# Vanishing of non-Abelian Berry phases in heavy solitonic baryons

H.K. Lee, M. Rho

*February 8, 1993*

# VANISHING OF NONABELIAN BERRY PHASES IN HEAVY SOLITONIC BARYONS

Hyun Kyu Lee* and Mannque Rho

*Service de Physique Théorique*
*C.E. Saclay*
*F-91191 Gif-sur-Yvette, France*

## ABSTRACT

In this article, we show – using a reasoning applicable to both the excitation spectrum in the chiral bag model and the hyperfine structure of diatomic molecules – that the generic form of a nonabelian Berry potential appears in heavy quark effective theory (HQET) and that the Berry potential vanishes for the soliton-heavy meson bound state in the heavy-quark limit. The vanishing of the Berry potential in HQET is shown to be related to the restoration of heavy-quark symmetry in infinite heavy-quark-mass limit in close analogy to diatomic molecules at infinite internuclear separation.

*Permanent address: Department of Physics, Hanyang University, Seoul 133-791, Korea. Supported in part by the KOSEF under Grant No.91-08-00-04 and by Ministry of Education (BSRI-92-231)

# 1 Introduction

In a series of recent papers[1], it has been shown that Berry structures[2] can appear naturally in hadron physics in the framework of the chiral bag model. More recently, the Berry structure was also identified in the spectrum of heavy-quark baryons[3, 4], providing an understanding of heavy-quark symmetry [5] in terms of a "symmetry restoration." In particular, it was shown in [3, 4] that when a baryon with a heavy quark is described in terms of the binding of a heavy meson with an $SU(2)$ soliton à la Callan and Klebanov (CK) [6], as the mass of the heavy quark increases, the Wess-Zumino term binding of Callan and Klebanov ceases to be operative as the Wess-Zumino term disappears but the binding survives with, however, the hyperfine coefficient $c$ that gives the splitting between, say, $\Sigma$ and $\Sigma^*$ vanishing as $1/m_H$ where $m_H$ is the heavy-meson mass.

In this paper, we present a simple argument that explains how a Berry structure[2] emerges in heavy quark effective theory[5] and how the Berry potential vanishes in the heavy-quark limit using the quantum mechanical binding mechanism of Manohar et al.[7]. We will use a reasoning completely analogous to that employed for the chiral bag[1] on the one hand and that used for the restoration of electronic rotational invariance in diatomic molecules [8] on the other. We will see that this provides yet another demonstration in support of previous arguments of Refs.[3, 4] that a generic Berry structure is playing a key role in the heavy-baryon structure.

# 2 Diatomic Molecules

In order to better understand the complex situation of strongly interacting systems that we are interested in, we first discuss a generic case of quantum mechanical system, namely the diatomic molecule. To do this, we begin by recalling here a well-known fact [9] that the conservation laws of a system coupled to a symmetric background gauge field persist in modified forms. The simplest example is the angular momentum of a system coupled to a Dirac ($U(1)$) magnetic monopole. Here the conserved angular momentum is modified to a sum of the usual mechanical angular momentum and a contribution from the magnetic field[10]. A more interesting phenomenon related to this observation is "spin-isospin" transmutation [11].

## 2.1 Non-abelian Berry Potential

As shown by Berry [2], in a quantum system, induced gauge fields naturally appear in

the space of slow variables when the fast variables are integrated out. They are referred to in the literature as Berry potentials. The Schrödinger equation resulting after fast variables are integrated out is given by the following form,

$$-\frac{1}{2m}(\vec{\nabla}_R - i\vec{\mathbf{A}})^2\mathbf{\Psi} = i\frac{\partial \mathbf{\Psi}}{\partial t} \tag{1}$$

where $\vec{\mathbf{A}}$ is defined by

$$\vec{\mathbf{A}}_{a,b} = i\langle a, \vec{R}|\vec{\nabla}|b, \vec{R}\rangle. \tag{2}$$

$|a, \vec{R}\rangle$ is a 'snap-shot' eigenstate for a given *slow variable* $\vec{R}$, which is related to the reference state $|a\rangle$ by a unitary operator $U(\vec{R})$ such that

$$|a, \vec{R}\rangle = U(\vec{R})|a\rangle. \tag{3}$$

Equation (1) is a matrix equation where $\mathbf{\Psi}$ is a column vector defined in a vector space described by $|a\rangle$.

It is convenient to introduce, following [13], a Grassmann variable $\theta_a$ for each $|a\rangle$ (say, an electronic state) and rewrite the Schrödinger equation Eq.(1) as

$$-\frac{1}{2m}(\vec{\nabla}_R - i\vec{\mathbf{A}}(\theta, \theta^\dagger, \vec{R}))^2\psi(\theta, \vec{R}) = i\frac{\partial \psi(\theta, \vec{R})}{\partial t}. \tag{4}$$

In the above equation, internal degrees of freedom are considered to be dynamical degrees of freedom treated classically in the form of anticommuting coordinates. Equation (4) can be obtained by quantizing the system described by the following Lagrangian

$$\mathcal{L} = \frac{1}{2}m\dot{\vec{R}}^2 + i\theta_a^\dagger(\frac{\partial}{\partial t} - i\vec{A}^\alpha T^\alpha_{ab}\cdot\dot{\vec{R}})\theta_b \tag{5}$$

where $\mathbf{T}^\alpha$ is a matrix representation in the vector space of $|a\rangle$'s for a generator $\mathcal{T}^\alpha$ of $U(\vec{R})$,

$$[\mathbf{T}^\alpha, \mathbf{T}^\beta] = if^{\alpha\beta\gamma}\mathbf{T}^\gamma. \tag{6}$$

Following the standard quantization procedure [13, 14], we obtain the following commutation relations,

$$[R_i, p_j] = i\delta_{ij}, \qquad \{\theta_a, \theta_b^\dagger\} = i\delta_{ab}. \tag{7}$$

It is then straightforward to obtain the Hamiltonian

$$H = \frac{1}{2m}(\vec{p} - \vec{\mathbf{A}})^2 \tag{8}$$

where

$$\begin{aligned} \vec{\mathbf{A}} &= \vec{A}^{\alpha} I^{\alpha}, \\ I^{\alpha} &= -i\theta_a^{\dagger} T_{ab}^{\alpha} \theta_b. \end{aligned} \tag{9}$$

Using the commutation relations, it can be verified that

$$[I^{\alpha}, I^{\beta}] = i f^{\alpha\beta\gamma} I^{\gamma}. \tag{10}$$

The Schrödinger equation

$$H\psi = i\frac{\partial \psi}{\partial t}, \tag{11}$$

with Eq.(8) leads to Eq.(4). It is clear that the Lagrangian, Eq.(5), is invariant under the gauge transformation

$$\vec{A}^{\alpha} \rightarrow \vec{A}^{\alpha} + f^{\alpha\beta\gamma}\Lambda^{\beta}\vec{A}^{\gamma} - \vec{\nabla}\Lambda^{\alpha}, \tag{12}$$

$$\theta_a \rightarrow \theta_a - i\Lambda^{\alpha} T_{ab}^{\alpha} \theta_b. \tag{13}$$

It should be noted that Eq.(13) corresponds to the gauge transformation on $|a\rangle$. This makes gauge invariance manifest in the Lagrangian. We should also point out that the Lagrangian (5) resembles closely *both* the Lagrangian obtained in [1] for the chiral bag model of baryon structure when the sea quarks are integrated out *and* the Lagrangian that emerges in heavy-quark effective theory discussed below.

### 2.2 Conserved angular momentum

Consider a particle coupled to an external gauge field of 't Hooft -Polyakov monopole[12] with a coupling constant $g$. The asymptotic form of the magnetic field is given by

$$\vec{\mathbf{B}} = -\frac{\hat{r}(\hat{r}\cdot\mathbf{T})}{gr^2} \tag{14}$$

which is obtained from the gauge field $\vec{\mathbf{A}}$

$$A_i^{\alpha} = \epsilon_{\alpha ij}\frac{r_j}{gr^2}, \tag{15}$$

$$\vec{\mathbf{B}} = \vec{\nabla}\times\vec{\mathbf{A}} - ig[\vec{\mathbf{A}}, \vec{\mathbf{A}}]. \tag{16}$$

Using the convention described in the previous section, the Hamiltonian of a particle coupled to a 't Hooft-Polyakov monopole can be written as

$$
\begin{aligned}
H &= \frac{1}{2m}(\vec{p} - \vec{\mathbf{A}})^2 \\
&= \frac{1}{2m}\vec{\mathbf{D}} \cdot \vec{\mathbf{D}}
\end{aligned} \tag{17}
$$

where $\vec{\mathbf{D}} = \vec{p} - \vec{\mathbf{A}}$. Here and in what follows we put $g = 1$ for close analogy with Eq.(8) Obviously the mechanical angular momentum $\vec{L}_m$ of a particle

$$
\vec{L}_m = m\vec{r} \times \dot{\vec{r}} = \vec{r} \times \vec{\mathbf{D}} \tag{18}
$$

does not satisfy the $SU(2)$ algebra after canonical quantization in Eq.(7) and moreover cannot be a symmetric operator that commutes with the Hamiltonian. Of course the conventional angular momentum, $\vec{L}_o = \vec{r} \times \vec{p}$, satisfies the usual angular momentum commutation rule, but it does not commute with the Hamiltonian and hence cannot be a conserved angular momentum of the system. This shows that the construction of a conserved angular momentum of a system coupled to a topologically nontrivial gauge field is not a trivial matter.

To construct the conserved angular momentum, we have to modify $\vec{L}_m$ to

$$
\vec{L} = \vec{L}_m + \vec{\mathbf{Q}}, \tag{19}
$$

with $\vec{\mathbf{Q}} = \vec{Q}^\alpha I^\alpha$ to be determined. The methods to determine $\vec{\mathbf{Q}}$ have been discussed in the literature [9, 10]. Here we adopt a rather straightforward method. The first condition required for $\vec{\mathbf{Q}}$ is the consistency condition that $\vec{L}$ satisfy the $SU(2)$ algebra

$$
[L_i, L_j] = i\epsilon_{ijk} L_k. \tag{20}
$$

This leads to an equation for $\vec{\mathbf{Q}}$,

$$
\vec{r}(\vec{r} \cdot \vec{\mathbf{B}}) + \vec{r}\vec{\mathcal{D}} \cdot \vec{\mathbf{Q}} - \vec{\mathcal{D}}(\vec{r} \cdot \vec{\mathbf{Q}}) = 0 \tag{21}
$$

where

$$
\vec{\mathcal{D}} = \vec{\nabla} - i[\vec{\mathbf{A}}, \quad ]. \tag{22}
$$

The second condition is obtained by requiring that $\vec{L}$ commute with H,

$$
[\vec{L}, H] = 0. \tag{23}
$$

Equation (23) can be replaced by a stronger condition

$$
[L_i, \mathbf{D}_j] = i\epsilon_{ijk} \mathbf{D}_k, \tag{24}
$$

which leads to

$$\mathcal{D}_i \mathbf{Q}_j + \delta_{ij} \vec{r} \cdot \vec{\mathbf{B}} - r_i \mathbf{B}_j = 0. \tag{25}$$

It is obvious that $\vec{L}$ satisfying Eq.(24) or (25) commutes with the Hamiltonian Eq.(17). Equation (25) is just the condition for "spherically symmetric potential" discussed by Jackiw[15]. Here we can verify it in a more straightforward way using Eq.(24). Moreover the meaning of spherical symmetry becomes clear from Eq.(23).

In the case of the 't Hooft-Polyakov monopole, Eqs.(14) and (15), it can be shown that

$$\vec{\mathbf{Q}} = \hat{r}(\hat{r} \cdot \mathbf{I}) \tag{26}$$

satisfies Eqs. (21) and (25). After inserting Eq.(26) into Eq.(19), we get

$$\begin{aligned} \vec{L} &= \vec{L}_m + \hat{r}(\hat{r} \cdot \mathbf{I}) && (27) \\ &= \vec{r} \times \vec{p} + \vec{I}, && (28) \end{aligned}$$

where

$$I_i = \delta_{i\alpha} I^\alpha. \tag{29}$$

Equation (28) with (29) shows clearly how the isospin-spin transmutation takes place in a system where a particle is coupled to a nonabelian monopole.

The same analysis can be applied to the abelian $U(1)$ monopole just by replacing $\hat{r} \cdot \vec{I}$ by -1 in Eqs.(26) and (27): We are considering a Dirac monopole with $e = g = 1$. Then

$$\begin{aligned} \vec{Q} &= \hat{r}, && (30) \\ \vec{L} &= m\vec{r} \times \dot{\vec{r}} - \hat{r}. && (31) \end{aligned}$$

One can rewrite Eq.(31) in a more familiar form seen in the literature

$$\vec{L} = \vec{r} \times \vec{p} - \vec{\Sigma}, \tag{32}$$

where

$$\vec{\Sigma} = \left( \frac{(1 - \cos\theta)}{\sin\theta} \cos\phi,\ \frac{(1 - \cos\theta)}{\sin\theta} \sin\phi,\ 1 \right). \tag{33}$$

### 2.3 Rotational symmetry of nonabelian Berry potential

So far our consideration has been rather general. Let us now focus on conserved angular momentum in a diatomic molecule in which a Berry potential couples to the dynamics of slow degrees of freedom, corresponding to the nuclear coordinate $\vec{R}$. This system has been studied by Zygelman[8].

The Berry potential is defined on the space spanned by the electronic states $\pi(|\Lambda| = 1)$ and $\Sigma(\Lambda = 0)$, where $\Lambda$'s are eigenvalues of the third component of the orbital angular momentum of the electronic states. The electronic states responding to the slow rotation $U(\vec{R})$ of $\vec{R}$ defined by

$$U(\vec{R}) = \exp(-i\phi L_z)\exp(i\theta L_y)\exp(i\phi L_z), \tag{34}$$

induce a Berry potential of the form

$$\begin{aligned} \vec{\mathbf{A}} &= i\langle\Lambda_a|U(\vec{R})\vec{\nabla}U(\vec{R})^\dagger|\Lambda_b\rangle & (35)\\ &= \frac{\mathbf{A}_\theta}{R}\hat{\theta} + \frac{\mathbf{A}_\phi}{R\sin\theta}\hat{\phi}, & (36)\end{aligned}$$

where

$$\begin{aligned} \mathbf{A}_\theta &= \kappa(R)(\mathbf{T}_y\cos\phi - \mathbf{T}_x\sin\phi),\\ \mathbf{A}_\phi &= \mathbf{T}_z(\cos\theta - 1) - \kappa(R)\sin\theta(\mathbf{T}_x\cos\phi + \mathbf{T}_y\sin\phi). \end{aligned} \tag{37}$$

Here $\vec{\mathbf{T}}$'s are spin-1 representations of the orbital angular momentum $\vec{L}$ and $\kappa$ measures the transition amplitude between the $\Sigma$ and $\pi$ states

$$\kappa(R) = \frac{1}{\sqrt{2}}|\langle\Sigma|L_x - iL_y|\pi\rangle|. \tag{38}$$

The nonvanishing field strength tensor is given by

$$\vec{\mathbf{B}} = \frac{F_{\theta\phi}}{R^2\sin\theta} = -\frac{(1-\kappa^2)}{R^2}T_z\hat{R}. \tag{39}$$

Following the procedure described in section 2.1, we introduce a Grassmann variable for each electronic state. Replacing $\mathbf{T}$ by $\mathbf{I}$ defined in Eq.(9) and quantizing the corresponding Lagrangian, we obtain the Hamiltonian

$$H = \frac{1}{2\mu}(\vec{p} - \vec{\mathbf{A}})^2, \tag{40}$$

where $\vec{\mathbf{A}} = \vec{A}^\alpha I^\alpha$ and $\mu$ is the reduced mass. The presence of the constant $\kappa$ – which is not quantized – in the Berry potential is a generic feature of nontrivial nonabelian Berry potentials as can be seen in many examples [16, 1].

To find a solution of Eq.(21) and Eq.(25), it is better to look into the Hamiltonian in detail. Exploiting the gauge invariance, the Hamiltonian can be rewritten in the most symmetric form. This can be done by subtracting a trivial (or pure gauge) part out of the Berry potential, which is equivalent to choosing a new gauge such that

$$\vec{\mathbf{A}}' = V^\dagger \vec{\mathbf{A}} V + iV^\dagger \vec{\nabla} V \tag{41}$$

$$\mathbf{F}' = V^\dagger \mathbf{F} V \tag{42}$$

where $V$ is an inverse operation of $U$ in Eq.(34), *i.e.*, $V = U^\dagger$. Then

$$\begin{aligned} \mathbf{A}'_\theta &= (1-\kappa)(I_x \sin\phi - I_y \cos\phi), \\ \mathbf{A}'_\phi &= (1-\kappa)\{-I_z \sin^2\theta + \cos\theta \sin\theta (I_x \cos\phi + I_y \sin\phi)\}, \end{aligned} \tag{43}$$

or more compactly

$$\vec{\mathbf{A}}' = (1-\kappa)\frac{\hat{R}\times\vec{I}}{R^2},$$

and

$$\vec{\mathbf{B}}' = -(1-\kappa^2)\frac{\hat{R}(\hat{R}\cdot\mathbf{I})}{R^2}. \tag{44}$$

A remarkable feature of the above Berry potential is that it has the same structure as the 't Hooft-Polyakov monopole, Eq.(14) and Eq.(15), but with different constant factors, $(1-\kappa)$ for vector potential and $(1-\kappa^2)$ for magnetic field. Because of these two different factors, one cannot simply take Eq.(26) as a solution of (25) for the case of nonabelian Berry potentials.

Using the following identities derived from Eq. (43),

$$\vec{R}\cdot\vec{\mathbf{A}}' = 0, \tag{45}$$

$$\vec{R}\times\vec{\mathbf{A}}' = -(1-\kappa)\{\vec{I} - (\vec{I}\cdot\hat{R})\hat{R}\}, \tag{46}$$

the Hamiltonian, Eq.(40), can be written as

$$H = -\frac{1}{2\mu R^2}\frac{\partial}{\partial R}R^2\frac{\partial}{\partial R} + \frac{1}{2\mu R^2}(\vec{L}_o + (1-\kappa)\vec{I})^2 - \frac{1}{2\mu R^2}(1-\kappa)^2(\vec{I}\cdot\hat{R})^2. \tag{47}$$

One sees from this Hamiltonian that the factor $(1-\kappa)$ controls hyperfine splitting. For this reason we call it "hyperfine coefficient." The corresponding quantity in heavy baryons is denoted $c$ in the later section.

Now one can show that the conserved angular momentum $\vec{L}$ is

$$\vec{L} = \vec{L}_o + \vec{I}, \tag{48}$$

$$= \mu\vec{R} \times \dot{\vec{R}} + \vec{\mathbf{Q}}, \tag{49}$$

with

$$\vec{\mathbf{Q}} = \kappa\vec{I} + (1-\kappa)\hat{R}(\hat{R}\cdot\vec{I}). \tag{50}$$

Hence, in terms of the conserved angular momentum $\vec{L}$, the Hamiltonian becomes

$$H = -\frac{1}{2\mu R^2}\frac{\partial}{\partial R}R^2\frac{\partial}{\partial R} + \frac{1}{2\mu R^2}(\vec{L}-\kappa\vec{I})^2 - \frac{1}{2\mu R^2}(1-\kappa)^2 \tag{51}$$

where $(\vec{I}\cdot\hat{R})^2 = 1$ has been used.

It is interesting to see what happens in the two extreme cases of $\kappa = 0$ and 1. For $\kappa = 0$, the $\Sigma$ and $\pi$ states are completely decoupled and only the $U(1)$ monopole field can be developed on the $\pi$ states[17]. Equation (50) becomes identical to Eq.(30) as $\kappa$ goes to zero and the Hamiltonian can be written as

$$H = -\frac{1}{2\mu R^2}\frac{\partial}{\partial R}R^2\frac{\partial}{\partial R} + \frac{1}{2\mu R^2}(\vec{L}\cdot\vec{L}-1) \tag{52}$$

which is a generic form for a system coupled to an $U(1)$ monopole field. Physically this corresponds to small internuclear distance at which the $\Sigma$ and $\pi$ states decouple.

For $\kappa = 1$, the degenerate $\Sigma$ and $\pi$ states form a representation of the rotation group and hence the Berry potential (and its field tensor) vanishes or becomes a pure gauge. Then $\vec{\mathbf{Q}} = \vec{I}$ and $\vec{L} = \mu\vec{R}\times\dot{\vec{R}} + \vec{I}$. Now $\vec{I}$ can be understood as the angular momentum of the electronic system which is decoupled from the spectrum. One can also understand this as the restoration of rotational symmetry in the electronic system. Physically $\kappa \to 1$ as $R \to \infty$. In the next section, we shall show that the same situation occurs in heavy-quark effective theory where the restoration of the heavy-quark symmetry for $m_Q \to \infty$ is manifested by a vanishing Berry potential.

# 3 Heavy-Quark Baryons

We shall now show that the same generic form of Berry potentials emerges in the spectrum of heavy-quark baryons (containing heavy quarks c, b etc) and that the heavy-quark symmetry discovered in QCD can be identified with the vanishing of nonabelian Berry potentials in the symmetry limit.

### 3.1 Emergence of Berry potentials

Consider the effective Lagrangian with chiral symmetry for light quarks and heavy quark symmetry for heavy quarks [7]

$$\begin{aligned}\mathcal{L} &= -iTr\overline{H}_a v^\mu \partial_\mu H_a + iTr\overline{H}_a H_b v^\mu \left(\Sigma^\dagger \partial_\mu \Sigma\right)_{ba} \\ &\quad + igTr\overline{H}_a H_b \gamma^\mu \gamma^5 \left(\Sigma^\dagger \partial_\mu \Sigma\right)_{ba} + \frac{F_\pi^2}{16} tr\left(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right) + \cdots \end{aligned} \tag{53}$$

which we rewrite in the rest frame of the heavy quark, $v^\mu = (1, 0)$,

$$\begin{aligned}\mathcal{L} &= -iTr\overline{H}_a \partial_t H_a + iTr\overline{H}_a H_b \left(\Sigma^\dagger \partial_t \Sigma\right)_{ba} \\ &\quad + igTr\overline{H}_a H_b \gamma^\mu \gamma^5 \left(\Sigma^\dagger \partial_\mu \Sigma\right)_{ba} + \frac{F_\pi^2}{16} Tr\left(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right) + \cdots \quad . \end{aligned} \tag{54}$$

Here $\Sigma$ is the usual chiral $SU(2)$ field, $H$ the heavy-meson field with the quark configuration $Q\bar{q}$ (where $Q$ denotes a heavy quark and $q$ a light quark) consisting of the pseudoscalar $P$ and the vector $P_\mu^*$ and $F_\pi$ the pion decay constant $\approx 186$ MeV. Throughout we shall follow the notation of [7].

To see how Berry potentials arise, we must identify "fast" and "slow" variables (or degrees of freedom) in the theory. We take the light anti-quark of a heavy meson as a "fast" variable to be "integrated out" in the presence of a slowly rotating soliton background constructed in the light meson sector which represents a "slow" variable, while the heavy quark is "integrated in" as a spectator. This can be visualized by assuming that the soliton-heavy meson bound system is composed of a heavy quark $Q$ sitting at the origin and a light antiquark in the heavy meson $H$, $\bar{q}_H$, moving in the background of a slowly rotating soliton. The heavy quark then is effectively decoupled from the soliton, so it is a spectator in the limit $m_Q \to \infty$. Therefore, as far as the rotation motion of the soliton is concerned, the effective degree of freedom is the light antiquark in the heavy meson that couples to the soliton. Thus one can take the "fast" variable in this system to be the light antiquark $\bar{q}_H$ instead of the whole heavy meson itself as in the CK picture used in [3, 4]. It is a "fast" variable in the sense that the excitation due to the motion of the slow variable (that is to say, the adiabatic rotation of the soliton) is of order $1/N_c$ while the energy splitting of the soliton-antiquark system is of order $N_c^0$ which depends on the grand spin $K$ as explained below. This means that the adiabatic theorem – which states that the fast degree of freedom continues to remain on its snapshot eigenstate up to a phase – can be used to obtain a Berry potential defined in the space of the slow variable. As we shall show below, there is a close analogy to the chiral bag case [1] where a quark (fast variable) moves in the background

field of a rotating soliton (slow variable). We shall exploit this analogy in simplifying our argument.

The next step in our argument is to find the spectrum of the "fast" variable – $H$ – in the solitonic background *before* rotating the soliton. The relevant part of the Lagrangian involving the $H$ field can be obtained by replacing $\Sigma$ by the time-independent soliton (hedgehog) field $\Sigma_o$ in Eq.(54),

$$\mathcal{L}_H = -iTr\overline{H}_a\partial_t H_a + igTr\overline{H}_a H_b\gamma^i\gamma^5\left(\Sigma_o^\dagger\partial_i\Sigma_o\right)_{ba} + \cdots \quad . \tag{55}$$

The second term of (55) is an interaction term which can be written in terms of the isospin $I_H$ and the angular momentum $S_l$ of the light degree of freedom, $\bar{q}_H$, in the heavy meson[7, 18]

$$H_I = V_I(\Sigma_o)\left(\vec{I}_H\cdot\vec{S}_l\right). \tag{56}$$

Hence the energy eigenstates are classified by the $K$-spin of $\bar{q}_H$, $K = I_H + S_l$ (recall that the hedgehog $K$-spin is zero) with an energy splitting $\Delta E = V_I^K$, which is of order $N_c^0$. The detailed calculations of $V_I(\Sigma_o)$ will be discussed later. What we wish to point out here is that the interaction Hamiltonian does not "see" the orbital angular momentum of $\bar{q}_H$. Therefore the orbital angular momentum is separately conserved.

We now rotate the chiral field with the slow variable $S(t)$

$$\Sigma = S(t)\Sigma_o S^\dagger(t) \tag{57}$$

in Eq.(54) but leave unrotated the heavy-meson field $H$ as proposed by Manohar et al.[7]. This contrasts with the quantization procedure of [3, 4] where the $H$ field is also rotated. The difference is just a matter of choosing frames, the former corresponding to the rotating inertial frame and the latter to the heavy-meson rest frame. Now with Eq. (57) and Eq.(54), the Lagrangian can be written in the form

$$\begin{aligned}\mathcal{L}_H = & -iTr\overline{H}\partial_t H \\ & +iTr\overline{H}_a H_b\left(\Sigma^\dagger\partial_t\Sigma\right)_{ba} \\ & +igTr\overline{H}_a H_b\gamma^0\gamma^5\left(\Sigma^\dagger\partial_t\Sigma\right)_{ba} \\ & +igTr\overline{H}_a H_b\gamma^i\gamma^5\left(S^\dagger(\Sigma_o^\dagger\partial_i\Sigma_o)S\right)_{ba}.\end{aligned} \tag{58}$$

If we assume that the bound state is formed at the origin of the soliton as discussed by Guralnik et al.[7], then the second and third terms of Eq. (58) vanish. This can be seen by the fact that

$$\begin{aligned}\Sigma^\dagger(0)\partial_t\Sigma(0) &= S\Sigma_0^\dagger(0)S^\dagger\partial_t S\Sigma_0(0)S^\dagger + \partial_t S^\dagger S \\ &= S^\dagger\partial_t S + \partial_t S^\dagger S = 0\end{aligned} \tag{59}$$

where the value of $\Sigma_0$ at $r = 0$, $\Sigma_0(0) = 1$, has been used. Then from the reduced Lagrangian

$$\begin{aligned}\mathcal{L}_H &= -iTr\overline{H}\partial_t H \\ &\quad +igTr\overline{H}_a H_b \gamma^i \gamma^5 \left(S^\dagger(\Sigma_o^\dagger \partial_i \Sigma_o)S\right)_{ba},\end{aligned} \tag{60}$$

the interaction Hamiltonian can be obtained as

$$H_I(t) = SH_I S^\dagger. \tag{61}$$

It seems natural to take $S(t)|K\rangle$ to be the snapshot eigenstate at t, $\Psi_K(t)$, where $|K\rangle$ is an energy eigenstate of the unrotating hedgehog soliton (*i.e.*, S(0)=1). The energy eigenstate of $H_I(t)$ evolves accordingly along the path determined by the snapshot eigenstate $\Psi_K(t)$ up to a phase, generating a Berry potential in Born-Oppenheimer approximation. The Berry potential can then be calculated in a standard way

$$\begin{aligned}\mathcal{A} &= i\langle KS^\dagger|\partial|SK\rangle \\ &= i\langle K|(S^\dagger \partial S)|K\rangle.\end{aligned} \tag{62}$$

The key feature of this procedure is that the Berry potential can be calculated in a reference state $|K\rangle$, which measures in fact how the $K$-states defined at t=0 get mixed during the adiabatic rotation. This also shows clearly that $\mathcal{A}$ is an induced gauge potential coupled to a slow variable which is the rotation of the soliton.

An equivalent but more instructive way to exhibit the Berry structure is to redefine the $H$ field in such a way that the rotation matrix $S$ acts on the heavy meson field rather than on the soliton. Let

$$H' = HS^\dagger, \quad \overline{H}' = S\overline{H}. \tag{63}$$

Now the action – which is a quantum mechanical Lagrangian for the slow variable $S(t)$ – becomes (dropping the prime of $H'$)

$$\begin{aligned}L &= \int d^3x \left[-i\mathrm{Tr}\overline{H}\partial_t H - i\mathrm{Tr}H(\partial_t S^\dagger S)\overline{H} + ig\mathrm{Tr}\overline{H}_a H_b \gamma^i\gamma^5 \left(\Sigma_o^\dagger \partial_i \Sigma_o\right)_{ba}\right] \\ &\quad + \frac{\mathcal{I}}{4}\mathrm{Tr}(S^\dagger \partial_t S)^2 + \cdots\end{aligned} \tag{64}$$

where we have now restored the kinetic energy term for the slow degree of freedom with $\mathcal{I}$ the moment of inertia of the rotating soliton. The third term of (64) involves no time derivative and hence is a potential which we denoted above as $(-H_I)$ on which we will have more to say below. The second term can be identified as the Berry potential which gives

the same $\mathcal{A}$ as in Eq. (62) when projected onto the $K$ state. That this identification is sensible can be seen by making an analogy to the chiral bag. In the case of the chiral bag discussed in [1], the adiabatic change of the skyrmion, $S(t)$, is incorporated in the following Lagrangian,

$$L^{CB} = \int_V d^3x \left[\overline{\psi} i\gamma^\mu \partial_\mu \psi - \frac{1}{2}\Delta_s \, \overline{\psi}\, S\, e^{i\gamma_5 \vec{\tau}\cdot\hat{r}F(r)}\, S^\dagger \, \psi\right] + \frac{\mathcal{I}}{4}\mathrm{Tr}(S^\dagger i\partial_t S)^2 \tag{65}$$

where $\psi$ is the confined light-quark field coupled to the rotating soliton field at the bag surface. This is the analog to Eq.(58) (plus the rotator kinetic energy term). Doing the field redefinition $\psi \to S\psi$, we have

$$L^{CB} = \int d^3x \left[\overline{\psi} i\gamma^\mu \partial_\mu \psi + \psi^\dagger S^\dagger i\partial_t S\, \psi \, - \frac{1}{2}\Delta_s \, \overline{\psi}\, e^{i\gamma_5 \vec{\tau}\cdot\hat{r}F(r)}\, \psi\right] + \frac{\mathcal{I}}{4}\mathrm{Tr}(S^\dagger i\partial_t S)^2. \tag{66}$$

This is the exact analog to Eq.(64). So the close analogy between the two systems, light quarks inside a bag wrapped by a soliton and a light antiquark of the heavy meson wrapped by a soliton, is established.

We note in passing that the same procedure could be applied to the CK scheme. The main difference will be the form of the induced gauge potential. As can be seen in Eqs.(58) and (64), it consists of two parts: the generic term that leads to a Berry potential and the terms (*i.e.*, the second and third terms of (58)) which, as a consequence of the profile of the bound heavy meson shrinking to the origin, vanish as $\sim 1/m_Q$ as the mass of the heavy quark $m_Q$ goes to infinity.

The heavy-quark effective Lagrangian written in the form of (64) lends itself also to a close analogy to the diatomic molecular system described by Eq.(5). To see this, we write

$$H = \theta_a(t) H_a, \qquad \overline{H} = \theta_a^\dagger(t)\overline{H}_a \tag{67}$$

where $\theta$ is the Grassmannian introduced before, with the index $a$ representing the flavor of the light antiquark $\bar{q}_H$. Now since the parameter space of slow rotation, $S(t)$, corresponds to the group manifold of $SU(2)$ which is isomorphic to $S^3$, it is convenient to use the left or right Maurer-Cartan forms as a basis for the vielbeins (one-form notation understood)

$$S^\dagger i dS = -\omega_a \tau_a = -v_a^c(\phi) d\phi^c \tau^a \tag{68}$$

where we expressed the "velocity" one-form $\omega$ in the basis of the vielbeins $v_a^c$, and $\phi$ denotes some arbitrary parametrization of the $SU(2)$, *e.g.* Euler angles. Using (67) and (68), we readily obtain

$$-i\int d^3x \mathrm{Tr}\overline{H}(\partial_t S^\dagger S) H = i\theta_b^\dagger(-iA_i^\alpha(\phi)T_{ba}^\alpha \dot{\phi}^i)\theta_a \tag{69}$$

with

$$\begin{aligned} -g_K T^\alpha_{ba} &\equiv \int d^3x \mathrm{Tr}(\overline{H}_b I^\alpha_H H_a), \\ A^\alpha_i &= 2g_K v^\alpha_i(\phi) \end{aligned} \tag{70}$$

where $T^a$ are $n \times n$-matrix representations of the generators of $S(t)$ in the $n$-degenerate $K$ space and $g_K$ is the corresponding charge. We have used the normalization

$$\int d^3x \mathrm{Tr}\overline{H}_a H_a = -1. \tag{71}$$

We also have

$$-i \int d^3x \mathrm{Tr}(\overline{H}\partial_t H) = i\theta^\dagger_a \partial_t \theta_a, \tag{72}$$

$$\mathrm{Tr}(S^\dagger i\partial_t S)^2 = g^{ij}(\phi)\dot{\phi}^i\dot{\phi}^j. \tag{73}$$

The Lagrangian with (69), (72) and (73) is identical in form to that of the diatomic molecule, Eq.(5).

### 3.2 The vanishing of the Berry potential and symmetry restoration

To understand the structure of the relevant Berry potential, we have to determine the ground state of the system consisting of a light antiquark and a skyrmion on which proper physical states are to be constructed in the collective coordinate quantization scheme. The interaction potential which induces a bound state is the third term in Eq. (58), which in the heavy meson rest frame is[19]

$$H_I = -\frac{gF'(0)}{2} \int d^3x \mathrm{Tr}\overline{H}H\sigma^j\tau^j. \tag{74}$$

This can be rewritten in terms of the light anti-quark spin operator $S_l$ and the heavy-meson isospin operator $I_H$ as [20]

$$H_I = 2gF'(0) \int d^3x \mathrm{Tr}\overline{H}\vec{I}_H \cdot \vec{S}_l H \tag{75}$$

For the $H$ classified by the $K$-spin, $\vec{K} = \vec{I}_H + \vec{S}_l$, with the normalization $\int d^3x \mathrm{Tr}\overline{H}H = -1$, we have

$$H_I = -gF'(0)(K^2 - \frac{3}{2}). \tag{76}$$

Therefore $K = 0$ is a bound state for $g > 0$ and $F'(0) < 0$. The $K = 1$ states are not bound unless one invokes higher-dimension terms. This is the key point in our reasoning.

Given that the relevant state has $K = 0$, the reason for the vanishing of the Berry potential in the heavy-meson (or heavy-quark) limit can be seen immediately. Since the $K = 0$ state is bound and all the physical baryon states are constructed after quantization on that state, *the relevant Berry potential must be calculated on the $K = 0$ state*. Although the $K = 0$ state is a singlet state with respect to the $K$-spin, the ground state consists of at least two degenerate states because of the two spin states of the heavy quark sitting at the origin. The quantum numbers of the ground state of a light antiquark and a heavy quark are the $K$-spin, the orbital angular momentum $l$ of $\bar{q}_H$, and the heavy-quark spin $S_Q$. There is degeneracy with respect to $S_Q$. Therefore the Berry potential of $K = 0$ for an S-wave bound state, if not zero, is *nonabelian* defined on two degenerate states ,

$$|K = 0\rangle|S_Q = +1/2\rangle, \quad \text{and} \quad |K = 0\rangle|S_Q = -1/2\rangle \tag{77}$$

where $|K = 0\rangle$ has the hedgehog configuration

$$|K = 0\rangle = \frac{1}{\sqrt{2}}\left(|\bar{d}\downarrow\rangle - |\bar{u}\uparrow\rangle\right). \tag{78}$$

However the Berry potential *vanishes* in the $K = 0$ state for the same reason that the $K = 0$ ground state in the chiral bag has a vanishing Berry phase [1, 21]. If $m_Q$ is not very large, then one should calculate also the contributions from the second and third terms of (58) which will come in as $1/m_Q$ corrections.

In the discussion presented up to this point, the role of heavy-meson symmetry is not *apparent*. To exhibit this, we first make the connection to the CK scheme as used in [3, 4]. In the CK picture, the spectrum of a heavy meson bound to a soliton is classified by the isospin and the orbital angular momentum (and the spin for vector mesons) of the mesons, $\phi$ and $\phi^*$, namely the grand spin $\Lambda = I + l + S$. The previous argument cannot be applied to this system directly. However, since we can construct the $K = 0$ state of $\bar{q}_H$ as a linear combination of both $\phi$ and $\phi^*$ which are in definite $\Lambda$ states, it is not difficult to see how the heavy mesons conspire to give a vanishing hyperfine coefficient, thereby making, say, the baryons $\Sigma_Q$ and $\Sigma_Q^*$ (with a heavy quark $Q$) degenerate. For the S-wave ground state with $l = 0$ given in Eq. (77), the relevant states with $\Lambda = 1/2$ are constructed by a direct product of $K$ of $\bar{q}_H$ and $S_Q$ of the heavy quark which can be decomposed into scalar and vector mesons as, for instance,

$$\begin{aligned}
\Phi_{\Lambda=1/2,\Lambda_3=1/2} &\equiv |K = 0\rangle|S_Q = 1/2\rangle = \frac{1}{2}\left(\sqrt{2}|B^{*-}(+)\rangle - |\overline{B^{*0}}(0)\rangle + |\overline{B^0}\rangle\right), \\
\Phi_{\Lambda=1/2,\Lambda_3=-1/2} &\equiv |K = 0\rangle|S_Q = -1/2\rangle = \frac{1}{2}\left(-\sqrt{2}|\overline{B^{*0}}(-)\rangle + |B^{*-}(0)\rangle + |B^-\rangle\right)
\end{aligned} \tag{79}$$

where $B^*$ and $B$ denoting vector and pseudoscalar $B$ mesons respectively are used explicitly. The spin state of the vector mesons is written in parenthesis as $+, 0, -$ for spin $1, 0, -1$ respectively. For example, $B^{*-}(+)$ represents the $B^{*-}$ meson of spin $S_3 = +1$. In writing Eq. (79), the standard recoupling of spin and isospin has been made. Now we can see the role of heavy-meson symmetry hidden in the $K$-spin states by replacing the $B$ fields in Eq. (79) by states with definite $\Lambda$ and $\Lambda_3$,

$$\begin{aligned} \Phi_{1/2,1/2} &= \frac{1}{2}\left(\sqrt{3}|\phi^*, 1/2, +1/2\rangle + |\phi, 1/2, +1/2\rangle\right), \\ \Phi_{1/2,-1/2} &= \frac{1}{2}\left(\sqrt{3}|\phi^*, 1/2, -1/2\rangle + |\phi, 1/2, -1/2\rangle\right) \end{aligned} \tag{80}$$

where

$$\begin{aligned} |\phi^*, 1/2, +1/2\rangle &= \frac{1}{\sqrt{3}}\left(\sqrt{2}|B^{*-}(+)\rangle - |\overline{B^{*0}}(0)\rangle\right), \\ |\phi, 1/2, +1/2\rangle &= |\overline{B^0}\rangle, \\ |\phi^*, 1/2, -1/2\rangle &= \frac{1}{\sqrt{3}}\left(|B^{*-}(0)\rangle - \sqrt{2}|\overline{B^{*0}}(-)\rangle\right), \\ |\phi^*, 1/2, -1/2\rangle &= |B^-\rangle. \end{aligned} \tag{81}$$

The conventions $|\phi^*, \Lambda, \Lambda_3\rangle$ and $|\phi^*, \Lambda, \Lambda_3\rangle$ are used in the above equations. If only the scalar meson is included, there is no way to construct a proper bound state of $K = 0$ and hence $c$ cannot vanish. One can verify this easily by calculating the matrix element of, say, $\tau_3$

$$\begin{aligned} \langle\phi^*, 1/2, \Lambda_3|\tau_3|\phi^*, 1/2, \Lambda_3\rangle &= -\frac{1}{3}\Lambda_3, \\ \langle\phi, 1/2, \Lambda_3|\tau_3|\phi, 1/2, \Lambda_3\rangle &= \Lambda_3. \end{aligned} \tag{82}$$

Using Eq. (82), we can see that the vanishing of matrix element of $\tau_3$ for the ground state with $K = 0$ is due to the *exact cancellation* of the contributions from $\phi$ and $\phi^*$,

$$\begin{aligned} \langle\Phi_{1/2,\Lambda_3}|\tau_3\ |\Phi_{1/2,\Lambda_3}\rangle &= \frac{1}{4}\left(3\langle\phi^*, 1/2, \Lambda_3|\tau_3|\phi^*, 1/2, \Lambda_3\rangle + \langle\phi, 1/2, \Lambda_3|\tau_3|\phi, 1/2, \Lambda_3\rangle\right) \\ &= 0 \end{aligned} \tag{83}$$

which has been obtained in [3, 4] in a different way. It is clear that both $\phi$ and $\phi^*$ are needed to cause the Berry potential to vanish for a soliton-heavy meson bound state.

The disappearance of Berry potentials naturally takes place when a symmetry is restored with a given set of states in a certain limit. In the diatomic molecular system discussed by Zygelman[8] and reanalyzed above, a Berry potential is obtained by slow rotation

of the internuclear radius $\vec{R}$ upon $\Sigma$ and $\pi$ states. For finite internuclear distances $R$, these states are not degenerate and cannot form a representation of the rotation group, hence developing a nonvanishing Berry (nonabelian) potential. In the limit $R \to \infty$, however, they become degenerate as $\kappa \to 1$ and form a representation of the rotation group and hence the Berry potential vanishes (or more precisely it becomes a pure gauge). In other words, when the rotational symmetry is restored, the Berry potential developed on the $\Sigma$ and $\pi$ states vanishes (or become pure gauge).

In the soliton-heavy meson bound system that we are considering, a similar reasoning can be made for the vanishing Berry potential. In the way formulated in this paper, the Berry potential vanishes because the relevant state has $K = 0$. The states with $K \neq 0$ have a nonvanishing Berry potential. It is in *making* the relevant state have $K = 0$ that the heavy-quark symmetry comes in. This symmetry is not "visible" while the electronic rotational symmetry of diatomic molecules is, so the way the symmetry is restored in the heavy-quark system is somewhat more subtle. Nonetheless the mechanism is quite similar.

# 4 Conclusion

When one considers the light antiquarks in heavy mesons as fast variables while the heavy quark sits at the origin as a spectator, the generic form of Berry potentials emerges as the soliton is slowly rotated. The bound state is composed of an antiquark and a skyrmion, characterized by the $K$ spin, $K = 0$. The Berry potential that develops on the $K = 0$ bound state is shown to vanish. This phenomenon can be understood as the cancellation of the contributions from scalar and vector mesons to the Berry potential or equivalently to the hyperfine coefficient $c$ that figures in the CK picture[3, 4]. It is argued that this cancellation is a consequence of the heavy-quark or heavy-meson symmetry which emerges in the heavy-quark limit and that the limiting behavior leading to a vanishing Berry potential is quite generic as discussed in the context of the restoration of an underlying symmetry in the dynamics.

**Acknowledgments**

We are grateful for valuable discussions with D.-P. Min, M.A. Nowak and I. Zahed. HKL acknowledges the hospitality of Service de Physique Théorique, CE-Saclay where part of this work was done.

## Appendix

The sign of the interaction Hamiltonian Eq.(76) differs from that of Ref.[7]. The reason for this is sketched in this appendix.

As done in Ref.[7], the isospin operator $I_H^k$ on the $H$ field is identified as

$$I_H^k H = H\left(-\frac{\tau^k}{2}\right). \tag{A.1}$$

The action of the spin operator for the light antiquark $S_l^k$ is more subtle. To see how it comes out, we define operationally

$$\vec{S}_l H(P^*, P) = H(\vec{S}_l P^*, \vec{S}_l P). \tag{A.2}$$

Let us consider for instance the action of $S_l^3$ on $H$. For this, we write explicitly the spin wave functions for $P$ and $P^*$ in terms of the heavy-quark spin labeled by $Q$ and the light anti-quark spin labeled by $l$

$$\begin{aligned} P &= \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_Q|\downarrow\rangle_l - |\downarrow\rangle_Q|\uparrow\rangle_l\right), \\ P^*_+ &= |\uparrow\rangle_Q|\uparrow\rangle_l, \\ P^*_0 &= \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_Q|\downarrow\rangle_l + |\downarrow\rangle_Q|\uparrow\rangle_l\right), \\ P^*_- &= |\downarrow\rangle_Q|\downarrow\rangle_l. \end{aligned} \tag{A.3}$$

Then the operation of $S_l^3$ yields

$$\begin{aligned} S_l^3 P^*_+ &= \frac{1}{2}P^*_+, \\ S_l^3 P^*_0 &= -\frac{1}{2}P, \\ S_l^3 P^*_- &= -\frac{1}{2}P^*_-, \\ S_l^3 P &= -\frac{1}{2}P^*_0. \end{aligned} \tag{A.4}$$

Then Eq. (A.2) for $S_l^3$ can be rewritten

$$S_l^3 H = \frac{1+\gamma_0}{2}\frac{1}{2}\left(P_+^*\gamma^- + P_0^*\gamma^5 - P_-^*\gamma^+ - P\gamma^3\right) \tag{A.5}$$

where

$$H = \frac{1+\gamma_0}{2}\left(P_+^*\gamma^- + P_0^*\gamma^3 + P_-^*\gamma^+ - P\gamma^5\right) \tag{A.6}$$

has been used. Therefore one gets after a short algebra

$$S_l^3 H(P^*, P) = H(+\frac{\sigma^3}{2}) \tag{A.7}$$

or more generally

$$\vec{S}_l = H\frac{\vec{\sigma}}{2}. \tag{A.8}$$

Similarly the heavy quark spin, $S_Q$, acting on $H$ gives

$$\vec{S}_Q H = -\frac{\vec{\sigma}}{2}H. \tag{A.9}$$

Thus the sign difference between Ref.[7] and this paper is in (A.7) for the identification of the light-antiquark spin operator $S_l$.

## References


[1] H.K. Lee, M.A. Nowak, M. Rho and I. Zahed, Phys. Lett. **B255**, 96 (1991); Phys. Lett. **B272**, 109 (1991); "Noabelian Berry phases in baryons," submitted to Annals of Physics (NY) (1993)

[2] M.V. Berry, Proc. Roy. Soc. (London) **A392**, 45 (1984); F. Wilczek and A. Zee, Phys. Rev. Lett. **52**, 2111 (1984)

[3] D. Min, Y. Oh, B. Park and M. Rho, "Soliton Structure of Heavy Baryons," SNUTP-92/76, 1992

[4] M.A. Nowak, M. Rho and I. Zahed, "Heavy Solitonic Baryons," Phys. Lett. **B**, in press.

[5] For a recent review, H. Georgi, *Proceedings of the Theoretical Advanced Study Institute*, ed. by R.K. Ellis, G.T. Hill and J.D. Lykken (World Scientific, Singapore, 1992) page 589

[6] C.G. Callan and I. Klebanov, Nucl. Phys. **B262**, 365 (1985); C.G. Callan, I. Klebanov and K. Hornbostel, Phys. Lett. **B202**, 260 (1988)

[7] E. Jenkins, A. V. Manohar and M. Wise, "Baryons Containing a Heavy Quark as Solitons," UCSD/PTH 92-17, 1992; Z. Guralnik, M. Luke and A. Monohar, "Properties of Baryons Containing a Heavy Quark in the Skyrme Model," UCSD/PTH 92-24, 1992; E. Jenkins and A. Manohar, Phys. Lett. **B294**, 273 (1992)

[8] B. Zygelman, Phys. Rev. Lett. **64**, 256 (1990)

[9] R. Jackiw, Acta Physica Austriaca, Suppl. **XXII**, 383 (1980);R. Jackiw and N.S. Manton, Annals of Phys. **127**, 257 (1980)

[10] C.N. Yang, *Lectures on Frontiers in Physics* (Korean Physical Society, Seoul, 1980)

[11] R. Jackiw and C. Rebbi, Phys. Rev. Lett.**36**, 1116 (1976); P. Hasenfratz and G. 't Hooft, Phys. Rev. Lett. **36**, 1119 (1976)

[12] G. 't Hooft, Nucl. Phys. **B79**, 276 (1974); A. Polyakov, JETP Lett. **20**, 194 (1974)

[13] R. Casalbuoni, Nuov. Cim. **33A**, 389 (1976); A.P. Balachandran, Per Salomonson, Bo-Sture Skagerstam, and Jan-Olov Winnberg, Phys. Rev. **D15**, 2308G (1977)

[14] P.A.M. Dirac, *Lectures on Quantum Mechanics* (Yeshiva Univ., New York, 1964)

[15] R. Jackiw, Phys. Rev. Lett. **56**, 2779 (1986)

[16] A. Shapere and F. Wilczek, eds, *Geometric phases in physics* (World Scientific, Singapore, 1989)

[17] J. Moody, A. Shapere and F. Wilczek, Phys. Rev. Lett. **56**, 893 (1986)

[18] In the CK scheme, the relevant quantum numbers are the isospin $I$ and the angular momentum $l$ of the kaon, not those of the strange quark. The grand spin is defined as $\Lambda = I + l$.

[19] As pointed out in [3], we also have the higher-dimension term $\sim \overline{H} v^\mu H B_\mu$ where $B_\mu$ is the topological baryon current which can induce binding. However addition of this term does not affect our arguments.

[20] We differ in sign from Ref.[7]. The reason for this difference is explained in Appendix. With this sign, we are consistent with $F(0) = \pi$ required for our definition of the baryon current.

[21] In fact, one can think of the antiquark $\bar{q}_H$ occupying the lowest-energy hedgehog quark state with $K = 0$ in the chiral bag model. In this way, it is even possible to "derive" the heavy-baryon structure in the manner that one constructs the Callan-Klebanov model from the chiral hyperbag as discussed in B.-Y. Park, D.-P. Min and M. Rho, Nucl. Phys. **A551**, 580 (1993).