FCNC in SUSY theories

C.A. Savoy

FCNC IN SUSY THEORIES

C. A. Savoy

Service de Physique Théorique, C.E. de Saclay, 91191 Gif-s/Yvette, France

Recent work on flavour changing neutral current effects in supersymmetric models is reviewed. The emphasis is put on new issues related to solutions to the flavour problem through new symmetries: GUTs, horizontal symmetries, modular invariances.

Invited Talk at the HEP95 Euroconference, Brussels, July 95.

1 Introduction

A rich literature is available about FCNC restrictions on supersymmetric extensions of the standard model. Nevertheless, both the LEP (and Tevatron) constraints on supersymmetric theories and some fresh insight on spontaneously broken supergravities from superstrings have encouraged a recent revival of this subject.

The basic supersymmetry induced FCNC (SFCNC) effects are produced by the analogues of the Standard Model loop diagrams for neutral current processes, with quarks and vector bosons replaced by squarks and gauginos. If quark(lepton) and squark(slepton) mass matrices are not diagonal in the same basis, even the the couplings to neutral gauginos to fermions and sfermions will not be diagonal and will induce FCNC effects. There are several sources of flavour mixing in gaugino couplings that we now turn to discuss. However, I want to keep in mind that supersymmetry must be a local symmetry, namely, a supergravity theory, at the fondamental level. This has implications on the structure of the low energy effective theory (and vice-versa, which is even more important!)

Within the general framework of supergravity, a theory is defined by the gauge and matter superfields, and by their couplings encoded in the Kähler potential and the superpotential. The low-energy theory is then fixed by the values of the auxiliary fields that provide supersymmetry breaking and their couplings to the light fields. The supersymmetric part of this effective theory contains the supersymmetrized gauge couplings and the supersymmetrized Yukawa couplings, encoded in an effective superpotential $W = \sum[Y^U_{ij} Q^i U^j H_2 + Y^D_{ij} Q^i D^i H_1 + Y^E_{ij} L^i E^j H_1]$, where H_1 and H_2 are the two Higgs superfields, and the matter superfields are as follows: Q, L, contain the $\mathrm{SU}(2)_{\mathrm{weak}}$ doublets of quarks and leptons, and U, D, E contain the right-handed quarks and leptons. The physical content of the three Yukawa coupling 3×3 matrices is given by their eigenvalues $Y_i (i = u, c, t; d, s, b; e, \mu, \tau)$ as well as the CKM matrix V. The observed quark masses and mixings and lepton masses reveal a strong hierarchy conveniently displayed in terms of a small parameter which we choose to be the Cabibbo angle, $\lambda = .22$: $Y_t : Y_c : Y_u = \lambda^8 : \lambda^4 : 1, Y_b : Y_s : Y_d = \lambda^4 : \lambda^2 : 1, Y_\tau : Y_\mu : Y_e = \lambda^4 : \lambda^2 : 1, V_{us} = \lambda, V_{cb} \sim \lambda^2, V_{ub} \sim \lambda^3$.

At the level of the effective theory, below the Planck scale, the supersymmetry breaking effects reduce to gaugino masses and the soft interactions in the scalar potential. The scalar $(\mathrm{mass})^2$ matrix depend on the Kähler potential and on the supersymmetry breaking auxiliary fields. The universality or flavour independence hypothesis assumes equal masses for all squarks at the unification. At lower energies, radiative corrections from Yukawa interactions split this degeneracy with flavour dependent shifts. The triscalar couplings are basically proportional to couplings in the superpotential. Again, if universality is assumed for the proportionality factors, referred to as A-parameters, their equality is spoilt at lower energies by the calculable radiative corrections.

Universality of soft terms is often assumed in SFCNC studies. Then, the most striking effects of radiative corrections are of two kinds. Gauge corrections are almost universal and attenuate loop effects by an overall rise in the squark masses if gluinos are relatively heavy. Yukawa corrections dominated by the top coupling, Y_t, tend to align the down squark mass eigenstates to the up quarks (if $\tan\beta$ is not too large). This reverses the pattern of gaugino couplings in comparison with the gauge boson ones. Chargino couplings to down squarks and up quarks are approximately flavour diagonal while gluino and neutralino couplings become proportional to the CKM matrix. However, the expected physical effects are either consistent with the present overall bounds on supersymmetric particles or they depend on unknown mixings and phases, but the $b \to s\gamma$ transition provides interesting information.

Thus, universality naturally suppresses SFCNC effects as it amounts to postulate the largest possible horizontal symmetry, $\mathrm{U}(3)^5$, for each of the 5 irreps of the Standard Model in the 3 fermion families, as an accidental symmetry, *i.e.*, a symmetry of the scalar potential in the limit where all Yukawa couplings vanish. This is justified if supergravityy couplings to the supersymmetry breaking are flavour independent. As we now turn to discuss, they are not necessarily so.

2 Flavour theories and supersymmetry

The fermion unit in the Standard Model is a family of 15 fermions that provide a non-trivial anomalous-free representation of the gauge group. GUTs are attempts to understand the fermion pattern by (vertical)unification of the elements of the family within a representation of a larger gauge theory at very high energies. The triplication of families is a puzzle. But these fermion replicas do not look as clones since they quite differ by the strong hierarchy in their Yukawa couplings. The natural explanation of this situation is to hypothetize that quarks/leptons of the same charge have different quantum numbers of some new symmetries at high energies (symmetries that commute with the Standard Model-symmetries have been called horizontal).

As in many particle physics issues, hints come from superstrings models, where one finds examples of compactifications with fermion families and neither vertical nor horizontal unification. Instead, there are in general additional abelian $U(1)$ symmetries that differentiate between fermions. Moreover, the superstring theory particle masses and couplings are field dependent dynamically determined quantities.

A conspicuous result of superstring studies is that the three families of quark superfields may couple to supergravity according to different terms in the Kähler potential. The relevant low energy limit of superstring models are described by a $N = 1$ suoergravities. The zero-mass string spectrum contain an universal dilaton S, moduli fields, related to the compactification of six superfluous dimensions, denoted by $T_\alpha (\alpha = 1..m)$, and matter chiral fields A^i. A crucial role is played by the target-space modular symmetries $SL(2, Z)$, transformations on the T_α that are invariances of the effective supergravity theory. In string models of the orbifold type, the matter fields A^i transform under $SL(2, Z)$ according to a set of numbers, $n_i^{(\alpha)}$, called the modular weights of the fields A^i with respect to the modulus T^α.

The dilaton superfield in these theories does have universal supergravity couplings to matter superfields. But the moduli couplings are fixed by modular invariances. Thus, the Kähler potential and the superpotential can have different dependences on the moduli for each flavour. On the other hand, these moduli correspond to flat directions of the scalar potential so that their vev's are fixed by quantum corrections. Assuming that the relevant ones come from the light sector, namely by the coupling of moduli to quarks and leptons in the low energy theory, it has been suggested that modular invariances can also provide a theory of flavour, by predicting the hierarchies in the moduli dependent Yukawa couplings. This interesting idea is discussed in more detail in the contributions[1] of E. Dudas and F. Zwirner. For this reason, it is not developped here.

Motivated by superstrings, as well as symmetries proposed to explain the structure of Yukawa couplings, new analyses[2,9] have been performed on FCNC transitions produced by non-universality in supergravitycouplings. Of course, the results are model dependent, one variable being the amount of the flavour independent supersymmetry breaking (in the dilaton direction) responsible for gaugino masses, that attenuates SFCNC. With this proviso the more important constraints in the quark sector are coming from K-physics. The lepton sector is less sensitive to gaugino masses, and lepton flavour violations put severe constraints on the parameters, but only as functions of unknown lepton mixing angles.

Nevertheless, in this talk I would like to focus on the SFCNC problem from the stand-point of different attempts to explain the origin of flavour, hence of fermion masses and mixings.

3 SFCNC effects from SUSY unification

Recently, the question of FCNC effects arising from SUSY GUTs has been analysed in detail in a series of papers[3] . This possibility was pointed out already some time ago, but the fact that the top Yukawa coupling is so large considerably enhances the effects. The idea is to estimate the renormalization correction from the running of the soft parameters in the theory from the supergravityscale (M_{Planck}) down to the GUT scale (M_{GUT}) in presence of very large Yukawa couplings, which is certainly the case for Y_t. In a GUT, above M_{GUT}, the following part of the superpotential give also rise to loop diagrams $\sum[Y_{ij}^U E^i U^j H_3 + Y_{ij}^D Q^i L^j H_3' + Y_{ij}^D D^i E^j H_3']$ involving the Higgs triplet partners. The coupling Y_t is allways large, while $Y_b = Y_\tau$ is large in O(10) unification or even for SU(5) with large $\tan\beta$. The effect of the running from M_{Planck} to M_{GUT} can be very important: the $\widetilde{\tau}_R$ is roughly reduced by a factor $(1 - Y_t^2/2Y_{max}^2)$, defined at M_{GUT}, where Y_{max}^2 is the value of Y_t for a Landau pole at M_{Planck}. The mass splitting with respect to $\widetilde{e}_R$ and $\widetilde{\mu}_R$ will remain at low energies and produce lepton flavour violating processes. Of course the results also depend on the angles defined by the diagonalizattion of the lepton and slepton masses. Assuming naive GUT relations for the lepton mixings - *cum grano salis* in view of the bad naive GUT predictions for the two light families - one gets sizeable FCNC effects in large regions of the parameter space. For large Y_b the effects are even bigger. The results can be illustrated by assuming universal boundary conditions at M_{Planck}, so that the slepton splitting is only due to the Higgs triplet. In this case, it is possible to present detailed predictions for the various lepton flavour violating processes (for quark FCNC, those are concealed by the analogous contributions from the MSSM superpotential).

Of course if one attempts a real theory of fermion masses based on GUTs, and O(10) has been preferred in this respect[4], for instance, by the introduction of non-renormalizable interactions and discrete symmetries, there will be corresponding constraints on the soft scalar masses and couplings. The framework will be similar to what is discussed herebelow in the case of abelian horizontal symmetries.

4 The pseudo-Goldstone approach

Dimopoulos and Giudice[5] invoke the pseudo-Goldstone phenomenon to enforce FCNC suppression. They assume a large $\Pi_{A=Q,U,D,L,E)}U(3)$'accidental' symmetry of the scalar potential, including the scalar masses, in the limit of vanishing Yukawa couplings. They introduce on-purpose multiplets, say in the $Adj(U(3)^5)$, whose vev's break $U(3)^5 \rightarrow U(1)^{15}$ or $\rightarrow [U(2)\times U(1)]^5$. The remaining symmetries entail the following form for each one of the sfermion mass matrices: $\widetilde{m}_A^2 = e^{-i\theta_A} diag(\widetilde{m}_{A1}^2,\ \widetilde{m}_{A2}^2\ \widetilde{m}_{A3}^2)e^{i\theta_A}$, where θ_A are matrices, each one containing five Goldstone fields living in the coset $U(3)/U(1)^3$ (the extension to $[U(2)\times U(1)]$ is obvious). These are massless states as the potential is flat along the θ_A directions. Actually, they are 'pseudo-Goldstone' states since the flavour symmetries are explicitly broken by the Yukawa couplings. The latter are taken *a priori* as given by the quark masses and CKM mixings. Then, at the quantum level, the hidden flavour symmetry is broken by loops with quarks that spoil the flatness along the θ_A directions. By minimization one obtains the θ_A vev's (and masses) in terms of the Yukawa couplings Y_A, such that the $\widetilde{m}_A^2$'s are all aligned to the Yukawa couplings Y_A but $\widetilde{m}_Q^2$ to the matrix $Y_U^2 + K^+ Y_D^2 K$. The quark squark alignment is as good as possible, still the $\widetilde{m}_Q^2$ disalignment could induce too much $K\bar{K}$ mixing. This is avoided if the remaining accidental symmetry is $U(2)\times U(1)$ so that $\widetilde{m}_{Q1}^2 = \widetilde{m}_{Q2}^2$. This can be implemented[5] by enlarging the accidental $U(3)^5$ symmetry to $O(8)$, spontaneously broken into $O(7)$.

In spite of its formal elegance, this approach does not address the flavour problem as far as the expected dependence of the Yukawa couplings on new fields is not envisaged while it might provide a prediction for quark masses as well. Also, the necessarily large number of *ad hoc* Goldstone fields could mitigate one's enthusiasm.

5 The supersymmetric Froggatt-Nielsen approach

The smalness of the mass ratios and mixing angles faces us with a problem of naturalness. The direction initiated by Froggatt and Nielsen [6] to understand such a hierarchical pattern goes as follows: (*i*) The key assumption is a gauged horizontal $U(1)_X$ symmetry violated by the small quark masses so that small Yukawa couplings are protected by this symmetry. The effective $U(1)_X$ symmetric theory below some scale M is supposed to be natural to the extent that all parameters are of $O(1)$. The scale M is the limit of validity of the effective theory, of $O(M_{Planck})$ if one adopts a superstring point of view. The X-charges of quarks, leptons and Higgses are free parameters to be fixed *a posteriori* and simply denoted q_i,u_i, d_i,l_i,e_i,h_1,h_2, for the different flavours, where $i = 1,2,3$ is the family index. (*ii*) One (or more) Froggatt-Nielsen field Φ, a Standard Modelgauge singletis introduced, and we normalize the $U(1)_X$ so that its charge is $X_\Phi = -1$. The effective (non-renormalizable) $U(1)_X$ allowed couplings are then of the form $g_{ij}^U(\Phi/M)^{q_i+u_j+h_2}Q^iU^jH_2$, with analogous expressions for the H_1 couplings to down quarks and leptons. The coefficients g_{ij}^U, etc, are taken to be natural, *i.e.*, of $O(1)$, unless they are required to vanish by the $U(1)_X$ symmetry. (*iii*) The small parameter λ is identified with the ratio $(\frac{<\Phi>}{M})$ as the $U(1)_X$ symmetry is broken by the Φ v.e.v.. Below the scale $<\Phi>= \lambda M$, one recovers the Standard Model with the effective Yukawa coupling matrices given by $Y_{ij}^U = \lambda^{|q_i+u_j+h_2|}$, $Y_{ij}^D = \lambda^{|q_i+d_j+h_1|}$, $Y_{ij}^E = \lambda^{|l_i+e_j+h_1|}$. The Yukawa matrix entries corresponding to negative total charge should vanish but these zeroes are filled by the diagonalization of the λ -dependent metrics.

The X-charges are now chosen to fit the hierarchy in the mass eigenvalues and mixing angles. The experimental masses (at $O(M_{Planck})$) of the third families give: $h_2+q_3+u_3 = 0$ and $x = h_1+q_3+d_3 = h_1+l_3+e_3$, where the parameter x depends on the assumed value for $\tan\beta$. With this restriction the Yukawa couplings depend only on the charge differences $q_i - q_3$, $u_i - u_3$,..., $e_i - e_3$ and x.

Recently, there has been an intensive investigation of this model[7,8], including a classification of the possible charge assignments[8]. But the question I would like to discuss here was first investigated by Leurer, Nir and Seiberg[9] in the Froggatt-Nielsen framework. Just like the Yukawa couplings, the soft supersymmetry breaking terms contain powers of the Φ -field to implement the $U(1)_X$ symmetry. The scalar mass matrices have a corresponding hierarchy among their elements, so that $(\widetilde{m}_A^2)_{i\bar{j}} = f_{Aij}\lambda^{|q_i-q_j|}$, A=Q,U,D,L,E, where, in the absence of any other symmetry principle, the coefficients f_{Aij} are all of the order of the supersymmetry breaking parameter $m_{3/2}^2$, where $m_{3/2}$ is the gravitino mass. Even in the flavour basis that diagonalizes quark mass matrices, the squark mass matrices will still be of the same non-diagonal form. Therefore large FCNC effects might be induced from loop diagrams with the exchange of neutral sfermions (gluino, photino,...) in possible disagreement with experiments. Indeed, with only one Φ -field , the acceptable $U(1)_X$ charge asignements yield $(\widetilde{m}^2_D)_{12}(\widetilde{m}^2_Q)_{12} \propto \frac{m_d}{m_s}$, which imply much too large FCNC effects in K-physics. One solution[9] is to double the Froggatt-Nielsen, with another abelian symmetry and a smaller scale. In this case it is possible to strongly suppress $(\widetilde{m}^2_D)_{12}$. Interestingly enough, the model predicts large $(\widetilde{m}^2_U)_{12}$ leading to sizeable $D\bar{D}$ mixing that could be experimentally tested.

Another solution[8] is to assume only one more singlet Φ' and an appropriate charge asignment so that $(\widetilde{m}^2_D)_{12}(\widetilde{m}^2_Q)_{12} \propto \frac{m_d^2}{m_s^2}$, which is just enough. Remarkably, in this model all anomalies related to $U(1)_X$ can be cancelled, while in the other models one has to rely upon the Green-Schwarz mechanism[7,8].

6 Horizontal symmetries in supergravity

On one hand, horizontal symmetries are a natural way to solve the family puzzle and the fermion mass hierarchy, and give some restrictions on squark masses as well. On the other hand, in string inspired supergravity, the sfermion masses depend on the modular properties of the matter fields and their modular dependence might well be related to the origin of flavour. What if one imposes both symmetries on a broken supergravity model? This has been recently investigated[10]. For definiteness, let us define the modular properties by some set of modular weights $n_i^{(\alpha)}$ associated to each of the matter fields, and their transformation under an abelian $U(1)_X$ symmetry implementing the Froggatt-Nielsen mechanism, by their charges X_i. Analogously, $n_\Phi^{(\alpha)}$ and X_Φ are introduced for the singlet field Φ. Now, let us require the supergravity theory to be invariant under these $SL(2,Z)$ and $U(1)_X$ transformations. Then, one shows the very interesting relation: $(q_i - q_j)n_\Phi^{(\alpha)} = X_\Phi(n_{q_i}^{(\alpha)} - n_{q_j}^{(\alpha)})$ between charge and modular weight differences. Though the results are easily generalized[10], let us keep only one modulus, say, the overall one, T. Through some mechanism that we do not quite understand yet, the dilaton S and the moduli T get their vev's that fix the gauge couplings and the compactified dimensions in string theory. Then, assume supersymmetry is broken by the auxiliary components of the S and T supermultiplets, F_S and F_T, and define the so-called gravitino angle[2], $\tan\theta = F_S/F_T$. The Φ vev, in this one-singlet case, is fixed by the Fayet-Iliopoulos term to be of $O(\lambda M_{Planck})$, and the supersymmetry breaking is precisely fixed in terms of n_Φ and X_Φ, with a F_Φ and a D_X components. Then the squark and slepton masses can be calculated, with a surprisingly simple expression, resulting of the coalescence of all sources of supersymmetry breaking. For instance, for diagonal entries one gets the relations: $\widetilde{m}^2_{i\bar{i}} - \widetilde{m}^2_{j\bar{j}} = (X_i - X_j)m^2_{3/2}$, where X_Φ is normalized to -1. For non-diagonal entries one has $\widetilde{m}^2_{i\bar{j}} \sim 3(X_i - X_j)m^2_{3/2}n_\Phi \cos^2\theta/\lambda^{|X_i - X_j|}$. Similar results also follow for triscalar couplings.

The consequences for SFCNC are an improvement with respect to those in the previous section. For instance, the contribution to $K\bar{K}$ mixing can be reduced by choosing models[8] with charges $d_1 = d_2$, and the same trick is possible to avoid too much lepton flavour violation.

7 Conclusion

Supersymmetry is the highway connection between flavour physics at low energies and flavour theories at the Planck scale. SFCNC phenomenology provide very selective constraints in this adventure.

8 References

1. E. Dudas, *these Proceedings*; F. Zwirner,*these Proceedings*.
2. A. Brignole, L. E. Ibáñez and C. Muñoz, *Nucl. Phys.* **B422** (1994) 235; D. Choudhury, F. Eberlein, A. König, J. Louis and S. Pokorski Phys. Lett.**B342** (1995) 180; P. Brax and M. Chemtob, *Phys. Rev.* **D51** (1995) 6550; J. Louis and Y. Nir, Nucl. Phys. **B447** (1995) 18. P. Brax and C. A. Savoy, Nucl. Phys. **B447** (1995) 227. A. Brignole, L. E. Ibáñez, C. Muñoz, and C. Scheich, Madrid preprint FTUAM-95-26, hep-ph/9508258.
3. R. Barbieri and L. J. Hall, *Phys. Lett.* **B338** (1994) 212; R. Barbieri, L. J. Hall, and A. Strumia, hep-ph/9501334.
4. S. Dimopoulos, L. Hall and S. Raby, *Phys. Rev. Lett.*, **68**(1992) 1984;*Phys. Rev.*, **D45**(1992) 4192; M. Carena, S. Dimopoulos, C.E. Wagner and S. Raby, CERN preprint TH95-053, hep-ph/9503488.
5. S. Dimopoulos and G. F. Giudice, CERN preprint TH95-90, hep-ph/9504296.
6. C. D. Froggatt and H. B. Nielsen, *Nucl. Phys.* **B147** (1979) 277; J. Bijnens and C. Wetterich, *Nucl. Phys.* **B283** (1987) 237.
7. L. E. Ibáñez and G. G. Ross, *Phys. Lett.* **B332** (1994) 100. P. Binétruy and P. Ramond, *Phys. Lett.* **B350** (1995) 49. V. Jain and R. Shrock, *Phys. Lett.* **B352** (1995). Y. Nir, *Phys. Lett.* **B354** (1995) 107.
8. E. Dudas, S. Pokorski and C. A. Savoy, *Phys. Lett.* **B356** (1995) 45.
9. M. Leurer, Y. Nir and N. Seiberg, *Nucl. Phys.* **B398** (1993) 319 and **B420** (1994) 468; Y. Nir and N. Seiberg, *Phys. Lett.* **B309** (1993) 337.
10. E. Dudas, S. Pokorski and C. A. Savoy, *hep-ph/9509410*