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Adaptive Fuzzy Proportional Integral Sliding Mode Control for Two-Tank Interacting System

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Highlights:

- A sliding mode control with proportional integral sliding surface (PISMC) was designed for tracking control of the liquid levels of two-tank interacting systems.
- The fuzzy inference system is used to approximate the uncertain function in the PISMC.
- Determination of the adaptive law.

Abstract. This paper presents an adaptive fuzzy proportional integral sliding mode control (AFPISMC) for two-tank interacting system (TTIS). In order to maintain the desired liquid level of the TTIS and meet the reference values for attenuated chattering problems, this paper proposes a combination of a sliding mode control (SMC) with a proportional integral (PI) sliding surface and a fuzzy inference system. Fuzzy logic and the universal approximation theorem of fuzzy systems are used to approximate the uncertain function in the PISMC. The stability of the control system is proved by the Lyapunov theory. The simulation results of the proposed method in MATLAB/Simulink were compared to a fuzzy control, a sliding mode control with conditional integrals, a fuzzy-PID control, and a conventional PID control. The comparison results showed that the proposed controller was most effective when the rising time reached 0.2375 s, the percent of overshoot was 0%, the steady state error converged to zero, the settling time was 0.4612 s, and chattering was reduced.

Keywords: *adaptive control; fuzzy logic; PI sliding surface; sliding mode control; two-tank interacting system.*

1 Introduction

Two-tank interacting systems (TTIS) are widely used in the petroleum, chemical, pharmaceutical and food processing industries [1], effluent treatment, water purification systems, nuclear power plants, and automatic liquid dispensing [2]. Several researchers around the world have designed and implemented liquid level controllers for TTIS. Several types of controllers can be used, including PID controllers, as presented by Miral & Ankitin [1], Mostafa & Aboubaker [3],

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Teena & Hepsiba [5]. The PID parameters were obtained using an appropriate gain method. However, the simulation in this study obtained a slow response with a rising time of about 15 s, a settling time of 560 s, and an overshoot of 27.27%. A fuzzy-PID controller was developed by Trinh [4]. This controller can improve the quality of the liquid level coupled-tank control system, increase process efficiency, and bring economic benefit to the end-user. However, the dynamics of the actuator and the sensor need to be studied in more detail on an actual device to obtain a more realistic control of the object model. A sliding mode control with saturation was developed by Teena & Hepsiba [5], which was used in a simulation to ensure that the control signal always remained within the bounds and the response was without overshoot. The settling time was about 7.6 s. A fuzzy controller was proposed by Miral & Ankit [1], for which the response with overshoot was 1.45%, the settling time was 47.2 s, and the rising time was 33 s. A sliding mode control using conditional integrators (CSMCCI) was developed by Sankata, *et al.* [2,6]. Its response with overshoot was 1.6%, the settling time was 330 s and the rising time was 87.184 s. A second-order sliding mode control was proposed by Reddy, *et al.* [7,8]. Its response with overshoot was 7.344%, the settling time was 51.576s, and the rising time was 30.010s. A comparison of these types of TTIS controllers for evaluation and selection of an appropriate control technique has not yet been done.

Sliding mode control (SMC) is a robust control technique, which possesses inherent advantages such as invariance to parametric uncertainty and disturbance rejection. However, if the amplitude of the sliding control law is not selected properly, it will cause chattering according to Hussein, *et al.* [9] and Jinkun [10]. Chattering is due to imperfections and time delays in switching caused by small time constant actuators. It makes the power circuit prone to overheating, leading to damage [10].

To reduce chattering in the control, the present study proposes a sliding mode control design with a proportional integral sliding surface (PISMC) to control the liquid level in TTIS. This provides a faster transient response with a minimum steady state error. The fuzzy logic technique is used to approximate the function in the sliding mode control with a PI sliding surface and determine the adaptive law. The main advantage of the fuzzy inference system approach is that it is model-free and is able to incorporate the available human knowledge about system operation. The adaptive fuzzy PI sliding mode control algorithm ensures that the actual liquid level converges to the desired liquid level in a finite time and reduces chattering around the sliding surface. The simulation results of the proposed method in MATLAB/Simulink were compared to a fuzzy control, a sliding mode control with conditional integrals, a fuzzy PID control, and a conventional PID. The comparison results showed that the proposed controller is robust with respect to both the parametric and unstructured uncertainties of the system.

This paper is organized in five sections: Section 2 presents the mathematical model of the TTIS. The adaptive fuzzy PI sliding mode control is presented in Section 3. The simulation and evaluation results are presented in Section 4, and Section 5 contains the conclusions.

2 Mathematical Model of the Two-Tank Interacting System

The process consisting of the two-tank interacting system (TTIS) shown in Figure 1 was presented by Miral & Ankit in [1]. The height of the liquid level is h_1 (cm) in tank 1 and h_2 (cm) is for tank 2. Volumetric flow into tank 1 is q_{in} (cm^3/min), the volumetric flow rate from q_1 (cm^3/min), and the volumetric flow rate from tank 2 is q_0 (cm^3/min). The cross-sectional area of tank 1 is A_1 (cm^2) and the area of tank 2 is A_2 (cm^2).

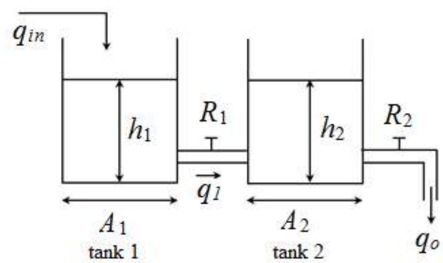

Figure 1 Model of the two tank interacting system.

The transfer function of the TTIS as Eq. (1) was taken from Miral & Ankit [1]:

$$G(s) = \frac{h_2(s)}{q_{in}(s)} = \frac{R_2}{T_1 T_2 s^2 + (T_1 + T_2 + A_1 R_2)s + 1} \tag{1}$$

where the time constant of tank 1 is $T_1 = A_1 R_1$ and the time constant of tank 2 is $T_2 = A_2 R_2$.

Let’s define $a = R_2, b = T_1 T_2,$ and $c = T_1 + T_2 + A_1 R_2$.

The set state variables are as follows:

$$x_1(t) = h_2(t) \tag{2}$$

$$x_2(t) = \dot{x}_1(t) = \dot{h}_2(t) \tag{3}$$

Taking the derivative of both sides of Eq. (3), we get Eq. (4):

$$\dot{x}_2(t) = \ddot{h}_2(t) = -\frac{1}{b}x_1(t) - \frac{c}{b}x_2(t) + \frac{a}{b}q_{in}(t) \tag{4}$$

or: $$\dot{x}_2(t) = -f(x) + \frac{a}{b}q_{in}(t) \tag{5}$$

where $f(x) = \frac{1}{b}x_1(t) + \frac{c}{b}x_2(t)$

We get the state space of the TTIS as follows:

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\frac{1}{b} & -\frac{c}{b}\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}0\\ \frac{a}{b}\end{bmatrix}q_{in}(t)$$

$$h_2(t) = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + 0q_{in}(t) \tag{6}$$

3 Adaptive Fuzzy PI Sliding Mode Control Design

The structure of the AFPISMC controller is presented in Figure 2.

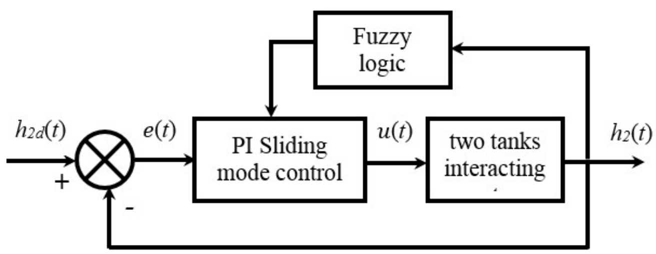

Figure 2 Structure of the adaptive fuzzy PI sliding mode control.

The PI sliding mode control algorithm ensures that the actual liquid level converges to the desired level in a finite time and reduces chattering around the sliding surface. The fuzzy logic technique is used to approximate the function in the sliding mode control with a PI sliding surface, and to determine the adaptive law.

3.1.1 Proportional Integral Sliding Mode Control Design

The sliding surface of the PISMC consists of two sliding surfaces, s_1 and s_2, the proportional sliding surface and the integral sliding surface is expressed in Eq. (7) was taken from Chien-Hong & Fu-Yuen [11]:

$$s = s_1 + s_2 = \left(\frac{d}{dt} + \lambda\right)e + \left(\frac{d}{dt} + \alpha\right)^2 \xi_e = 2\dot{e} + (\lambda + 2\alpha)e + \alpha^2 \int_0^t e(\tau)d\tau \tag{6}$$

where $\lambda > 0, \alpha > 0$. The error and derivative of error are as follows:

$$e = h_{2d} - h_2 \tag{7}$$

$$\dot{e} = \dot{h}_{2d} - \dot{h}_2 \tag{8}$$

where h_{2d} is the reference level, h_2 is the real height.

Taking the derivative of Eq. (7), we get Eq. (10):

$$\dot{s} = 2\ddot{e} + (\lambda + 2\alpha)\dot{e} + \alpha^2 e \tag{9}$$

Substituting Eq. (5) into Eq. (10), we get:

$$\dot{s} = 2\left[\ddot{h}_{2d} + f(x)\right] + (\lambda + 2\alpha)\dot{e} + \alpha^2 e - \frac{2a}{b} q_{in} \tag{10}$$

The reaching law with a constant rate as in Eq. (12), was taken from Jinkun [10]:

$$\dot{s}(t) = -\eta sign(s) \tag{11}$$

We get the following PI sliding mode control equation:

$$u_{PISMC} = \frac{b}{a}\left[\ddot{h}_{2d} + f\right] + \frac{b}{2a}\left[(\lambda + 2\alpha)\dot{e} + \alpha^2 e + \eta sign(s)\right] \tag{12}$$

Eq. (13) ensures that the actual liquid level converges to the desired level in a finite time and reduces chattering around the sliding surface.

3.2 Fuzzy Approximation

Using the universal approximation theorem of the fuzzy system, we designed the fuzzy system $\hat{f}(x|\theta)$ to approximate $f(x)$ as given by Jinkun in [12].

Consider input x_1 and x_2. By designing five membership functions, we get $n = 2, i = 1,2, p_1 = p_2 = 5$, so we can have $p_1 \times p_2 = 25$ fuzzy rules.

We use two steps to construct the fuzzy system $\hat{f}(x|\theta)$ as follows:

Step 1: For the variable $x_i (i = 1,2)$, define p_i fuzzy sets.

Step 2: Use $\prod_{i=1}^{n} p_i = p_1 \times p_2 = 25$ fuzzy rules to construct fuzzy system $\hat{f}(x|\theta)$. The j^{th} fuzzy rule is expressed as follows:

$$R^{(j)}: \text{if } x_1 \text{ is } A_1^{l_1} \text{ and } x_2 \text{ is } A_1^{l_2} \text{ then } \hat{f} \text{ is } B^{l_1 l_2} \tag{14}$$

where $l_i = 1,2,3,4,5; i = 1,2; j = 1,2, \ldots, 25; B^{l_1 l_2}$ are the fuzzy sets of $\hat{f}$.

Then, the first and the twenty-fifth fuzzy rule can be expressed as:

$R^{(1)}$: if x_1 is A_1^1 and x_2 is A_1^1 then $\hat{f}$ is E^1

$R^{(25)}$: if x_1 is A_1^5 and x_2 is A_1^5 then $\hat{f}$ is E^{25}

The fuzzy inference is designed as follows:

1. Using a product inference engine for the fuzzy rule premise, we get $\prod_{i=1}^{2} \mu_{A_i^{l_i}}(x_i)$.
2. Use the singleton fuzzifier to get $\bar{y}_f^{l_1 l_2}$, where $\bar{y}_f^{l_1 l_2} = f(x_1, x_2)$ is the point $[x_1, x_2]$, at which $\mu_{B^{l_1 l_2}}\left(\bar{y}_f^{l_1 l_2}\right)$ achieves its maximum value, and we assume that $\mu_{B^{l_1 l_2}}\left(\bar{y}_f^{l_1 l_2}\right) = 1.0$.
3. Using a product inference engine for the premise and the conclusion of the fuzzy rule, we get $\bar{y}_f^{l_1 l_2}\left(\prod_{i=1}^{2} \mu_{A_i^{l_i}}(x_i)\right)$, and using the union operator for different fuzzy rules, we can get the output of the fuzzy system as $\sum_{l_1=1}^{5}\sum_{l_2=1}^{5}\left[\bar{y}_f^{l_1 l_2}\right]\left(\prod_{i=1}^{2} \mu_{A_i^{l_i}}(x_i)\right)$.
4. Using the center average defuzzifier, we can get the output of the fuzzy system as follows:

$$\hat{f}(x|\theta) = \frac{\sum_{l_1=1}^{5}\sum_{l_2=1}^{5} \bar{y}_f^{l_1 l_2}\left(\prod_{i=1}^{2} \mu_{A_i^{l_i}}(x_i)\right)}{\sum_{l_1=1}^{5}\sum_{l_2=1}^{5}\left(\prod_{i=1}^{2} \mu_{A_i^{l_i}}(x_i)\right)} \tag{13}$$

where $\mu_{A_i^j}(x_i)$ is the membership function of x_i.

Let $\bar{y}_f^{l_1 l_2}$ be the freedom parameter, $\theta = \left[\bar{y}_f^1 \quad \cdots \quad \bar{y}_f^{25}\right]^T$ be the parameter vector. Introducing the fuzzy basis vector $\xi(x)$, Eq. (13) becomes:

$$\hat{f}(x|\theta) = \hat{\theta}^T \xi(x) \tag{14}$$

where $\xi(x)$ is the fuzzy basis vector with $\prod_{i=1}^{n} p_i = p_1 \times p_2 = 25$ elements, its $l_1 l_2$ is as follows:

$$\xi_{l_1 l_2}(x) = \frac{\prod_{i=1}^{2} \mu_{A_i^{l_i}}(x_i)}{\sum_{l_1=1}^{5}\sum_{l_2=1}^{5}\left(\prod_{i=1}^{2} \mu_{A_i^{l_i}}(x_i)\right)} \tag{15}$$

3.3 Adaptive Fuzzy Control Design and Analysis

We set the optimum parameter as Eq. (18), as given by Jinkun in [12]:

$$\theta^* = arg\,min\left[\sup_{x\in R^2}\left|\hat{f}(x|\theta)\right| - f(x)\right] \tag{16}$$

Then, $f(x) = \theta^{*T}\xi(x) + \varepsilon$ (17)

where ε is the approximation error.

$$f(x) - \hat{f}(x) = \theta^{*T}\xi(x) + \varepsilon - \hat{\theta}\xi(x) = -\tilde{\theta}^T\xi(x) + \varepsilon \tag{18}$$

Define the Lyapunov function as follows: $V = \frac{1}{2}s^2 + \frac{1}{2\gamma}\tilde{\theta}^T\tilde{\theta}$ (19)

where $\gamma > 0, \tilde{\theta} = \hat{\theta} - \theta^*$, then, $\dot{\tilde{\theta}} = \dot{\hat{\theta}}$ and

$$\dot{V} = \frac{1}{2}s\dot{s} + \frac{1}{\gamma}\tilde{\theta}^T\dot{\hat{\theta}} = s\left(c\dot{e} + f(x) + \frac{a}{b}q_{in} - \ddot{h}_{2d}\right) + \frac{1}{\gamma}\tilde{\theta}^T\dot{\hat{\theta}} \tag{20}$$

We define the adaptive fuzzy PI sliding mode control law as follows:

$$u_{AFPISMC} = \frac{b}{a}\left[\ddot{h}_{2d} + \hat{f}\right] + \frac{b}{2a}\left[(\lambda + 2\alpha)\dot{e} + \alpha^2 e + \eta sign(s)\right] \tag{21}$$

Then,

$$\begin{aligned}\dot{V} &= \frac{1}{2}s\dot{s} + \frac{1}{\gamma}\tilde{\theta}^T\dot{\hat{\theta}} = s\left(2\left(f - \hat{f}\right) - \eta sign(s)\right) + \frac{1}{\gamma}\tilde{\theta}^T\dot{\hat{\theta}} \\ &= s\left(-2\tilde{\theta}^T\xi(x) + 2\varepsilon - \eta sign(s)\right) + \frac{1}{\gamma}\tilde{\theta}^T\dot{\hat{\theta}} \\ &= 2\varepsilon s - \eta|s| + \tilde{\theta}^T\left(\frac{1}{\gamma}\dot{\hat{\theta}} - 2s\xi(x)\right)\end{aligned} \tag{22}$$

Choosing $\eta > |\varepsilon|_{max} + \eta_0, \eta_0 > 0$, then the adaptive law is:

$$\dot{\hat{\theta}} = 2\gamma s\xi(x) \tag{23}$$

Then $\dot{V} = 2\varepsilon s - \eta|s| \le -\eta_0|s| \le 0$

From the above analysis, we can see that the fuzzy system approximation error can be overcome by the robust term $\eta sign(s)$.

From $\dot{V} \le -\eta_0|s| \le 0$, we have $\int_0^t \dot{V}dt \le -\eta_0\int_0^t |s|dt$, i.e., $V(t) - V(0) \le -\eta_0\int_0^t |s|dt$

Then, V is limited, s and $\tilde{\theta}$ are limited, from expression $\dot{s}$, $\dot{s}$ is limited, and $\int_0^\infty |s|dt$ is limited. From Barbalat's lemma [11], when $t \to \infty$, we have $s \to 0$. Then $e \to 0, \dot{e} \to 0$.

Since V is limited as $t \to \infty$, thus $\hat{\theta}$ is limited. Because when $\dot{V} \equiv 0$, we cannot get $\tilde{\theta} \equiv 0$, so $\hat{\theta}$ will not converge to θ^*.

Based on the Lyapunov stability criterion, the sliding surface s will reach zero in a finite time and stay zero under the control law in Eq. (23) and the adaptive law in Eq. (25). Once $s = 0$ and $\dot{s} = 0$ are established, the system in Eq. (6) will converge from any initial condition $x(0) \neq 0$ to $x(t) = [x_1(t) \quad x_2(t)]^T = [0 \quad 0]^T$ along the sliding-mode surface in a finite time, and the adaptive error will also converge to zero.

4 Simulation Results and Evaluation

The proposed controller simulation diagram (AFPISMC) is presented in Figure 3. The parameters of the TTIS are presented in Table 1. Table 2 presents the parameters of the proposed controller.

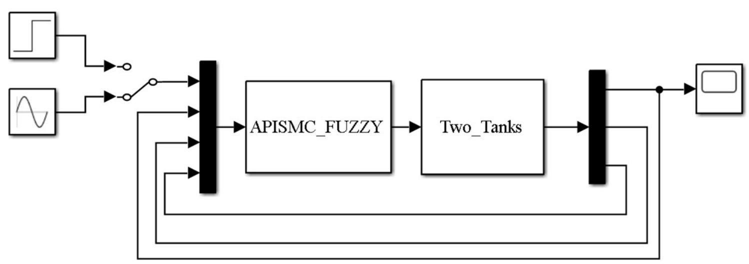

Figure 3 Schematic diagram of the AFPISMC.

Table 1 The parameters of the two tank interacting system.

Parameters	A_1	A_2	R_1	R_2
Value	0.0145	0.0145	1478.57	642.86
Unit	m^2	m^2	sec/m^2	sec/m^2

Table 2 The parameters of the proposed controller.

Parameters	α	λ	γ	η
Value	15	25	5000	50

The step response and error of the system with the AFPISMC controller are presented as Figure 4, in which we can see that the actual liquid level response of the TTIS converged to the reference level of 0.055 m with the rising time reached 0.2375 s, eliminating the steady-state error and reducing overshoot, while the settling time was 0.4612 s. These criteria are presented in Table 3 and compared with the fuzzy control, the sliding mode control with condition integrals, the fuzzy-PID control, and the traditional PID control. The results presented in Table 3 show that the efficiency of the AFPISMC controller was better than that of the fuzzy control, the sliding control with condition integrals, the fuzzy-PID control, and the traditional PID control.

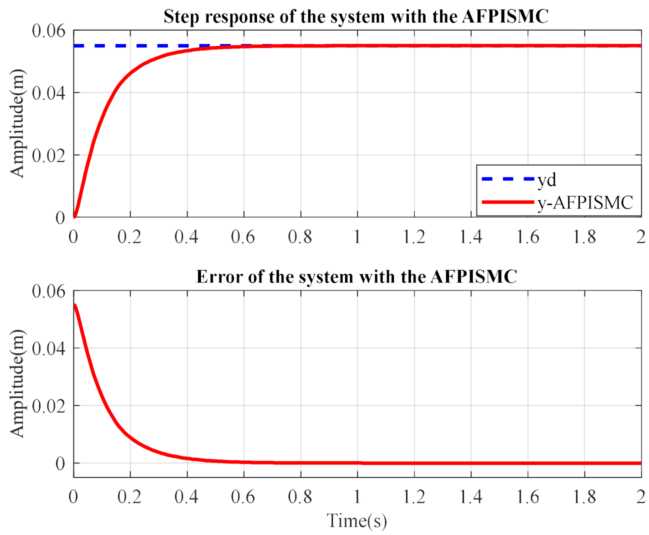

Figure 4 The step response and error of the system with AFPISMC.

Table 3 The achieved quality criteria of the AFPISMC controller.

Controller	AFPIS MC	Fuzzy controller [1]	Sliding mode control [5]	Fuzzy PID controller [4]	Sliding control with condition integrals [2]
Rise time s	0.2375	33	-	1.5	87.184
Overshoot (%)	0	1.45	0	0	1.6
Steady state error (m)	0	-	-	0	-
Settling time s	0.4612	47.2	7.6	3.1	330

The control signal with the step function illustrated in Figure 5 shows that the chattering phenomenon was reduced, with the amplitude converging to zero. This result demonstrates the effectiveness of the AFPISMC algorithm in controlling the TTIS. Figure 6 shows the step response of the controller in case of white noise (assuming sensor noise) acting on the system output. The actual response of the system still converges to the reference signal in a finite time.

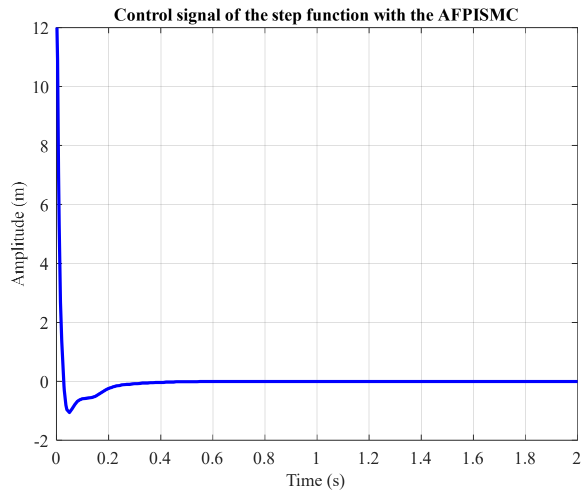

Figure 5 Control signal of the step function with AFPISMC.

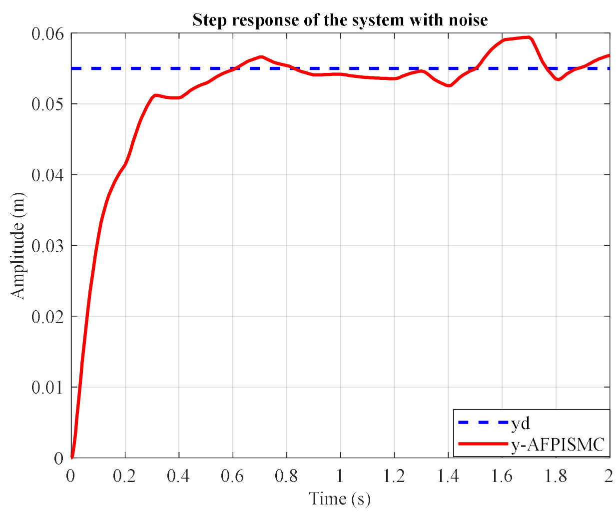

Figure 6 The step response of the controller with white noise.

The sine response and error of the system with AFPISMC controller are shown in Figure 7. The actual response of the system from Figure 7 follows the reference with an error approaching 0. Figure 8 shows the result of the $f(x)$ function approximation using fuzzy logic and the control signal with sine input. The fuzzy logic can approximate the function $f(x)$ as well, with the error converging to zero.

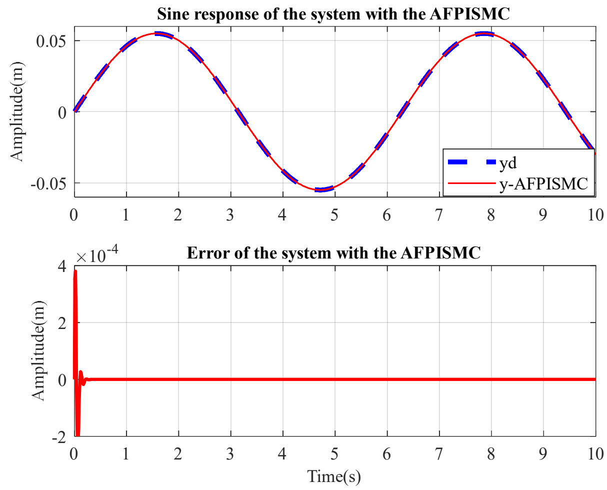

Figure 7 The sine response and error of the system with AFPISMC.

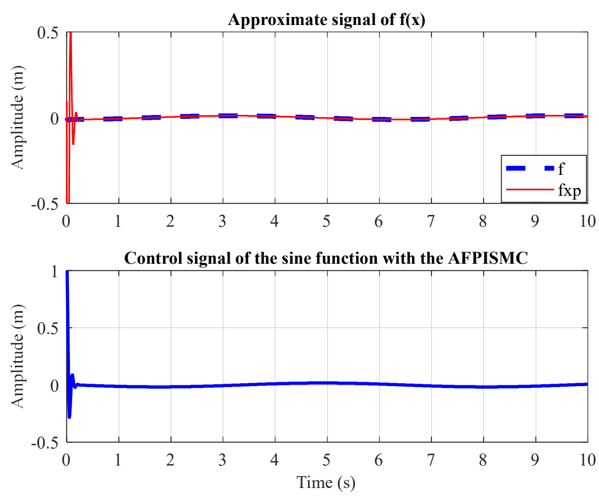

Figure 8 The result of the function *f(x)* approximation and the control signal with sine input.

Figure 9 shows the response to the sine input of the proposed controller in the case of white noise affecting the system output. The actual response of the system still converged to the reference signal in a finite time.

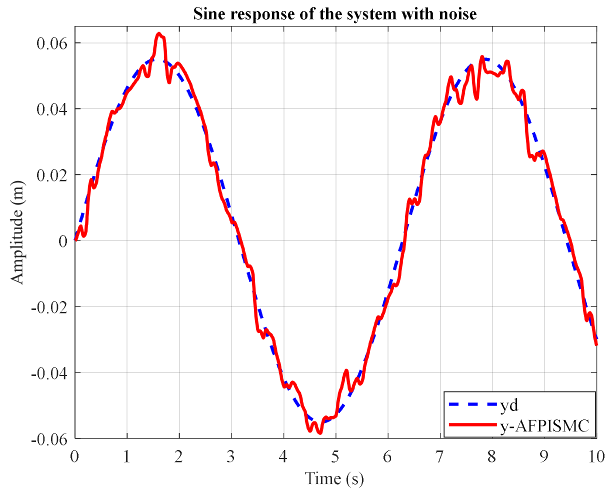

Figure 9 The sine response of the controller with white noise.

The results presented above show the effectiveness of the proposed control method in liquid level control of the TTIS.

5 Conclusions

This study designed an adaptive fuzzy PI sliding mode controller to control the liquid level of two tank interacting systems. The PI sliding mode control ensures that the actual liquid level reaches the desired level in a finite time and reduces chattering around the sliding surface. The fuzzy inference system is effective with the approximation error converging to zero, and the adaptive law ensuring that the real responses track to the reference signal with different inputs. The simulation results in MATLAB/Simulink showed the effectiveness of the proposed control method. The achieved quality criteria of the controller used in this study were better than a fuzzy control, a sliding mode control with condition integrals, a fuzzy-PID control, and a traditional PID control. However, the values of alpha, lambda and gamma in the control law were still chosen by using a trial-and-error method. Also, the fuzzy logic needs an enormous amount of fuzzy rules to control complicated and large-scale systems without the confirmation of closed-loop stability.

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