# Flat beams and application to the mass separation of radioactive beams

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# FLAT BEAMS AND APPLICATION TO THE MASS SEPARATION OF RADIOACTIVE BEAMS

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*Abstract*

The notion of a "flat beam" is now well established and has been proven theoretically and experimentally with applications for linear colliders. In this paper, we propose a new and simple demonstration of the "flat beam theorem", and a possible application in the frame of radioactive ion beam (RIB) production. It consists of using a magnetized multi-specie heavy ion beam extracted from a high frequency ECR source, decoupling the transverse phase planes in such a way as to obtain a very small horizontal emittance, and using a dipole to separate the isotopes. A design of such a transport and separation line is proposed and commented upon.

## INTRODUCTION

The transformation of a round beam, with two equal transverse emittances, into a "flat beam", with one emittance drastically decreased, was noted by Hagedoorn [1] and Derbenev [2]. This transformation is possible if the beam is created inside a solenoid and extracted with some increase of energy. In such a case, the emittances are a combination of the initial thermal emittance created in the plasma and an (x,y') and (y,x') coupling created by the solenoid fringe field. Then a suitable decoupling operates the "round-to-flat beam" transformation.

We propose a new application using a heavy ion beam from an ECR source associated with a mass separator.

## SOME PROPERTIES OF ECR SOURCES

ECR source are used worldwide to ionise either stable or radioactive atoms. In all cases it is convenient to obtain as pure a beam as possible: these ECR sources are generally used in association with a mass separator consisting of a bending magnet and slits for selection of the desired specie.

For stable ion beams, the mass resolution needed is generally about 150, but for RIB applications a resolution of several 1000s is necessary to separate pollutants.

In order to increase the beam intensity, the RF frequency of heavy ion ECR sources is often chosen to be very high (18, 28 or 60Mhz): the consequence is a possible increase of the transverse emittance, due to the very high solenoidal magnetic field as mentioned in [3] and measured in [4]. (However, this increase can be limited in the case of high charge states, at least for some ECR source implementations). We can profit from this beam magnetisation to make the mass separation easier.

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## THE FLAT BEAM THEOREM

A pedagogical explanation of the flat beam effect can be found for example in K.J.Kim [5] and related papers.

The demonstration of the theorem uses two invariant properties of symplectic transformations: one is the 4D phase space volume; the other one is very difficult to establish mathematically [6].

Here is a new very simple demonstration of this theorem. Let's consider a 4D beam matrix corresponding to an angular-momentum-dominated beam:

$$\sigma_0 = \begin{bmatrix} \sigma_{11} & \sigma_{12} & 0 & au \\ \sigma_{21} & \sigma_{22} & -au & 0 \\ 0 & -au & \sigma_{11} & \sigma_{12} \\ au & 0 & \sigma_{21} & \sigma_{22} \end{bmatrix} \qquad (1)$$

$$= \varepsilon_{tot} \begin{bmatrix} \beta & -\alpha & 0 & \sqrt{1-\lambda^2} \\ -\alpha & \gamma & -\sqrt{1-\lambda^2} & 0 \\ 0 & -\sqrt{1-\lambda^2} & \beta & -\alpha \\ \sqrt{1-\lambda^2} & 0 & -\alpha & \gamma \end{bmatrix}$$

$$\sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21} = \varepsilon_{tot}^2 = \varepsilon_{th}^2 + a^2u^2$$

$$u = \frac{\beta}{1+\alpha^2}\varepsilon_{tot}$$

$$a = \frac{qB}{2p_0} = \frac{\sqrt{1-\lambda^2}\left(1+\alpha^2\right)}{\beta}$$

$$\lambda = \frac{\varepsilon_{th}}{\varepsilon_{tot}} \quad ; \quad 0 \le \lambda \le 1$$

$B$ is the ECR solenoidal field, ($q$, $m_0$) is the ion specie with momentum $p_0$ at the ECR exit and $\lambda$ is the ratio between the thermal emittance generated inside the ECR plasma and the total emittance after magnetisation.

Let's now consider *any* beam transfer line composed of a succession of symplectic elements. Such a line can always be described by a unique 4D symplectic matrix $T$ which satisfies:

$$T^t JT = J \quad ; \quad J = \begin{bmatrix} J_2 & 0 \\ 0 & J_2 \end{bmatrix} \quad ; \quad J_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Suppose that the line is able to decouple the two phase planes of the beam matrix $\sigma_0$. We don't know yet if such a transfer line exists, but here we just want to know the properties of the final emittances $\varepsilon_1$ and $\varepsilon_2$. We obtain:

$$\sigma = T\sigma_0 T^t = \begin{bmatrix} s_{11} & 0 & 0 & 0 \\ 0 & s_{22} & 0 & 0 \\ 0 & 0 & s_{33} & 0 \\ 0 & 0 & 0 & s_{44} \end{bmatrix} \quad (2)$$

$$\varepsilon_{xx'}^2 = s_{11}s_{22}$$
$$\varepsilon_{yy'}^2 = s_{33}s_{44}$$

Here is the key point: for any real number $\mu$, and using the fact that the determinant of a symplectic matrix is equal to 1, we obtain:

$$T(\sigma_0 - \mu J)T^t = \sigma - \mu TJT^t = \sigma - \mu J$$
$$\det(\sigma_0 - \mu J) = \det(\sigma - \mu J)$$

This eliminates the matrix $T$ and gives two polynomials in $\mu$. By calculating all the coefficients, we deduce:

$$\varepsilon_{xx'}\varepsilon_{yy'} = \varepsilon_{th}^2$$
$$\varepsilon_{xx',yy'} = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21}} \pm au = \varepsilon_{tot}\left(1 \pm \sqrt{1-\lambda^2}\right)$$

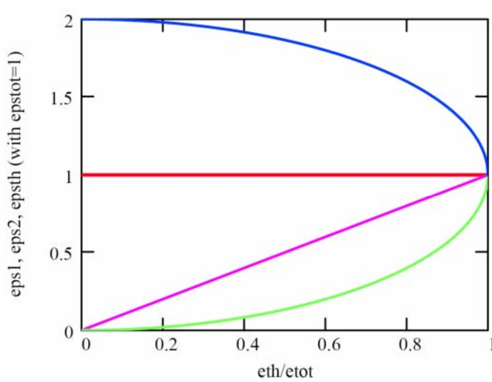


Figure 1: Flat beam emittances as function of $\lambda$

The figure 1 shows that the emittance $\varepsilon_1$ (green) is smaller than $\varepsilon_{tot}$ (red) and even smaller than the thermal emittance $\varepsilon_{th}$ (pink), while $\varepsilon_2$ (blue) remains smaller than twice the initial emittance.

In practice, we can indeed *construct* such a beam transfer line which decouples the two phase planes, by using a set of 3 or 4 skew quadrupoles.

## USING FLAT BEAMS FOR RIBS

Consider now an ECR source associated with a mass separation beam line composed as follows:

- A high frequency ECR source
- A set of 3 skew quadrupoles giving the round-to-flat beam transformation
- A set of 4 quadrupoles to match the beam to the dipole by creating the object point
- A double-focusing dipole with associated hexapoles to minimize the non linearities
- A slit system for mass selection at the image point
- An optional set of one solenoid and 3 skew quadrupoles to re-establish equal emittances. (This part is optional because it can be useful for some physics experiments to use a flat beam direcly)

As an example, let's consider a 90° double-focusing dipole, with radius $\rho$ and magnetic edge angle 26.55°. If the beam is tuned in such a way that the image and object points are both located at a distance of $2\rho$, with a maximum envelope inside the dipole of $x_{max}$, the theoretical resolution is given by:

$$R_{es} = \frac{\sqrt{2}}{4}\frac{x_{max}}{\varepsilon_{xx'}} \approx \frac{\sqrt{2}}{4}\frac{x_{max}}{\varepsilon_{tot}}\frac{2}{\lambda^2}$$

The resolution is inversely proportional to the emittance, which emphasizes clearly the usefulness of the flat beam for RIB mass separation: for a ratio of 20% between thermal and total emittances, $R_{es}$ is multiplied by 50.

## BEAM DYNAMICS EXAMPLE

We consider the case of an ion specie with $q/A$=1/6, with the following initial beam matrix:

$$\sigma_0 = 194.\begin{bmatrix} 0.162 & 0 & 0 & 0.9796 \\ 0 & 6.173 & -0.9796 & 0 \\ 0 & -0.9796 & 0.162 & 0 \\ 0.9796 & 0 & 0 & 6.173 \end{bmatrix}$$

This corresponds to a reasonable ratio $\lambda$ = 0.2, with a moderate magnetic field of 1 Tesla. The source extraction voltage is 60kV ($\beta_c$=0.00463) and the effective radius of the beam is 5.6mm. For the simulation, a Gaussian distribution truncated to $3\sigma$ is used, with an emittance of 0.1π.mm.mrad RMS normalised. The transfer line is the one mentioned above, with a dipole magnet radius of 1m. Matching and calculations are performed with the code TRACEWIN [7].

We see in figure 2 the objects and image points obtained at the distance $2\rho$ from the dipole. Moreover, the vertical beam envelope is kept small.

Figure 3 shows the evolution of both emittances, the horizontal emittance being decreased before the mass selection, and we can check that:

$$\frac{\varepsilon_{yy'}}{\varepsilon_{xx'}} = \frac{1+\sqrt{1-\lambda^2}}{1-\sqrt{1-\lambda^2}} \approx \frac{4}{\lambda^2} = 100$$
$$\frac{\varepsilon_{tot}}{\varepsilon_{xx'}} = \frac{1}{1-\sqrt{1-\lambda^2}} \approx \frac{2}{\lambda^2} = 50$$

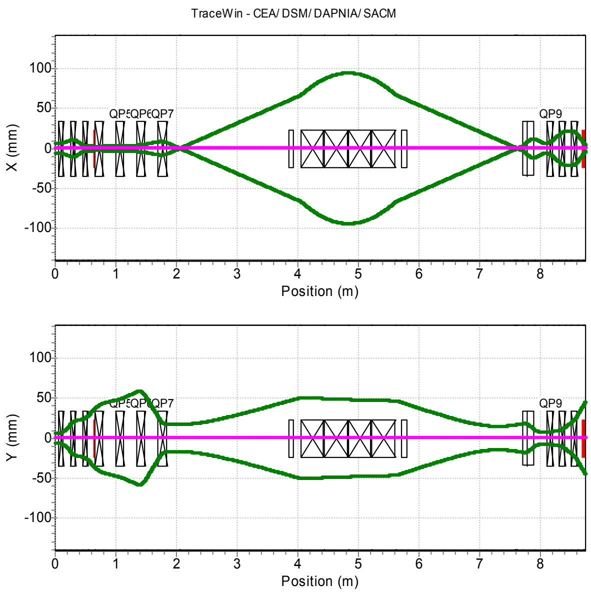


Figure 2: x and y beam envelopes with flat beam operation.

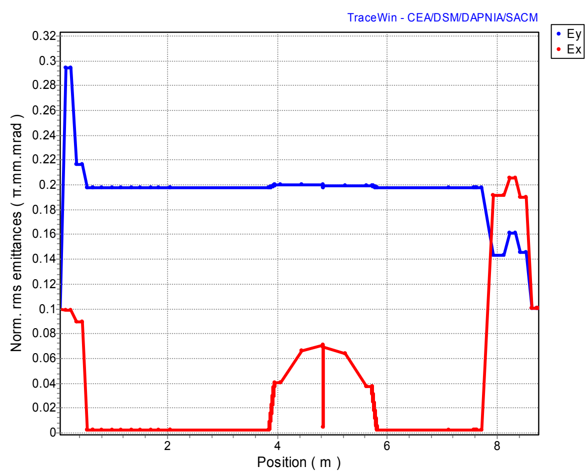


Figure 3: $\varepsilon_{x,x'}$ and $\varepsilon_{yy'}$ rms-norm emittance evolution

Figure 4 shows that a reasonable total slit aperture of 0.15mm is necessary at the image point, for an elimination of 99% of the closest pollutant, with 90% of the beam of interest accepted. This corresponds to the theoretical formula:

$$x_{slit} = \pm \frac{4\rho}{\sqrt{2}} \frac{\varepsilon_{tot}}{x_{\max}} \frac{\lambda^2}{2}$$

We take into account the non-linearities of the dipoles which are minimised by using 3 hexapoles. The resolution obtained is approximately 10000.

This resolution could be increased by considering a greater ECR magnetic field and by increasing the radius and pole width of the separator. We could also choose a 120° dipole instead of 90°.

Building such a transfer line could be challenging: in particular, the source extraction voltage and the magnetic fields should be very stable. The quadrupole rotation angles are less critical: rotation errors lower than 0.2° do not damage the final resolution.

We note also that the formula giving the theoretical resolution can be expressed as follows:

$$R_{es} \approx \frac{\sqrt{2}}{4} \frac{x_{\max}}{\varepsilon_{tot}} \frac{2}{\lambda^2} \approx K\, x_{\max} B \beta_c ,$$

where $K$ is a constant depending only upon the plasma temperature $kT$, $B$ is the ECR source magnetic field and $\beta_c$ is the relativistic factor.

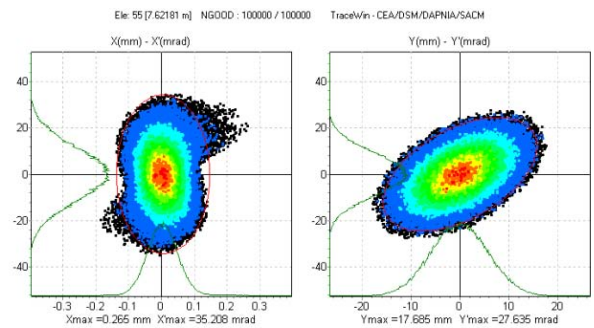


Figure 4: Transverse emittance portraits at the image point (with very different x and y scales)

## CONLUSION

We have shown that the flat beam transformation could present an interest for radioactive ion beam applications. Although the flat beam effect has already been demonstrated using electron beams, a bench test with a heavy ion ECR source could be very promising.

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