, 1]	[0.779, 1]	[1, 1]	[1.314, 1.696]	[1.270, 1.636]
<i>D</i>	[0.653, 0.842]	[0.653, 0.842]	[0.590, 0.761]	[1, 1]	[0.639, 0.821]
<i>E</i>	[1.258, 1.616]	[1.177, 1.512]	[0.611, 0.787]	[1.218, 1.565]	[1, 1]

Table 1.7 Normalized weight Matrix

Main criteria		Weight
Physical Spec	<i>A</i>	0.231
Economic	<i>B</i>	0.186
Performance	<i>C</i>	0.232
Environmental	<i>D</i>	0.112
Payload Capacity	<i>E</i>	0.239

Table 1.8 Base case final rankings for criterion and sub-criterion

Main Criteria	Weight	Sub Criteria	Weight
Physical Spec	0.231	UAS Overall Size (A_1)	0.365
		Weight (A_2)	0.275
		Drone Type (A_3)	0.196
		Fuel Type (A_4)	0.164
Economic	0.186	Internal Computing Components (B_1)	0.237
		Location Accuracy (B_2)	0.270
		Communication and Data Quality (B_3)	0.120
		Traceability (B_4)	0.198
		Reliability (B_5)	0.175
Performance	0.232	Repair Cost (C_1)	0.212
		Total Unit Cost (C_2)	0.116
		Total Lifecycle Cost (C_3)	0.205
		Operating Cost (C_4)	0.243
		Training Cost (Ec) (C_5)	0.224
Environmental	0.112	Adaptability to Extreme Weather Conditions (D_1)	0.375 0.200
		Environmental Impact (D_2)	0.161
		Recharge (D_3)	0.264
		Required Delivery Distance (D_4)	

Table 1.8 (continued)

Main Criteria	Weight	Sub Criteria	Weight
Payload	0.239	Max Flight Time Without Recharge (E_1)	0.121
Capacity		Total Recharge (E_2)	0.039
		Charge/ Fuel Usage Rate (E_3)	0.035
		Maximum Load (E_4)	0.243
		Maximum Carry Dimensions (E_5)	0.093
		Maximum Reachable Altitude (E_6)	0.093
		Drone Speed (E_7)	0.057
		Adaptability to Dynamic Assignment (E_8)	0.068
		Packaging Handling Flexibility (E_9)	0.169
		Delivery Flexibility (E_{10})	0.082

Table 1.9 Decision matrix for the alternatives and definition of *fuzzy positive ideal solution* and *fuzzy negative ideal solution*

Criterion	Drone A	Drone B	Drone C	Drone D	IS	IS2	Weight
A_1	MH	VH	H	MH	[0.60,0.70],[0.15,0.30]	[0.50, 0.60],[0.25, 0.40]	0.084
A_2	AE	H	MH	AE	[0.55,0.65],[0.20, 0.35]	[0.50, 0.50],[0.50, 0.50]	0.064
A_3	H	H	MH	AE	[0.55,0.65],[0.20, 0.35]	[0.50, 0.50],[0.50, 0.50]	0.045
A_4	H	H	H	ML	[0.55,0.65],[0.20, 0.35]	[0.25, 0.40],[0.50, 0.60]	0.038
B_1	L	H	AE	H	[0.55,0.65],[0.20, 0.35]	[0.20, 0.35],[0.55, 0.65]	0.044
B_2	L	H	MH	VH	[0.60,0.70],[0.15,0.30]	[0.20, 0.35],[0.55, 0.65]	0.050

Table 1.9 (continued)

Criterion	Drone A	Drone B	Drone C	Drone D	IS	IS2	Weight
B_3	L	H	H	MH	[0.55,0.65], [0.20, 0.35]	[0.20, 0.35], [0.55, 0.65]	0.022
B_4	AE	H	H	MH	[0.55,0.65], [0.20, 0.35]	[0.50, 0.50], [0.50, 0.50]	0.037
B_5	AE	H	H	MH	[0.55,0.65], [0.20, 0.35]	[0.50, 0.50], [0.50, 0.50]	0.033
C_1	H	VL	AE	VL	[0.55,0.65], [0.20, 0.35]	[0.15, 0.30], [0.60, 0.70]	0.049
C_2	VH	VL	L	AL	[0.60,0.70], [0.15,0.30]	[0.15, 0.30], [0.60, 0.70]	0.027
C_3	H	L	ML	VL	[0.55,0.65], [0.20, 0.35]	[0.15, 0.30], [0.60, 0.70]	0.048
C_4	VH	H	ML	AE	[0.60,0.70], [0.15,0.30]	[0.25, 0.40], [0.50, 0.60]	0.056

Table 1.9 (continued)

C_5	VH	H	L	ML	[0.60,0.70], [0.15,0.30]	[0.20, 0.35], [0.55, 0.65]	0.052
D_1	ML	VH	H	MH	[0.60,0.70], [0.15,0.30]	[0.25, 0.40], [0.50, 0.60]	0.042
D_2	VH	H	H	VL	[0.60,0.70], [0.15,0.30]	[0.15, 0.30], [0.60, 0.70]	0.023
D_3	VH	VH	ML	H	[0.60,0.70], [0.15,0.30]	[0.25, 0.40], [0.50, 0.60]	0.018
D_4	VL	VH	VH	AH	[0.65,0.75], [0.10,0.25]	[0.15, 0.30], [0.60, 0.70]	0.030
E_1	AL	H	H	VH	[0.60,0.70], [0.15,0.30]	[0.10, 0.25], [0.65, 0.75]	0.029
E_2	MH	AE	AE	AH	[0.65,0.75], [0.10,0.25]	[0.50, 0.50], [0.50, 0.50]	0.009

Table 1.9 (continued)

Criterion	Drone A	Drone B	Drone C	Drone D	IS	IS2	Weight
E_3	H	MH	AE	ML	[0.55,0.65], [0.20, 0.35]	[0.25, 0.40], [0.50, 0.60]	0.009
E_4	VL	L	AE	VH	[0.60,0.70], [0.15,0.30]	[0.15, 0.30], [0.60, 0.70]	0.058
E_5	VL	L	MH	VH	[0.60,0.70], [0.15,0.30]	[0.15, 0.30], [0.60, 0.70]	0.022
E_6	L	MH	AH	AH	[0.65,0.75], [0.10,0.25]	[0.20, 0.35], [0.55, 0.65]	0.022
E_7	ML	ML	VH	H	[0.60,0.70], [0.15,0.30]	[0.25, 0.40], [0.50, 0.60]	0.014
E_8	VL	VH	MH	AE	[0.60,0.70], [0.15,0.30]	[0.15, 0.30], [0.60, 0.70]	0.016
E_9	AE	MH	ML	VII	[0.60,0.70], [0.15,0.30]	[0.25, 0.40], [0.50, 0.60]	0.040

Table 1.9 (continued)

Criterion	Drone A	Drone B	Drone C	Drone D	IS	IS2	Weight
E_{10}	AE	MH	AE	VL	[0.50, 0.60], [0.25, 0.40]	[0.15, 0.30], [0.60, 0.70]	0.020

After obtaining the overall aggregated rankings for criterion and sub-criterion, decision matrices are collected for all experts. These matrices are generated following equation (13) where all four drone options are compared using the linguistic scale provided in Table 1. One example decision matrix obtained from the first decision-maker is shown in Table 9. Additionally, following Step 10, we obtain the *fuzzy positive ideal solution* ($\tilde{I}S^+$) and *fuzzy negative ideal solution* $\tilde{I}S^-$ for each criterion and list them in two separate columns in Table 9. The separation measure between the j^{th} alternative and $\tilde{I}S^+$ for k^{th} decision-maker (D_j^{*k}) is calculated using equation (16). Similarly, we calculate (D_j^{-k}) using equation (17) and the separation measures obtained from all three decision-makers are listed in Table 10. Note that we used the weights in Table 9 to calculate the separation measures for the alternatives. Now, we use Table 10 information in (18)-(20) to obtain the final ranking of the alternatives (base case result). The summary of the base case results is presented in Figure 4. The results demonstrate that Drone B is a preferred selection under base case settings. However, several other drones also received high weights and, in fact, very close to the weight of Drone B (e.g., Drones C and D). This is because, in an ideal case, many different types of drones can be used to perform similar tasks (i.e., last-mile delivery tasks). Note that this result would be more specific under different specialized drone deployment scenarios which are discussed in detail in the following subsections.

Table 1.10 Final separation measures from all decision-makers

Separation measures	Drone A	Drone B	Drone C	Drone D
D_j^{*1}	0.352	0.231	0.259	0.257
D_j^{-1}	0.303	0.339	0.312	0.352
D_j^{*2}	0.364	0.234	0.248	0.262
D_j^{-2}	0.312	0.341	0.324	0.347
D_j^{*3}	0.367	0.226	0.245	0.257
D_j^{-3}	0.310	0.348	0.325	0.344

Drone	Weight	Base Ranking
Drone A	0.461	4
Drone B	0.597	1*
Drone C	0.561	3
Drone D	0.573	2

*Preferred Drone



Figure 1.4 Base case ranking

1.4.2 Sensitivity Analysis

This subsection identifies how drone selection decisions are impacted under different possible yet realistic scenarios and draws key managerial insights. The first scenario represents the last mile drone delivery case in an urban area where a drone collects the parcels from a base station and delivers them to the destination. The next scenario is almost similar to the first one; however, in this case, the drone collects parcels from a delivery truck that is already out for delivery. The

third scenario is similar to the second one in the sense that the drone deploys from a delivery vehicle but takes place in rural areas where the distribution of customers is generally less than that of urban areas. The last scenario investigates the appropriate drone option to be selected in the event of possible disruptions, such as a natural disaster event that would limit accessibility to a group of people in need. Under this scenario, it is challenging to utilize the routing truck for drone delivery since as the infrastructure of the disaster impacted area might totally be collapsed. For example, hurricane damage may limit road access needed to get time-sensitive medicine to disaster survivors. Therefore, the only delivery possibility in this scenario is with a stationary-based drone that collects parcels from a base and delivers them to the disaster-affected areas. Following these scenarios, the IVIF TOPSIS methodology is applied to obtain the best possible drone options in different scenarios. Note that decision-makers' preference or criterion-wise choice of alternatives depends significantly on the scenario under consideration. The input decision matrix obtained from the first decision-maker for each of these four scenarios is listed in Tables 11-14 in **Appendix A**. The next few subsections detail the rationale behind investigating these scenarios and report the results obtained from the performed analysis.

1.4.2.1 Scenario 1: Stationary Urban Canyon Scenario

According to the most recent census by the United States Census Bureau, about 80.7% of the total population of the United States resides in urban areas (United States Census Bureau., 2018). The number of customers per unit area is also very dense in this type of environment, requiring more delivery activities per square mile than in most other demographic region types. Hence, the drone solution for this scenario has to climb higher altitudes to cover the customers' locations for delivering parcels. A drone has to carry the parcel from the base station, deliver the product/parcel, and then come back to the base station. Therefore, the base station should be

located somewhere in the center of the urban area such that the drones can cover the customers at the urban area borders as well as ones in the more densely populated central regions. The results of this scenario can be seen in Figure 5. Similar to the base case, in this scenario, Drone B shows the highest preference ranking, among others. Drone A can also be used, but drone type D will not be a viable option in this specific case.

Drone	Weight	New	Base
Drone A	0.597	2	4
Drone B	0.693	1*	1
Drone C	0.532	3	3
Drone D	0.456	4	2
	*Preferred Drone		




Figure 1.5 Final ranking of the last mile delivery drones under scenario 1

1.4.2.2 Scenario 2: Truck Based Urban Canyon Scenario

The next scenario simulates a situation within a large urban area where maintaining quick deliveries would be difficult if the base station is located at the center of the urban canyon. In this case, the drone would be operated from one or multiple delivery trucks moving in predefined routes while carrying the parcels to be delivered. The trucks themselves are considered to be the delivery bases/hubs. Whenever the truck approaches a delivery destination, the drone is used to carry the parcel, delivering them to the destination. After the delivery, the drone returns to the base truck and prepares for the next delivery. This method is more effective because a drone does not need to visit the actual base station every time for package delivery; instead, it can collect packages from the truck and perform the last-mile delivery. The results (shown in Figure 6) for this scenario identify drone type B as the top preference. This is because, considering the delivery type, small

drones such as Drone B would be more effective as they are designed to fly short distances. Drone type A is also small, and it also shows convincing results compared to Drone type C and D. But, Drone A did not get the highest preference due to having a manual control system. To be specific, Drone type B is the most effective option for this scenario.

Drone	Weight	New	Base
Drone A	0.623	2	4
Drone B	0.774	1*	1
Drone C	0.545	3	3
Drone D	0.432	4	2
*Preferred Drone			



Figure 1.6 Final rankings of the last mile delivery drones under scenario 2

1.4.2.3 Scenario 3: Truck Based Rural Area

This scenario shifts focus from urban to rural areas. In rural areas, houses are more sparsely located; hence, the delivery locations are more scattered than those of urban areas. Therefore, collecting packages from a base station and delivering them to the destination becomes less practical than in an urban setting. As with the last scenario, in this use case, the delivery process to be controlled from a truck base is considered. The results for this scenario (see Figure 7) show that type C and D drones outperform types A and B. Between type C and D, type C shows slightly better performance. The result is realistic considering that rural areas are not always well developed, and the routing optimization might put the truck too far from the actual delivery destination requiring bigger drones to be deployed. Type C and D drones are comparatively bigger in size, having better range hence obtaining better results for this scenario.

1.4.2.4 Scenario 4: Stationary Based Disaster Affected Area Drone Delivery

The last scenario considers disaster-affected areas as potential delivery locations. The infrastructures for the areas where time-sensitive deliveries need to be made are most likely to be damaged, thereby making standard road-based transportation impossible. Therefore, the last mile delivery drone can play a useful role in these areas to enable immediate delivery. This scenario is analyzed with the proposed IVIF TOPSIS technique. The results (shown in Figure 8) suggest that drone type D seems to be the most appropriate option under this specific scenario. Drone D has higher capacity and longer flight distance which are critical for this use case. On the contrary, type A drone demonstrates the least viable option primarily due to the lower flying range, poor altitude adjustment capacity, and extensive need for manual control.

Drone	Weight	New	Base
Drone A	0.376	4	4
Drone B	0.494	3	1
Drone C	0.644	1*	3
Drone D	0.619	2	2

*Preferred Drone




Figure 1.7 Final rankings of the last mile delivery drones under scenario 3

Drone	Weight	New	Base
Drone A	0.099	4	4
Drone B	0.357	3	1
Drone C	0.629	2	3
Drone D	0.906	1*	2

*Preferred Drone




Figure 1.8 Final rankings of the last mile delivery drones under scenario 4

1.5 Conclusion

This paper proposes a standardized methodology for selecting last-mile delivery drones. A comprehensive list of criteria, which will be applicable for comparing between any set of last-mile delivery drones, are proposed. A systematic multi-criteria, multi-personnel decision-making approach, Interval Valued Inferential Fuzzy TOPSIS method is proposed to perform this analysis. The proposed methodology can handle the fuzziness in decision making, remove judgmental biases, and provide the best result depending on the use cases. A simple demonstration of the decision-making model, designing a real-life case study, is demonstrated. Further, multiple sensitivity analyses are performed, and the results from these analyses provide insightful grounds for the decision-makers involved in this process. Overall, these analyses provide a clear idea about which type of drones might be more applicable in different use cases based on different situations and delivery needs. This work can be extended in different directions. The demographic impact and cultural differences in drone selection based on specific drone deployment situations can be addressed in the future. Besides, more detailed drone deployment studies can be conducted considering different types of disruption (natural and artificial) situations. Further, to handle the uncertainty in decision making different types of fuzzy sets can be employed, such as hesitant fuzzy sets, non-stationary fuzzy sets, fuzzy multisets. The usability and competitive benefits of using different types of fuzzy sets in terms of robust decision-making are another essential elements requiring crucial investigation. We will address these issues in future studies.

CHAPTER II

MULTI-CRITERIA DECISION FOR SCHOOLS SHOOTING SOLUTION

2.1 Introduction

During the post-coronavirus period, there has been a change in most aspects of social life. The school shootings seem to take pause judging by the latest events that they remind us have not gone away. With various school shootings, the issue has again taken the consciousness of the public (Daniels et al., 2010). When school shootings happen, attention is placed on the likely causes and prospective solutions that might assist the country in addressing the problem.

Considering the concern regarding the school shootings and their sudden increase, the policymakers are searching for answers on why such events happen and how to prevent them. In the 1990s, the federal government commenced working on the assessments of school shooters and ways of preventing them (Lawrence & Birkland, 2004). Much planning has often happened before the school shooting, and the shooters often showed concerning behaviors beforehand. However, there is a lack of a profile that would sufficiently define the spectrum of the school shooters.

Solution for school shootings has become a significant point of research in recent years. The stakeholders in the education sector have been researching their effectiveness in preventing a school shooting. They have begun exploring how to impede school shootings. For instance, school-based programs have been introduced to reduce the aggression of learners. The U.S. Department of Education has been instrumental in brainstorming strategies to get rid of the school shooting. Many countries, especially the United States, have explored different criteria for choosing the

appropriate solution for school shootings. Whereas the ideation of and execution with school shootings might be rising in popularity, the selection of the best school shooting solution is constrained by various factors (Arcus, 2002). Choosing the appropriate school shooting solution necessitates an in-depth exploration to make the process of selection more structured.

With the rise of social issues, there is a need for prompt prevention of school shootings (Gerard et al., 2016). In the previous years, various studies have been carried out to test the feasibility of using school shooting solutions to remove the vice in society. It is challenging to select suitable school shooting solutions. Some school shooting solutions are effective, and others are less effective. Thus, it is vital to have a structured methodology for school shooting solutions selection. However, there is a lack of research that has been carried out to offer a standard, well-structured methodology to perform school shooting solutions selection that would be suitable for any academic institution. To fill this gap, this paper suggests a standardized method to choose the school shooting solutions option among the list of prospective school shooting solutions that would be suitable for any academic institution.

To address the literature gap, this exploration suggests a comprehensive methodology for choosing school shooting solutions. First, a comprehensive list of criteria requires to be developed to describe the solution for a school shooting. Then, a multi-criterion decision-making approach, namely, Interval Valued Inferential Fuzzy (IVIF) TOPSIS method, is suggested to choose a solution from the list of alternatives. The school shooting problem scenarios are made to derive managerial insights and to show the application of the proposed methodology. The contradicting attributes are the building blocks of the school shooting selection problem. The decision-makers offer linguistic evaluation, which can be converted to their numerical value and used in the MCDM technique via the fuzzy set theory. The MCDM methods, including the fuzzy TOPSIS, fuzzy

VIKOR, and fuzzy AHP, use the normal fuzzy sets. The extension of frequent fuzzy sets includes the type-2 fuzzy sets and intuitionistic fuzzy sets. There was the use Interval Valued Inferential Fuzzy (IVIF) TOPSIS technique for school shooting solutions selection problem. The method utilizes the IVIF sets to offer a more robust description of membership functions that can address the judgmental biases.

The paper is broken down into different chapters. Chapter 2 offers the description of the school shootings solutions selection problem and suggests the important factors that can significantly impact the process of selection. Section 2.2 familiarizes the audience with the IVIF TOPSIS decision-making model by indicating the steps of the model. In section 2.3 , a real-life case study indicating the distinct deployment of school shooting solutions scenarios is presented that shows the appropriateness of using the distinct school shooting solutions under the different deployment scenarios whereas disclosing the vital managerial insights of these cases. In section 5, there is a summarization and presentation of the further research.

2.2 Problem Statement And Criterion Selection

From the perspective of school administration, school shooting solutions offer an option for handling social vices in academic institutions. The loss of innocent lives among students is forcing the stakeholders in the education sector to consider deploying the solutions to addressing the social vice. U.S. Department of Education has recommended the many variants of addressing school shootings. These school shooting solutions are defined by distinguishing characteristics that can address the social vice in academic institutions. Thus, choosing a suitable school shooting solution is crucial for its efficiency. The choosing of effective school shooting solutions relies on various factors. The paper recommended a list of the main factors that influence the school shooting solutions selection. A total of twenty-eight sub-criteria have been identified and

categorized under the main criteria, namely, (i) physical specification, (ii) economic, (iii) performance, (iv) environmental, and (v) payload capacity. Notably, the criteria are sourced from literature. The in-depth descriptions for every sub-criterion are offered in this area. Figure indicates the hierarchical representation of sub-criterion under every criterion. Figure shows the methodology flow diagram for the application of the IVIF TOPSIS method.

2.3 Methodology

As mentioned in the beginning, a survey was conducted with a focus group. A group of well-educated stakeholders received a group of questions discussion. The survey that was conducted by asking a set of open-ended questions to a small group of the target population. This population was all stakeholders in the education. Some participants were teachers, paraeducators, principals, and parents. The small group is derived from the fact that the selected groups are “concentrated” on a given topic.

Thus, the selection criteria for participants in the group of survey takers include having domain knowledge on the topic, being within the age range, and being comfortable taking the survey. Compared to individual surveys, the main advantage of the selected group is in that the group could promote a collaboration, which results in more than the sum of each individual’s output. In order to achieve the objective of this study, a series of online focus group surveys were conducted, which could adapt geographically circulated populations in a virtual space.

The participants of the focus group survey were found through professional organizations and societies that focus on education safety, as well as experts from leading schools. Purposive variety was used to select participants with expertise in the two key areas, understanding the student population and building security. These are the following requirements to be included in the survey, participants must (1) over 18 years old, (2) be D in English speaking, (3) be able to

hear and talk remotely, and (4) have expertise in relevant areas, such as terrorism/active shooter incidents, school/office building design, security engineering in buildings, and a stakeholder in the school. The participants' professions were engineers, operations directors, parents, students, schools' teachers, and counselors (5). All of the participants had professional experience and expertise closely related to education and being in the school environment. Specifically, 4 participants have been involved in decision-making for school safety. The other participants mainly focus on students' and students' safety, including electronics security, anti-terrorism design, and crime prevention through environmental design.

The survey was designed to rank the best alternatives/solutions according to the participant's knowledge and expertise. The first five questions were related to demographics. More than 50% of the participants were female. This number indicates that there is a larger female population in the education industry. More than 70% of the participants hold a degree of a master's or higher. This indicates that participants are very educated and rounded in the field of education. The largest age group was between 30 to 45. This also correlates with the degree and level of experience of participants

The first part covers the beginning of the survey, includes welcome messages, explanation of the survey purpose and use of data, obtainment of participants' consent to participate in the study to be used, and self-introduction of the participants. In the second part, the survey asks the participants a demographic question, 'How old are you' 'what is your highest level of education', "What gender do you identify as?". All these questions assisted us with the right sample. All participants had great knowledge. The third part of the questions was ranking the alternatives according to groups. The first part of this section included an overall ranking of the groups. The

solutions were ranked into four main categories: 1-outside device, 2- trainings, 3-Increase Security, and 4- Distractions.

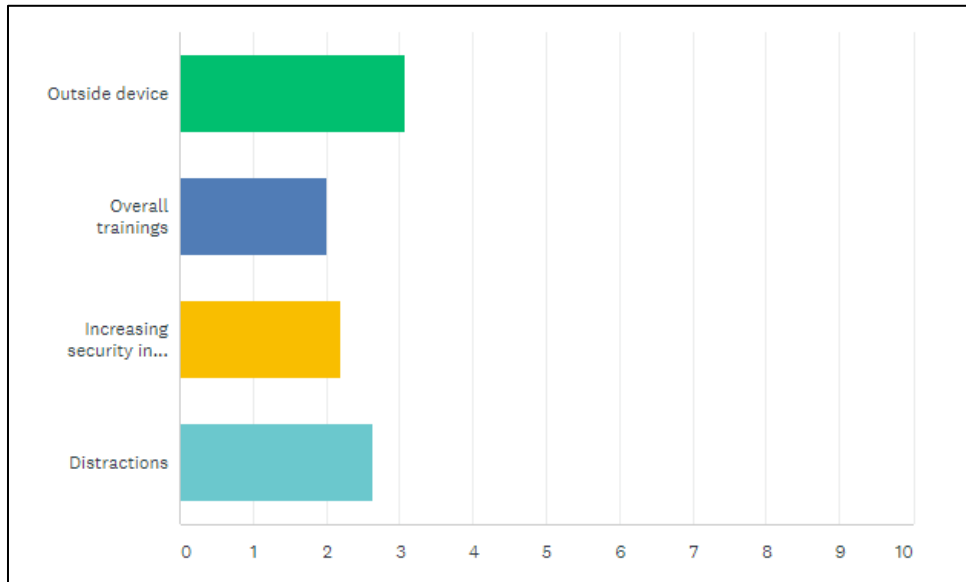


Figure 2.1 All categories as solutions

According to Figure 2.1 outside devices ranked the highest than came distractions. This can be an indicator that stakeholders do prefer having assists with some tool they can personally use. As you can see in Table 2.1 Training ranked the least wanted from participants. This indicates the lack of wanting knowledge.

Table 2.1 Analysis of responses for main categories

	1	2	3	4	TOTAL	SCORE
Outside device	34.62% 9	46.15% 12	11.54% 3	7.69% 2	26	3.08
Overall trainings	22.22% 6	3.70% 1	25.93% 7	48.15% 13	27	2.00
Increasing security in buildings	11.11% 3	29.63% 8	25.93% 7	33.33% 9	27	2.19
Distractions	29.63% 8	18.52% 5	37.04% 10	14.81% 4	27	2.63

2.3.1 Interval Valued Intuitionistic Fuzzy Multi-Criterion Decision Making Model:

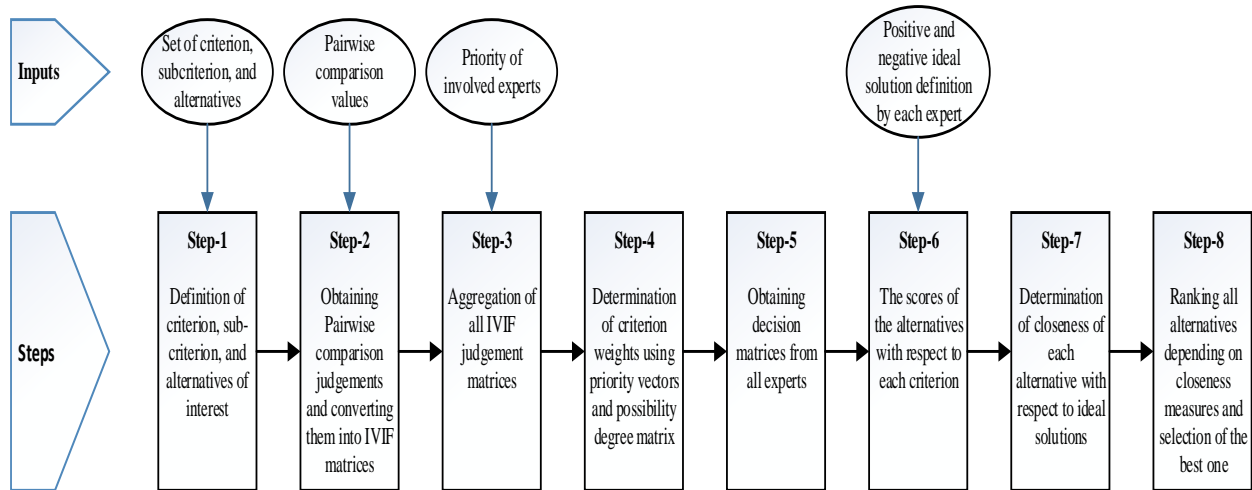


Figure 2.2 Methodology flow diagram with inputs.

Table 2.2 List of school safety alternatives

The alternative list for school shootings solutions selection.			
Outside device	Overall Trainings	Increasing security in buildings	Distractions
All teachers carry guns	Students and teachers do onsite training related to school shootings	Increase Security guards at school	Loud noise to distract the shooter
All administration carries guns	Teachers and Administration do onsite training	Doors will have special lockdown devices	Strong beam to distract the shooter
Any administration or teaching will have the option to carry or not to carry guns	Teachers and students do online training	Increase metal detectors	Loud noise and beam together
All teachers carry teasers	Teachers do online training	Create an Emergency app	Smoke to distract a shooter
All administration carries a taser	Teachers and Admin do online training	All classrooms will have bulletproof jackets	

Table 2.2 (continued)

Everyone does online training			
	Everyone does onsite training		

2.3.2 Criteria and Sub-criteria

1. *Cost of implementation*

This criterion highlights the resources required to implement the solution and whether the resources are readily available to the organization

2. *Effectiveness*

This criterion establishes whether the chosen intervention will yield the desired result in addressing the shooting menace in the school environment.

3. *Feasibility*

This criterion establishes the practicability of the proposed solution and whether it can be implemented within the stipulated timelines while yielding the desired results

4. *Safety*

This criterion establishes the safety of the intervention and whether it is likely to predispose the people in the school environment to further pressure and anxiety.

2.3.3 Interval Valued Intuitionistic Fuzzy Multi-Criterion Decision Making Model:

In this section, we introduce a robust hierarchical group decision-making model that can be applied to any complex multi-criteria decision-making problem. This model (Oztaysi et al., 2017) used the concept of interval-valued intuitionistic fuzzy (IVIF) preference relations along with the IVIF-TOPSIS decision-making method. The methodology begins with pairwise comparisons between different criteria and alternatives followed by several sequential steps for obtaining the final set of decisions with the overall weightage of different alternatives. For creating a better understanding of the method, in this section, we provided some basic definitions of IVIF sets and relevant calculations of IVIFS along with step by step illustration of procedures for applying IVIF sets in the multi-criterion decision-making process. Interested readers are suggested to follow the article Oztaysi et al. (2017) for learning more about IVIF numbers and their basic calculations.

Definition 3.1: Interval Valued Intuitionistic Fuzzy Numbers

Interval-valued inferential fuzzy set \tilde{I} over the universe of discourse X can be defined as:

$\tilde{I} = \{ \langle x, \mu_I(x), \nu_I(x) \rangle \mid x \in X \}$, where $C \subseteq [0,1]$ is the set of all closed subintervals of intervals, where $C \subseteq [0,1]$ is the set of all closed intervals of the interval $\mu_I \longrightarrow C \subseteq [0,1], \nu_I(x) \longrightarrow C \subseteq [0,1]$ with the conditions $0 \leq \sup \mu_i(x) + \sup \nu_i(x) \leq 1, \forall x \in X$.

Definition 3.2: Hesitancy degree of IVIF numbers

The unknown degree or hesitancy degree of IVIF numbers of $x \in X$ in $\tilde{I} = \left(\left[\mu_I^-, \mu_I^+ \right], \left[\nu_I^-, \nu_I^+ \right] \right)$ can be represented as $\pi_I(x) = 1 - \mu_I(x) - \nu_I(x) = (1 - \mu_I^+(x) - \nu_I^+(x), 1 - \mu_I^-(x) - \nu_I^-(x)) = (\pi_I^l(x), \pi_I^u(x))$.

Here, $\mu_I(x) = [\mu_I^-, \mu_I^+]$ and $\nu_I(x) = [\nu_I^-, \nu_I^+]$

Definition 3.3: Score function

For any IVIFN \tilde{I} the score function defined in equation (1)

$$S(\tilde{I}) = \frac{\mu_I^- + \mu_I^+ - \nu_I^- - \nu_I^+}{2} \quad (2.1)$$

Definition 3.4: Accuracy function

For any IVIFN \tilde{I} , the accuracy function is determined following equation (2).

$$H(\tilde{I}) = \frac{\mu_I^- + \mu_I^+ + \nu_I^- + \nu_I^+}{2} \quad (2.2)$$

The step by step procedure for conducting the proposed MCDA approach is described as follows:

Step 1: In the first step, we should define the MCDA problem of interest and represent it in a hierarchical structure with n criteria, sub-criterion (if any), and m decision alternatives. In addition, the experts' inputs should be collected, and pairwise comparison matrices should be formed following the linguistic comparison scale shown in table 1.14(Oztaysi et al., 2015). **Table 2** represents a simple form of data accumulation table having pairwise comparison values in it. This kind of table should be formed for all criteria and sub-criteria levels for every expert providing inputs to the study.

Table 2.3 Linguistic scale with corresponding IVIFS (Oztaysi et al., 2015).

Linguistic terms	IVIFS
Absolutely Low (AL)	$([0.10, 0.25],[0.65, 0.75])$
Very Low (VL)	$([0.15, 0.30],[0.60, 0.70])$
Low (L)	$([0.20, 0.35],[0.55, 0.65])$
Medium Low (ML)	$([0.25, 0.40],[0.50, 0.60])$
Exactly Equal (EE)	$([0.50, 0.50],[0.50, 0.50])$
Approximately Equal (AE)	$([0.45, 0.55],[0.30, 0.45])$
Medium High (MH)	$([0.50, 0.60],[0.25, 0.40])$
High (H)	$([0.55, 0.65],[0.20, 0.35])$
Very High (VH)	$([0.60, 0.70],[0.15, 0.30])$
Absolutely High (AH)	$([0.65, 0.75],[0.10, 0.25])$

Table 2.4 Example of a basic pairwise comparison matrix using linguistic scales

k^{th} decision maker	1 st criterion	2 nd criterion	3 rd criterion	...	n^{th} criterion
1 st criterion	EE	AL	MH		EE
2 nd criterion		EE	VH		MH
3 rd criterion			EE		ML
....					
n^{th} criterion					EE

Step 2: The linguistic data obtained in the previous step should be converted to their corresponding IVIFS according to table 3. This conversion will yield individual IVIF judgement matrices \tilde{P} for each decision-maker. Since this is a pairwise comparison matrix, the dimension of this matrix is $n \times n$. The definition of this IVIF judgement matrix $\tilde{P} = (\tilde{p}_{ij})_{n \times n}$ can be seen in equation (3) where $i(i = 1,2,3, \dots \dots n)$ and $j(j = 1,2,3, \dots \dots n)$ represents the criterion number.

$$\tilde{P} = \begin{bmatrix} ([\mu_{11}^-, \mu_{11}^+], [v_{11}^-, v_{11}^+]) & \dots & ([\mu_{1n}^-, \mu_{1n}^+], [v_{1n}^-, v_{1n}^+]) \\ \vdots & \ddots & \vdots \\ ([\mu_{n1}^-, \mu_{n1}^+], [v_{n1}^-, v_{n1}^+]) & \dots & ([\mu_{nn}^-, \mu_{nn}^+], [v_{nn}^-, v_{nn}^+]) \end{bmatrix} \quad (2.3)$$

Note that the reciprocal value of any IVIF number $[\mu_{ij}^-, \mu_{ij}^+], [v_{ij}^-, v_{ij}^+]$ will be simply $[v_{ji}^-, v_{ji}^+], [\mu_{ji}^-, \mu_{ji}^+]$.

Step 3: After constructing all the pairwise matrices and using IVIFS for each expert, all the corresponding matrices are aggregated into a single judgement matrix \tilde{P}_g (equation 4). This

aggregation is done by following equation (5) where $\tilde{\alpha}_i = \langle [\mu_i^-, \mu_i^+], [v_i^-, v_i^+] \rangle$ ($i = 1, 2, \dots, n$) is a collection of IVIF numbers and w represents the set of experts weights $w = (w_1, w_2, \dots, w_n)$ such that $w_i \in [0, 1], \sum_{i=1}^n w_i = 1$.

$$WA_w(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = \left\langle \left[1 - \prod_{i=1}^n (1 - \mu_i^-)^{w_i}, 1 - \prod_{i=1}^n (1 - \mu_i^+)^{w_i} \right], \left[\prod_{i=1}^n (v_i^-)^{w_i}, \prod_{i=1}^n (v_i^+)^{w_i} \right] \right\rangle \quad (2.4)$$

Step 4: In this step, the score judgement matrix $\tilde{S} = (\tilde{s}_{ij})_{n \times n}$ and interval multiplicative matrix $\tilde{A} = (\tilde{a}_{ij})_{n \times n}$ should be constructed by using equations 6 and 7, respectively. The values of this obtained matrix \tilde{A} is between 0 and 1.

$$\tilde{S} = \begin{bmatrix} [\mu_{g_{11}}^- - v_{g_{11}}^+, \mu_{g_{11}}^+ - v_{g_{11}}^-] & \dots & [\mu_{g_{1n}}^- - v_{g_{1n}}^+, \mu_{g_{1n}}^+ - v_{g_{1n}}^-] \\ \vdots & \ddots & \vdots \\ [\mu_{g_{n1}}^- - v_{g_{n1}}^+, \mu_{g_{n1}}^+ - v_{g_{n1}}^-] & \dots & [\mu_{g_{nn}}^- - v_{g_{nn}}^+, \mu_{g_{nn}}^+ - v_{g_{nn}}^-] \end{bmatrix} \quad (2.5)$$

$$\begin{aligned} \tilde{A} &= \begin{bmatrix} [e^{(\mu_{g_{11}}^- - v_{g_{11}}^+)}, e^{(\mu_{g_{11}}^+ - v_{g_{11}}^-)}] & \dots & [e^{(\mu_{g_{1n}}^- - v_{g_{1n}}^+)}, e^{(\mu_{g_{1n}}^+ - v_{g_{1n}}^-)}] \\ \vdots & \ddots & \vdots \\ [e^{(\mu_{g_{n1}}^- - v_{g_{n1}}^+)}, e^{(\mu_{g_{n1}}^+ - v_{g_{n1}}^-)}] & \dots & [e^{(\mu_{g_{nn}}^- - v_{g_{nn}}^+)}, e^{(\mu_{g_{nn}}^+ - v_{g_{nn}}^-)}] \end{bmatrix} \\ &= \begin{bmatrix} [\tilde{a}_{11}^-, \tilde{a}_{11}^+] & \dots & [\tilde{a}_{1n}^-, \tilde{a}_{1n}^+] \\ \vdots & \ddots & \vdots \\ [\tilde{a}_{n1}^-, \tilde{a}_{n1}^+] & \dots & [\tilde{a}_{nn}^-, \tilde{a}_{nn}^+] \end{bmatrix} \end{aligned} \quad (2.6)$$

Step 5: After computing the interval multiplicative matrix \tilde{A} , the priority vector of this exponential matrix can be computed as follows.

$$\tilde{w}_i = \left[\frac{\sum_{j=1}^n \tilde{a}_{ij}^-}{\sum_{i=1}^n \sum_{j=1}^n \tilde{a}_{ij}^-}, \frac{\sum_{j=1}^n \tilde{a}_{ij}^+}{\sum_{i=1}^n \sum_{j=1}^n \tilde{a}_{ij}^+} \right] = [w_i^-, w_i^+], i = 1, 2, \dots, n \quad (2.7)$$

Step 6: In this step, the weights \tilde{w}_i obtained in **step 5** should be compared, and the possibility degree matrix is obtained following equations (9) and (10).

$$\psi(\tilde{w}_i > \tilde{w}_j) = \psi_{ij} = \frac{\max(0, w_i^+ - w_j^-) - \max(0, w_i^- - w_j^+)}{(w_i^+ - w_i^-) + (w_j^+ - w_j^-)} \quad (2.8)$$

$$\psi(\tilde{w}_j > \tilde{w}_i) = \psi_{ji} = \frac{\max(0, w_j^+ - w_i^-) - \max(0, w_j^- - w_i^+)}{(w_i^+ - w_i^-) + (w_j^- - w_j^+)} \quad (2.9)$$

where $\psi_{ij} \geq 0, \psi_{ij} + \psi_{ji} = 1, \psi_{ii} = 1/2$. The possibility matrix $\psi = (\psi_{ij})_{n \times n}$ prioritization is done according to (11).

$$w_i = \frac{\sum_{j=1}^n \psi_{ij} - 1}{n} + 0.5 \quad (2.10)$$

Step 7: Next, the vector w_i comprising of weights for criteria and sub criteria are normalized, and this normalized matrix is obtained by the following equation.

$$w_i^T = \frac{w_i}{\sum_{i=1}^n w_i} \quad (2.11)$$

Step 8: The weight calculation should be done for all criteria and sub-criterion of the hierarchy following step 1 to 7. After performing this step, we should have all the weights for criteria and sub-criteria listed as vectors that will be used in the alternative evaluation.

Step 9: From this step, we should start the incorporation of IVIF-TOPSIS method into our model (Ye, 2010). First, we obtain the decision matrix $[\tilde{D}^k]$ from each decision-maker using the same

table 1.14 as a basis for comparison. Equation 11 depicts the anatomy of the decision matrix $[\tilde{D}_k]$ where n and m stands for the number of criterion ($i = 1, 2, \dots, n$) and number of alternatives ($j = 1, 2, \dots, m$) respectively. The overall IVIF-TOPSIS method is elaborated in **Step 10**.

$$\tilde{D}_k = \begin{matrix} & \begin{matrix} A_1 & & A_2 & & \dots & & A_m \end{matrix} \\ \begin{matrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{matrix} & \begin{bmatrix} (\mu_{11k}^-, \mu_{11k}^+) [v_{11k}^-, v_{11k}^+] \\ (\mu_{21k}^-, \mu_{21k}^+) [v_{21k}^-, v_{21k}^+] \\ \vdots \\ (\mu_{n1k}^-, \mu_{n1k}^+) [v_{n1k}^-, v_{n1k}^+] \end{bmatrix} & \begin{bmatrix} (\mu_{12k}^-, \mu_{12k}^+) [v_{12k}^-, v_{12k}^+] \\ (\mu_{22k}^-, \mu_{22k}^+) [v_{22k}^-, v_{22k}^+] \\ \vdots \\ (\mu_{n2k}^-, \mu_{n2k}^+) [v_{n2k}^-, v_{n2k}^+] \end{bmatrix} & \dots & \begin{bmatrix} (\mu_{1mk}^-, \mu_{1mk}^+) [v_{1mk}^-, v_{1mk}^+] \\ (\mu_{2mk}^-, \mu_{2mk}^+) [v_{2mk}^-, v_{2mk}^+] \\ \vdots \\ (\mu_{nmk}^-, \mu_{nmk}^+) [v_{nmk}^-, v_{nmk}^+] \end{bmatrix} \end{matrix} \quad (2.12)$$

Here, C_1, C_2, \dots, C_n represents the weight vector of element n .

Step 10: The TOPSIS method requires the definition of positive and negative ideal solutions for

each criterion, and these inputs are obtained from experts. The fuzzy positive ideal solution $I\tilde{S}_k^+$

and fuzzy negative ideal solution $I\tilde{S}_k^-$ for k^{th} decision maker is obtained by using equations (2.13)

and (2.14) along with the score function (1) and accuracy functions (2) as appropriate.

$$I\tilde{S}_k^+ = (([\mu_{1^*k}^-, \mu_{1^*k}^+], [v_{1^*k}^-, v_{1^*k}^+]), ([\mu_{2^*k}^-, \mu_{2^*k}^+], [v_{2^*k}^-, v_{2^*k}^+]), \dots, ([\mu_{n^*k}^-, \mu_{n^*k}^+], [v_{n^*k}^-, v_{n^*k}^+])) \quad (2.13)$$

$$I\tilde{S}_k^- = (([\mu_{1-k}^-, \mu_{1-k}^+], [v_{1-k}^-, v_{1-k}^+]), ([\mu_{2-k}^-, \mu_{2-k}^+], [v_{2-k}^-, v_{2-k}^+]), \dots, ([\mu_{n-k}^-, \mu_{n-k}^+], [v_{n-k}^-, v_{n-k}^+])) \quad (2.14)$$

Here, $([\mu_{1^*k}^-, \mu_{1^*k}^+], [v_{1^*k}^-, v_{1^*k}^+])$ and $([\mu_{1-k}^-, \mu_{1-k}^+], [v_{1-k}^-, v_{1-k}^+])$ are representatives of maximum and minimum intuitionistic fuzzy sets respectively among the alternatives' values for i^{th} criterion.

Next, the separation measures between j^{th} alternative and $I\tilde{S}_k^+$ and $I\tilde{S}_k^-$ for each decision maker can be obtained as follows.

$$D_j^{*k} = \sqrt{\frac{1}{2} \sum_{i=1}^n w_i^T \left[(\mu_{ijk}^- - \mu_{i^*k}^-)^2 + (\mu_{ijk}^+ - \mu_{i^*k}^+)^2 + (v_{ijk}^- - v_{i^*k}^-)^2 + (v_{ijk}^+ - v_{i^*k}^+)^2 + (\pi_{ijk}^l - \pi_{i^*k}^l)^2 + (\pi_{ijk}^u - \pi_{i^*k}^u)^2 \right]} \quad (2.15)$$

$$D_j^{-k} = \sqrt{\frac{1}{2} \sum_{i=1}^n w_i^T \left[(\mu_{ijk}^- - \mu_{i-k}^-)^2 + (\mu_{ijk}^+ - \mu_{i-k}^+)^2 + (v_{ijk}^- - v_{i-k}^-)^2 + (v_{ijk}^+ - v_{i-k}^+)^2 + (\pi_{ijk}^l - \pi_{i-k}^l)^2 + (\pi_{ijk}^u - \pi_{i-k}^u)^2 \right]} \quad (2.16)$$

where π_{ijk}^l, π_{ijk}^u and $\pi_{i^*k}^l, \pi_{i^*k}^u$ are calculated following the hesitancy degree equation (**Definition 3.2**).

Finally, the separation measures are aggregated following (18) and (19). Equation (20) provides the overall closeness co-efficient for all alternatives. The values of this closeness coefficient U_j dictates the preference order of alternatives where the larger value means more preference. The final ranking of alternatives is done based of this closeness coefficient U_j .

$$D_j^* = \sum_{k=1}^k (\lambda_k D_j^{*k}) \quad (2.17)$$

$$D_j^- = \sum_{k=1}^k (\lambda_k D_j^{-k}) \quad (2.18)$$

$$U_j = \frac{D_j^-}{D_j^- + D_j^+} \quad (2.19)$$

2.4 Case Study

This paper showcases a comprehensive case study familiarizing an actual issue experienced by tertiary institutions with rude and impolite students. As the corporations pursue the decision to apply the solutions for a school shooting, they require choosing at least one solution relying on the requirements among a diversity of available solutions with distinct capabilities. After thoroughly examining the characteristics of the available solutions, four alternatives were chosen. For each of the solutions for school shootings is perceived as distinguishing factor. Every category has distinct combinations of these features to help in the proof of notion. The chosen solutions for school shootings have been suggested by different tertiary institutions are selected grounded on the common characteristics for the purpose of visualization.

First, the input parameters of the characters have illuminated the summary of the attributes of the chosen solutions for school shootings. We applied the IVIF TOPSIS method, a robust MCDA methodology, to get the outcomes of the base case. By assessing the distinct actual requirements of the school, multiple sensitivity analyses producing distinct realistic scenarios are performed. The outcomes of the sensitivity analysis disclose the major managerial insights that can be used to offer the policymakers a baseline to make school shooting selection relying on distinct use cases.

Table 2.5 Weights for criteria

Criteria	Weight
Cost of Implementation	0.25
Effectiveness	0.25
Feasibility	0.25
Safety	0.25

Table 2.6 Alternatives Scores

Alternatives	Score	Weight
Outside Device	3.08	0.311
Training	2	0.202
Increasing Security	2.19	0.221
Distractions	2.63	0.266

Table 2.7 Pairwise matrix

Pairwise Comparison Matrix				
	Cost of Implementation	Effectiveness	Feasibility	Safety
Cost of implementation	EE	L	MH	L
Effectiveness		EE	H	VH
Feasibility			EE	MH
Safety				EE

Table 2.8 Score Judgment matrix

	Cost of Implementation	Effectiveness	Feasibility	Safety
Cost of implementation	(0,0)	(-0.35,-0.30)	(0.25,0.20)	(0.35,-0.3)
Effectiveness		(0,0)	(0.35,0.30)	(.45,0.40)
Feasibility			(0,0)	(.25,0.20)
Safety				(0,0)

Table 2.9 Interval Exponential Matrix

	Cost of Implementation	Effectiveness	Feasibility	Safety
Cost of implementation	[1,1]	[0.70468809, 0.740818221]	[1.284025417, 1.221402758]	[0.70468809, 0.740818221]
Effectiveness		[1,1]	[1.419067549, 1.349858808]	[1.568312185, 1.491824698]
Feasibility			[1.1]	[1.284025417, 1.221402758]
Safety				[1,1]

Table 2.10 Decision Maker one rankings and solutions

Criteria	Outside device	Training	Increase security	Distractions
Cost of Implementation	ML	MH	VH	VL
Effectiveness	ML	H	H	VL
Feasibility	H	VH	MH	ML
Safety	L	VH	VH	VL
Pairwise Comparison Matrix				
Criteria	Outside device	Training	Increase security	Distractions
Cost of Implementation	([0.25, 0.40], [0.50, 0.60])	([0.50, 0.60], [0.25, 0.40])	([0.60, 0.70], [0.15, 0.30])	([0.15, 0.30], [0.60, 0.70])
Effectiveness	([0.25, 0.40], [0.50, 0.60])	([0.55, 0.65], [0.20, 0.35])	([0.55, 0.65], [0.20, 0.35])	([0.15, 0.30], [0.60, 0.70])
Feasibility	([0.55, 0.65], [0.20, 0.35])	([0.60, 0.70], [0.15, 0.30])	([0.50, 0.60], [0.25, 0.40])	([0.25, 0.40], [0.50, 0.60])
Safety	([0.20, 0.35], [0.55, 0.65])	([0.60, 0.70], [0.15, 0.30])	([0.60, 0.70], [0.15, 0.30])	([0.15, 0.30], [0.60, 0.70])
Score Judgement Matrix				
Criteria	Outside device	Training	Increase security	Distractions
Cost of Implementation	(-0.25,-0.2)	(0.25,0.20)	(0.45,0.40)	(-0.45,-0.40)
Effectiveness	(-0.25,-0.2)	(0.35,0.30)	(0.35,0.30)	(-0.45,-0.40)
Feasibility	(0.35,0.30)	(0.45,0.40)	(0.25,0.20)	(-0.25,-0.2)
Safety	(-0.35,-0.30)	(0.45,0.40)	(0.45,0.40)	(-0.45,-0.40)

Table 2.11 Decision Maker two rankings and solutions

Criteria	Outside device	Training	Increase security	Distractions
Cost of Implementation	ML	H	AH	L
Effectiveness	MA	MH	EE	AL
Feasibility	MH	H	ML	ML
Safety	EE	AH	MH	ML
	Pairwise Comparison Matrix			
Criteria	Outside device	Training	Increase security	Distractions
Cost of Implementation	([0.25, 0.40], [0.50, 0.60])	([0.55, 0.65], [0.20, 0.35])	([0.65, 0.75], [0.10, 0.25])	([0.20, 0.35], [0.55, 0.65])
Effectiveness	([0.50, 0.60], [0.25, 0.40])	([0.50, 0.60], [0.25, 0.40])	([0.50, 0.50], [0.50, 0.50])	([0.10, 0.25], [0.65, 0.75])
Feasibility	([0.50, 0.60], [0.25, 0.40])	([0.55, 0.65], [0.20, 0.35])	([0.25, 0.40], [0.50, 0.60])	([0.25, 0.40], [0.50, 0.60])
Safety	([0.50, 0.50], [0.50, 0.50])	([0.65, 0.75], [0.10, 0.25])	([0.50, 0.60], [0.25, 0.40])	([0.25, 0.40], [0.50, 0.60])
	Score Judgemnt Matrix			
Criteria	Outside device	Training	Increase security	Distractions
Cost of Implementation	(-0.25,-0.20)	(0.35,0.30)	(0.55,0.50)	(-0.35,-0.30)
Effectiveness	(0.25,0.20)	(0.25,0.20)	(0,0)	(-0.55,-0.50)
Feasibility	(0.25,0.20)	(0.35,0.30)	(-0.25,-0.20)	(-0.25,-0.20)
Safety	(0,0)	(0.55,0.50)	(0.25,0.20)	(-0.25,-0.20)

Table 2.12 Decision Maker three rankings and solutions

Criteria	Outside device	Training	Increase security	Distractions
Cost of Implementation	L	VH	AH	ML
Effectiveness	VL	AE	H	AL
Feasibility	MH	H	AE	L
Safety	VL	VH	VH	ML
Pairwise Comparison Matrix				
Criteria	Outside device	Training	Increase security	Distractions
Cost of Implementation	([0.20, 0.35], [0.55, 0.65])	([0.60, 0.70], [0.15, 0.30])	([0.65, 0.75], [0.10, 0.25])	([0.25, 0.40], [0.50, 0.60])
Effectiveness	([0.15, 0.30], [0.60, 0.70])	([0.45, 0.55], [0.30, 0.45])	([0.55, 0.65], [0.20, 0.35])	([0.10, 0.25], [0.65, 0.75])
Feasibility	([0.50, 0.60], [0.25, 0.40])	([0.55, 0.65], [0.20, 0.35])	([0.45, 0.55], [0.30, 0.45])	([0.20, 0.35], [0.55, 0.65])
Safety	([0.15, 0.30], [0.60, 0.70])	([0.60, 0.70], [0.15, 0.30])	([0.60, 0.70], [0.15, 0.30])	([0.25, 0.40], [0.50, 0.60])
Score Judgement Matrix				
Criteria	Outside device	Training	Increase security	Distractions
Cost of Implementation	(-0.35,-0.30)	(0.45,0.40)	(0.55,0.50)	(-0.25,-0.2)
Effectiveness	(-0.45,-0.4)	(-0.15,-0.10)	(0.35,0.30)	(-0.55,-0.50)
Feasibility	(0.25,0.2)	(0.35,0.30)	(-0.15,-0.10)	(-0.35,-0.30)
Safety	(-0.45,-0.4)	(0.45,0.40)	(0.45,0.40)	(-0.25,-0.2)

Table 2.13 Decision Maker three rankings and solutions

Criteria	Outside device	Training	Increase security	Distractions
Cost of Implementation	ML	MH	H	MH
Effectiveness	VL	MH	VH	L
Feasibility	AE	H	H	L
Safety	L	AE	H	AL
Pairwise Comparison Matrix				
Criteria	Outside device	Training	Increase security	Distractions
Cost of Implementation	([0.25, 0.40], [0.50, 0.60])	([0.50, 0.60], [0.25, 0.40])	([0.55, 0.65], [0.20, 0.35])	([0.50, 0.60], [0.25, 0.40])
Effectiveness	([0.15, 0.30], [0.60, 0.70])	([0.50, 0.60], [0.25, 0.40])	([0.60, 0.70], [0.15, 0.30])	([0.20, 0.35], [0.55, 0.65])
Feasibility	([0.45, 0.55], [0.30, 0.45])	([0.55, 0.65], [0.20, 0.35])	([0.55, 0.65], [0.20, 0.35])	([0.20, 0.35], [0.55, 0.65])
Safety	([0.20, 0.35], [0.55, 0.65])	([0.45, 0.55], [0.30, 0.45])	([0.55, 0.65], [0.20, 0.35])	([0.10, 0.25], [0.65, 0.75])
Score Judgement Matrix				
Criteria	Outside device	Training	Increase security	Distractions
Cost of Implementation	(-0.25,-0.20)	(0.25,0.20)	(0.35,0.30)	(0.25,0.20)
Effectiveness	(-0.45,-0.40)	(0.25,0.20)	(0.45,0.40)	(-0.35,-0.30)
Feasibility	(0.15,0.10)	(0.35,0.30)	(0.35,0.30)	(-0.35,-0.30)
Safety	(-0.35,-0.30)	(0.15,0.10)	(0.35,0.30)	(-0.55,-0.50)

2.4.1 Scenario 1: Large School Setting

According to the Department of Education, school shooting has become a hurdle towards academic brilliance in many tertiary institutions. Hence, the stakeholders prefer having assistance with some tools they can use. The base station should be located at the school such that the solutions can cover the students in the most affected areas. In this scenario, the outside device shows the highest preference ranking compared to others.

Table 2.14 Preferred solution for large school setting

School Shooting Solution	Weight	Score
Outside Device	0.311	3.08*
Training	0.202	2
Increasing Security	0.211	2.19
Distraction	0.266	2.63
Distraction *Preferred School Shooting Solution		

2.4.2 Scenario 2: Small school Setting

The different stakeholders in the education sector choose the solution that students and teachers do onsite training related to school shootings. In this scenario, distractions show the highest preference ranking compared to others.

Table 2.15 Preferred solution in small school setting

School Shooting Solution	Weight	Score
Outside Device	0.111	1.6
Training	0.202	2
Increasing Security	0.211	2.19
Distraction	0.266	2.63*
Distraction *Preferred School Shooting Solution		

2.4.3 Scenario 3: K-12 schools

The different stakeholders in the education sector choose the solution; increasing security will decrease the level of school shootings. In this scenario, the outside device shows the highest preference ranking compared to distraction and outside devices.

Table 2.16 Preferred solution for K-12

School Shooting Solution	Weight	Score
Outside Device	0.311	3.08*
Training	0.202	2
Increasing Security	0.211	2.19
Distraction	0.266	2.63
Distraction *Preferred School Shooting Solution		

2.5 Conclusion

The paper recommends a methodology for choosing the school shooting solutions. A list of criteria will be suitable for comparing the solutions for school shooting are proposed. A multi-criterion Interval Valued Inferential Fuzzy TOPSIS method is suggested to carry out this assessment. The suggested methodology can address the fuzziness in making decisions and offer the best outcomes relying on the use cases. The use of the decision-making model by crafting an actual case study is shown. Also, the various sensitivity analyses are carried out, and the outcomes from the analyses offer the knowledge for the decision-makers who engage in the process. Overall, the analyses provide a clear notion regarding the appropriate solution grounded on distinct circumstances. This work can be elongated into distinct directions. To address the uncertainty in decision-making, distinct kinds of fuzzy sets can be applied. The use of distinct kinds of fuzzy sets in terms of making decisions forms an important element that necessitates further investigation. The best solution for multi-criteria solution for a school shooting is outside devices. Continuous seminars on using outside devices among students are effective because they will help develop insight among the students on the need to avoid school shootings in academic institutions and embrace peace.

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APPENDIX A
DECISION MATRIX

Table A.1 Decision matrix for stationary urban canyon scenario (Scenario 1)

Criterion	Detail	Drone A	Drone B	Drone C	Drone D
A_1	UAS overall size	VH	VH	VL	AL
A_2	Weight	VH	H	ML	AL
A_3	Drone Type	VH	VH	VL	AL
A_4	Fuel Type	VH	VH	VH	AL
B_1	Internal Computing Components	ML	MH	AE	ML
B_2	Location and Proximity Accuracy	ML	VH	H	AE
B_3	Communication and Data Quality	ML	H	H	MH
B_4	Traceability	ML	H	H	MH
B_5	Reliability	ML	VH	H	MH
C_1	Repair Cost	H	VL	AE	VL
C_2	Total Unit Cost	VH	AE	ML	L
C_3	Total Lifecycle Cast	H	L	ML	VL
C_4	Operating Cost	VH	H	ML	AE
C_5	Training Grit	MH	VH	L	L
D_1	Adaptability	H	VH	MH	ML
D_2	Environmental Impact	VH	H	H	VL
D_3	Recharge/ Refuel location	VH	VH	ML	H
D_4	Required Delivery Distance	ML	VH	VH	AH
E_1	Max Flight Time	ML	VH	VH	VH
E_2	Total Recharge/ fuel Time	MH	AE	AE	AH
E_3	Charge/Fuel Usage Rate	H	MN	AE	ML
E_4	Maximum Load	AE	H	H	VH
E_5	Maximum Carry Dimensions	AE	H	H	VH
E_6	Maximum Reachable Altitude	VL	H	AH	AH
E_7	Drone Speed	ML	ML	VH	H
E_8	Adaptability to Dynamic Assignment	VL	VH	MH	AE
E_9	Package Handling Flexibility	AE	MH	ML	VH
E_{10}	Delivery Flexibility	VH	AH	VL	AL

Table A.2 Decision matrix for truck based urban canyon scenario (Scenario 2)

Criterion	Detail	Drone A	Drone B	Drone C	Drone D
A_1	UAS overall size	AH	VH	V L	AL
A_2	Weight	AH	H	ML	AL
A_3	Drone Type	AH	AH	V L	AL
A_4	Fuel Type	AH	AH	AH	AL
B_1	Internal Computing Components	L	H	H	AE
B_2	Location and Proximity Accuracy	ML	VH	H	AE
B_3	Communication and Data Quality	ML	H	H	MH
B_4	Traceability	ML	H	H	MH
B_5	Reliability	ML	VH	H	MH
C_1	Repair Cost	VH	MH	AE	L
C_2	Total Unit Cost	VH	MH	AE	L
C_3	Total Lifecycle Cost	VH	AE	AE	L
C_4	Operating Cost	VH	H	ML	L
C_5	Training Cost	H	VH	L	VL
D_1	Adaptability	VH	AH	AE	L
D_2	Environmental Impact	VH	VH	H	AL
D_3	Recharge/ Refuel Location	AH	AH	V L	AL
D_4	Required Delivery Distance	ML	VH	VH	AH
E_1	Max Flight Time	ML	VH	VH	VH
E_2	Total Recharge/Refuel Time	H	MH	MH	AH
E_3	Charge/Fuel Usage Rate	AE	H	H	AE
E_4	Maximum Load	ML	H	H	VH
E_5	Maximum Carry Dimensions	ML	H	H	VH
E_6	Maximum Reachable Altitude	AE	AH	AH	AH
E_7	Drone Speed	ML	ML	VH	H
E_8	Adaptability to Dynamic Assignment	VL	VH	MH	AE
E_9	Package Handling Flexibility	AE	MH	ML	VH
E_{10}	Delivery Flexibility	VH	AH	L	V L

Table A.3 Decision matrix for truck-based rural area scenario (Scenario 3)

Criterion	Detail	Drone A	Drone B	Drone C	Drone D
A_1	UAS overall size	AH	VH	VL	AL
A_2	Weight	AH	H	ML	AL
A_3	Drone Type	AH	AH	H	ML
A_4	Fuel Type	A H	AH	AH	AE
B_1	Internal Computing Components	VL	H	H	MH
B_2	Location and Proximity Accuracy	L	AH	H	MH
B_3	Communication and Data Quality	L	AH	H	MH
B_4	Traceability	L	H	H	MH
B_5	Reliability	ML	VH	H	MH
C_1	Repair Cost	H	AE	ML	AL
C_2	Total Unit Cost	AE	ML	ML	VL
C_3	Total Li recycle Cost	H	MH	MH	ML
C_4	Operating Cost	VH	H	ML	L
C_5	Training Cast	H	H	ML	ML
D_1	Adaptability	VH	AH	MH	AE
D_2	Environmental Impact	VH	V H	H	AL
D_3	Recharge/ Refuel Location	AH	H	MH	ML
D_4	Required Delivery Distance	AL	VH	VH	AH
E_1	Max Flight Time	AL	VH	VH	VH
E_2	Total Recharge/Refuel Time	H	MH	MH	AH
E_3	Charge /Fuel Usage Rate	AE	H	H	AE
E_4	Maximum Load	L	MH	VH	AH
E_5	Maximum Carry Dimensions	L	MH	VH	AH
E_6	Maximum Reachable Altitude	ML	H	AH	AH
E_7	Drone Speed	ML	ML	VH	H
E_8	Adaptability to Dynamic Assignment	VL	V H	MH	AE
E_9	Package Handling Flexibility	ML	MH	H	VH
E_{10}	Delivery Flexibility	AE	AH	H	MH

Table A.4 Decision matrix for disaster affected area scenario (Scenario 4)

Criterion	Detail	Drone A	Drone B	Drone C	Drone D
A_1	UAS overall size	VL	L	AE	VH
A_2	Weight	VL	AE	AE	VH
A_3	Drone Type	AE	AE	AE	AE
A_4	Fuel Type	L	L	L	H
B_1	Internal Computing Components	AE	H	H	MH
B_2	Location and Proximity Accuracy	AE	H	H	H
B_3	Communication and Data Quality	AE	H	H	H
B_4	Traceability	AE	H	H	H
B_5	Reliability	L	ML	AE	H
C_1	Repair Cost	AE	AE	AE	AE
C_2	Total Unit Cost	AE	AE	AE	AE
C_3	Total Lifecycle Cost	AE	AE	AE	AE
C_4	Operating Cost	ML	L	VL	AE
C_5	Training Cost	MH	MH	AE	AE
D_1	Adaptability	VL	L	MH	VH
D_2	Environmental Impact	VH	H	H	AE
D_3	Recharge/ Refuel Location	L	L	L	VH
D_4	Required Delivery Distance	AL	AE	AE	AH
E_1	Max Flight Time	AL	AE	AE	VH
E_2	Total Recharge /Refuel Time	AE	AE	AE	AH
E_3	Charge/Fuel Usage Rate	AE	AE	AE	AE
E_4	Maximum Load	AL	L	ML	VH
E_5	Maximum Carry Dimensions	AL	ML	MH	VH
E_6	Maximum Reachable Altitude	AL	VL	AH	AH
E_7	Drone Speed	ML	ML	VH	H
E_8	Adaptability to Dynamic Assignment	AE	VH	H	MH
E_9	Package Handling Flexibility	AL	ML	MH	VH
E_{10}	Delivery Flexibility	AL	MH	VH	AH