Chiral Dynamics and Heavy-Fermion Formalism in Nuclei: Exchange Axial Currents

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Chiral Dynamics and Heavy-Fermion Formalism in Nuclei: I. Exchange Axial Currents

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ABSTRACT

Chiral perturbation theory in heavy-fermion formalism is developed for meson-exchange currents in nuclei and applied to nuclear axial-charge transitions. Calculation is performed to the next-to-leading order in chiral expansion which involves graphs up to one loop. The result turns out to be very simple. The previously conjectured notion of "chiral filter mechanism" in the time component of the nuclear axial current and the space component of the nuclear electromagnetic current is verified to that order. As a consequence, the phenomenologically observed soft-pion dominance in the nuclear process is given a simple interpretation in terms of chiral symmetry in nuclei. In this paper we focus on the axial current, relegating the electromagnetic current which can be treated in a similar way to a separate paper. We discuss the implication of our result on the enhanced axial-charge transitions observed in heavy nuclei and clarify the relationship between the phenomenological meson-exchange description and the chiral Lagrangian description.

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1 Introduction

By now there exist a large number of unambiguous experimental evidences [1] for meson-exchange currents in nuclear responses to electroweak probes. We also have available a rather satisfactory and successful theory to describe the large bulk of experimental observations [2]. While inherently phenomenological in character, the approaches taken so far to describe meson-exchange currents have been commensurate with the ingredients that account for our progressive understanding of nuclear forces and to the extent that nucleon-nucleon interactions are now fairly accurately understood, one can have a great deal of confidence in the theoretical tool with which the effect of exchange currents is calculated. There remains however the fundamental question as to how our phenomenological understanding of nuclear forces and associated meson currents can be linked to the fundamental theory of strong interactions, QCD.

In this paper we make a first step towards answering this question by applying chiral perturbation theory (ChPT) to nuclear electroweak processes. To start with, we assume that at low energies dominated by infrared properties of QCD, the most important aspect of QCD is the spontaneously broken chiral symmetry and hence that in nuclear dynamics, it is chiral symmetry that plays a predominant role [3]. The important role of chiral symmetry in nuclear physics was recognized early on by Chemtob and Rho for exchange currents [4] but this issue was recently given a stronger impetus and a more modern meaning by Weinberg in connection with nuclear forces [5] and by Rho [6] in connection with what is known "nuclear chiral filter phenomenon" (for definition, both intuitive and more rigorous, see later).

The key question addressed here is this: To what extent can nuclear processes be described by QCD or equivalently at low energies by chiral perturbation theory? Weinberg approaches this issue by studying nuclear many-body forces. We propose here to do the same by looking at nuclear response functions responding to slowly varying electroweak fields [7]. We suggest that chiral perturbation theory can be made – under certain conditions specified below – considerably more powerful and predictive for response functions than for nuclear forces. In calculating nuclear forces to loop orders in chiral perturbation theory, one encounters a plethora of counter terms to renormalize the theory, most of which are not accessible by experiments [8]; furthermore there are contact four-fermion interactions in the Lagrangian – most of which are again unknown parameters – that have to be carefully examined and treated. In principle, this may be feasible, perhaps with the help of lattice QCD calculations but in practice it may not be possible to make clear and useful predictions because of many uncontrolled parameters. While the tree order chiral theory justifies *à posteriori* the current nuclear physics practice of using two-body static forces [5], it appears that chiral symmetry will be unable to make any truly significant statement on the structure of nuclear forces for sometime to come. A major new development will be required before one can make a prediction that goes beyond the accuracy of the phenomenological approach which has been strengthened by the wealth of experimental data. On the contrary, as we will show in this paper, when the formalism is applied to nuclear response functions, in particular, to exchange currents, it can make a highly nontrivial and potent prediction. This is because nuclear short-range correlations generated by nuclear interactions at short distance – which while poorly understood of their mechanism, are nonetheless operative in nuclear medium – screen all the contact interactions, both intrinsic and induced and consequently *all* of the four-fermion (and higher) counter term contributions, effectively "filtering" off the ill-understood short-range operators: Given phenomenological information on nuclear wave functions at short distance, the short-range suppression helps in simplifying nuclear response functions. In addition, in certain kinematic conditions, higher order chiral corrections are found to be *naturally* suppressed. The suppression of the many-fermion counter term contributions at the one-loop order that we are studying is, as will be stated more precisely later, a consequence of the fact that such terms occurring at high orders in the chiral expansion reflect the degrees of freedom that enter directly neither in nuclear forces nor in nuclear currents at the chiral order considered. The combination of these two phenomena lead to the "chiral filtering" proposed previously [10]. In this paper, we will establish this chiral filtering to one-loop order in chiral perturbation theory. This will provide, in our opinion, the very first compelling explanation of the pion-exchange dominance observed in axial charge transitions (considered in this paper) as well as in radiative np capture or in threshold electrodisintegration of the deuteron.

As stressed by Weinberg, chiral perturbation theory is useful in nuclear physics *only for "irreducible" diagrams* that are by choice free of infrared divergences. This means that both in nuclear forces and in exchange currents, reducible graphs are to be taken care of by a Schrödinger equation or its relativistic generalization with the irreducible graphs entering as potentials. This also implies that in calculating exchange currents in ChPT, we are to use the wave functions so generated to calculate matrix elements to obtain physical amplitudes. This is of course the standard practice in the theory of meson-exchange currents but it is also in this sense that ChPT is predictive in nuclei. Clearly this precludes what one might call "fully consistent chiral perturbation theory" where nuclear forces, nuclear currents and wave functions are all calculated to the same order of chiral perturbation expansion. Such a calculation even if feasible is likely to make no sense. A little thought would persuade the reader that it is a futile exercise.

We will here focus on the irreducible diagrams contributing to exchange currents. We will calculate next-to-leading order terms in the chiral counting involving one-loop graphs. In doing this, we will employ the recently developed heavy fermion formalism (HFF) [11]. The standard ChPT [12] arranges terms in power of (∂/Λ_χ) and/or of (m_π/Λ_χ) where ∂ is four-derivative acting on the Goldstone boson (*viz*, pion) field, m_π the pion mass (≈ 140 MeV) and $\Lambda_\chi \approx 1$ GeV, the chiral expansion scale. It has been established that this expansion works well at low energies for such processes as $\pi\pi$ scattering. However the situation is different when baryons are involved. The dynamically generated masses of the baryons are of $O(\Lambda_\chi)$ and hence when the baryon field is acted upon by time-derivative, it gives an $O(1)$ term. Therefore a straightforward derivative expansion fails. (Incidentally this is also the reason why a chiral Lagrangian describing pion interactions well with low-order derivative terms does not necessarily describe well skyrmion properties.) The HFF circumvents this difficulty in rearranging the derivative expansion. Indeed the principal virtue of the HFF is that it provides a consistent chiral expansion in Q/Λ_χ where Q is four-derivative on pion field or pion mass or space derivative on baryon field; it avoids time derivative on baryon field which is of order Λ_χ which is not small. The standard ChPT involving baryons [13] can in principle be arranged to give a similar expansion. However it requires a laborious reshuffling of terms avoided in the HFF. The distinct advantage of the HFF is that the multitude of diagrams that appear in such calculations as ours in the standard ChPT involving baryons get reduced to a handful of manageable terms, thus alleviating markedly the labor involved. We will see that there is an enormous simplification in the number of terms and in their expressions. The potential disadvantage might be that the HFF is not fully justified for the mass corresponding to that of the nucleon and hence higher order "$1/m$" corrections may have to be systematically included. We will examine the class of approximations we make in the calculation by looking at the next order terms. It turns out that the leading "$1/m$" correction is absent in our calculation. We shall discuss this matter in the concluding section.

While the procedure is practically the same, the resulting expression for electromagnetic (EM) current is somewhat more involved. We will therefore not treat it here although we shall give a general treatment of the theory applicable to both axial and EM currents. The detailed analysis on the EM currents, together with an application to threshold np capture, will be reported in a separate paper [14]. Both currents are intricately connected even at low energy through current algebras and we will need some vertices involving the EM current.

It is perhaps obvious but we should stress that for both vector and axial-vector currents, relevant symmetries (*i.e*, conserved vector current and partially conserved axial-vector current) are preserved to the chiral order considered since both nuclear forces and currents are treated on the same footing with the same effective Lagrangian. More on this point later.

This paper is organized as follows. In Section 2, we state our basic assumption in applying ChPT to nuclear dynamics. In Section 3, we describe the effective chiral Lagrangian with which we develop heavy-fermion formalism including "$1/m$" corrections. We also define the relevant kinematics we will consider. The chiral counting rules are given in Section 4. In Section 5, the renormalization of n-point vertices that enter in the calculation is detailed. For the sake of making this paper as self-contained as possible and to define notations, we also list the renormalized quantities for the pion and the nucleon following from the Lagrangian. Readers familiar with renormalization of heavy-fermion chiral Lagrangian could proceed directly to subsection 5.4. Two-body exchange currents are calculated in

Section 6. Both momentum-space and coordinate-space formulas are given. Numerical analyses are described in Section 7. In Section 8, we explain why there are *no other graphs* that can contribute to the same order and point out in what circumstances they can show up in physical observables. Concluding remarks including those on the observed enhanced axial-charge transitions in heavy nuclei are made in Section 9. The Appendices A-I list all the formulas needed in the calculation.

This paper is written in as a self-contained way as possible so as to be readable by those who are not familiar with the recent development in the field. Some of the material are quite standard and readily available in the literature. Most of them however serve as a check of our calculation.

2 Strategies in Nuclear Physics

We wish to calculate operators effective in nuclei for transitions induced by the vector and axial vector currents of electroweak interactions, denoted respectively by V_μ and A_μ associated with the electroweak fields $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$. In principle there will be n-body currents for $N \geq n > 1$ in N-body systems. Here we will focus only on one- and two-body currents, ignoring those with $n > 2$. The reasons for so doing are given in the literature [2, 4] but we will later show that n-body currents for $n > 2$ are suppressed to the order considered for long wavelength probes.

The diagrams we wish to calculate are generically given by Fig. 1. They correspond to the standard definition of single-particle and two-particle exchange currents entering in the description of nuclear response functions to the external electroweak fields. These have been calculated before in terms of phenomenological Lagrangians. Here we wish to do so using chiral perturbation theory (ChPT), starting with a chiral Lagrangian that is supposed to model QCD at low energies. Following the chiral counting rule we will derive later, we will restrict our consideration to one-loop order, which corresponds to going to the next order in the chiral counting to the leading soft-pion limit. Although one-loop calculations have been done before for nucleon properties [15] and for infinite nuclear matter [16], they have up to date not been performed in finite nuclear processes. We believe this work constitutes the first attempt to implement consistently chiral symmetry in nuclear processes.

In dealing with divergences encountered in calculating loop graphs, in particular the loops involving two-pion exchange, we will need a certain prescription for handling operators that are short-ranged in coordinate space. This prescription does not follow from chiral symmetry alone and will have to be justified on a more general ground. Specifically, we argue that consistency with ChPT demands that zero-range interactions be "killed" by nuclear short-range correlations:

- Firstly, the zero-range operators that come from finite counter terms appearing in four-fermion interactions figure neither directly nor indirectly – but importantly – in the successful phenomenological nucleon-nucleon potentials and hence must represent the degrees of freedom unimportant for the length scale involved. In fact, one can show (see Appendix I) that the counter terms we need to introduce (denoted $\kappa_4^{(1,2)}$ later) *cannot* arise, unlike in the better understood π-π scattering [17], from an approximation of taking an infinite mass limit of the strong interaction resonances such as the vector mesons ρ, ω etc. which have a scale comparable to the chiral scale and play an important role in boson-exchange potential models.

- Secondly, ChPT by its intrinsic limitation cannot possibly provide a nuclear force that can account for the interactions shorter-ranged than two pion or one vector-meson range at most. Thus the truly short-range interaction known to be present in the nucleon-nucleon interaction must involve elements that are not calculable by means of finite-order chiral expansion even if such an expansion existed. Thus it would be inconsistent to put a part of such interactions into the currents in the context of ChPT without a similar account in the nuclear force. It is known that even to one-loop order, the number of counter terms is so large in the calculation of nuclear forces that it is highly unlikely that one can make a meaningful prediction based strictly on low-order chiral perturbation expansion [8]. As suggested in Ref.[9], one should implement ChPT calculations with phenomenological informations whenever available.

- Applied to the "irreducible diagrams" that enter in the definition of exchange currents, ChPT screens out the short-range part of the interaction which originates from dynamics of possibly non-chiral origin. When the matrix elements of the operators arising from the irreducible graphs are calculated with wave functions suitably computed in the presence of two-nucleon potentials, the short-range correlation built into nuclear wavefunctions must therefore suppress strongly interactions that occur at an internuclear distance ≤ 0.6 fm, automatically "killing" the δ function interactions associated with finite counter terms. This fact will be kept in mind when we derive two-body operators in coordinate space.

We will present, at several places in the paper, arguments to justify the above procedure which purports to establish that *the only unknown parameters in the theory must be (a) negligible in magnitude and (b) further suppressed by nuclear correlations when embedded in nuclear medium.*

There is nothing very much new in our calculation of the one-body operators except for its consistency with chiral invariance. As for the two-body exchange currents, our results are new. There are two graphs to consider: One-pion exchange (Fig. 2a) and two-pion exchange (Fig. 2b). Both involve one-loop order graphs. Note that we are to calculate only "irreducible graphs."

3 Effective Chiral Lagrangian

We begin with the effective chiral Lagrangian that consists of pions and nucleons involving lowest derivative terms[13] relegating the role of other degrees of freedom such as vector mesons and nucleon resonances Δ to a later publication[#1],

$$\begin{aligned}\mathcal{L}_0 &= \overline{N}\left[i\gamma^\mu(\partial_\mu+\Gamma_\mu)-m+ig_A\gamma^\mu\gamma_5\Delta_\mu\right]N-\frac{1}{2}C_a\left(\overline{N}\Gamma_a N\right)^2\\ &+\frac{F^2}{4}\mathrm{Tr}\left(\nabla_\mu\Sigma^\dagger\nabla^\mu\Sigma\right)+\frac{1}{2}M^2F^2\mathrm{Tr}(\Sigma)+\cdots+\mathcal{L}_{\mathrm{CT}},\end{aligned} \tag{1}$$

where $m\simeq 939$MeV is the nucleon mass, $g_A\simeq 1.25$ is the axial coupling constant and $F\simeq 93$ MeV is the pion decay constant. The ellipsis stands for higher derivative and/or symmetry-breaking terms which will given later as needed. We have written the Lagrangian with the renormalized parameters m, g_A, F and M with suitable counter terms $\mathcal{L}_{\mathrm{CT}}$ to be specified later. Under the chiral SU(2)×SU(2) transformation[#2], the chiral field $\Sigma=\exp(i\frac{\vec{\tau}\cdot\vec{\pi}}{F})$ transforms as $\Sigma\to g_{\mathrm{R}}\Sigma g_{\mathrm{L}}^\dagger$ ($g_{\mathrm{R}},g_{\mathrm{L}}\in$ SU(2)) and the covariant derivative of the chiral field transforms as Σ does,

$$\begin{aligned}\nabla_\mu\Sigma &= \partial_\mu\Sigma-i(\mathcal{V}_\mu+\mathcal{A}_\mu)\Sigma+i\Sigma(\mathcal{V}_\mu-\mathcal{A}_\mu)\\ &\to g_{\mathrm{R}}\,\nabla_\mu\Sigma\,g_{\mathrm{L}}^\dagger\end{aligned} \tag{2}$$

where the external gauge fields $\mathcal{V}_\mu=\vec{\mathcal{V}}_\mu\cdot\frac{\vec{\tau}}{2}$ and $\mathcal{A}_\mu=\vec{\mathcal{A}}_\mu\cdot\frac{\vec{\tau}}{2}$ transform locally

$$\begin{aligned}\mathcal{V}_\mu+\mathcal{A}_\mu\to\mathcal{V}'_\mu+\mathcal{A}'_\mu=g_{\mathrm{R}}(\mathcal{V}_\mu+\mathcal{A}_\mu)g_{\mathrm{R}}^\dagger-i\partial_\mu g_{\mathrm{R}}\cdot g_{\mathrm{R}}^\dagger,\\ \mathcal{V}_\mu-\mathcal{A}_\mu\to\mathcal{V}'_\mu-\mathcal{A}'_\mu=g_{\mathrm{L}}(\mathcal{V}_\mu-\mathcal{A}_\mu)g_{\mathrm{L}}^\dagger-i\partial_\mu g_{\mathrm{L}}\cdot g_{\mathrm{L}}^\dagger.\end{aligned}$$

In our work, only the electroweak $(SU(2)\times U(1))$ external fields will be considered. The Lagrangian of course has global $SU(2)\times SU(2)$ invariance in the absence of the pion mass term. Non-linear realization of chiral symmetry is expressed in terms of $\xi=\sqrt{\Sigma}=\exp(i\frac{\vec{\tau}\cdot\vec{\pi}}{2F})$ and $U=U(\xi,g_{\mathrm{L}},g_{\mathrm{R}})$ defined with ξ

$$\xi\to g_{\mathrm{R}}\xi U^\dagger=U\xi g_{\mathrm{L}}^\dagger.$$

[#1]While the vector mesons and the nucleon resonances (in particular, the Δ) play an important role in nuclear phenomenology – and they can be easily implemented in ChPT at least in low orders, they are unimportant for the process we discuss in this paper. It is not difficult to see which processes require such degrees of freedom but we will not pursue this matter, for a treatment of such processes goes beyond the framework of ChPT.

[#2]We are using a slightly unconventional notation of Ref. [13] which we will follow in this paper. This facilitates checking our results on single-nucleon properties against those derived in [13] using standard (relativistic) chiral perturbation expansion. The more familiar transformation of the chiral field used in the literature is gotten by replacing Σ by $\Sigma^\dagger$. We are also working with the exponentiated (Sugawara) form of chiral Lagrangian instead of Weinberg's [5] used previously. They are of course equivalent. For the rest we will follow the Bjorken-Drell metric and convention.

Now nucleon field N transforms as $N \to UN$, and covariant derivatives of nucleon field and chiral field transform as nucleon field does, $D_\mu N \to U D_\mu N$ and $\Delta_\mu \to U \Delta_\mu U^\dagger$ where[#3]

$$\begin{aligned}
D_\mu N &= (\partial_\mu + \Gamma_\mu) N, \\
\Gamma_\mu &= \frac{1}{2}\left[\xi^\dagger, \partial_\mu \xi\right] - \frac{i}{2}\xi^\dagger(\mathcal{V}_\mu + \mathcal{A}_\mu)\xi - \frac{i}{2}\xi(\mathcal{V}_\mu - \mathcal{A}_\mu)\xi^\dagger, \\
\Delta_\mu &= \frac{1}{2}\xi^\dagger\left(\nabla_\mu \Sigma\right)\xi^\dagger = \frac{1}{2}\left\{\xi^\dagger, \partial_\mu \xi\right\} - \frac{i}{2}\xi^\dagger(\mathcal{V}_\mu + \mathcal{A}_\mu)\xi + \frac{i}{2}\xi(\mathcal{V}_\mu - \mathcal{A}_\mu)\xi^\dagger.
\end{aligned} \tag{3}$$

The U can be expressed as a complicated local function of ξ, $g_{\rm L}$ and $g_{\rm R}$. The explicit form of U is not needed for our discussion.

Note that we have included the four-fermion non-derivative contact term studied recently by Weinberg[5]. We will ignore possible four-fermion contact terms involving derivatives (except for counter terms encountered later) and quark mass terms since they are not relevant to the chiral order (in the sense defined precisely later) that we are working with. The explicit chiral symmetry breaking is included minimally in the form of the pion mass term. Higher order symmetry breaking terms do not play a role in our calculation.

3.1 Heavy-fermion formalism

For completeness – and to define our notations, we sketch here the basic element of the heavy-fermion formalism (HFF)[18] applied to nuclear systems as developed by Jenkins and Manohar[11] wherein the nucleon is treated as a heavy fermion. As stressed in Introduction, the relativistic formulation of ChPT works well when only mesons are involved but it does not work when baryons are involved since while space derivatives on baryons fields can be arranged to appear on the same footing as four-derivatives on pion fields, the time derivative on baryon fields picks up a term of order of the chiral symmetry breaking scale and hence cannot be used in the chiral counting. This problem is avoided in the HFF. To set up the HFF, the fermion momentum is written as

$$p^\mu = m v^\mu + k^\mu \tag{4}$$

where v^μ is the 4−velocity with $v^2 = 1$, and k^μ is the small residual momentum. (In the practical calculation that follows, we will choose the heavy-fermion rest frame $v^\mu = (1, \vec{0})$.) We define heavy fermion field $B_v(x)$ for a given four-velocity v^μ, by[#4]

$$B_v(x) = {\rm e}^{im\, v\cdot x} N(x). \tag{5}$$

The field B_v is divided into two parts which are eigenstates of $\not v$,

$$B_v = B_v^{(+)} + B_v^{(-)} \equiv \frac{1+\not v}{2} B_v + \frac{1-\not v}{2} B_v \equiv P_+ B_v + P_- B_v. \tag{6}$$

[#3] We have defined two covariant derivatives involving chiral fields, $\nabla_\mu \Sigma$ and Δ_μ. We can express one in terms of the other, but it is convenient as done frequently in the literature to use $\nabla_\mu \Sigma$ for the meson sector and Δ_μ for the meson-nucleon sector.

[#4] Another familiar field redefinition is $B_v(x) = {\rm e}^{im\gamma\cdot v\; v\cdot x} N(x)$, with $v^2 = 1$. This definition gives exactly the same physics to the lowest order in $\frac{1}{m}$ expansion.

As defined, $B_v^{(+)}(B_v^{(-)})$ can be identified as positive (negative) energy solution. As will be justified in the following subsection, the negative energy solution is suppressed for large baryon mass and its contribution can subsequently be incorporated as higher-order corrections in the inverse mass expansion. Thus to the leading order, the fermion loops can be ignored. With the neglect of the negative energy solutions, we have a useful relation for gamma matrices sandwiched between spinors which holds for any Γ,

$$\overline{B}_v \Gamma B_v = \overline{B}_v \not{v} \Gamma B_v = \overline{B}_v \Gamma \not{v} B_v = \overline{B}_v \frac{1}{2}\{\Gamma, \not{v}\} B_v \,. \tag{7}$$

It follows from this identity that

$$\overline{B}_v \gamma_5 B_v = 0, \quad \overline{B}_v \gamma^\mu B_v = v^\mu \overline{B}_v B_v. \tag{8}$$

Let us define spin operators S_v^μ by

$$\overline{B}_v \gamma_5 \gamma^\mu B_v = -2\overline{B}_v S_v^\mu B_v \tag{9}$$

or explicitly

$$S_v^\mu = \frac{1}{4}\gamma_5 \left[\not{v}, \gamma^\mu\right] \,. \tag{10}$$

The spin operators have the following identities,

$$\begin{aligned} \{S_v^\mu, S_v^\nu\} &= \frac{1}{2}(v^\mu v^\nu - g^{\mu\nu})\,, && (11)\\ [S_v^\mu, S_v^\nu] &= i\epsilon^{\mu\nu\alpha\beta} v_\alpha S_\beta \quad \text{with} \quad \epsilon_{0123} = 1\,. && (12) \end{aligned}$$

From the anti-commutation rule, we have

$$\begin{aligned} S_v \cdot S_v &= \frac{1}{4}(1-d) = -\frac{3-2\epsilon}{4} \simeq -\frac{3}{4}\,, && (13)\\ S_v^\alpha S_v^\mu S_{v\alpha} &= \frac{1}{4}(d-3) S_v^\mu \simeq \frac{1}{4} S_v^\mu, && (14)\\ (q\cdot S_v)^2 &= \frac{1}{4}\left[(q\cdot v)^2 - q^2\right] && (15) \end{aligned}$$

where d is the dimension of the space-time, $d = g_\mu^\mu$ and we have defined $\epsilon = (4-d)/2$. Between spinors, we have the approximate relations

$$\begin{aligned} \overline{B}_v\, S_v^\mu\, B_v &\simeq \left(\frac{1}{2}\vec{\sigma}\cdot\vec{v},\ \frac{1}{2}\vec{\sigma}\right), && (16)\\ \overline{B}_v \left[S_v^0, \vec{S}_v\right] B_v &\simeq -\frac{i}{2}\vec{v}\times\vec{\sigma} && (17) \end{aligned}$$

with $\vec{\sigma}$ the usual Pauli spin matrices. We see that S_v^0 and $\left[S_v^0, \vec{S}_v\right]$ are suppressed by a factor of $\vec{v} = \mathcal{O}\left(\frac{Q}{m}\right)$ where Q is the characteristic small momentum scale for processes with small three-velocity. Since

$$\begin{aligned} [S_v^\mu, S_v^\nu] &= \frac{i}{4}\left(\sigma^{\mu\nu} + \not{v}\sigma^{\mu\nu}\not{v}\right)\\ &= \frac{i}{2}\sigma^{\mu\nu} + \left(v^\mu S_v^\nu - v^\nu S_v^\mu\right)\gamma_5 \end{aligned}$$

where $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$, we also have

$$\overline{B}_v \left(\frac{i}{2}\sigma^{\mu\nu}\right) B_v = \overline{B}_v \left[S_v^\mu, S_v^\nu\right] B_v \tag{18}$$

$$\overline{B}_v \left(\sigma^{\mu\nu}\gamma_5\right) B_v = 2i\overline{B}_v \left(v^\mu S_v^\nu - v^\nu S^\mu\right) B_v. \tag{19}$$

We are now in position to write down the chiral Lagrangian (1) in HFF. The nucleon part of the Lagrangian becomes

$$\overline{N}(i\partial\!\!\!/ - m)N = \overline{B}_v iv\cdot\partial B_v \tag{20}$$

and the corresponding nucleon propagator $S(mv+k)$ is[#5]

$$iS(mv+k) = \frac{i}{v\cdot k + i0^+}. \tag{21}$$

Our chiral Lagrangian (1) expressed in terms of the heavy-fermion field to leading (*i.e*, zeroth) order in $\frac{1}{m}$ takes the form

$$\begin{aligned}\mathcal{L}_0 &= \overline{B}_v \left[iv^\mu(\partial_\mu + \Gamma_\mu) + 2ig_A S_v^\mu \Delta_\mu\right] B_v - \frac{1}{2}C_a\left(\overline{B}_v\Gamma_a B_v\right)^2 \\ &+ \frac{F^2}{4}\mathrm{Tr}\left(\nabla_\mu\Sigma^\dagger\nabla^\mu\Sigma\right) + \frac{1}{2}M^2F^2\mathrm{Tr}(\Sigma).\end{aligned} \tag{22}$$

In practical calculations, the chiral field Σ or ξ is expanded in power of the pion field. The explicit form resulting from such expansion as well as the vector and axial-vector currents calculated via Noether's theorem are given in Appendix A.

3.2 $\frac{1}{m}$ corrections

As mentioned, the HFF is based on simultaneous expansion in the chiral parameter and in "$1/m$". We have so far considered leading-order terms in $1/m$, namely, $O((1/m)^0)$. We now discuss $\frac{1}{m}$ corrections following closely the discussion of Grinstein [19]. We choose to do this in a perhaps more general way than needed for our purpose. Consider the following Lagrangian

$$\begin{aligned}\mathcal{L} &= \overline{N}\left[iD\!\!\!/ - m + \gamma^\mu\gamma_5 A_\mu\right] N - \frac{1}{2}C_a(\overline{N}\Gamma_a N)^2 \\ &= \overline{B}\left[iD\!\!\!/ - m(1 - v\!\!\!/) + \gamma^\mu\gamma_5 A_\mu\right] B - \frac{1}{2}C_a(\overline{B}\Gamma_a B)^2\end{aligned}$$

[#5] Although we do not actually encounter it in our calculation, it might be worthwhile to point out one technical subtlety. The infinitesimal $i0^+$ is inserted to define the singularity structure of the propagator. When we encounter a $d-$dimensional loop integral we first perform the Wick-rotation to put it in the Euclidean metric. In doing this, we assume that the first and third quadrants (in the plane of real l^0 vs. imaginary l^0) contain no poles. If we can take the flow direction of the loop-momentum to be the direction of the fermion momentum, there is no problem. However for some graphs it is impossible to do this. For instance consider a two-nucleon box diagram. In this case, we have one fermion line in which the loop-momentum flow direction is opposite to that of the fermion arrow. In this case, the fermion propagator is of the form

$$\frac{1}{v\cdot k - i0^+} = \frac{1}{v\cdot k + i0^+} + 2i\pi\delta(v\cdot k).$$

where we have included Weinberg's four-fermion contact term with Γ_a an arbitrary hermitian operator which we assume to contain no derivatives. We have also introduced an arbitrary "axial" field A_μ which we take to be hermitian and free of gamma matrices. (Here and in what follows, we shall omit the subscript v in B_v, $\overline{B}_v$ and S_v^μ.) The equation of motion satisfied by B is

$$[G - m(1-\not v)]\, B = 0 \tag{23}$$

with

$$G \equiv g - C_a \Gamma_a (\overline{B}\Gamma_a B), \quad g \equiv i\not D + \gamma^\mu\gamma_5 A_\mu. \tag{24}$$

Multiplying (23) on the left by P_-, we obtain

$$P_-\, G\, B - 2m B^{(-)} = 0$$

which leads to

$$B^{(-)} = \frac{1}{2m} P_-\, G\, B = \frac{1}{2m} P_-\, G\, B^{(+)} + \mathcal{O}\left(\frac{1}{m^2}\right). \tag{25}$$

Now multiplying P_+ to (23), we get

$$P_+\, G\, B = 0$$

which gives

$$P_+ \left[G + \frac{1}{2m} G\, P_-\, G\right] B^{(+)} = \mathcal{O}\left(\frac{1}{m^2}\right). \tag{26}$$

Given this, it is now a simple matter to write down the Lagrangian that gives rise to the equation of motion to the desired order [#6]. The result correct to $O(1/m)$ is

$$\begin{aligned} \mathcal{L} &= \overline{B}\left[g + \frac{1}{2m} g\, P_-\, g\right] B \\ &\quad -\frac{1}{2} C_a \left\{\overline{B}\left[\Gamma_a + \frac{1}{2m}\left(\Gamma_a\, P_-\, g + g\, P_-\, \Gamma_a\right)\right] B\right\}^2 \\ &\quad +\frac{1}{2m} C_a C_b \left(\overline{B}\Gamma_a B\right)\left(\overline{B}\Gamma_a\, P_-\, \Gamma_b B\right)\left(\overline{B}\Gamma_b B\right) \end{aligned} \tag{27}$$

with g defined in (24). To put this into a more standard form, we use the identities

$$\begin{aligned} P_+\, g\, P_+ &= P_+ \frac{1}{2}\{\not v, g\}\, P_+\,, \\ P_+\, g\, P_-\, g'\, P_+ &= P_+\, \{P_-\,, g\} g'\, P_+ = P_+\, g\, \{P_-\,, g'\}\, P_+\,, \\ P_+\, g\, P_-\, g\, P_+ &= P_+ \left[\frac{1}{2}\left\{\not v, g^2\right\} - \frac{1}{4}\{g, g\,\{\not v, g\}\}\right] P_+\,. \end{aligned} \tag{28}$$

One can show from these identities that

$$\left(\overline{B}^{(+)}\,\Gamma_a\, B^{(+)}\right) \otimes \overline{B}^{(+)}\,\Gamma_a\, P_- = 0 \tag{29}$$

[#6] We remind the reader that one should not insert the solution of $B^{(-)}$ into the original Lagrangian, since in Lagrangian approach, what is important is the form, not the value. However in Hamiltonian approach, the insertion of the solution for $B^{(-)}$ into the original Hamiltonian is allowed.

for any $\Gamma_a = \{1,\ \gamma_5,\ \gamma_\mu,\ \gamma_\mu\gamma_5,\ \sigma_{\mu\nu}\}$. This allows us to simplify the Lagrangian further to the form

$$\begin{aligned}\mathcal{L} &= \overline{B}\,(iv\cdot D + 2S\cdot A)\,B - \frac{1}{2}C_a\left(\overline{B}\Gamma_a B\right)^2 + \frac{1}{2m}\overline{B}\left(-D^2 + (v\cdot D)^2\right.\\ &\quad \left. + [S^\mu, S^\nu]\,[D_\mu, D_\nu] - (v\cdot A)^2 - 2i\,\{v\cdot A, S\cdot D\}\right) B, \qquad (30)\end{aligned}$$

for general Γ_a allowed by symmetries. In our case, $A_\mu = ig_A\Delta_\mu = ig_A\frac{1}{2}\xi^\dagger(\nabla_\mu\Sigma)\xi^\dagger$, so our $\frac{1}{m}$ term Lagrangian is

$$\begin{aligned}\delta\mathcal{L} &= \frac{1}{2m}\overline{B}\left(-D^2 + (v\cdot D)^2 + [S^\mu, S^\nu]\,[D_\mu, D_\nu] + g_A^2(v\cdot\Delta)^2 + 2g_A\,\{v\cdot\Delta, S\cdot D\}\right) B\\ &\quad + \mathcal{O}\left(\frac{1}{m^2}\right). \qquad (31)\end{aligned}$$

While Eq.(31) is the first "$1/m$" correction, it is not the entire story to the order considered. One can see that it is also the next order in the chiral counting in derivatives and it is expected in any case, independently of the inverse baryon mass corrections. Thus in a practical sense, the coefficients that appear in each term are parameters rather than fixed by chiral symmetry in HFF. We also note that in the derivation given above, neither $O(1/m)$ corrections to the quartic fermion term nor sixth-order fermion terms are generated by the $(1/m)$ expansion. This of course does not mean that such terms cannot contribute on a general ground. We have however checked that no such terms arise from exchanges of single heavy mesons that are formally "integrated out" which means that our calculations are not affected by such terms, at least to the order we are concerned with.

4 Counting Rules

In this section, we rederive and generalize somewhat Weinberg's counting rule[5] using HFF. Although we do not consider explicitly the vector-meson degrees of freedom, we include them here in addition to pions and nucleons. Much of what we obtain turn out to be valid in the presence of vector mesons. Now in dealing with them, their masses – which are comparable to the chiral scale Λ_χ – will be regarded as heavy compared to the momentum probed Q – say, scale of external three momenta or m_π.

In establishing the counting rule, we make the following key assumptions: Every intermediate meson (whether heavy or light) carries a four-momentum of order of Q. In addition we assume that for any loop, the effective cut-off in the loop integration is of order of Q. We will be more precise as to what this means physically when we discuss specific processes, for this clarifies the limitation of the chiral expansion scheme.

An arbitrary Feynman graph can be characterized by the number $E_N(E_H)$ of external – both incoming and outgoing – nucleon (vector-meson) lines, the number L of loops, the number $I_N(I_\pi,\ I_H)$ of internal nucleon (pion, vector-meson) lines. Each vertex can in turn be characterized by the number d_i of derivatives and/or of m_π factors and the number n_i

(h_i) of nucleon (vector-meson) lines attached to the vertex. Now for a nucleon intermediate state of momentum $p^\mu = mv^\mu + k^\mu$ where $k^\mu = \mathcal{O}(Q)$, we acquire a factor Q^{-1} since

$$S_F(mv+k) = \frac{1}{v\cdot k} = \mathcal{O}(Q^{-1}). \tag{32}$$

An internal pion line contributes a factor Q^{-2} since

$$\Delta(q^2; m_\pi^2) = \frac{1}{q^2 - m_\pi^2} = \mathcal{O}(Q^{-2}) \tag{33}$$

while a vector-meson intermediate state contributes Q^0 ($\sim O(1)$) as one can see from its propagator

$$\Delta_F(q^2; m_V^2) = \frac{1}{q^2 - m_V^2} \simeq \frac{1}{-m_V^2} = \mathcal{O}(Q^0) \tag{34}$$

where m_V represents a generic mass of vector mesons. Finally a loop contributes a factor Q^4 because its effective cut-off is assumed to be of order of Q. We thus arrive at the counting rule that an arbitrary graph is characterized by the factor Q^ν with

$$\nu = -I_N - 2I_\pi + 4L + \sum_i d_i \tag{35}$$

where the sum over i runs over all vertices of the graph. Using the identities, $I_\pi + I_H + I_N = L + V - 1$, $I_H = \frac{1}{2}\sum_i h_i - \frac{E_H}{2}$ and $I_N = \frac{1}{2}\sum_i n_i - \frac{E_N}{2}$, we can rewrite the counting rule

$$\nu = 2 - \frac{E_N + 2E_H}{2} + 2L + \sum_i \nu_i, \qquad \nu_i \equiv d_i + \frac{n_i + 2h_i}{2} - 2. \tag{36}$$

We recover the counting rule derived by Weinberg [5] if we set $E_H = h_i = 0$.

The situation is different depending upon whether or not there is external gauge field (*i.e.*, electroweak field) present in the process. In its absence (as in nuclear forces), ν_i is non-negative

$$d_i + \frac{n_i + 2h_i}{2} - 2 \geq 0. \tag{37}$$

This is guaranteed by chiral symmetry [5]. This means that the leading order effect comes from graphs with vertices satisfying

$$d_i + \frac{n_i + 2h_i}{2} - 2 = 0\,. \tag{38}$$

Examples of vertices of this kind are: $\pi^k NN$ with $k \geq 1\,(d_i = 1,\ n_i = 2,\ h_i = 0)$, $hNN\,(d_i = 0,\ n_i = 2,\ h_i = 1)$, $(\overline{N}\Gamma N)^2\,(d_i = 0,\ n_i = 4,\ h_i = 0)$, $h\pi^k$ with $k \geq 1\,(d_i = 1,\ n_i = 0,\ h_i = 1)$, etc where h denotes vector-meson fields.

In NN scattering or in nuclear forces, $E_N = 4$ and $E_H = 0$, and so we have $\nu \geq 0$. The leading order contribution corresponds to $\nu = 0$, coming from three classes of diagrams; one-pion-exchange, one-vector-meson-exchange and four-fermion contact graphs. In πN

scattering, $E_N = 2$ and $E_H = 0$, we have $\nu \geq 1$ and the leading order comes from nucleon Born graphs, seagull graphs and one-vector-meson-exchange graphs.#7

In the presence of external fields, the condition becomes [6]

$$\left(d_i + \frac{n_i + 2h_i}{2} - 2\right) \geq -1\,. \tag{39}$$

The difference from the previous case comes from the fact that a derivative is replaced by a gauge field. The equality holds only when $h_i = 0$, $n_i = 2$ or $h_i = 0$, $n_i = 0$. We will later show that this is related to the "chiral filter" phenomenon. The condition (39) plays an important role in determining exchange currents. Apart from the usual nucleon Born terms which are in the class of "reducible" graphs and hence do not enter into our consideration, we have two graphs that contribute in the leading order to the exchange current: the "seagull" graphs and "pion-pole" graphs #8, both of which involve a vertex with $\nu_i = -1$. On the other hand, a vector-meson-exchange graph involves a $\nu_i = +1$ vertex. This is because $d_i = 1$, $h_i = 2$ at the $J_\mu hh$ vertex. Therefore vector-exchange graphs are suppressed by power of Q^2. This counting rule is the basis for establishing the chiral filtering even when vector mesons are present (see Appendix I). Thus the results we obtain without explicit vector mesons are valid more generally.

5 Renormalization in Heavy-Fermion Formalism

In this section, we discuss renormalization in heavy fermion formalism. Most of the renormalized quantities that we will write down here have been obtained by others in standard ChPT [13]. We rederive them for completeness and as a check of our renormalization procedure.

For reasons stated above, fermion loops are suppressed in HFF. Our basic premise is that antiparticle solutions should be irrelevant to physical processes in large-mass and low-energy situations. Their effects can however be systematically taken into account in "$1/m$" expansion.

We shall denote "bare" quantities by $\mathring{m}$, $\mathring{M}$, $\mathring{F}$, $\mathring{g}_A$ and the corresponding "renormalized (to their physical values)" quantities by $m = \mathring{m} + \delta m$, M, F, g_A, respectively, for nucleon mass, pion mass, pion decay constant ($\simeq 93$ MeV) and axial coupling constant ($\simeq 1.25$).

5.1 Dimensional regularization

We adopt the dimensional regularization scheme to handle ultraviolet singularities in our loop calculations. It has the advantage of avoiding power divergences like $\delta(0) \sim \Lambda^4_{cut}$

#7 We note here that scalar glueball fields χ play only a minor role in πN scattering because the $\chi\pi\pi$ vertex ($d_i = 2,\ n_i = 0,\ h_i = 1$) acquires an additional Q power.

#8 These are standard jargons in the literature. See [2, 4].

where Λ_{cut} is the cut-off mass. In $d = 4 - 2\epsilon$ dimensions, all the infinities are absorbed in $\frac{1}{\epsilon}$. When heavy fields are involved, somewhat different parametrization schemes and integral formulas are needed. The relevant ones for our calculation are

$$\frac{1}{A^n B} = 2^n \frac{\Gamma(n+1)}{\Gamma(n)} \int_0^\infty d\lambda \frac{\lambda^{n-1}}{(2\lambda A + B)^{n+1}}, \tag{40}$$

$$\frac{1}{A^n BC} = 2^n \frac{\Gamma(n+2)}{\Gamma(n)} \int_0^1 dz \int_0^\infty d\lambda \frac{\lambda^{n-1}}{[2\lambda A + zB + (1-z)C]^{n+2}} \tag{41}$$

and

$$\Gamma(n) \int_0^\infty d\lambda \lambda^k (\lambda^2 + \Box^2)^{-n} = \frac{1}{2} \Box^{1+k-2n} \Gamma\left(n - \frac{k+1}{2}\right) \Gamma\left(\frac{k+1}{2}\right). \tag{42}$$

This integral is singular when

$$n - \frac{k+1}{2} = 0,\ -1,\ -2, \cdots \tag{43}$$

so ϵ must be kept finite until the integration in λ is performed. Some relevant integral identities needed in this paper are given in Appendix C.

As is customary in the dimensional regularization, we introduce an arbitrary mass scale μ. After renormalization, the results should, of course, be independent of the scale μ. Here some comments are in order regarding the one-loop renormalization scheme. First, all the divergences of our theory can be classified in two classes by their degrees of divergence: quadratic and logarithmic divergences. The quadratic divergences are removed by counter terms that are of *exactly the same form as the lowest chiral-order Lagrangian*, $\mathcal{L}_0$ as given by (22). Thus these quadratic singularities can be absorbed into the renormalization process of the basic quantities, namely, g_A, F, M, m. To take care of the logarithmic singularities, we include counter terms that are higher chiral order by Q^2 than $\mathcal{L}_0$. As it is an arduous task [13] to write down all possible counter terms, we shall write down only the counter terms needed for the calculation. All the quadratic divergences can be written in terms of $\Delta(M^2)$ and all the logarithmic divergences in terms of η defined by

$$\begin{aligned}\Delta(M^2) &= \frac{\mu^{4-d}}{i} \int \frac{d^d l}{(2\pi)^d} \frac{1}{l^2 - M^2} = -\frac{M^2}{16\pi^2} \Gamma(-1+\epsilon) \left(\frac{M^2}{4\pi\mu^2}\right)^{-\epsilon} \\ &= \frac{M^2}{16\pi^2} \left(\frac{1}{\epsilon} + 1 + \Gamma'(1) - \ln \frac{M^2}{4\pi\mu^2}\right) + \mathcal{O}(\epsilon),\end{aligned} \tag{44}$$

$$\eta = \frac{1}{16\pi^2} \Gamma(\epsilon) \left(\frac{M^2}{4\pi\mu^2}\right)^{-\epsilon} = \frac{1}{16\pi^2} \left(\frac{1}{\epsilon} + \Gamma'(1) - \ln \frac{M^2}{4\pi\mu^2}\right) + \mathcal{O}(\epsilon) \tag{45}$$

where $\epsilon = (4-d)/2$ and $\Gamma'(1) \simeq -0.577215$. Note that both Δ and η are scale-dependent and singular. But the coefficients of the counter terms (written down below) are also scale-dependent and singular. To remove the scale dependence and singularity, one must adjust the coefficients of the counter terms. This procedure however is not unique since finite parts of the coefficients of the counter terms are totally arbitrary. There are many ways to eliminate this non-uniqueness leading to a variety of "subtraction schemes." In this paper, we use the scheme whereby renormalization is made at the on-shell point for the nucleon and at zero four-momentum for the pion and the current. Thus the quantities g_A, F and the coefficients of the counter terms are defined at zero momentum of the axial current and the pion.

To make our discussion on renormalization streamlined, we list below *all* the counter terms needed in our work (for the meson sector, see [12]) to which we will refer back as required;

$$\begin{aligned}
\mathcal{L}_{\rm CT} &= \overline{B}\left[-\delta m + (Z_N - 1) i v\cdot D + i\frac{c_1}{F^2}(v\cdot D)^3\right] B \\
&+ i g_A \frac{c_2}{F^2}\overline{B}\, S^\mu v^\nu v^\alpha \left(\Delta_\mu D_\nu D_\alpha - \overleftarrow{D}_\nu\, \Delta_\mu D_\alpha + \overleftarrow{D}_\nu \overleftarrow{D}_\alpha\, \Delta_\mu\right) B \\
&+ i\frac{c_3}{F^2}\overline{B} v^\alpha \left[D^\beta, [D_\alpha, D_\beta]\right] B \\
&+ i\frac{c_4}{F^2}\overline{B}\left[S^\alpha, S^\beta\right]\{v\cdot D, [D_\alpha, D_\beta]\}\, B \\
&+ i\frac{c_5}{F^2}\overline{B}\,[v\cdot\Delta, [v\cdot D, v\cdot\Delta]]\, B \\
&+ i\frac{c_6}{F^2}\overline{B}\,[S\cdot\Delta, [v\cdot D, S\cdot\Delta]]\, B \\
&+ \Big\{-\frac{g_A}{8F^4} d_4^{(1)}\, \overline{B}\tau_a D_\mu B\cdot \overline{B}\,[v\cdot\Delta, \tau_a]\, S^\mu B \\
&\quad - \frac{g_A}{4F^4} d_4^{(2)} \left(\overline{B}\,[S^\mu, S^\nu]\, D_\mu B\cdot \overline{B} v\cdot\Delta\, S_\nu B + \overline{B}\,[S^\mu, S^\nu]\, v\cdot\Delta\, D_\mu B\cdot \overline{B} S_\nu B\right) \\
&\quad + \text{ h.c.}\Big\} \qquad (46)
\end{aligned}$$

with

$$\begin{aligned}
Z_N &= 1 + \frac{3(d-1)g_A^2}{4}\frac{\Delta(M^2)}{F^2}, \\
\delta m &= \frac{3 g_A^2 M^3}{32\pi F^2}, \\
c_1 &= \frac{3}{2} g_A^2 \eta + c_1^R, \\
c_2 &= \frac{d-3}{3} g_A^2 \eta + c_2^R, \\
c_3 &= -\frac{1}{6}\left[1 + (d+1) g_A^2\right]\eta + c_3^R, \\
c_4 &= -2 g_A^2 \eta + c_4^R, \\
c_5 &= \eta + c_5^R, \\
c_6 &= -(d-3) g_A^4 \eta + c_6^R, \\
d_4^{(1)} &= \kappa_4^{(1)} + \left[(d-1) g_A^2 - 2\right]\eta, \\
d_4^{(2)} &= \kappa_4^{(2)} - 8 g_A^2 \eta \qquad (47)
\end{aligned}$$

where c_i^R $(i = 1, 2, \cdots, 6)$ and $\kappa_4^{(1,2)}$ are finite renormalized constants that we will refer to as "finite counter terms." Since these finite counter terms and the finite parts of loop contributions are scale-independent, our final results also are scale-independent and regular. The chiral counting is immediate from the counter-term Lagrangian. We should mention for later purpose that the two-derivative four-fermion counter terms proportional to $d_4^{(1,2)}$ cannot be gotten from single low-mass resonance exchanges and hence do not figure in long-range as well as intermediate-range NN potentials.

We should note here that although the above counter-term Lagrangian contains the isospin matrix τ, chiral invariance of the Lagrangian is preserved. This can be verified by noting that

$$\left(R^{-1}\tau_a R\right)_{ij}\left(R^{-1}\tau_a R\right)_{kl} = (\tau_a)_{ij}\,(\tau_a)_{kl} \tag{48}$$

where $(i,j,k,l=1,2)$ and R is the SU(2) matrix for chiral transformation, $B\to RB$.#9 Also note that S^μ is hermitian while Δ_μ and $[S^\mu, S^\nu]$ are antihermitian and $\gamma^0 D_\mu^\dagger\gamma^0 = \overleftarrow{D}_\mu \equiv \overleftarrow{\partial}_\mu - \Gamma_\mu$ where Γ_μ is the antihermitian operator defined by $D_\mu = \partial_\mu + \Gamma_\mu$.

5.2 Pion properties to one loop

Since due to pair suppression fermion loops can be ignored, renormalization in the pion properties is rather simple. The wavefunction renormalization Z_π, renormalized mass M and pion decay constant F (here as well as in what follows renormalized at $q^2=0$ with $M\neq 0$) are given by [12]#10

$$\begin{aligned} Z_\pi &= 1-\frac{2}{3}\frac{\Delta(M^2)}{F^2}, && (49)\\ M^2 &= \overset{\circ}{M}{}^2\left[1-\frac{\Delta(M^2)}{2F^2}\right], && (50)\\ F &= \overset{\circ}{F}\left[1+\frac{\Delta(M^2)}{F^2}\right] && (51)\end{aligned}$$

with $\Delta(M^2)$ defined in Eq.(44).

5.3 Nucleon properties to one loop

One-loop graphs for nucleon propagator are given in Fig. 3. Fig. 3*b* vanishes due to isospin symmetry, so only Fig. 3*a* survives to contribute to the nucleon self-energy Σ,

$$\Sigma(v\cdot k) = -3\frac{g_A^2}{F^2}S^\alpha S^\beta \int_l \frac{l_\alpha l_\beta}{v\cdot(l+k)\,(l^2-M^2)} \tag{52}$$

where (and in what follows)

$$\int_l \equiv \frac{\mu^{4-d}}{i}\int\frac{d^dl}{(2\pi)^d}\,. \tag{53}$$

#9 Actually this equation can be simply understood if one uses an $O(3)$ representation. In this case, R is an orthogonal real matrix and $(\tau_a)_{ij} = -i\epsilon_{aij}$, $a,i,j=1,2,3$.

#10 We have not put in the counter terms that appear at $O(Q^4)$ in the pure meson sector, so the L_i^r terms of Gasser and Leutwyler [12] are missing from our expression. Since the meson sector proper does not concern us here, we will leave out such terms from now on.

From this, we get

$$Z_N \;=\; 1+\Sigma'(0) \;=\; 1+\frac{3(d-1)g_A^2}{4}\frac{\Delta(M^2)}{F^2} \tag{54}$$

where the prime on Σ stands for derivative with respect to $v\cdot k$. In our case confined to irreducible graphs, there is no nucleon pole, so we can set $v\cdot k=0$ in the denominator of the nucleon propagator. For an *off-shell* nucleon, we have

$$\Sigma(v\cdot k)=(d-1)\frac{3g_A^2}{4F^2}h(v\cdot k) \tag{55}$$

with the function $h(v\cdot k)$ defined by

$$\int_l \frac{l_\alpha l_\beta}{v\cdot(l+k)\,(l^2-M^2)} = g_{\alpha\beta}h(v\cdot k)+v_\alpha v_\beta(\cdots) \tag{56}$$

where $(\cdots)$ stands for a function that does not concern us. The evaluation of $h(v\cdot k)$ is described in Appendix D. For $(v\cdot k)^2\le M^2$, we have

$$\Sigma(y)=\frac{3(d-1)g_A^2}{4F^2}\Delta(M^2)\,y-\frac{3g_A^2}{2F^2}\eta\,y^3+\frac{3g_A^2}{4F^2}(M^2-y^2)\,\overline{h_0}(y), \tag{57}$$

where $y=v\cdot k$ and η is a singular quantity given by (45) and $\overline{h_0}$ is a finite function, the explicit form of which is given in Appendix D. We note that the above equation contains the $\frac{1}{\epsilon}$ divergence in the coefficient of $(v\cdot k)^3$ as well as in the coefficient of $(v\cdot k)$ (*i.e*, in $\Delta(M^2)$). This additional singularity arises also in the conventional method. See the paper by Gasser *et. al.*[13]. The counter term needed to remove this divergence, as given in (46), is

$$\Sigma_{\rm CT}(y)=\delta m-(Z_N-1)\,y+\frac{c_1}{F^2}\,y^3\,. \tag{58}$$

In order to regularize the propagator subject to the condition $\Sigma(0)=\Sigma'(0)=0$, we choose

$$\begin{aligned}
Z_N &= 1+\frac{3(d-1)g_A^2}{4}\frac{\Delta(M^2)}{F^2},\\
\delta m &= \frac{3g_A^2M^3}{32\pi F^2},\\
c_1 &= \frac{3}{2}g_A^2\eta+c_1^R.
\end{aligned}$$

The result is

$$\Sigma(y)=\delta m+\frac{c_1^R}{F^2}\,y^3+\frac{3g_A^2}{4F^2}\left(M^2-y^2\right)\overline{h_0}(y). \tag{59}$$

Here the finite constant c_1^R is in principle to be determined from experiments. To see its physical meaning, we should look at a process involving an off-shell nucleon. For instance, when $v\cdot k=\pm M$, we have

$$\Sigma(\pm M)=\pm\frac{c_1^R}{F^2}M^3+\delta m. \tag{60}$$

Finally, the $\frac{1}{m}$ correction is readily seen to be

$$\delta\Sigma(k)=-\frac{1}{2m}\left[k^2-(v\cdot k)^2\right]. \tag{61}$$

5.4 Renormalization of 3-Point Vertex Functions

In this subsection, we shall calculate three-point vertex functions to one-loop order, in particular, $J_\mu NN$ and πNN given in Fig. 4, where $J_\mu = A_\mu(V_\mu)$ denotes the axial vector (vector) current. We treat the vector current simultaneously since some vertices involving it figure in our calculation. Each vertex function is a sum of contributions from tree graphs, one-loop graphs, wavefunction renormalization, higher-order counter term insertion and $\mathcal{O}\left(\frac{1}{m}\right)$ corrections, if needed. The tree-graph contribution to $J_\mu NN$ is

$$\begin{aligned}\Gamma^{\mu,a}_{ANN}(\mathrm{tree}) &= g_A\,\tau_a\,S^\mu,\\ \Gamma^{\mu,a}_{VNN}(\mathrm{tree}) &= \frac{\tau_a}{2}\,v^\mu.\end{aligned} \tag{62}$$

Wavefunction renormalization produces multiplicative coefficients Z, *i.e.*, Z_N for $\Gamma^{\mu,a}_{ANN}$ and $Z_\pi^{\frac{1}{2}}Z_N$ for $\Gamma^a_{\pi NN}$ etc.

Unless noted otherwise we will always set the momentum flow of all pions and currents to be outgoing. The current-off-shell-nucleon couplings that we will consider are of the type

$$\begin{aligned}N(mv+k) &\to N(mv+k-q)+J^\mu_a(q),\\ N(mv+k) &\to N(mv+k-q)+\pi_a(q)\end{aligned}$$

with the relevant momenta indicated in the parentheses.

For completeness, we list in Appendix F all the contributions to the three-point functions of each Feynman graph.

5.4.1 Axial vertex function $\Gamma^{\mu,a}_{ANN}$

For off-shell nucleons, we find

$$\Gamma^{\mu,a}_{ANN} = g_A\,\tau_a\,S^\mu\left[1+\frac{\Delta(M^2)}{F^2}+\frac{3(d-1)g_A^2}{4F^2}\Delta(M^2)+\frac{d-3}{4}\frac{g_A^2}{F^2}h_3(v\cdot k,v\cdot q)\right] \tag{63}$$

where $h_3(v\cdot k,v\cdot q)\equiv\frac{1}{v\cdot q}\left[h(v\cdot k-v\cdot q)-h(v\cdot k)\right]$ is evaluated in Appendix D. The singularity in the above equation is removed by the counter term contribution

$$[\Gamma^{\mu,a}_{ANN}]_{\mathrm{CT}} = g_A\,\tau_a\,S^\mu\left(-\frac{c_2}{2F^2}\right)\left[3(v\cdot k)^2-3v\cdot k\,v\cdot q+(v\cdot q)^2\right] \tag{64}$$

with

$$c_2 = \frac{d-3}{3}g_A^2\eta + c_2^R \tag{65}$$

where c_2^R is a finite renormalized coupling constant. Adding the counter term contribution to the loop contribution, we obtain a renormalized axial coupling constant by $g_A = g_A(k=0,q=0)$ where $g_A(k,q)$ is defined by

$$\Gamma^{\mu,a\;R}_{ANN}(k,q)\equiv g_A(k,q)\,\tau_a\,S^\mu. \tag{66}$$

Physically $g_A(k,q)$ is just the axial charge form factor for the incoming nucleon of momentum $mv^\mu + k^\mu$ and the axial current carrying the momentum q^μ. Explicitly it is given by

$$\frac{g_A(k,q)}{g_A} = 1 + \frac{g_A^2}{4F^2}\overline{h}_3(v\cdot k, v\cdot q) - \frac{c_2^R}{2F^2}\left[3(v\cdot k)^2 - 3v\cdot k\, v\cdot q + (v\cdot q)^2\right] \qquad (67)$$

where $\overline{h}_3(v\cdot k, v\cdot q)$ is a finite function defined by

$$\overline{h}_3(v\cdot k, v\cdot q) \equiv h_3(v\cdot k, v\cdot q) + \Delta(M^2) - \frac{2}{3}\eta\left[3(v\cdot k)^2 - 3v\cdot k\, v\cdot q + (v\cdot q)^2\right] \qquad (68)$$

and

$$g_A = \mathring{g}_A\left[1 + \frac{\Delta(M^2)}{F^2}\left(1 + \frac{d}{2}g_A^2\right)\right] \simeq 1.25 \qquad (69)$$

where we have equated the renormalized g_A to the experimental value. Finally, the $\frac{1}{m}$ correction is

$$\delta\Gamma_{ANN}^{\mu,a} = -\frac{1}{2m}g_A\,\tau_a\, v^\mu\,(2k-q)\cdot S. \qquad (70)$$

For on-shell nucleons, $v\cdot k = v\cdot q = 0$ or in Breit frame, $v\cdot k = \frac{1}{2}v\cdot q$, so we find

$$g_A(k,q) = g_A. \qquad (71)$$

Note that we have neither momentum dependence nor $\frac{1}{m}$ corrections for an on-shell nucleon or in Breit frame. This means that there will be no one-loop correction to the πNN vertex in the exchange currents calculated below.

Given an experimental axial charge form factor of the off-shell nucleon, one can fix the constant c_2^R. The vertex $\Gamma_{\pi NN}^a$ can be obtained by a direct calculation or by means of Lehmann-Symanzik-Zimmermann (LSZ) formulation: Both give the same result

$$\Gamma_{\pi NN}^{a\;R}(k,q) = -i\frac{g_A(k,q)}{F}\,\tau_a\, q\cdot S. \qquad (72)$$

5.4.2 Vector vertex function $\Gamma_{VNN}^{\mu,a}$

The full form of $\Gamma_{VNN}^{\mu,a}$ is rather involved,

$$\Gamma_{VNN}^{\mu,a} = \frac{\tau_a}{2}\left(v^\mu F_1^V + \frac{1}{m}[q\cdot S, S^\mu]\, F_2^V - q^\mu F_3^V\right) \qquad (73)$$

where

$$\begin{aligned}
F_1^V &= 1 - \frac{1}{F^2}\left[f_1(q^2) - \Delta(M^2)\right] + \frac{g_A^2}{F^2}\otimes \\
&\quad \left\{(d-1)\left[\frac{3}{4}\Delta(M^2) - \frac{1}{4}h_3(v\cdot k, v\cdot q) - B_2(k,q)\right]\right. \\
&\quad\quad \left. +2\left[(v\cdot q)^2 - q^2\right]\frac{\partial}{\partial q^2}B_2(k,q) + 4v\cdot q B_1(k,q)\right\}, \\
F_2^V &= 4m\frac{g_A^2}{F^2}B_0(k,q), \\
F_3^V &= \frac{v\cdot q}{F^2}f_3(q^2) + \frac{g_A^2}{F^2}\left[(d+1) - 2\left((v\cdot q)^2 - q^2\right)\frac{\partial}{\partial q^2}\right]B_1(k,q)
\end{aligned} \qquad (74)$$

with the functions f_i, h_i and B_i given explicitly in Appendix B, D and E, respectively. Although the above equations appear to have quadratic divergences, they actually have only logarithmic divergences as can be seen below :

$$\begin{aligned}
F_1^V &= 1-\frac{9}{4}(2v\cdot k-v\cdot q)\frac{g_A^2}{F^2}\frac{M}{6\pi}+\frac{\eta}{6F^2}q^2+\frac{d+1}{6}\frac{g_A^2}{F^2}\eta q^2 \\
&\quad +\frac{d-1}{2}\eta\frac{g_A^2}{F^2}\left[3(v\cdot k)^2-3v\cdot k\,v\cdot q+(v\cdot q)^2\right]+\cdots, \\
F_2^V &= -4m\frac{M}{16\pi}\frac{g_A^2}{F^2}+2m\frac{g_A^2}{F^2}(2v\cdot k-v\cdot q)\eta+\cdots, \\
F_3^V &= \left[1+(d+1)g_A^2\right]\frac{v\cdot q}{6F^2}\eta+\cdots
\end{aligned} \tag{75}$$

where the ellipsis denotes finite and $O(Q^n)|_{n>2}$ terms. Quadratic divergences disappear because of EM gauge invariance. We see that $\Gamma^{\mu,a}_{VNN}(k=q=0)=\frac{1}{2}\tau_a\,v^\mu$. The remaining logarithmic divergences are removed by the counter term in Eq.(46) of the form

$$\begin{aligned}
[\Gamma^{\mu,a}_{VNN}]_{\rm CT} &= -\frac{c_1}{2F^2}\tau_a v^\mu\left[3(v\cdot k)^2-3v\cdot q\,v\cdot k+(v\cdot q)^2\right] \\
&\quad +\frac{c_3}{2F^2}\tau_a\left(q^2\,v^\mu-v\cdot q\,q^\mu\right) \\
&\quad +\frac{c_4}{2F^2}\tau_a\,v\cdot(2k-q)\,[q\cdot S,S^\mu]
\end{aligned} \tag{76}$$

with

$$\begin{aligned}
c_3 &= -\frac{1}{6}\left[1+(d+1)g_A^2\right]\eta+c_3^R, \\
c_4 &= -2g_A^2\eta+c_4^R.
\end{aligned} \tag{77}$$

When $v\cdot k=v\cdot q=0$, we have $F_3^V=0$ and

$$F_1^V(q^2) = 1+\frac{c_3^R}{F^2}q^2-\frac{q^2}{16\pi^2F^2}\left[\frac{1+3g_A^2}{2}K_0(q^2)-2(1+2g_A^2)K_2(q^2)\right], \tag{78}$$

$$F_2^V(q^2) = -\frac{g_A^2 m}{4\pi F^2}\int_0^1 dz\sqrt{M^2-z(1-z)q^2}+1 \tag{79}$$

where we have added the $\frac{1}{m}$ correction appearing in the second term of F_2^V [#11]. It is easy to see in (78) that the counter term constant c_3^R can be related directly to the isovector charge radius of the nucleon. We will give the precise relation later. $K_i(q^2)$ are the finite pieces of the functions $f_i(q^2)$ defined in Appendix B,

$$\begin{aligned}
K_0(q^2) &= \int_0^1 dz\ln\left[1-z(1-z)\frac{q^2}{M^2}\right], \\
K_2(q^2) &= \int_0^1 dz\,z(1-z)\ln\left[1-z(1-z)\frac{q^2}{M^2}\right].
\end{aligned} \tag{80}$$

[#11] Although the loop contribution to the Pauli form factor F_2^V is finite and hence requires no infinite counter term, there is a finite counter term contributing to it which we did not – but should – include in our formula. As pointed out by Bernard *et al.* [20], the finite counter term can be considered as coming from the ρ exchange as in vector dominance picture. This point will be addressed more precisely in [14].

5.5 Renormalization of 4-Point Vertex Function

In this section, we study the 4-point vertex functions denoted $\Gamma^{\mu,ab}_{\pi A}$ and $\Gamma^{\mu,ab}_{\pi V}$ to one loop as given in Fig. 5, corresponding to the process

$$N(mv+k) \to N(mv+k-q_a-q_b) + J^\mu_a(q_a) + \pi_b(q_b)$$

where the isospin components of the current and the pion are denoted by the subscripts a and b, respectively. Here $v\cdot k$ represents how much off-shell the incoming nucleon is and $v\cdot(k-q_a-q_b)$ the same for the outgoing nucleon. For tree graphs, we have

$$\Gamma^{\mu,ab}_{\pi A}(\text{tree}) = -\frac{1}{2F}\epsilon_{abc}\tau_c\, v^\mu. \tag{81}$$

The full formulas for non-vanishing graphs for off-shell nucleons are given in Appendix G. Here we limit ourselves only to the on-shell nucleon case. For axial-charge transitions, only the six graphs Fig.5 $(a)-(f)$ survive. Figures 5 $(g),(h)$ are proportional to S_μ, so suppressed for the time component and Figures 5 $(i)-(n)$ are proportional to $v\cdot S=0$. Figures 5 $(o)-(r)$ do not contribute to the axial current. (We have included them for later purpose, see [14].)

Adding the loop contributions and tree graphs with their wavefunction renormalization constant, we have

$$\Gamma^{\mu,ab}_{\pi A} = \epsilon_{abc}\tau_c\,\Gamma^{\mu,-}_{\pi A} + i\delta_{ab}\,\Gamma^{\mu,+}_{\pi A} \tag{82}$$

with

$$\begin{aligned}
\Gamma^{\mu,-}_{\pi A} &= -\frac{v^\mu}{2F}\left\{1-\frac{1}{F^2}\left[\left(1+(d-1)g_A^2\right)\overline{f_1}(q^2) - 8g_A^2(q\cdot S)^2 f_2(q^2) - v\cdot q_a\, h_0^A(v\cdot q_a)\right]\right\}\\
&\quad -\frac{2g_A^2}{F^3}\left[q\cdot S, S^\mu\right]B_0(q^2)\\
&\quad +\frac{g_A^4}{F^3}\left(\frac{1-d}{4}\left\{q_b\cdot S, S^\mu\right\}h_4^A(v\cdot q_a) + \frac{d-3}{4}\left[q_b\cdot S, S^\mu\right]h_4^S(v\cdot q_a)\right), \qquad (83)\\
\Gamma^{\mu,+}_{\pi A} &= -(q_a+3q_b)^\mu\frac{4g_A^2}{3F^3}\left[\frac{1-d}{4} + 2(q\cdot S)^2\frac{\partial}{\partial q^2}\right]B_0(q^2) + \frac{v^\mu}{F^3}v\cdot q_a\, h_0^S(v\cdot q_a)\\
&\quad -3\frac{g_A^4}{F^3}\left(\frac{1-d}{4}\left\{q_b\cdot S, S^\mu\right\}h_4^S(v\cdot q_a) + \frac{d-3}{4}\left[q_b\cdot S, S^\mu\right]h_4^A(v\cdot q_a)\right) \qquad (84)
\end{aligned}$$

where $q=q_a+q_b$, $\overline{f_1}(q^2)=f_1(q^2)-f_1(0)$ and $h_i^{S,A}(y)=\frac{1}{2}\left[h_i(y)\pm h_i(-y)\right]$. The integrals defining the functions $f_i(q^2)$ for $i=0,1,2,3$ are listed and evaluated in Appendix B. (h_i's are defined in Appendix D and B_0 in Appendix E.) The log divergences contained in these vertices are removed by the counter term contribution,

$$\left[\Gamma^{\mu,ab}_{\pi A}\right]_{\rm CT} = -\epsilon_{abc}\frac{1}{F}\left[\Gamma^{\mu,c}_{VNN}\right]_{\rm CT}$$

$$
\begin{aligned}
&+ \epsilon_{abc}\tau_c \frac{c_5}{F^2} v^\mu (v\cdot q_a)^2 \\
&+ \frac{c_6}{2F^2}\frac{v\cdot q_a}{F}\left(i\delta_{ab}\left[S^\mu, q_b\cdot S\right] - \epsilon_{abc}\tau_c \left\{S^\mu, q_b\cdot S\right\}\right)
\end{aligned}
\tag{85}
$$

with

$$
\begin{aligned}
c_5 &= \eta + c_5^R, \\
c_6 &= -(d-3)g_A^4\eta + c_6^R
\end{aligned}
\tag{86}
$$

where c_i^R are renormalized finite constants listed in Eq.(46) and $[\Gamma^{\mu,c}_{VNN}]_{\rm CT}$ is given by (76).

With *soft* momentum, we have $v\cdot q_a = 0$ for which we obtain a surprisingly simple expression, *viz*,

$$
\Gamma^{\mu=0,ab}_{\pi A} = -\epsilon_{abc}\tau_c \frac{1}{2F} F_1^V(t) \tag{87}
$$

where $t \equiv q^2 = (q_a+q_b)^2$ and F_1^V is the isovector Dirac form factor Eq.(78). The one-loop renormalization of the πANN vertex corresponds to the isovector charge radius obtainable from the form factor F_1^V for which the finite counter term c_3^R plays a key role. We see that $\Gamma^{\mu,ab}_{\pi A}$ and $\Gamma^{\mu,a}_{ANN}$ are related, respectively, to $\Gamma^{\mu,a}_{VNN}$ and $\Gamma^{\mu,ab}_{\pi V}$ calculated in Appendix G. That the $A_\mu \pi NN$ vertex for a soft pion is simply given by F_1^V has of course been understood since a long time via current algebra and also in terms of the ρ-meson exchange.

Finally for the $\frac{1}{m}$ correction, one can readily obtain the corrections to the vertices

$$
\begin{aligned}
\delta\Gamma^{\mu,ab}_{\pi A} &= -\epsilon_{abc}\tau_c \frac{1}{4mF}\left(2k^\mu - q^\mu - v^\mu(2v\cdot k - v\cdot q) + 2\left[q\cdot S, S^\mu\right]\right) \\
&\quad - i\delta_{ab}\, v^\mu\, v\cdot q_a \frac{g_A^2}{4mF}, \\
\delta\Gamma^{\mu,ab}_{\pi V} &= -i\delta_{ab}\, v\cdot q_a \frac{g_A}{2mF} S^\mu
\end{aligned}
\tag{88}
$$

where k is the residual momentum of the incoming nucleon and $k-q = k - q_a - q_b$ is that of the outgoing nucleon. An important point to note from Eq.(88) is that *for the case $v\cdot q_a = 0$ and on-shell nucleons, we have no contribution from $\frac{1}{m}$ corrections to the time component of the axial current and the space component of the vector current.* This is the basis for the pair suppression we will exploit in the application to axial-charge transitions in nuclei.

The complete list of the four-point functions involving the vector and axial-vector currents needed here and in [14] is given in Appendix G.

6 Two-Body Exchange Currents

So far we have computed one-loop corrections to the graphs involving one nucleon. They are extractable from experimental data on nucleon properties. In this section we incorporate the above corrections into – and derive – two-body exchange currents in heavy-fermion chiral perturbation theory. As shown previously [7], the time component of the axial-vector current (and also the space component of the vector current [14]) in the long-wavelength limit is best amenable to chiral perturbation loop calculations.#12 We will work out the computation to one loop order corresponding to the next-to-leading order in the chiral counting rules as derived in Section 4. The process of interest is

$$N(p_1)+N(p_2)\to N(p_1')+N(p_2')+J_a^\mu(k),$$

where we have indicated the relevant kinematics with $q_2=p_2'-p_2$, $q_1=p_1'-p_1$ and the energy-momentum conservation $q_1+q_2+k=0$. The process is *soft* in the sense that

$$v\cdot k\simeq v\cdot q_i=v\cdot(p_i'-p_i)\simeq\mathcal{O}\left(\frac{Q^2}{m}\right)\simeq 0$$

where m is the nucleon mass or chiral scale. This kinematics markedly simplifies the calculation. Clearly this kinematics does not hold, say, for energetic real photons.

It is convenient to classify graphs by the current vertex involved. The graphs that contain $J_\mu\pi NN$ play a dominant role since $J_\mu\pi NN\sim v^\mu$ for the axial current (and $J_\mu\pi NN\sim S^\mu$ for the vector current). The graphs which contain $J_\mu NN$ (and $J_\mu\pi\pi NN$ for the vector current) can be ignored to the relevant order because the role of the vector and axial currents is interchanged. In what follows, we discuss the axial current only. The argument for the vector current goes almost in parallel and will be detailed in [14].

What we are particularly interested in is the time component of the axial current, with axial-charge transitions in nuclei in mind. This is where the "pion dominance" is particularly cleanly exhibited. The space component is also interesting both theoretically and phenomenologically. Theoretically Gamow-Teller transitions – observed to be quenched – represent the other side of the coin relating to the chiral filter phenomenon discussed above, namely that chiral symmetry alone or more precisely *soft* mechanisms associated with it cannot make statements on this quantity [21]. Empirically the quenching phenomenon is closely associated with the missing strength of giant Gamow-Teller resonances in nuclei. Since the treatment of the space part of the axial current requires going beyond chiral perturbation theory, we will not pursue this issue any further in this paper.

For convenience, we define an "axial-charge" operator $\mathcal{M}$ by#13

$$\vec{A}^{\mu=0}\equiv\frac{g_A}{4F^2}\vec{\mathcal{M}}. \tag{89}$$

#12This is the crucial point in using ChPT in nuclei that quantifies the general discussion given in Section 2. Since this point is often misunderstood by nuclear physics community, we would like to stress it once more although to some it may sound obvious and repetitive. The chiral counting on which our analysis is based is meaningful only if $\frac{Q^2}{m^2}<<1$ where Q is the characteristic momentum or energy scale involved in the process. Therefore ChPT cannot describe processes that involve energy or momentum scale exceeding that criterion. This means that processes probing short-distance interactions are not accessible by finite-order ChPT. In particular two-body currents describing an internuclear distance $r_{12}\le 0.6$ fm cannot be probed by the expansion we are using. We argued before – and will make use of the fact – that there is a natural cut-off provided by short-range nuclear interactions that go beyond the strategy of ChPT which allows a meaningful use of the small Q expansion.

#13The operator $\mathcal{M}$ is an isovector but in what follows we will not explicitly carry the isospin index.

We decompose $\mathcal{M}$ into $\mathcal{M} = \mathcal{M}_{tree} + \mathcal{M}_{loop}$ where $\mathcal{M}_{tree}$ denotes the axial charge operator coming from the one-pion-exchange tree graph (*i.e*, the soft-pion term) and $\mathcal{M}_{loop}$ is what comes from loop corrections. Further we decompose $\mathcal{M}_{loop}$ into

$$\mathcal{M}_{loop} = \mathcal{M}_{1\pi} + \mathcal{M}_{2\pi} \tag{90}$$

where $\mathcal{M}_{1\pi}$ denotes the loop correction to the one-pion-exchange axial charge operator (also referred in the literature to as "seagull graph") and $\mathcal{M}_{2\pi}$ the contribution from two-pion-exchange graphs and *tree graphs* involving four-fermion contact terms with counter-term insertions. We will later argue that the latter does not contribute. One-loop graphs involving four-fermion contact interactions, while allowed in the relevant chiral order, do not however contribute either.

6.1 Results in momentum space

As stated above, the non-zero contributions to the time component of the axial current come only from the graphs that contain a $J_\mu \pi NN$ vertex. The tree seagull graph supplemented with a vertex form factor – and properly renormalized – leads to[#14]

$$\vec{A}^\mu_{tree} + \vec{A}^\mu_{1\pi} = i\vec{\tau}_1 \times \vec{\tau}_2 \frac{g_A}{2F^2}\, v^\mu\, q_2 \cdot S_2\, \frac{1}{M^2 - q_2^2} F_1^V(q_1^2) + (1 \leftrightarrow 2) \tag{91}$$

where F_1^V is the Dirac isovector form factor of the nucleon. To the order considered, there are no further corrections. The present formalism allows us to calculate within the scheme the form factor F_1^V. The tree-graph (or soft-pion) contribution corresponds to taking $F_1^V = 1$. The difference $(F_1^V - 1)$ is given by Figs. $5(a) - (f)$. Taking $k^\mu \to 0$, we encounter two spin-isospin operators,

$$\mathcal{T}^{(1)} \equiv -2i\vec{\tau}_1 \times \vec{\tau}_2\, q \cdot S_1 + (1 \leftrightarrow 2) \simeq i\vec{\tau}_1 \times \vec{\tau}_2\, \vec{q} \cdot (\vec{\sigma}_1 + \vec{\sigma}_2), \tag{92}$$

$$\mathcal{T}^{(2)} \equiv 2(\vec{\tau}_1 + \vec{\tau}_2)\, [q \cdot S_2, S_1 \cdot S_2] + (1 \leftrightarrow 2) \simeq i(\vec{\tau}_1 + \vec{\tau}_2)\, \vec{q} \cdot \vec{\sigma}_1 \times \vec{\sigma}_2 \tag{93}$$

with $q^\mu \equiv q_2^\mu \simeq -q_1^\mu$. With the help of these operators, we can rewrite (91) as

$$\mathcal{M}_{1\pi} = -\mathcal{T}^{(1)} \frac{1}{M^2 - q^2} \left[F_1^V(q^2) - 1\right] \tag{94}$$

with $\mathcal{M}_{tree} = -\mathcal{T}^{(1)} \frac{1}{M^2 - q^2}$.

As for two-pion-exchange and four-fermion contact interaction contributions, the relevant diagrams are those given in Fig.$6(a) - (k)$ and their symmetrized ones. Before going

[#14] We denote particle indices by $i = 1, 2$ without expliciting heavy fermion fields. For instance, S_1 should be understood as the spin operator sandwiched between $\overline{B}_v$ and B_v of particle 1 with velocity v. In this section, q^2 is the four-momentum squared of the pion but we are concerned with the situation where $\frac{q_0}{|\mathbf{q}|} << 1$, so the static approximation $q^2 \approx -|\mathbf{q}|^2$ will be made in practical calculations and also in Fourier-transforming to coordinate space later. In fact, the static approximation is not only natural for the chiral counting but also essential for suppression of n-body forces and currents for $n > 2$. More on this later.

into any details, one can readily see that each graph in Fig. 6 contributes a term of at least $O(Q)$. This can be shown both in the conventional method and in HFF by observing that their contributions vanish if we set $M = q_i^\mu = k^\mu = 0$. This assures us that our counting rule is indeed correct. Therefore we can neglect all the graphs proportional to S^μ since the axial-charge operator involves $S^0 \sim O(Q/m_N)$ as stated before. Figures (f), (g), (h) and (j) belong to this class. Now Fig. (e) is identically zero because of the isospin symmetry and Fig. (i) is proportional to $v \cdot S = 0$. The graph (k), involving time-ordered pion propagators, are the so-called "recoil graphs" [4] which as we shall argue in Section 8 will be cancelled by similar recoil terms in reducible graphs. So we are left with only the four graphs (a), (b), (c) and (d) to calculate. Without any further approximation than using HFF, the full expression of the four graphs comes out to be

$$\begin{aligned}
\vec{A}^\mu(a) &= -(2\vec{\tau}_2 - i\vec{\tau}_1 \times \vec{\tau}_2)\,\frac{g_A}{8F^4}\,(v^\mu\, q_2 \cdot S)_1\; f_0(q_2^2),\\
\vec{A}^\mu(b) &= (2\vec{\tau}_2 + i\vec{\tau}_1 \times \vec{\tau}_2)\,\frac{g_A}{8F^4}\,(q_2 \cdot S\, v^\mu)_1\; f_0(q_2^2),\\
\vec{A}^\mu(c) &= (-2\vec{\tau}_1 - 2\vec{\tau}_2 + i\vec{\tau}_1 \times \vec{\tau}_2)\frac{g_A^3}{2F^4}\,(v^\mu\, S^\alpha)_1 \left(S^\beta S^\nu\right)_2 I_{\nu,\alpha\beta}(q_2),\\
\vec{A}^\mu(d) &= (2\vec{\tau}_1 + 2\vec{\tau}_2 + i\vec{\tau}_1 \times \vec{\tau}_2)\frac{g_A^3}{2F^4}\,(S^\alpha\, v^\mu)_1 \left(S^\nu S^\beta\right)_2 I_{\nu,\alpha\beta}(q_2),
\end{aligned} \tag{95}$$

with $f_0(q^2)$ given in detail in Appendix B and $I_{\mu,\alpha\beta}(q)$ defined by

$$I_{\mu,\alpha\beta}(q) = \int_l \frac{(l+q)_\mu l_\alpha l_\beta}{v\cdot l\; v\cdot(l+q)\,(l^2 - M^2)\,[(l+q)^2 - M^2]}. \tag{96}$$

This integral is evaluated in Appendix E. Using the conditions $v \cdot q_i = 0$ and $k^\mu \simeq 0$, we can rewrite them in a symmetrized form

$$\vec{A}^\mu(a+b) = \frac{v^\mu}{16\pi^2}\frac{g_A}{8F^4}\left[K_0(q^2) - 16\pi^2\eta\right]\mathcal{T}^{(1)}, \tag{97}$$

$$\begin{aligned}
\vec{A}^\mu(c+d) &= -\frac{v^\mu}{16\pi^2}\frac{g_A^3}{16F^4}\Big\{\left[-(d-1)16\pi^2\eta + 3K_0(q^2) + 2K_1(q^2)\right]\mathcal{T}^{(1)}\\
&\quad -8\left[K_0(q^2) - 16\pi^2\eta\right]\mathcal{T}^{(2)}\Big\},\\
\left[\vec{A}^\mu\right]_{\rm CT} &= -v^\mu\,\frac{g_A}{16F^4}\left(d_4^{(1)}\mathcal{T}^{(1)} + d_4^{(2)}\mathcal{T}^{(2)}\right).
\end{aligned} \tag{98}$$

where $q^\mu \equiv \frac{1}{2}(q_2 - q_1)^\mu$ and $\left[\vec{A}^\mu\right]_{\rm CT}$ is the contribution from the counter-term Lagrangian (46). The $K_i(q^2)$ are finite pieces of the functions $f_i(q^2)$ defined in Appendix B, *i.e.*,

$$K_0(q^2) = \int_0^1 dz \ln\left[1 - z(1-z)\frac{q^2}{M^2}\right],$$

$$K_1(q^2) = \int_0^1 dz \frac{-z(1-z)q^2}{M^2 - z(1-z)q^2}.$$

The expressions (97) and (98) contain singularities in η, which are removed by the counter term contribution with

$$\begin{aligned}
d_4^{(1)} &= \kappa_4^{(1)} + \left[(d-1)g_A^2 - 2\right]\eta,\\
d_4^{(2)} &= \kappa_4^{(2)} - 8g_A^2\eta
\end{aligned} \tag{99}$$

where the renormalized constants κ_4's are finite and scale-independent.

The resulting two-body axial-charge operator including finite counter-term contributions is

$$\mathcal{M}_{2\pi} = \frac{1}{16\pi^2 F^2}\left\{\left[-\frac{3g_A^2-2}{4}K_0(q^2) - \frac{1}{2}g_A^2 K_1(q^2)\right]\mathcal{T}^{(1)} + 2g_A^2 K_0(q^2)\mathcal{T}^{(2)}\right\} - \frac{1}{4F^2}\left(\kappa_4^{(1)}\mathcal{T}^{(1)} + \kappa_4^{(2)}\mathcal{T}^{(2)}\right). \tag{100}$$

The two-body axial-charge operator due to loop correction is then the sum of (94) and (100)

$$\mathcal{M}_{loop} = \mathcal{M}_{1\pi} + \mathcal{M}_{2\pi}. \tag{101}$$

As it stands, the constants κ_4's are the only unknowns in the theory as they cannot be determined from nucleon-nucleon interactions as mentioned before. They could in principle be extracted from two-nucleon processes like $N+N \to N+N+\pi$ but they appear as higher-order corrections and it is inconceivable to obtain an information on these presumably small constants from such processes. However as argued above, we expect the constants $\kappa_4^{(i)}$ to be numerically small and what is more significant, when we go to coordinate space as we shall do below to apply the operator to finite nuclei, they become δ functions and will be completely suppressed as we discussed in Section 2. In momentum space, such constant terms have also to be removed as done for the celebrated Lorentz-Lorenz effect (or more generally for the Landau-Migdal g_0') in pion-nuclear scattering [22]. It should be stressed that *once the constant counter terms are removed, no unknown parameters enter at next to the leading order in the chiral expansion in nuclei.* It is also noteworthy that to the order considered, the loop contributions are renormalization-scale independent.

6.2 Going to coordinate space

Applications in nuclear transitions are made more readily in configuration space. Furthermore considerations based on ranges of nucleon-nucleon interactions which seem necessary for rendering chiral symmetry meaningful in nuclei are more transparent in this space. Therefore we wish to Fourier-transform the operators (97) and (98) into a form suitable for calculations with realistic nuclear wave functions. In doing this, we will treat the pion propagator in static approximation, namely, $q^2 \approx -|\mathbf{q}|^2$. In Appendix B, we show how the highly oscillating integrals involved in the calculation can be converted into integrals of smooth functions by performing the Fourier transform *before* doing the parametric integration. Since the spin-isospin operators $\mathcal{T}^{(1)}$ and $\mathcal{T}^{(2)}$ contain $\vec{q}$ – which is a derivative operator in configuration space, it is convenient to define

$$\begin{aligned}\tilde{\mathcal{T}}^{(1)} &= \vec{\tau}_1 \times \vec{\tau}_2\ \hat{r}\cdot(\vec{\sigma}_1 + \vec{\sigma}_2),\\ \tilde{\mathcal{T}}^{(2)} &= (\vec{\tau}_1 + \vec{\tau}_2)\ \hat{r}\cdot(\vec{\sigma}_1 \times \vec{\sigma}_2).\end{aligned} \tag{102}$$

Writing Eqs.(94) and (100) in coordinate space which we will denote by $\tilde{\mathcal{M}}$ to distinguish from the momentum-space expression, we obtain – modulo δ function terms mentioned above – the principal result of this paper:

$$\begin{aligned}
\tilde{\mathcal{M}}_{tree}(r) &= \tilde{\mathcal{T}}^{(1)} \frac{d}{dr}\left[-\frac{1}{4\pi r}\mathrm{e}^{-Mr}\right], \qquad (103)\\
\tilde{\mathcal{M}}_{1\pi}(r) &= c_3^R \frac{M^2}{F^2}\tilde{\mathcal{M}}_{tree}\\
&+ \frac{\tilde{\mathcal{T}}^{(1)}}{16\pi^2F^2}\frac{d}{dr}\left\{-\frac{1+3g_A^2}{2}\left[K_0(r)-\tilde{K}_0(r)\right]+(2+4g_A^2)\left[K_2(r)-\tilde{K}_2(r)\right]\right\}, \qquad (104)\\
\tilde{\mathcal{M}}_{2\pi}(r) &= \frac{1}{16\pi^2F^2}\frac{d}{dr}\left\{-\left[\frac{3g_A^2-2}{4}K_0(r)+\frac{1}{2}g_A^2K_1(r)\right]\tilde{\mathcal{T}}^{(1)}+2g_A^2K_0(r)\tilde{\mathcal{T}}^{(2)}\right\}, \qquad (105)\\
\tilde{\mathcal{M}}_{loop}(r) &= \tilde{\mathcal{M}}_{1\pi}(r)+\tilde{\mathcal{M}}_{2\pi}(r). \qquad (106)
\end{aligned}$$

As defined, $\mathcal{M}_{n\pi}$ are $n\pi$ exchange corrections to the soft-pion (tree) term. $\mathcal{M}_{loop}$ is therefore the total loop correction we wish to calculate. The explicit forms of the functions $K_i(r)$ and $\tilde{K}_i(r)$ are given in Appendix B.

As noted above, the constant c_3^R can be extracted from the isovector Dirac form factor of the nucleon, *i.e.*,

$$c_3^R\frac{M^2}{F^2} = \frac{M^2}{6}\langle r^2\rangle_1^V \simeq 0.04784. \qquad (107)$$

It is interesting to separate what we might call "long wavelength contribution" from $\tilde{\mathcal{M}}_{1\pi}$,

$$\tilde{\mathcal{M}}_{1\pi}(r) = \delta_{soft}\,\tilde{\mathcal{M}}_{tree}(r)\ +\ (\text{short range part})$$

where

$$\delta_{soft} = c_3^R\frac{M^2}{F^2}+\frac{M^2}{16\pi^2F^2}\left[\frac{1+3g_A^2}{2}\left(2-\frac{\pi}{\sqrt{3}}\right)-(1+2g_A^2)\left(\frac{17}{9}-\frac{\pi}{\sqrt{3}}\right)\right] \simeq 0.051 \qquad (108)$$

and compare this one-loop prediction for δ_{soft} [#15] to what one would expect from the phenomenological dipole form factor

$$F_1^V(q^2) = \left(\frac{\Lambda^2}{\Lambda^2-q^2}\right)^2 \qquad (109)$$

with $\Lambda = 840$ MeV. This form factor leads to the following one-pion exchange contribution to $\mathcal{M}_{loop}$, corresponding to (104):

$$\tilde{\mathcal{M}}_{1\pi}^{dipole} = \tilde{\mathcal{T}}^{(1)}\left\{\frac{M^2}{4\pi x_\pi}Y_1(x_\pi)\left[\left(\frac{\Lambda^2}{\Lambda^2-M^2}\right)^2-1\right]\right.$$

[#15] It is worth noting that this contribution is generic in the sense that it is more or less model-independent: It is of the same form and magnitude whether it is given by chiral one-loop graphs or by the vector dominance (see Appendix I) or by the phenomenological dipole form factor. This may have to do with the fact that it is controlled entirely by chiral symmetry. It is curious though that this longest wavelength effect is an *enhancement* rather than quenching usually associated with form factors.

$$-\frac{1}{4\pi}\left[\frac{1}{2}\left(\frac{\Lambda^2}{\Lambda^2-M^2}\right)\Lambda^2+\left(\frac{\Lambda^2}{\Lambda^2-M^2}\right)^2\left(\frac{1}{r^2}+\frac{\Lambda}{r}\right)\right]e^{-\Lambda r}\Bigg\}. \quad (110)$$

Identifying the first term of (110) with the first term of (104), we see that δ_{soft} corresponds to

$$\left(\frac{\Lambda^2}{\Lambda^2-M^2}\right)^2-1\simeq 0.0571.$$

It is remarkable – and pleasing – that the one-loop calculation of δ_{soft} is so close to the empirical value. Furthermore the remaining term in (104) involving the functions $K_i^\prime$ corresponds – and when applied to the process of interest, is numerically close – to the second (short-ranged) term in (110).

7 Numerical Results

In order to get a qualitative idea of the size involved, let us first look at the magnitude of the relevant terms given in momentum space. For this purpose we set $q^2\approx -|\mathbf{q}|^2\sim -Q^2$, where Q is taken to be a characteristic small momentum scale probed in the process which we take to be of order of m_π at most. For convenience we shall factor out the tree contribution from the expression (101) and write it as

$$\mathcal{M}=\mathcal{M}_{tree}(1+\delta_{\rm M}+O(Q^3)) \quad (111)$$

where $\delta_{\rm M}$ is the chiral correction of $O(Q^2)$ that we have computed (relative to the tree contribution). We obtain

$$\delta_{\rm M}=\delta_{1\pi}+\delta_{2\pi}$$

where, setting $\langle\mathcal{T}^{(1)}\rangle=\langle\mathcal{T}^{(2)}\rangle$ in nuclear medium #16 and dropping the κ_4's #17

$$\begin{aligned}\delta_{1\pi} &\approx \frac{Q^2}{4F^2}\left[-\frac{2}{3}F^2\langle r^2\rangle_1^V+\frac{1+3g_A^2}{8\pi^2}K_0(Q^2)-\frac{1+2g_A^2}{2\pi^2}K_2(Q^2)\right],\\ \delta_{2\pi} &\approx -\frac{Q^2+M^2}{4F^2}\left[\frac{5g_A^2+2}{16\pi^2}K_0(Q^2)-\frac{g_A^2}{8\pi^2}K_1(Q^2)\right].\end{aligned} \quad (112)$$

For $Q\sim m_\pi\approx 140$ MeV, $g_A=1.25$ and $F\approx 93$ MeV, we get

$$|\delta_{1\pi}|\sim 0.045, \qquad |\delta_{2\pi}|\sim 7.5\times 10^{-3}. \quad (113)$$

This is consistent with the notion that at the relevant scale Q, the chiral correction remains *small.*

#16 One can show in fermi-gas model, Wigner's $SU(4)$ supermultiplet model or even jj-coupling shell model of nucleus with one particle outside of closed core, $\langle\mathcal{T}^{(1)}\rangle=\langle\mathcal{T}^{(2)}\rangle$. This relation will be assumed in all numerical calculations that follow. We would like to thank Kuniharu Kubodera for his help on this relation.

#17 Dropping the constant terms in momentum space is not fully justified unless all other terms of the same nature are removed as well. This problem is avoided in coordinate space. We give only the absolute values for the $\delta_{n\pi}$ for the same reason. See below for more on this matter.

We now turn to a more realistic estimate of the chiral correction appropriate to the actual situation in finite nuclei. Calculating nuclear transition matrix elements in momentum space is cumbersome and delicate. There are several reasons for this. The most serious problem is the implementation of the short-range correlation. In the well-studied case as in the π-nuclear scattering, we know how to proceed, obtaining the celebrated Lorentz-Lorenz effect. Roughly the argument goes as follows [22]. Consider a term of the form $\vec{q}^2/(\vec{q}^2+M^2)$ that figures in the p-wave pion-nuclear scattering amplitude, or more specifically in the interaction between the particle-hole states excited by the pion. Rewrite this as $1-M^2/(\vec{q}^2+M^2)$. Removing the constant 1 corresponds to suppressing a δ function in coordinate space and leads to the Lorentz-Lorenz factor. Note that this procedure of accounting for short-range correlations can even change the sign. Unfortunately our case does not lend itself to a simple treatment of this kind because of the nonanalytic terms coming from the loop contributions: there is no economical way of "removing δ functions" from them. This task is much simpler and more straightforward in coordinate space.

Let us therefore turn to the coordinate space operators (103) and (106). In Fig. 7, we plot $\tilde{\mathcal{M}}_{tree}$ (103) and $\tilde{\mathcal{M}}_{loop}$ (106) as function of the internuclear distance $r=|\vec{r}_1-\vec{r}_2|$ setting $\tilde{\mathcal{T}}^{(1)}=\tilde{\mathcal{T}}^{(2)}=1$. Some of the important features discussed in the preceding sections can be seen in this plot. While negligible at large distance, say, $r>1$ fm, the loop corrections get progressively significant at shorter distances and at $r\sim 0.4$ fm, they are comparable to the soft-pion result. There is nothing surprising or disturbing about this feature at short distances. At shorter distances which are probed by the momentum scale approaching the chiral scale, there is no reason to ignore the degrees of freedom *integrated out* from the theory. Low-order calculations with higher chiral-order degrees of freedom eliminated cannot possibly describe the short-distance physics properly. This may be construed as a sign that ChPT is not predictive in nuclei. We claim that this is not so. The point is that as long as the scale Q probed by experiments is much less than the chiral scale, truncating higher chiral-order and shorter wavelength degrees of freedom as done in ChPT can be meaningful provided short-range nuclear correlations are implemented in the way discussed above.

Calculations of the nuclear matrix elements with sophisticated wave functions in finite (light and heavy) nuclei – and comparison with experimental data – will be made and reported in a separate paper. Here for our purpose of getting a semi-quantitative idea, the fermi-gas model as used by Delorme [23] will suffice. One could incorporate accurate correlation functions – and this will be done for specific transitions in finite nuclei. Here we will not do so. We shall instead take the simplest correlation function, namely $\hat{g}(r,d)=\theta(r-d)$ with the cut-off distance $d\approx 0.7$ fm as used by Towner [24]. Since this is a rather crude approach, we will consider the range of d values between 0.5 and 0.7 fm.

Specifically we are interested in the ratio of the matrix elements $\langle\mathcal{M}_{loop}\rangle/\langle\mathcal{M}_{tree}\rangle$

which in fermi-gas model takes the form (see Appendix H)

$$R(d,\rho) \equiv \frac{\langle \mathcal{M}_{loop} \rangle}{\langle \mathcal{M}_{tree} \rangle} = \frac{\int_d^\infty dr\, r\, [j_1(p_F r)]^2\, \tilde{\mathcal{M}}_{loop}(r)}{\int_d^\infty dr\, r\, [j_1(p_F r)]^2\, \tilde{\mathcal{M}}_{tree}(r)} \tag{114}$$

where p_F and $\rho = \frac{2}{3\pi^2} p_F^3$ are, respectively, the fermi-momentum and density of the system, $j_1(x) = \frac{\sin x}{x^2} - \frac{\cos x}{x}$ and $\tilde{\mathcal{M}}_{loop}(r) \equiv \tilde{\mathcal{M}}_{1\pi}(r) + \tilde{\mathcal{M}}_{2\pi}(r)$. Note that $w(p_F, r) \equiv 4\pi r\, [j_1(p_F r)]^2 / p_F^2$ can be viewed as a weighting function. Since this calculation is straightforward, we shall not go into details here. For completeness, however, we sketch the calculation in Appendix H.

In Fig. 8 are plotted the functions $w(p_F, r)\tilde{\mathcal{M}}_{tree}(r)$ and $w(p_F, r)\tilde{\mathcal{M}}_{loop}(r)$ with $\tilde{\mathcal{T}}^{(1)} = \tilde{\mathcal{T}}^{(2)} = 1$ for $p_F \simeq 1.36\ \mathrm{fm}^{-1}$ corresponding to nuclear matter density. The ratio $R(d,\rho)$ is plotted in Figure 9 for $d = 0.5,\ 0.7$ fm. For $d = 0.7$ fm which was used by Towner[24], the loop correction is at most of the order of 10% of the soft-pion term at nuclear matter density. There are two important points to note in the result. The first is that *separately* the loop corrections to the one-pion term (*i.e*, $\mathcal{M}_{1\pi}$) and the two-pion term (*i.e*, $\mathcal{M}_{2\pi}$) can be substantial but the *sum* is small. The second point is that the resulting loop contribution has a remarkably weak density dependence. The first is a consequence of chiral symmetry reminiscent of the tree-order cancellation in linear σ model of the nucleon pair term and the σ-exchange term in the S-wave πN scattering amplitude. The second observation has a significant ramification on the mass dependence of axial-charge transitions in heavy nuclei to which we will return shortly.

8 Other Contributions

Here we briefly discuss what other graphs could potentially contribute and the reason why they are suppressed in our calculation. Consider the two-body graphs given by Figure 6(k) where the pion propagators are *time-ordered*. They belong to what one calls "recoil graphs" in the literature [4]. To $O(Q^2)$ relative to the soft-pion term, these graphs – and more generally *all* recoil graphs including one-pion exchange – do not contribute. The reason is identical to the suppression of three-body forces as discussed by Weinberg[9]: the graphs in Fig. 10 are exactly cancelled by the recoil corrections to the iterated one-pion exchange graphs that are included in the class of reducible graphs. Thus to the extent that the static approximation is used in defining the one-pion exchange potential, these graphs should not be included as corrections. Incidentally this justifies the standard practice of ignoring recoil graphs in calculating exchange contributions in both weak and electromagnetic processes in nuclei.

We have ignored in our calculation three-body and higher-body contributions such as Figure 11. The reason for ignoring these graphs is identical to that used for proving the suppression of three-body and other multi-body forces [9]. As in nuclear forces, they can contribute at $O(Q^3)$ relative to the soft-pion term[8].

An interesting question to ask is in what situations the approximations that justify dropping the graphs considered here *break down* in nuclei. It is clear that the static approximation – one of the essential ingredients of the heavy-fermion formalism – must break down when the energy transfer involved is large. Imagine that one is exciting a Δ resonance in nuclei by electroweak field. The energy transfer is of the order of 300 MeV, so the static approximation for the pion propagator involving a $\pi\Delta N$ vertex cannot be valid. In such cases, one would expect that multi-body forces and currents suppressed in this work could become important. This suggests specifically that in electron scattering from nuclei with sufficiently large energy transfer, n-body currents (for $n > 2$) will become progressively more important in heavier nuclei. Combined with the dropping mass effect (*i.e*, "Brown-Rho" scaling mentioned below), one expects a large deviation from the standard mean-field description used currently.

9 Conclusions and Discussions

We have used heavy-baryon chiral perturbation formalism to calculate the leading corrections to the soft-pion axial-charge operator in nuclei. Exploiting short-range suppression of the counter terms and other short-range components of the two-body operator, we have shown that the chiral filter mechanism holds in nuclear matter with a possible uncertainty of no more than 10%, thus confirming the dominance of the soft-pion exchange. In a separate paper, we will show that the same holds in electromagnetic responses in nuclei. *Since the currents (both vector and axial-vector) are calculated consistently with the symmetries involved, they are fully consistent with nuclear forces that are calculated to the same chiral order*: Ward-Takahashi identities will be formally satisfied although in practice approximations made for calculations may disturb them. The final consistency will of course have to be checked *à posteriori* case-by-case.

Taking this result as a statement of chiral symmetry of QCD in nuclei, what can one learn from this concerning the phenomenological models popular in nuclear physics where one uses exchanges of all the low-lying bosons in fitting nucleon-nucleon scattering (such as the Bonn potential) as well as calculating the exchange currents? Suppose we denote the axial-charge two-body operator from one-pion exchange with form factors by $A_{1\pi}$, one-heavy-meson exchange with form factors by A_H, the axial current form factor by A_{FF}, all calculated within a phenomenological model, then our result implies that for the model to be consistent with chiral symmetry, then the total *must* sum to

$$A_{total} = A_{1\pi} + A_H + A_{FF} + \cdots \approx A_{soft}(1+\delta), \qquad |\delta| << 1 \tag{115}$$

where A_{soft} is the soft-pion term as defined in this paper and δ is the next-to-leading term of $O(Q^2)$. Our calculation illustrates how individually significant terms conspire to give a small $O(Q^2)$ correction which is insensitive to nuclear density.

One other outcome of our result is that while a subset of graphs can have a substantial density dependence, the small net chiral correction from the totality of the graphs does not have an appreciable nuclear density dependence, at least in fermi-gas model. We see no reason why this weak density dependence should not persist in more realistic nuclear models. Thus assuming that n-th order chiral corrections for $n \geq 3$ (relative to the leading soft-pion term) are not anomalously large, we come to the conclusion that meson-exchange axial-charge contributions to nuclear matrix elements cannot be substantially enhanced in heavy nuclei over that in light nuclei. The question arises then as to what could be the explanation for Warburton's recent observation that while the mesonic effect is about 50% in light nuclei, it is required to be 100% in heavy nuclei such as in lead region [25]. One suggestion [26] was that the parameters of the basic chiral Lagrangian have to be modified in the presence of nuclear matter consistent with trace anomaly of QCD [27]. It predicted that hadron masses and pion decay constants that appear in the single-particle and one-pion exchange two-body operators are scaled by a universal factor Φ that depends on matter density. Another suggestion [24, 28] was that exchanges of heavy mesons σ, ρ, ω, a_1 etc could become important in heavy nuclei while relatively unimportant in light systems. The latter mechanism relied on nucleon-antinucleon pair terms in phenomenological Lagrangians. Both mechanisms seemed to qualitatively account for the enhancement.

We wish to understand the possible link, if any, between the chiral Lagrangian approach and the phenomenological approach that includes pair terms involving heavy mesons. Since within the chiral approach developed in this paper the pair is naturally suppressed as required by chiral symmetry and multi-body currents are also suppressed as discussed above, the scaling mass effect of [26, 27] is the only plausible mechanism left within low-order chiral expansion for the medium enhancement noted by Warburton. Needless to say, we cannot rule out – though we deem highly unlikely – the possibility that higher order chiral terms supply the needed density effect. Incorporating the possible 10% loop correction calculated above in the two-body operator and the scaling factor $\Phi = m_N^*/m_N \approx 0.8$, one gets in the scheme of Ref.[26] the enhancement in heavy nuclei (at nuclear matter density) $\epsilon_{\rm MEC} \approx 2.1$ which is reasonable in the lead region compared with the experimental value 2.01 ± 0.05 [25]. Within the scheme, this is the entire story and a surprisingly simple one. Of course more detailed finite nuclei calculations will be needed to make a truly meaningful test of the theory.

In the phenomenological approach studied by the authors in [24] and [28], there is no fundamental reason to suppress $N\bar{N}$ pairs, so that heavy mesons could contribute through the pair term. However the exchange of heavy mesons, particularly that of vector mesons, is suppressed by short-range correlations in nuclear wave functions. Furthermore in the model of Towner[24], a large cancellation takes place in the sum such that the relation (115) seems to hold well[29]: Towner finds $\delta < 10\%$ over a wide range of nuclei. It is naturally tempting to suggest that Towner's model gives a result close to ours *because it is consistent with chiral symmetry,* at least to the same order of chiral expansion as ours, with higher-order terms implicit in Towner's model which need not be consistent with ChPT somehow cancelling out[#18]. There is however one aspect that needs to be clarified: in the models of [24] and [28], there is a pair term associated with a scalar meson (σ) exchange. In the chiral Lagrangian used in this paper, there is no equivalent scalar field. We have however the scalar field χ associated with the trace anomaly of QCD which plays a role in the Brown-Rho scaling[27]. We believe that these two effects are roughly related in the sense discussed in Ref.[30]. In this sense we would say that the pair term involving the σ meson is simulating the density-dependence of the nucleon mass in the *one-body* axial-charge operator. There is no mechanism in Ref.[24, 28], however, for the density dependence of Ref.[26] in the two-body operator. We suggest that this can be generated by taking three-body terms with an $N\bar{N}$ pair coupled to a σ-exchange.

An obvious omission in our treatment of the axial current is the space component of the current governing Gamow-Teller transitions in nuclei (and the time part of the electromagnetic current in [14]). The reason for this was already stated at several points in the paper: this part of the current is *not* dominated by a soft-pion exchange and indeed as noted many years ago [21] it is rather the very short-ranged part of nuclear interactions (roughly equivalent to the removal of the δ functions associated with the counter terms in the spin-isospin channel) that plays an important role, *e.g.* in quenching the axial-vector coupling constant from the free-space value $g_A = 1.25$ to $g_A^* \approx 1$ in nuclear matter. (For a similar situation with the isoscalar axial-charge transition mediated by a neutral weak current where soft-pion exchange is forbidden, see Ref.[31].) Furthermore, three-body operators for the space component of the axial current may not be negligible. For instance, as one can see from Appendix A, the three-body Gamow-Teller operator involving one nucleon with an $A_\mu\pi\pi NN$ vertex with the pions absorbed by two other nucleons is not trivially suppressed as it is for the axial-charge operator. This suggests that low-order chiral perturbation theory may have little to say about this aspect of nuclear interactions. It is intriguing that in nuclei, both chiral and non-chiral aspects of QCD seem to coexist in the same low-energy domain. This makes QCD in nuclei quite different from and considerably more intricate than QCD in elementary particles studied by particle physicists.

Finally we mention a few additional issues we have not treated in this paper but we consider to be important topics for future studies.

- It would be interesting to see what two-loop (and hopefully higher-loop) and corresponding chiral corrections do to the chiral filter phenomenon. Two-loop calculations are in general a horrendous task but the situation in nuclear axial-charge transitions

[#18] The following observation may be relevant to our argument that the counter terms $\kappa_4^{(i)}$ *must* be ignored. Suppose one constructs a purely phenomenological theory based on meson-exchange picture by fitting experiments but conform to the symmetries of the strong interactions. Towner's model is one such example. One can convince oneself that in such a model, it is not possible to generate counter terms of the κ_4 type in infinite mass limit. Therefore if such terms existed, then they must be due to degrees of freedom that are *not relevant* at the accuracy required.

might be considerably simpler than in other processes.

- It would be important to see whether ChPT is predictive for processes involving larger momentum transfers as well as large energy transfer. From our experience with the electrodisintegration of the deuteron at large momentum transfers where the naive soft-pion approximation seems still to work fairly well, we conjecture that the chiral filter mechanism holds still in some channels even in processes probed at shorter distances or at larger momentum transfers. But as mentioned above, large energy transfer electron scattering might require multi-body currents in heavy nuclei.

- In this paper, we worked with an effective Lagrangian in which *all other degrees of freedom* than pions and nucleons have been integrated out. It would be important to reformulate ChPT using a Lagrangian that contains vector mesons incorporated à la hidden gauge symmetry (HGS) [32][#19] and also nucleon resonances (such as Δ). As mentioned before, we believe that the chiral filter argument presented in this paper is not modified in the presence of these resonances in the Lagrangian. In Appendix I, we show that the presence of vector mesons does not modify our prediction on the axial-charge operator. Furthermore it is not difficult to see that the baryon resonances – in particular the Δ resonances – do not contribute to the axial-charge transitions to the order considered. However as is known for Gamow-Teller transitions in nuclei [21], certain processes in nuclei might require, even at zero momentum transfer, an explicit role of some of these heavier particles. As recently shown by Harada and Yamawaki [34], vector mesons introduced via HGS can easily be quantized, so their implementation in ChPT would pose no great difficulty.

- A systematic higher-loop chiral perturbation approach using the same heavy-fermion formalism to kaon-nuclear interactions and kaon condensation has not yet been worked out. This is an important issue for hypernuclear physics, relativistic heavy-ion physics and stellar collapse [35].

- Finally if the parameters of effective chiral Lagrangians scale as a function of nuclear matter density as suggested by Brown and Rho[27], then one expects that as matter density increases, many-body currents will become increasingly important even at small energy transfer. This was already noticed in [26] where the soft-pion exchange charge operator became stronger in heavier nuclei. We already noted that this effect will show up more prominently in nuclear electromagnetic responses with large energy transfer. Future accurate experiments in electron scattering off nuclei will test this prediction. Of course this issue has to be treated together with many-body forces that enter into such processes.

[#19] As stressed by Georgi, the HGS is an approach most suited to a systematic chiral counting when vector mesons are explicitly present. See [33].

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Appendix A: Chiral Lagrangian Eq.(22) Expanded

For completeness we expand $\mathcal{L}_0$ in powers of the pion field and in external fields. We will group $\mathcal{L}_0$ by the number of external gauge field, $\mathcal{L}_0 = \mathcal{L}_0^0 + \mathcal{L}_0^1 + \mathcal{L}_0^2$,

$$
\begin{aligned}
\mathcal{L}_0^0 &= \frac{1}{2}(\partial_\mu\vec{\pi})^2 - \frac{1}{2}M^2\vec{\pi}^2 - \frac{1}{6F^2}\left[\vec{\pi}^2\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} - (\vec{\pi}\cdot\partial_\mu\vec{\pi})^2\right] + \frac{M^2}{4!\,F^2}\left(\vec{\pi}^2\right)^2 \\
&+ \overline{B}iv\cdot\partial B \\
&+ \overline{B}\left\{-\frac{v^\mu}{4F^2}\vec{\tau}\cdot\vec{\pi}\times\partial_\mu\vec{\pi} - \frac{g_A}{F}S^\mu\vec{\tau}\cdot\left[\partial_\mu\vec{\pi} + \frac{1}{6F^2}\left(\vec{\pi}\,\vec{\pi}\cdot\partial_\mu\vec{\pi} - \partial_\mu\vec{\pi}\,\vec{\pi}^2\right)\right]\right\}B \\
&- \frac{1}{2}C_a\left(\overline{B}\Gamma_a B\right)^2 + \cdots, \qquad \text{(A.1)} \\
\mathcal{L}_0^1 &= \vec{\mathcal{V}}^\mu\cdot\left[\vec{\pi}\times\partial_\mu\vec{\pi} - \frac{1}{3F^2}\vec{\pi}\times\partial_\mu\vec{\pi}\,\vec{\pi}^2\right] - F\vec{\mathcal{A}}^\mu\cdot\left[\partial_\mu\vec{\pi} - \frac{2}{3F^2}\left(\partial_\mu\vec{\pi}\,\vec{\pi}^2 - \vec{\pi}\,\vec{\pi}\cdot\partial_\mu\vec{\pi}\right)\right] \\
&+ \frac{1}{2}\overline{B}\left(v^\mu\vec{\mathcal{V}}_\mu + 2g_AS^\mu\vec{\mathcal{A}}_\mu\right)\cdot\left[\vec{\tau} + \frac{1}{2F^2}\left(\vec{\pi}\,\vec{\tau}\cdot\vec{\pi} - \vec{\tau}\,\vec{\pi}^2\right)\right]B \\
&+ \frac{1}{2}\overline{B}\left(v^\mu\vec{\mathcal{A}}_\mu + 2g_AS^\mu\vec{\mathcal{V}}_\mu\right)\cdot\left[\frac{1}{F}\vec{\tau}\times\vec{\pi} - \frac{1}{6F^3}\vec{\tau}\times\vec{\pi}\,\vec{\pi}^2\right]B + \cdots, \qquad \text{(A.2)} \\
\mathcal{L}_0^2 &= \frac{1}{2}F^2\vec{\mathcal{A}}_\mu^2 + F\vec{\pi}\cdot\vec{\mathcal{V}}_\mu\times\vec{\mathcal{A}}^\mu + \frac{1}{2}\left[\vec{\pi}^2\left(\vec{\mathcal{V}}_\mu^2 - \vec{\mathcal{A}}_\mu^2\right) - (\vec{\pi}\cdot\vec{\mathcal{V}}_\mu)^2 + (\vec{\pi}\cdot\vec{\mathcal{A}}_\mu)^2\right] + \cdots. \qquad \text{(A.3)}
\end{aligned}
$$

From the (A.2), we extract Noether currents,

$$
\begin{aligned}
\vec{A}^\mu &= -F\left[\partial_\mu\vec{\pi} - \frac{2}{3F^2}\left(\partial_\mu\vec{\pi}\,\vec{\pi}^2 - \vec{\pi}\,\vec{\pi}\cdot\partial_\mu\vec{\pi}\right)\right] \\
&+ \frac{1}{2}\overline{B}\left\{2g_AS^\mu\left[\vec{\tau} + \frac{1}{2F^2}\left(\vec{\pi}\,\vec{\tau}\cdot\vec{\pi} - \vec{\tau}\,\vec{\pi}^2\right)\right] + v^\mu\left[\frac{1}{F}\vec{\tau}\times\vec{\pi} - \frac{1}{6F^3}\vec{\tau}\times\vec{\pi}\,\vec{\pi}^2\right]\right\}B + \cdots, \qquad \text{(A.4)} \\
\vec{V}^\mu &= \left[\vec{\pi}\times\partial_\mu\vec{\pi} - \frac{1}{3F^2}\vec{\pi}\times\partial_\mu\vec{\pi}\,\vec{\pi}^2\right] \\
&+ \frac{1}{2}\overline{B}\left\{v^\mu\left[\vec{\tau} + \frac{1}{2F^2}\left(\vec{\pi}\,\vec{\tau}\cdot\vec{\pi} - \vec{\tau}\,\vec{\pi}^2\right)\right] + 2g_AS^\mu\left[\frac{1}{F}\vec{\tau}\times\vec{\pi} - \frac{1}{6F^3}\vec{\tau}\times\vec{\pi}\,\vec{\pi}^2\right]\right\}B + \cdots. \qquad \text{(A.5)}
\end{aligned}
$$

Appendix B: Functions $f_i(q^2)$

The functions $f_i(q^2)$ $(i = 0, 1, 2, 3)$ figuring in subsections (5.4.2) and (5.5) are defined by

$$
\begin{aligned}
f_0(q^2) &= \int_l \frac{1}{(l^2 - M^2)\,[(l+q)^2 - M^2]}, \\
\frac{1}{2}g_{\alpha\beta}f_1(q^2) + q_\alpha q_\beta f_2(q^2) &= \int_l \frac{(l+q)_\alpha l_\beta}{(l^2 - M^2)\,[(l+q)^2 - M^2]} \\
f_3(q^2) &= 2f_2(q^2) + \frac{1}{2}f_0(q^2) \qquad \text{(B.1)}
\end{aligned}
$$

where we have defined $f_3(q^2)$ through

$$\int_l \frac{l_\alpha\,(2l+q)_\beta}{(l^2-M^2)\;[(l+q)^2-M^2]} \equiv g_{\alpha\beta} f_1(q^2) + q_\alpha q_\beta f_3(q^2). \tag{B.2}$$

Here and in what follows, the mass M could be thought of as the pion mass M. One can verify that

$$\frac{\partial f_1(q^2)}{\partial q^2} = f_2(q^2).$$

After performing the parametric integration, we have

$$\begin{aligned}
f_0(q^2) &= \eta - \frac{1}{16\pi^2} K_0(q^2), \\
f_1(q^2) &= \Delta(M^2) - \frac{\eta}{6} q^2 - \frac{q^2}{16\pi^2}\left[2K_2(q^2) - \frac{1}{2}K_0(q^2)\right], \\
f_2(q^2) &= -\frac{\eta}{6} + \frac{1}{16\pi^2} K_2(q^2), \\
f_3(q^2) &= \frac{\eta}{6} + \frac{1}{16\pi^2}\left[2K_2(q^2) - \frac{1}{2}K_0(q^2)\right]
\end{aligned} \tag{B.3}$$

where

$$\eta = \frac{1}{16\pi^2}\Gamma(\epsilon)\left(\frac{M^2}{4\pi\mu^2}\right)^{-\epsilon}$$

and $K(q^2)$'s are finite functions given explicitly by

$$K_0(q^2) = \int_0^1 dz \ln\left[1 - z(1-z)\frac{q^2}{M^2}\right] = -2 + \sigma \ln\left(\frac{\sigma+1}{\sigma-1}\right), \tag{B.4}$$

$$K_1(q^2) = \int_0^1 dz \frac{-z(1-z)q^2}{M^2 - z(1-z)q^2} = 1 - \frac{\sigma^2-1}{2\sigma}\ln\left(\frac{\sigma+1}{\sigma-1}\right), \tag{B.5}$$

$$\begin{aligned}
K_2(q^2) &= \int_0^1 dz\, z(1-z) \ln\left[1 - z(1-z)\frac{q^2}{M^2}\right] \\
&= -\frac{4}{9} + \frac{\sigma^2}{6} + \frac{\sigma(3-\sigma^2)}{12}\ln\left(\frac{\sigma+1}{\sigma-1}\right),
\end{aligned} \tag{B.6}$$

with

$$\sigma \equiv \left(\frac{4M^2 - q^2}{-q^2}\right)^{\frac{1}{2}}. \tag{B.7}$$

We should note that all the functions given above are positive definite for negative q^2 and vanish when $q^2 = 0$. For $-q^2 \ll M^2$, we have

$$\begin{aligned}
K_0(q^2) &= K_1(q^2) = \frac{1}{6}\frac{-q^2}{M^2} + \mathcal{O}\left(\frac{q^4}{M^4}\right), \\
K_2(q^2) &= \frac{1}{30}\frac{-q^2}{M^2} + \mathcal{O}\left(\frac{q^4}{M^4}\right).
\end{aligned}$$

In the chiral limit ($-q^2 \gg M^2$), they simplify to

$$\begin{aligned}
K_0(q^2) &= \ln\frac{-q^2}{M^2} - 2, \\
K_1(q^2) &= 1, \\
K_2(q^2) &= \frac{1}{6}\ln\frac{-q^2}{M^2} - \frac{5}{18}.
\end{aligned}$$

In order to go to the r−space, we must Fourier-transform $K_i(q^2)$ and $M^2/(M^2 - q^2)\, K_i(q^2)$. This involves an integration[#20] of highly oscillating functions. Instead of introducing a regulating function which kills contributions from large $\vec{q}^2$ and performing a tricky numerical calculation, we transform the problem into an integral of a smooth function with the use of residue calculation. The point is that we do the parametric integration (of variable z) at the last step. To see how this work, let us rewrite $K_i(q^2)$,

$$\begin{aligned}
K_0(-Q^2) &= \int_0^1 dx \ln\left(1 + \frac{Q^2}{E^2}\right), \\
K_1(-Q^2) &= \int_0^1 dx\, \frac{Q^2}{Q^2 + E^2}, \\
K_2(-Q^2) &= \int_0^1 dx\, \frac{1 - x^2}{4} \ln\left(1 + \frac{Q^2}{E^2}\right).
\end{aligned} \tag{B.8}$$

where $Q = |\mathbf{q}| = \sqrt{-q^2}$ and $E = E(x) = 2M/\sqrt{1 - x^2}$.[#21] First we Fourier-transform algebraically the integrands of the above integrals and then do the parametric integral numerically. The Fourier transform of K_1 becomes an elementary residue calculation with a pole at $Q = iM$,

$$K_1(r) = \delta(\mathbf{r}) - \frac{1}{4\pi r}\int_0^1 dx\, E^2\, \mathrm{e}^{-Er} \tag{B.9}$$

while K_0 and K_2 are somewhat involved due to the logarithmic function. We rewrite the logarithmic functions (of K_0 and K_2) into a simple pole form by integration by part

$$\int_0^1 dx \left[1, \frac{1 - x^2}{4}\right] \ln\left(1 + \frac{Q^2}{E^2}\right) = \int_0^1 dx\, \frac{2x^2}{1 - x^2}\left[1, \frac{1}{4} - \frac{x^2}{12}\right] \frac{Q^2}{Q^2 + E^2}. \tag{B.10}$$

With the above equation and

$$\int \frac{d^3\mathbf{q}}{(2\pi)^3}\, \mathrm{e}^{i\,\mathbf{q}\cdot\mathbf{r}}\, \frac{Q^2}{Q^2 + E^2}\, \frac{\Lambda^2}{Q^2 + \Lambda^2} = \frac{1}{4\pi r}\, \frac{\Lambda^2}{\Lambda^2 - E^2}\left(\Lambda^2 \mathrm{e}^{-\Lambda r} - E^2 \mathrm{e}^{-Er}\right), \tag{B.11}$$

we obtain the expressions for $K(r, \Lambda)$ defined by

$$K_i(r, \Lambda) \equiv \int \frac{d^3\mathbf{q}}{(2\pi)^3}\, \mathrm{e}^{i\,\mathbf{q}\cdot\mathbf{r}}\, K_i(-Q^2)\, \frac{\Lambda^2}{Q^2 + \Lambda^2}, \tag{B.12}$$

[#20] Recall that $K_i(q^2)$ does not go to zero when $\vec{q}$ goes to infinity.

[#21] We have made the change of variable, $x = 2z - 1$, to render the expressions more symmetric.

$$
\begin{aligned}
K_0(r,\Lambda) &= \frac{1}{4\pi r}\int_0^1 dx\, \frac{2x^2}{1-x^2}\,\frac{\Lambda^2}{\Lambda^2-E^2}\left(\Lambda^2 \mathrm{e}^{-\Lambda r}-E^2\mathrm{e}^{-Er}\right), \\
K_2(r,\Lambda) &= \frac{1}{4\pi r}\int_0^1 dx\, \frac{2x^2}{1-x^2}\left(\frac{1}{4}-\frac{x^2}{12}\right)\frac{\Lambda^2}{\Lambda^2-E^2}\left(\Lambda^2 \mathrm{e}^{-\Lambda r}-E^2\mathrm{e}^{-Er}\right).
\end{aligned} \quad \text{(B.13)}
$$

Here the parameter Λ is introduced to regularize the integrals near the origin of the configuration space. When $\Lambda=\infty$, we get the expressions for the $K_i(r)$ needed in the paper,

$$
K_i(r) \equiv \lim_{\Lambda\to\infty} K_i(r,\Lambda). \quad \text{(B.14)}
$$

The above integrals are non-singular even near $x=1$, since E increases so as to make the integrals regular. However the expressions contain highly singular terms near the origin of configuration space when Λ goes to infinity. To see this, note

$$
K_0(r) = -\frac{1}{4\pi r}\int_0^1 dx\,\frac{2x^2}{1-x^2}\,E^2\,\mathrm{e}^{-Er} + \lim_{\Lambda\to\infty}\frac{1}{4\pi r}\,\Lambda^2 K_0(q^2=\Lambda^2)\,\mathrm{e}^{-\Lambda r},
$$

and

$$
\lim_{\Lambda\to\infty}\frac{1}{4\pi r}\,\Lambda^2\mathrm{e}^{-\Lambda r} = \delta(\mathbf{r}).
$$

Now $K_0(\Lambda^2)$ goes to infinity logarithmically when Λ^2 goes to infinity. So, roughly, the second term of (9) behaves in the limit of infinite Λ as

$$
\lim_{\Lambda^2\to\infty}\ \ln\frac{\Lambda^2}{M^2}\,\delta(\mathbf{r}).
$$

The mathematical reason for this behavior is not hard to see : For large Q^2, both K_0 and K_2 increase logarithmically with a gentle slope. Thus they can be viewed as a constant at large Q^2 with their value increasing to infinity. The constant behavior leads to the $\delta(\mathbf{r})$ function. This term singular at the origin is of course "killed" by the short-range correlation present in nuclear wave functions.

In terms of the function so defined, we can immediately obtain $\tilde{K}_i(r)$ defined by

$$
\tilde{K}_i(r) \equiv \int \frac{d^3\mathbf{q}}{(2\pi)^3}\,\mathrm{e}^{i\,\mathbf{q}\cdot\mathbf{r}}\,K_i(-Q^2)\,\frac{M^2}{Q^2+M^2} \;=\; K_i(r,M). \quad \text{(B.15)}
$$

In summary we have the expressions for $K_i(r)$ (valid for $r>0$)

$$
\begin{aligned}
K_0(r) &= -\frac{1}{4\pi r}\int_0^1 dx\,\frac{2x^2}{1-x^2}\,E^2\mathrm{e}^{-Er}, \\
K_2(r) &= -\frac{1}{4\pi r}\int_0^1 dx\,\frac{2x^2}{1-x^2}\left(\frac{1}{4}-\frac{x^2}{12}\right)E^2\mathrm{e}^{-Er}
\end{aligned} \quad \text{(B.16)}
$$

and the expressions for $\tilde{K}_i(r)$ (valid in the whole region)

$$
\begin{aligned}
\tilde{K}_0(r) &= \frac{1}{4\pi r}\int_0^1 dx\,\frac{2x^2}{1-x^2}\,\frac{M^2}{M^2-E^2}\left(M^2\mathrm{e}^{-Mr}-E^2\mathrm{e}^{-Er}\right), \\
\tilde{K}_2(r) &= \frac{1}{4\pi r}\int_0^1 dx\,\frac{2x^2}{1-x^2}\left(\frac{1}{4}-\frac{x^2}{12}\right)\frac{M^2}{M^2-E^2}\left(M^2\mathrm{e}^{-Mr}-E^2\mathrm{e}^{-Er}\right).
\end{aligned} \quad \text{(B.17)}
$$

Appendix C: Integral Identities

In this Appendix, we list some useful identities for the integrals we need to evaluate. Consider the following integral without imposing the condition $v^2 = 1$,

$$I_\alpha(v, M^2) \equiv \int_l \frac{l_\alpha}{v \cdot l\,(l^2 - M^2)},$$

then Lorentz covariance implies that its most general form is

$$I_\alpha(v, M^2) = v_\alpha I_0(v^2, M^2)\,.$$

Now multiplying v^α to both sides, we get

$$\Delta(M^2) = v^2 I_0(v^2, M^2)$$

or

$$\int_l \frac{l_\alpha}{v \cdot l\,(l^2 - M^2)} = \frac{v_\alpha}{v^2}\Delta(M^2)\,. \tag{C.1}$$

Using this expression, we obtain the following identities by successive differentiation with respect to v,

$$\int_l \frac{l_\alpha l_\beta}{(v\cdot l)^2\,(l^2 - M^2)} = -\left(\frac{g_{\alpha\beta}}{v^2} - 2\frac{v_\alpha v_\beta}{(v^2)^2}\right)\Delta(M^2) \tag{C.2}$$

$$\int_l \frac{l_\alpha l_\beta l_\mu}{(v\cdot l)^3\,(l^2 - M^2)} = \left[4\frac{v_\alpha v_\beta v_\mu}{(v^2)^3} - \frac{v_\mu g_{\alpha\beta} + v_\alpha g_{\beta\mu} + v_\beta g_{\alpha\mu}}{(v^2)^2}\right]\Delta(M^2) \tag{C.3}$$

and so on.

Appendix D: Functions $h(v \cdot k)$'s

In this Appendix, we define – and give explicit forms of – the function $h(v\cdot k)$ that figures in the self-energy for the off-shell nucleon discussed in section (4.3). The basic one is $h_0(v\cdot k)$,

$$\begin{aligned} h_0(v\cdot k) &= \int_l \frac{1}{v\cdot(l+k)\ (l^2 - M^2)} \\ &= 2y\eta + \frac{1}{8\pi^2}\left[\pi M_* + 2y - 2M_* \sin^{-1}\left(\frac{y}{M}\right)\right], \quad \text{for} \quad |y| \le M, \qquad \text{(D.1)} \\ &= 2y\eta + \frac{1}{8\pi^2}\left[2y - 2\tilde{M}\sinh^{-1}\left(\frac{y}{M}\right) - \theta(y-M)2i\pi\tilde{M}\right], \ \text{for} \ \ |y| \ge M \end{aligned}$$

(D.2)

where $y \equiv v\cdot k$, $M_* \equiv \sqrt{M^2 - y^2}$, $\tilde{M} \equiv \sqrt{y^2 - M^2}$ and $-\frac{\pi}{2} \le \sin^{-1}(x) \le \frac{\pi}{2}$. We observe that the imaginary part appears only when $v \cdot k \ge M$. The even part of h_0 has a very simple form

$$h_0^S(y) \equiv \frac{h_0(y) + h_0(-y)}{2} = -\frac{1}{8\pi}\sqrt{M^2 - y^2}. \tag{D.3}$$

For special values of y, we have $h_0(0) = -\frac{M}{8\pi}$, $h_0'(0) = 2\eta$, $h_0(M) = -h_0(-M) = \frac{2M}{1-2\epsilon}\eta$. The finite function $\overline{h_0}(y)$ is defined by

$$\overline{h_0}(y) = h_0(y) - 2y\eta. \tag{D.4}$$

We now examine the function $h(v\cdot k)$ defined by

$$\int_l \frac{l_\alpha\, l_\beta}{v\cdot(l+k)\,(l^2-M^2)} \equiv g_{\alpha\beta}h(v\cdot k) + v_\alpha v_\beta(\cdots). \tag{D.5}$$

If we multiply the above equation by $g^{\alpha\beta}$ and v^β, we obtain the following identity

$$h(y) = \frac{1}{d-1}\left[y\,\Delta(M^2) + (M^2-y^2)h_0(y)\right]. \tag{D.6}$$

Note that the even part has a very simple form,

$$h^S(y) \equiv \frac{h(y)+h(-y)}{2} = -\frac{1}{24\pi}\left(M^2-y^2\right)^{\frac{3}{2}}. \tag{D.7}$$

Let us define $h_3(v\cdot k, v\cdot q)$

$$\int_l \frac{l_\alpha l_\beta}{v\cdot(l+k-q)\,v\cdot(l+k)\,(l^2-M^2)} = g_{\alpha\beta}h_3(v\cdot k, v\cdot q) + v_\alpha v_\beta(\cdots)$$

or

$$h_3(v\cdot k, v\cdot q) = \frac{1}{v\cdot q}\left[h(v\cdot k - v\cdot q) - h(v\cdot k)\right]. \tag{D.8}$$

When the nucleon is on-shell, that is, $v\cdot k = v\cdot q = 0$, then h_3 becomes $h_3(0,0) = -\Delta(M^2)$. More generally, for small momentum, we have

$$h_3(v\cdot k, v\cdot q) = -\Delta(M^2) - \frac{M}{16\pi}(2v\cdot k - v\cdot q) + \frac{2}{3}\eta\left[3(v\cdot k)^2 - 3v\cdot k\, v\cdot q + (v\cdot q)^2\right] + \cdots \tag{D.9}$$

where the ellipsis denotes finite and higher momentum terms. Finally consider $h_4(v\cdot q)$ defined by

$$\int_l \frac{l_\alpha\, l_\beta}{(v\cdot l)^2\, v\cdot(l-q)\,(l^2-M^2)} \equiv g_{\alpha\beta}h_4(v\cdot q) + v_\alpha v_\beta(\cdots). \tag{D.10}$$

The h_4 is a somewhat complicated function,

$$\begin{aligned} h_4(y) &= \frac{h(-y)-h(0)}{y^2} + \frac{\Delta(M^2)}{y} \\ &= \frac{M}{16\pi} + \frac{2}{3}v\cdot q\,\eta + \cdots \end{aligned} \tag{D.11}$$

with $h(y)$ given by (D.6) and the ellipsis again denotes finite and higher momentum terms.

Appendix E: Integrals for Two-Pion Exchange Currents

Consider the integrals of the form

$$\int_l \frac{l\text{'s}}{v\cdot(l+k)\,(l^2-M^2)\,[(l+q)^2-M^2]}$$

which figure in two-pion exchange currents. For most of the cases, we do not need the terms proportional to v^μ as they appear multiplied by the spin operator S_μ and vanish. To utilize this, we assume that the spin operator is multiplied to the numerator. Now we have

$$\int_l \frac{(l+q)^\alpha\, l^\beta}{v\cdot(l+k)\,(l^2-M^2)\,[(l+q)^2-M^2]} \;=\; \left[g^{\alpha\beta}+2q^\alpha q^\beta\frac{\partial}{\partial q^2}\right]B_0(k,q), \tag{E.1}$$

$$\begin{aligned}\int_l \frac{(l+q)^\alpha\, l^\beta\,(2l+q)^\mu}{v\cdot(l+k)\,(l^2-M^2)\,[(l+q)^2-M^2]} \;&=\; \left(g^{\alpha\beta}+2q^\alpha q^\beta\frac{\partial}{\partial q^2}\right)[q^\mu B_1(k,q)+v^\mu B_2(k,q)]\\ +\left(q^\alpha g^{\beta\mu}+q^\beta g^{\alpha\mu}\right)B_1(k,q) \;&+\; \left(q^\alpha g^{\beta\mu}-q^\beta g^{\alpha\mu}\right)B_0(k,q)\end{aligned} \tag{E.2}$$

where we have neglected terms proportional to v^α or v^β. After some algebra, we can get the following relations,

$$q^2B_1(k,q)+v\cdot qB_2(k,q)=h(v\cdot k)-h(v\cdot k-v\cdot q),$$
$$B_2(k,q)=f_1(q^2)+v\cdot q\,[B_0(k,q)-B_1(k,q)]-2v\cdot kB_0(k,q).$$

When $v\cdot k=v\cdot q=0$, they become elementary functions,

$$\begin{aligned}B_0(q^2) &= -\frac{1}{16\pi}\int_0^1 dz\sqrt{M^2-z(1-z)q^2},\\ B_1(q^2) &= 0,\\ B_2(q^2) &= f_1(q^2)\end{aligned} \tag{E.3}$$

For small but nonzero momentum, they become

$$\begin{aligned}B_0(k,q) &= -\frac{M}{16\pi}+\left(v\cdot k-\frac{1}{2}v\cdot q\right)\eta+\cdots,\\ B_1(k,q) &= \frac{v\cdot q}{6}\eta+\cdots,\\ B_2(k,q) &= \Delta(M^2)+(2v\cdot k-v\cdot q)\frac{M}{16\pi}-\left[2(v\cdot k)^2-2v\cdot k\,v\cdot q+\frac{2}{3}(v\cdot q)^2+\frac{q^2}{6}\right]\eta\\ &\quad+\cdots.\end{aligned} \tag{E.4}$$

For two pion exchange graphs, we need to evaluate

$$I_{\nu,\alpha\beta}\equiv\int_l\frac{(l+q)_\nu\, l_\alpha\, l_\beta}{v\cdot l\, v\cdot(l+q)\,(l^2-M^2)\,[(l+q)^2-M^2]}. \tag{E.5}$$

In evaluating this function, we neglect terms proportional to v_ν, v_α or v_β because they vanish when multiplied by $S^\nu S^\alpha S^\beta$. With the parametrization explained in the text, we have, in the limit of $v\cdot q=0$,

$$I_{\nu,\alpha\beta}(q)=-\frac{1}{32\pi^2}(-q_\nu g_{\alpha\beta}+q_\alpha g_{\nu\beta}+q_\beta g_{\nu\alpha})\left[K_0(q^2)-16\pi^2\eta\right]-\frac{1}{16\pi^2}\frac{q_\nu q_\alpha q_\beta}{q^2}K_1(q^2), \tag{E.6}$$

where $K_1(q^2)$ is defined at (B.5),

$$K_1(q^2) = \int_0^1 dz \frac{-z(1-z)q^2}{M^2 - z(1-z)q^2}. \tag{E.7}$$

For completeness, we also list results for vector currents for which we need the following integrals

$$\int_l \frac{(l+q)^\alpha\, l^\beta\, (2l+q)^\mu}{v\cdot l\, v\cdot (l+q)\, (l^2-M^2)\, [(l+q)^2 - M^2]}. \tag{E.8}$$

We first look at its low momentum behavior,

$$-v^\mu \frac{M}{8\pi} g^{\alpha\beta} - \left(q^\alpha g^{\beta\mu} - q^\beta g^{\alpha\mu}\right)\eta + \mathcal{O}\left(q^2\right). \tag{E.9}$$

In the limit of $v\cdot q = 0$, we have

$$\int_l \frac{(l+q)^\alpha\, l^\beta\, (2l+q)^\mu}{(v\cdot l)^2\, (l^2-M^2)\, [(l+q)^2 - M^2]}$$
$$= -\frac{v^\mu}{8\pi}\left(g^{\alpha\beta} + 2q^\alpha q^\beta \frac{\partial}{\partial q^2}\right)\int_0^1 dz \sqrt{M^2 - z(1-z)q^2} - \left(q^\alpha g^{\beta\mu} - q^\beta g^{\alpha\mu}\right) f_0(q^2). \tag{E.10}$$

Again we dropped terms proportional to v^α or v^β.

Appendix F: Three-point Vertices (Figure 4)

In this section as well as in the next, we classify graphs into Class A, Class V and Class AV. The graphs in Class V appear only with the vector current while the graphs in Class AV appear both for the vector and axial-vector currents. The graphs in Class A involving the axial-vector current do not figure in three-point vertices. We define two operators for the graphs in Class AV: $T_1^\mu = v^\mu$ and $T_2 = 2g_A S^\mu$. We write the expressions only for the axial-vector current for graphs in Class AV. The expressions for the vector current is obtained by interchanging T_1^μ and T_2^μ. Fig.4*c* and 4*d* vanish because they are proportional to $v\cdot S$.

Class AV[#22]

$$\Gamma^{\mu,a}_{ANN}(a) = \frac{\tau_a}{2}\frac{T_2^\mu}{F^2}\Delta(M^2), \tag{F.1}$$

$$\Gamma^{\mu,a}_{ANN}(b) = \frac{\tau_a}{2}\frac{g_A^2}{F^2} S_\alpha T_2^\mu S_\alpha\, h_3(v\cdot k, v\cdot q), \tag{F.2}$$

$$i\Gamma^a_{\pi NN}(a) = \frac{g_A}{3F^3}\tau_a\, q\cdot S\,\Delta(M^2), \tag{F.3}$$

$$i\Gamma^a_{\pi NN}(b) = \frac{d-3}{4}\tau_a\, q\cdot S\,\frac{g_A^3}{F^3}\, h_3(v\cdot k, v\cdot q), \tag{F.4}$$

[#22] The figure label $a, b, c...$ is given in the parenthesis.

Class V

$$\Gamma^{\mu,a}_{VNN}(e) = -\frac{\tau_a}{2F^2}\left[v^\mu f_1(q^2) + v\cdot q\, q^\mu f_3(q^2)\right], \tag{F.5}$$

$$\begin{aligned}\Gamma^{\mu,a}_{VNN}(f) &= 2\tau_a\frac{g_A^2}{F^2}\left\{\left[S\cdot S + 2(q\cdot S)^2\frac{\partial}{\partial q^2}\right][q^\mu B_1(k,q) + v^\mu B_2(k,q)]\right. \\ &\quad \left. + \{q\cdot S, S^\mu\}\, B_1(k,q) + [q\cdot S, S^\mu]\, B_0(k,q)\right\}\end{aligned} \tag{F.6}$$

$$\Gamma^{\mu,abc}_{V\pi\pi}(g) = i\epsilon_{abc}(q_c - q_b)^\mu \frac{5}{3}\frac{\Delta(M^2)}{F^2}, \tag{F.7}$$

$$\Gamma^{\mu,abc}_{V\pi\pi}(h) = i\epsilon_{abc}\frac{1}{F^2}\left[-(q_c - q_b)^\mu f_1(q_a^2) + q_a^\mu\,(q_c^2 - q_b^2) f_3(q_a^2)\right]. \tag{F.8}$$

Here the index a labels the isospin of the photon with four-momentum q_a, the indices b and c the isospin of the pions with their momenta q_b and q_c and the momentum conservation is $q_a + q_b + q_c = 0$.

Appendix G: Four-point Vertices (Figure 5)

Here we define $q^\mu = q_a^\mu + q_b^\mu$. For other notations, see Appendix F. Figures $5(i) - (n)$ vanish because they are proportional to $v\cdot S$. Here we restrict ourselves to the case of on-shell nucleons, $v\cdot k = v\cdot(q_a + q_b) = 0$.

Class A

$$\Gamma^{\mu,ab}_{\pi A}(a) = \frac{1}{2F^3}\epsilon_{abc}\tau_c\, v^\mu f_1(q^2), \tag{G.1}$$

$$\begin{aligned}\Gamma^{\mu,ab}_{\pi A}(b) &= -i\delta_{ab}(2q_a + 6q_b)^\mu \frac{2g_A^2}{3F^3}\left[S\cdot S + 2(q\cdot S)^2\frac{\partial}{\partial q^2}\right] B_0(q^2) \\ &\quad -\epsilon_{abc}\tau_c\,\frac{2g_A^2}{F^3}\left\{v^\mu\left[S\cdot S + 2(q\cdot S)^2\frac{\partial}{\partial q^2}\right] f_1(q^2) + [q\cdot S, S^\mu]\, B_0(q^2)\right\},\end{aligned} \tag{G.2}$$

Class AV

$$\Gamma^{\mu,ab}_{\pi A}(c) = -\frac{5}{12}\epsilon_{abc}\tau_c\frac{T_1^\mu}{F^3}\Delta(M^2), \tag{G.3}$$

$$\Gamma^{\mu,ab}_{\pi A}(d) = \frac{g_A^2}{2F^3}\epsilon_{abc}\tau_c\, S_\alpha T_1^\mu S^\alpha\, \Delta(M^2), \tag{G.4}$$

$$\Gamma^{\mu,ab}_{\pi A}(e+f) = \frac{T_1^\mu}{4F^3}\left\{4i\delta_{ab}\, v\cdot q_a\, h_0^S(v\cdot q_a) + \epsilon_{abc}\tau_c\left[\Delta(M^2) - 2v\cdot q_a h_0^A(v\cdot q_a)\right]\right\}, \tag{G.5}$$

$$\begin{aligned}\Gamma^{\mu,ab}_{\pi A}(g+h) &= (-3i\delta_{ab} + \epsilon_{abc}\tau_c)\frac{g_A^3}{2F^3} S_\alpha\, q_b\cdot S\, T_2^\mu\, S^\alpha\, h_4(v\cdot q_a) \\ &\quad +(-3i\delta_{ab} - \epsilon_{abc}\tau_c)\frac{g_A^3}{2F^3} S_\alpha\, T_2^\mu\, q_b\cdot S\, S^\alpha\, h_4(-v\cdot q_a),\end{aligned} \tag{G.6}$$

Class V

$$\Gamma^{\mu,ab}_{\pi V}(o+p+q) = \frac{g_A}{2F^3} v\cdot q_a \left\{ (-2i\delta_{ab} - \epsilon_{abc}\tau_c) \int_l \frac{l\cdot S\,(2l+q_a)^\mu}{v\cdot l\,(l^2-M^2)\,[(l+q_a)^2-M^2]} \right.$$
$$\left. -(2i\delta_{ab} - \epsilon_{abc}\tau_c) \int_l \frac{l\cdot S\,(2l-q_a)^\mu}{v\cdot l\,(l^2-M^2)\,[(l-q_a)^2-M^2]} \right\}, \quad \text{(G.7)}$$

$$\Gamma^{\mu,ab}_{\pi V}(r) = -2i\delta_{ab}\frac{g_A^3}{F^3}\int_l \frac{(l+q_a)\cdot S\, q_b\cdot S\, l\cdot S(2l+q_a)^\mu}{v\cdot(l+q_a)\, v\cdot l\,(l^2-M^2)\,[(l+q_a)^2-M^2]} \quad \text{(G.8)}$$

where

$$B_0(q^2) = -\frac{1}{16\pi}\int_0^1 dz\,\sqrt{M^2 - z(1-z)q^2}, \quad \text{(G.9)}$$

$$h_0^S(v\cdot q) = \frac{1}{2}\left[h_0(v\cdot q) + h_0(-v\cdot q)\right] = -\frac{1}{8\pi}\sqrt{M^2-(v\cdot q)^2}, \quad \text{(G.10)}$$

$$h_0^A(v\cdot q) = \frac{1}{2}\left[h_0(v\cdot q) - h_0(-v\cdot q)\right]. \quad \text{(G.11)}$$

Now we study *low-momentum expansion for on-shell nucleons with* $v\cdot k = v\cdot(q_a+q_b) = 0$. To second order in external momentum;

Class A

$$\Gamma^{\mu,ab}_{\pi A}(a) = \frac{1}{2F^3}\epsilon_{abc}\tau_c\, v^\mu \left[\Delta(M^2) - \frac{\eta}{6}q^2 + \cdots\right],$$

$$\Gamma^{\mu,ab}_{\pi A}(b) = -i\delta_{ab}(q_a+3q_b)^\mu \frac{g_A^2}{F^3}M'$$
$$-\epsilon_{abc}\tau_c\,\frac{2g_A^2}{F^3}\left\{v^\mu\, S\cdot S\Delta(M^2) - v^\mu\frac{\eta}{6}\left[S\cdot S\, q^2 + 2(q\cdot S)^2\right] - [q\cdot S, S^\mu]\, M'\right\}$$
$$+\ \cdots.$$

Class AV

$$\Gamma^{\mu,ab}_{\pi A}(c) = -\frac{5}{12}\epsilon_{abc}\tau_c\frac{T_1^\mu}{F^3}\Delta(M^2),$$

$$\Gamma^{\mu,ab}_{\pi A}(d) = \frac{g_A^2}{2F^3}\epsilon_{abc}\tau_c\, S_\alpha T_1^\mu S^\alpha\,\Delta(M^2),$$

$$\Gamma^{\mu,ab}_{\pi A}(e+f) = \frac{T_1^\mu}{4F^3}\left\{-8i\delta_{ab}\, v\cdot q_a\, M' + \epsilon_{abc}\tau_c\left[\Delta(M^2) - 4(v\cdot q_a)^2\,\eta\right]\right\}\cdots,$$

$$\Gamma^{\mu,ab}_{\pi A}(g+h) = (-3i\delta_{ab} + \epsilon_{abc}\tau_c)\frac{g_A^3}{2F^3}S_\alpha\, q_b\cdot S\, T_2^\mu\, S^\alpha\,\left[M' + \frac{2}{3}\,v\cdot q_a\,\eta\right]$$
$$+(-3i\delta_{ab} - \epsilon_{abc}\tau_c)\frac{g_A^3}{2F^2}S_\alpha\, T_2^\mu\, q_b\cdot S\, S^\alpha\,\left[M' - \frac{2}{3}\,v\cdot q_a\,\eta\right] + \cdots.$$

Class V

$$\Gamma^{\mu,ab}_{\pi V}(o+p+q) = \frac{g_A}{F^3}\,v\cdot q_a\left[4i\delta_{ab}M'S^\mu + \epsilon_{abc}\tau_c(S^\mu v\cdot q_a + v^\mu q_a\cdot S)\eta\right] + \cdots,$$

$$\Gamma^{\mu,ab}_{\pi V}(r) = -2i\delta_{ab}\frac{g_A^3}{F^3}\left\{-2v^\mu M' S_\alpha q_b\cdot S\, S^\alpha - (q_a\cdot S\, q_b\cdot S\, S^\mu - S^\mu\, q_b\cdot S\, q_a\cdot S)\eta\right\}$$
$$+\cdots.$$

We have used the notations $M' = \frac{M}{16\pi}$ and $\eta = \frac{1}{16\pi^2}\left(\frac{M^2}{4\pi\mu^2}\right)^{-\epsilon}\Gamma(\epsilon)$.

Appendix H: Fermi-Gas Model for Two-Body Axial-Charge Operator

Let $|F\rangle$ be the ground state of Fermi-gas model whose fermi-momentum is p_F and $|ph\rangle = b_p^\dagger b_h|F\rangle$ be the one-particle (labeled by p) one-hole (labeled by h) excited state, where $b_\alpha(b_\alpha^\dagger)$ is the annihilation(creation) operator of a fermion state characterized by α. Consider the matrix element $\langle ph|\mathcal{M}|F\rangle$ (or its *effective one body operator* $\mathcal{M}_{\text{eff}}$),

$$\langle ph|\mathcal{M}|F\rangle = \langle p|\mathcal{M}_{\text{eff}}|h\rangle = \sum_{\beta\in F} \frac{1}{g_\beta}\langle p,\beta|\mathcal{M}|h,\beta\rangle \tag{H.1}$$

where g_β is defined by $\left\{b_\alpha, b_\beta^\dagger\right\} = g_\beta\delta_{\alpha,\beta}$. In computing this, it is convenient to define the antisymmetrized wave function $|\alpha,\beta\rangle$ in terms of the simple two-particle state $|\alpha,\beta)$

$$|\alpha,\beta\rangle = \frac{|\alpha,\beta) - |\beta,\alpha)}{\sqrt{2}}, \tag{H.2}$$

so the matrix element $\langle p,\beta|\mathcal{M}|h,\beta\rangle$ is of the form

$$\langle p,\beta|\mathcal{M}|h,\beta\rangle = (p,\beta|\mathcal{M}|h,\beta) - (p,\beta|\mathcal{M}|\beta,h). \tag{H.3}$$

The first term is the Hartree term and the second the Fock term. Rewriting $|\alpha\rangle$ as $|\mathbf{p}_\alpha m_\alpha t_\alpha\rangle$ where $\mathbf{p}_\alpha$ is the momentum of the state labeled by α and m_α (t_α) the third component of the spin (isospin) of the state α, we may write the axial charge operator as

$$(1',2'|\mathcal{M}|1,2) = (t_1'm_1', t_2'm_2'|\left[\mathcal{T}^{(1)}\phi_1(q) + \mathcal{T}^{(2)}\phi_2(q)\right]|t_1m_1, t_2m_2) \tag{H.4}$$

where $\mathcal{T}^{(1)} = i\,\vec{\tau}_1\times\vec{\tau}_2(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\mathbf{q}$, $\mathcal{T}^{(2)} = i\,(\vec{\tau}_1+\vec{\tau}_2)\vec{\sigma}_1\times\vec{\sigma}_2\cdot\mathbf{q}$ and $\mathbf{q} = \mathbf{p}_2' - \mathbf{p}_2 = \mathbf{p}_1 - \mathbf{p}_1'$, $q \equiv |\mathbf{q}|$.

It is trivial to see that the Hartree term must vanish. Thus we are to calculate the Fock term. First we shall show that the matrix element of $\mathcal{T}^{(1)}$ is equal to that of $\mathcal{T}^{(2)}$ when summed over spin and isospin of occupied states. Doing the sum, we get

$$\sum_{m_\beta}\sum_{t_\beta}(m_pt_p, m_\beta t_\beta|\mathcal{T}^{(1)}|m_\beta t_\beta, m_ht_h) = -4\,\langle m_pt_p|\vec{\tau}\otimes\vec{\sigma}|m_ht_h\rangle\cdot\mathbf{q}. \tag{H.5}$$

For $\mathcal{T}^{(2)}$, we simply interchange the spin and isospin operators and get the same result. It then follows that the effective one-body operator of the axial-charge operator becomes

$$\langle p|\mathcal{M}_{\text{eff}}|h\rangle = 4\vec{\tau}_{ph}\vec{\sigma}_{ph}\int_{|\mathbf{p}|<p_F}\frac{d\mathbf{p}}{(2\pi)^3}\,(\mathbf{p}-\mathbf{p}_h)\,\phi(|\mathbf{p}-\mathbf{p}_h|) \tag{H.6}$$

where $\phi(q) = \phi_1(q) + \phi_2(q)$, p_F stands for the fermi-momentum of the ground state $|F\rangle$ and $\vec{\tau}_{ph}(\vec{\sigma}_{ph})$ is $\langle t_p|\vec{\tau}|t_h\rangle$ ($\langle m_p|\vec{\sigma}|m_h\rangle$). This form is particularly useful when the particle is on the fermi surface $|\mathbf{p}_h| = p_F$,

$$\int_{|\mathbf{p}|<p_F}\frac{d\mathbf{p}}{(2\pi)^3}\,(\mathbf{p}-\mathbf{p}_h)\,\phi(|\mathbf{p}-\mathbf{p}_h|) = -\frac{\hat{\mathbf{p}}_h}{4\pi^2}8p_F^4\int_0^1 dx\,(x^3-x^5)\,\phi(2xp_F). \tag{H.7}$$

In order to give a meaning to this expression, we have to account for short-range correlations. Otherwise we can get erroneous results. For instance, a constant in momentum space (say, $\phi(q) =$ constant) gives a contribution whereas it should be suppressed in reality. One way to assure a correct behavior at short-distance is to subtract the constant as one does for the Lorentz-Lorenz effect in π-nuclear scattering. However this procedure is not always practicable if one is dealing with non-polynomial terms. It is therefore preferable to go to coordinate space by Fourier-transform. For this, define $f(r)$ by

$$f(r) = \int \frac{d\mathbf{q}}{(2\pi)^3} \mathrm{e}^{i\mathbf{q}\cdot\mathbf{r}} \phi(q) . \tag{H.8}$$

Using

$$\int_{|\mathbf{p}|<p_{\mathrm{F}}} \frac{d\mathbf{p}}{(2\pi)^3} \mathrm{e}^{i\mathbf{p}\cdot\mathbf{r}} = \frac{p_{\mathrm{F}}^2}{2\pi^2 r} j_1(p_{\mathrm{F}} r), \quad j_1(x) = \frac{\sin x}{x^2} - \frac{\cos x}{x}$$

and

$$\int d\Omega\, \hat{r} \mathrm{e}^{i\mathbf{p}\cdot\mathbf{r}} = i\, 4\pi \hat{\mathbf{p}} j_1(pr),$$

we obtain

$$\langle ph|\mathcal{M}|F\rangle = \vec{\tau}_{ph}\, \vec{\sigma}_{ph} \cdot \hat{\mathbf{p}}_h \frac{8p_{\mathrm{F}}^2}{\pi} \int_0^\infty dr\, r\, j_1(p_{\mathrm{F}} r) j_1(p_h r) \frac{d}{dr} f(r). \tag{H.9}$$

Introducing a correlation function $\hat{g}(r,d)$ where d is a parameter of $\hat{g}$, we get the final expression

$$\langle ph|\mathcal{M}|F\rangle = \vec{\tau}_{ph}\, \vec{\sigma}_{ph} \cdot \hat{\mathbf{p}}_h \frac{8p_{\mathrm{F}}^2}{\pi} \int_d^\infty dr\, r\, j_1(p_{\mathrm{F}} r) j_1(p_h r) \frac{d}{dr} f(r). \tag{H.10}$$

In the numerical results discussed in the next, we have used the simplest correlation function, $\hat{g}(r,d) = \theta(r-d)$.

Appendix I: The Role of Vector Mesons

In this Appendix, we describe briefly the role that vector mesons play in the axial-charge transitions in chiral perturbation theory. We will in particular establish that vector mesons can contribute only at $\mathcal{O}(\frac{1}{m^2})$ and hence their contributions are suppressed to the chiral order we are concerned with. For simplicity, we shall consider the vector field V_μ only. The axial vector field $a_{1\mu}$ could also be included but we shall leave it out since it plays even less significant role in our case. Let

$$V_\mu = t_a V_\mu^a, \qquad \mathrm{Tr}(t_a t_b) = \frac{1}{2}\delta_{ab} \tag{I.1}$$

denote the spin-1 field. The index a and b are $(1,2,3)$ for SU(2) with $\vec{t} = \frac{\vec{\tau}}{2}$, and $(0,1,2,3)$ for U(2) with $t_0 = \frac{1}{2}$. The $a = (1,2,3)$ components correspond to the ρ mesons and $a = 0$ to the ω meson. We write the relevant part of the Lagrangian as#23

$$\mathcal{L} = \bar{N}\left[\gamma^\mu \left(i\partial_\mu + gV_\mu + g_A\gamma_5\, i\Delta_\mu\right) - m\right] N + \frac{F^2}{2}\langle i\Delta_\mu\, i\Delta^\mu\rangle + \frac{1}{4}M^2F^2\langle\Sigma\rangle$$

#23 We have not included the axial-vector field a_1, although it is not difficult to do so. For the axial-charge process we are considering the a_1 field does not play an important role. For the Gamow-Teller operator, however, the axial field may not be ignorable.

$$+ \quad \frac{1}{2}M_V^2\langle\left(V_\mu - \frac{i}{g}\Gamma_\mu\right)^2\rangle - \frac{1}{4}\langle V_{\mu\nu}V^{\mu\nu}\rangle + \mathcal{L}_{an} \tag{I.2}$$

where

$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - ig\,[V_\mu, V_\nu]\,, \tag{I.3}$$

and Γ_μ (Δ_μ) were given in Section 3 and explicitly take the form

$$\begin{aligned} i\Gamma_\mu &= \vec{t}\cdot\left[\vec{\mathcal{V}}_\mu + \frac{1}{F}\vec{\pi}\times\vec{\mathcal{A}}_\mu - \frac{1}{2F^2}\vec{\pi}\times\partial_\mu\vec{\pi} + \cdots\right], \\ i\Delta_\mu &= \vec{t}\cdot\left[\vec{\mathcal{A}}_\mu + \frac{1}{F}\vec{\pi}\times\vec{\mathcal{V}}_\mu - \frac{1}{F}\partial_\mu\vec{\pi} + \cdots\right] \end{aligned} \tag{I.4}$$

where $\mathcal{V}_\mu$ ($\mathcal{A}_\mu$) is the external vector (axial-vector) field and the ellipsis denotes terms involving more than three fields. Here $\mathcal{L}_{an}$ is an "anomalous parity" piece involving the totally antisymmetric ϵ tensor which we do not explicit here as it does not contribute. Also four-fermion interaction terms do not figure in the discussion. In (I.2), the constants g and M_V can be identified as the VNN coupling constant and the mass of the V meson respectively. We are using the short-hand notation

$$\langle X\rangle \equiv 2\,\mathrm{Tr}(X) \tag{I.5}$$

for any X. This convention is convenient due to the normalization of t_a, $\langle XY\rangle = X_a Y_a$ for any $X = t_a X_a$ and $Y = t_a Y_a$.

Before proceeding, let us note a few characteristics of this Lagrangian:

- It is vector gauge-invariant (or hidden gauge invariant) provided that gV_μ transforms as $i\Gamma_\mu$ does, $V_\mu \to UV_\mu U^\dagger - \frac{i}{g}\partial_\mu U\cdot U^\dagger$. It is also invariant under (global) chiral transformation apart from the pion mass term.
- It has vector-meson dominance.
- When M_V goes to infinity, we recover our previous chiral Lagrangian involving only π's and nucleons N.
- There is a $V\gamma$ mixing but the mixing is trivial in the sense that the photon field appears only as an external (non-propagating) field.

For the reasons spelled out in the main text, we wish to transform the Lagrangian to a form appropriate for heavy fermion formalism. Including the "$1/m$" terms, we have

$$\begin{aligned} \mathcal{L} &= \bar{B}\,(iv\cdot D + 2g_A\,S\cdot i\Delta)\,B + \frac{F^2}{2}\langle i\Delta_\mu\, i\Delta^\mu\rangle + \frac{1}{4}M^2F^2\langle\Sigma\rangle \\ &+ \frac{1}{2m}\bar{B}\left(-D^2 + (v\cdot D)^2 + [S^\mu, S^\nu]\,[D_\mu, D_\nu] - g_A^2(v\cdot i\Delta)^2 - 2ig_A\,\{v\cdot i\Delta, S\cdot D\}\right)B \\ &+ \frac{1}{2}M_V^2\langle\left(V_\mu - \frac{i}{g}\Gamma_\mu\right)^2\rangle - \frac{1}{4}\langle V_{\mu\nu}V^{\mu\nu}\rangle + \mathcal{L}_{an} \end{aligned} \tag{I.6}$$

where $D_\mu = \partial_\mu - igV_\mu$. Now let us calculate the tree-order contribution of the vector mesons to the two-body axial charge operator. Three types of graphs contribute. The relevant graphs are given in Figure 11.

First we find that the graph (c) does not contribute. To see this, note that G-parity does not allow the couplings $\rho\rho\mathcal{A}_\mu$ and $\omega\omega\mathcal{A}_\mu$. The coupling for $\mathcal{A}_\mu\rho\omega$ is of the form $\epsilon^{\mu\nu\alpha\beta}\omega_{\nu\alpha}\rho_\beta$ coming from the anomalous-parity term $\mathcal{L}_{an}$ of (I.6). In the figure (c), each vector meson brings in v_μ as one can see in (I.6), so that we have $\epsilon^{\mu\nu\alpha\beta}v_\alpha v_\beta = 0$.

Working out the graphs (a) and (b) #24 we get

$$\begin{aligned}\vec{A}^\mu(1) &= i\vec{\tau}_1\times\vec{\tau}_2\frac{g_A}{2F^2}\frac{M_V^2}{M_V^2-q_1^2}\frac{1}{M^2-q_2^2}\left(q_2\cdot S_2 - \frac{v\cdot q_2}{m_N}S_2\cdot P_2\right)\\ &\otimes\left(v_1^\mu + \frac{1}{m_N}[S_1^\mu, q_1\cdot S_1] - \frac{v_1\cdot q_1}{M_V^2}q_1^\mu\right) + (1\leftrightarrow 2) \qquad (I.8)\\ \vec{A}^\mu(2) &= (\vec{\tau}_1+\vec{\tau}_2)\frac{g_A}{2F^2}\frac{M_V^2}{M_V^2-q_2^2}\frac{v^\mu}{4m_N}\left(v_2\cdot S_2 + \frac{1}{m_N}[S_1\cdot S_2, q_2\cdot S_2] - \frac{v_2\cdot q_2}{M_V^2}q_2\cdot S_2\right)\\ &+ (1\leftrightarrow 2) \qquad (I.9)\end{aligned}$$

where we have used the KSRF relation $M_V^2 = 2g^2F^2$ and defined

$$\begin{aligned}P_i^\mu &= \frac{1}{2}(p_i+p_i')^\mu,\\ v_i^\mu &= v^\mu + \frac{1}{m_N}\left(P_i^\mu - v^\mu\, v\cdot P_i\right) \qquad (I.10)\end{aligned}$$

and $q_i = p_i' - p_i$, $i=1,2$. Now noting that $v\cdot q \simeq v_i\cdot q_j = \mathcal{O}\left(\frac{Q^2}{m_N}\right)$ and $S_i^0 = \mathcal{O}\left(\frac{Q}{m_N}\right)$, we have (setting $q_2 = -q_1 \equiv q$)

$$\begin{aligned}\vec{A}^0(1) &= i\vec{\tau}_1\times\vec{\tau}_2\frac{g_A}{2F^2}\frac{M_V^2}{M_V^2-M^2}\left(\frac{1}{M^2-q^2} - \frac{1}{M_V^2-q^2}\right) q\cdot S_2\left[1+\mathcal{O}\left(\frac{Q^2}{m_N^2}\right)\right]\\ &+ (1\leftrightarrow 2) \qquad (I.11)\\ \vec{A}^0(2) &= (\vec{\tau}_1+\vec{\tau}_2)\frac{g_A}{2F^2}\left[\mathcal{O}\left(\frac{Q^2}{m_N^3}\right) + \mathcal{O}\left(\frac{Q^3}{m_N^2M_V^2}\right)\right]. \qquad (I.12)\end{aligned}$$

The leading part of Equation (I.11) is nothing but the one-pion exchange current with one-loop radiative corrections (91) expressed now in terms of a vector-dominated Dirac form factor F_1^V. (In fact, with the Lagrangian (I.6), the *soft-pion* contribution corresponds to (I.11) in the limit $M_V \to \infty$.) There is no further correction to what has already been obtained with our Lagrangian given in its full glory in Appendix A. This corroborates our

#24 In the spin-1 propagator

$$D_{ab}^{\mu\nu}(q) = \frac{\delta_{ab}}{q^2-M_V^2}\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{M_V^2}\right). \qquad (I.7)$$

the term proportional to $\frac{1}{M_V^2}q^\mu q^\nu$ is a correction term of $\mathcal{O}\left(\frac{Q^2}{M_V^2}\right)$ relative to the leading term ($\propto g^{\mu\nu}$). It is further suppressed in the vector-meson exchange between two nucleons since the VNN vertex function is proportional to v^μ and $v\cdot q = \mathcal{O}\left(\frac{Q^2}{m}\right)$. So effectively this term is of order $\frac{Q^3}{mM_V^2}$ and hence can be dropped.

argument that the counter terms $\kappa_4^{(1,2)}$ cannot come from one vector-meson exchange in the limit $m_V \to \infty$. This also establishes our assertion that vector mesons do not modify our result on the "chiral filter mechanism."

For completeness, we give the corresponding axial-charge operator in coordinate space

$$\tilde{\mathcal{M}}_{tree}^{\rm VMD} + \tilde{\mathcal{M}}_{1\pi}^{\rm VMD} = \tilde{\mathcal{T}}^{(1)}\,(1+\delta_{soft})\,\frac{1}{4\pi r}\left[\left(M+\frac{1}{r}\right){\rm e}^{-Mr} - \left(M_V+\frac{1}{r}\right){\rm e}^{-M_V r}\right] \quad \text{(I.13)}$$

where $\delta_{soft} = \frac{M^2}{M_V^2 - M^2}$ and we have dropped the terms of order m_N^{-2}. This is the vector-dominated form of $\mathcal{M}_{1\pi}$, in place of (104): The second term of vector-meson range in (I.13) is the counterpart to the shorter-ranged loop correction in (104). In fermi-gas model (I.13) predicts roughly the same quenching as the loop calculation (104).

FIGURE CAPTIONS

Figure 1

Generic nuclear electroweak currents up to two body. The solid line represents the nucleon, the blob with a cross the coupling of electroweak fields and the shaded blob without cross stands for the strong interactions.

Figure 2

Two-body exchange currents: (a) One-pion exchange; (b) two-pion exchange. The solid blob represents a strong-interaction vertex and the shaded blob with a cross the vertex involving an external field and strong interactions. The solid line represents the nucleon and broken line the pion.

Figure 3

One-loop graphs contributing to the nucleon self-energy Σ. As in Figure 2, the solid line represents the nucleon, the broken line the pion.

Figure 4

One-loop graphs contributing to the three-point $G_\mu NN$ vertex where $G_\mu = \mathcal{A}_\mu$ ($\mathcal{V}_\mu$) is the external axial-vector (vector) field, the encircled cross representing the field coupling. Here and in Fig. 5, vector-field couplings are also drawn for comparison and for later use in [14].

Figure 5

One-loop graphs contributing to the four-point $G_\mu \pi NN$ vertex. For axial-charge transitions, only the graphs (a)-(f) contribute.

Figure 6

One-loop graphs contributing to two-body two-pion exchange currents ($(a)-(h)$), four-fermion-field contact interaction currents ($(i)-(j)$) and "recoil" current (k). The pion propagator appearing in $(a)-(j)$ is the Feynman one while that in (k) is a time-ordered one. Only the graphs (a), (b), (c) and (d) survive for the axial-charge operator.

Figure 7

$4\pi r^2\tilde{\mathcal{M}}_{tree}$ (solid line) and $4\pi r^2\tilde{\mathcal{M}}_{loop}$ (broken line) defined in Eqs.(103) and (106) vs. r in fm. Here and in Fig. 8, we have set $\tilde{\mathcal{T}}^{(1)} = \tilde{\mathcal{T}}^{(2)} = 1$.

Figure 8

$r[j_1(p_F r)]^2\tilde{\mathcal{M}}_{tree}$ (solid line) and $r[j_1(p_F r)]^2\tilde{\mathcal{M}}_{loop}$ (broken line) vs. r with $p_F \approx 1.36\ \mathrm{fm}^{-1}$ (corresponding to nuclear matter density). See the caption for Fig. 7.

Figure 9

The ratios of the matrix elements $\frac{\langle\mathcal{M}_X\rangle}{\langle\mathcal{M}_{tree}\rangle}$ in fermi-gas model vs. ρ/ρ_0 for $d = 0.5, 0.7$ fm for $X = 1\pi,\ 2\pi,\ 1\pi + 2\pi$ corresponding to one-loop correction to the one-pion exchange graph, one-loop two-pion exchange graph and the sum of the two, respectively.

Figure 10

Three-body currents: a) Genuine three-body current with Feynman pion propagators; b) "recoil" three-body currents with time-ordered pion propagators; the ellipsis stands for other time-orderings and permutations. Both (a) and (b) are of order $O(Q^3)$ relative to the leading soft-pion term.

Figure 11

Vector-meson contribution with the Lagrangian (I.6) to the two-body axial charge operator. V and V' stand for vector mesons of mass M_V. For the axial current, $V = \rho$ and $V' = \omega$.