Time-varying non linear modeling of electrodynamic loudspeakers

Romain Ravaud, Guy Lemarquand, Tangi Roussel

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R. Ravaud [a], G. Lemarquand [a,*] and T. Roussel

[a] *Laboratoire d'Acoustique de l'Universite du Maine, UMR CNRS 6613, Avenue Olivier Messiaen, 72085 Le Mans Cedex 9, France*

Abstract

This paper deals with the time-varying nonlinear analytical modeling of the electrodynamic loudspeaker. We propose a model which takes into account the variations of Small signal parameters. The six Small signal parameters (R_e, L_e, Bl, R_{ms}, M_{ms}, C_{ms}) depend on both time and input current. The electrodynamic loudspeaker is characterized by the electrical impedance which, precisely measured, allows us to construct polynomial functions for each Small signal parameter. By using this analytical model, we propose to compare two identical electrodynamic loudspeakers. One of them is supposed to be run in and the other one is not. The experimental methodology is based on a precise measurement. In all the paper, the time scale is assumed to be much longer than one period of the harmonic excitation.

Key words: Loudspeaker, Electrodynamic, Electrical Impedance

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* Corresponding author.

Email address: guy.lemarquand@univ-lemans.fr (G. Lemarquand).

1 Introduction

The reference model describing the electrodynamic loudspeaker designed by Thiele and Small [1] predicts that the electrodynamic loudspeaker is both a linear system and a stationary one. This analytical model is very useful since it is simple to use. However, an electrodynamic loudspeaker exhibits nonlinearities which depend on time. Some authors, such as A.J.M Kaiser [2] and W.Klippel [3] [4], have studied the nonlinearities of electrodynamic loudspeakers. These nonlinearities have become better and better known [5] [6] and some authors have proposed a new structure of loudspeaker with an ironless motor and without any outer rims and spider [7] in order to eliminate these nonlinearities.

The other drawback of an electrodynamic loudspeaker is that it is an time-varying system [8]. Indeed, the electrical resistance R_e increases in time due to the heat produced by the voice coil. Then, the compliance C_{ms} depends on time since the outer rim and the spider become more elastic because of the heat produced by the resistance. The Small signal model using lumped parameters does not forecast these time-varying phenomena, and such an time-varying analytic model taking into account these properties does not exist. In this paper, we put forward a way of characterizing experimentally the time dependence and the level dependence of the Small signal parameters. This experimental characterization allows us to compare two identical electrodynamic loudspeakers. One of them is supposed to be run in and the other one is not. The knowledge of the time necessary to break-in an electrodynamic loudspeaker is very important because this element of information gives indications about the physical properties of both the mechanical stiffness

k and the mechanical damping parameter R_{ms}. The first section presents the Small signal model using lumped parameters and the main nonlinearities of an electrodynamic loudspeaker. The second section presents the experimental methodology to identify the variations of the Small signal parameters. In the third section, the time dependence of the Small signal parameters and its consequences are discussed. The last section presents an analytical model which takes into account the variations of the Small signal parameters in time and according to the input current.

2 The Small signal model using lumped parameters and its limits

2.1 The Small signal model using lumped parameters

According to the Small signal model using lumped parameters, two coupled differential equations are necessary to describe the electrodynamic loudspeaker. One of them is called the electrical differential equation and is given by:

$$u(t) = R_e i(t) + L_e \frac{di(t)}{dt} + Bl \frac{dx(t)}{dt} \tag{1}$$

The other one is called the mechanical differential equation and is given by:

$$M_{ms} \frac{d^2x(t)}{dt^2} = Bli(t) - R_{ms} \frac{dx(t)}{dt} - \frac{1}{C_{ms}} x(t) \tag{2}$$

The parameters used in Eqs.(1) and (2) are the following:
$i(t)$=coil current $[A]$
$u(t)$=input voltage $[V]$

$x(t)$=position of voice coil $[m]$

Bl=electrodynamic driving parameter $[T.m]$

R_{ms}=mechanical damping parameter and drag force $[N.s.m^{-1}]$

C_{ms}=mechanical compliance of suspension(spider, outer rim)$[m.N^{-1}]$

M_{ms}=equivalent mass of moving voice coil, cone, air$[Kg]$

R_e=electrical resistance of voice coil$[\Omega]$

L_e=inductance of voice coil $[H]$

Eqs. (1) and (2) allow us to define the electrodynamic loudspeaker electrical impedance Z_e which is expressed as follows:

$$Z_e = R_e + jL_e w + \frac{Bl^2}{R_{ms} + jM_{ms}w + \frac{1}{jC_{ms}w}} \tag{3}$$

Eq. (3) is well known and is often used to describe the electrodynamic loudspeaker. However, Eq.(3) does not forecast the distortions created by an electrodynamic loudspeaker and the time dependence of the Small signal parameters. Moreover, if we take into account the eddy currents [9] which occur when the input frequency increases, the electrical impedance Z_e should be written as follows:

$$\begin{aligned} Z_e(i,t) = R_e(i,t) + \frac{jR_\mu(i,t)L_e(i,t)w}{jL_e(i,t)w + R_\mu(i,t)} \\ + \frac{Bl(i,t)^2}{R_{ms}(i,t) + jM_{ms}(i,t)w + \frac{1}{jC_{ms}(i,t)w}} \end{aligned} \tag{4}$$

where $R_\mu(i,t)$ is the eddy current resistance. $Z_e(i,t)$ is a time-varying nonlinear transfer function; at each time and for different input currents, its value changes. In Eq. (4), we assume all the parameters depend on both time and input current. Strictly speaking, these dependences exist but it is very difficult to find them experimentally and to predict them analytically. All these parameters have not the same sensitivity both to input current and to time. Moreover, some parameters vary a lot with the input current but do not create important distortions

2.2 Nonlinearities of electrodynamic loudspeakers

The nonlinearities that produce distortion phenomena can be classified into three categories. The first type corresponds to the motor nonlinearities and is described in section (2.2.1). The second type corresponds to the suspension nonlinearities and is described in section (2.2.4). The third type corresponds to the acoustical nonlinearities [10] and is not described here since these nonlinearities are not directly produced by the electrodynamic loudspeaker.

2.2.1 The motor structure

The force factor Bl is not uniform in the air gap. First, the force factor depends on the voice coil position. Indeed, the magnetic field induction B is the superposition of two fields. One of them is created by the permanent magnet and is time independent. This field crosses through the yoke pieces but only thirty per cents serves to move the coil. The other one is created by the coil and is time dependent. Klippel [3] proposed to model the force factor by using a polynomial writing.

$$Bl(x) = Bl_0 + Bl_1 x + Bl_2 x^2 \tag{5}$$

2.2.2 The voice coil inductance

The coil self inductance depends on the moving part position. This dependence generates a reluctant force. This reluctant force is given by:

$$F_{rel}(t) = \frac{1}{2}i(t)^2\frac{dL_e(x)}{dx} \tag{6}$$

We see that when L_e does not depend on the voice coil position x, the reluctant force $F_{rel}(t)$ equals zero, it is one of the assumptions of the Small signal model using lumped parameters.

2.2.3 Eddy currents

The electrical conductivity of the iron is high enough to let the eddy currents appear in the iron yoke pieces of the motor. Vanderkooy [9] proposed a model which takes this phenomenon into account, the electrical impedance varies like $L_e\sqrt{w}$. The interaction between the eddy currents and the current in the coil generates a drag force F_{drag} which can be written as follows:

$$F_{drag} = \eta(i,x)\frac{dx(t)}{dt}^{1,7} \tag{7}$$

where $\eta(i,x)$ can be defined as the sensitivity of the drag force according to the eddy currents ; this one depends on input current and the position of the voice coil.

2.2.4 The suspension

A classical suspension is mostly made of rubber, impregnated fabric or molded plastic. The Small signal model using lumped parameters describes a suspension as an ideal spring but an actual suspension shows non linear behaviour. In consequence, its compliance C_{ms} depends on the movement amplitude and the induced damping parameter R_{ms} depends greatly on both the amplitude and frequency. More generally, many authors use the mechanical stiffness k which is defined by:

$$k = \frac{1}{C_{ms}} \tag{8}$$

Like the force factor Bl, k can be written in terms of a polynomial function.

$$k(x) = k_0 + k_1 x + k_2 x^2 \tag{9}$$

Such a model has been used by Klippel [3] to model the non linear behaviour of both the outer rim and the spider. However, such a model cannot take into account the effect of the hysteretic response of elastomers.

2.3 Time varying properties of the electrodynamic loudspeakers

2.3.1 The electrical resistance R_e

Many authors studied the non stationnarities of electrodynamic loudspeakers as M.Gander [11],[8] and showed that the Small signal parameters depend on time. The parameter which seems to be the most sensitive to time is the electrical resistance R_e. The electrical resistance R_e increases in time due to the heat produced by the coil:

$$R_e(t) = \rho \frac{l}{S}(1 + \alpha \Delta T + ...) \tag{10}$$

where $\alpha = 4.10^{-2}K^{-1}$ for the copper, l is the electric wire length, S is the electric wire cross section area and ΔT is the temperature elevation due to the heat produced by the coil. The electrical resistance variation can modify both the outer rim and the spider properties. The heat produced by the electrical resistance due to the heat produced by the coil passes through to both the outer rim and the spider. Consequently, their temperature increases. The increase in the temperature of the spider and the outer rim generates a modification of their mechanical behaviour.

2.3.2 Time dependence of the mechanical stiffness k

Although analytical models taking into account the time dependence of the mechanical stiffness k do not exist, the properties of the outer rim change in time on account of the heat produced by the electrical resistance due to the Joule effect. Experimentally, this dependence is visible on the electrical impedance and this phenomenon is discussed in this paper. The outer rim and the spider exhibit both viscous and elastic characteristics. The type of viscoelasticity which occurs in the case of an actual electrodynamic loudspeaker is non linear. In consequence, a volterra equation cannot be used to connect stress and strain and a simple model to describe such a behaviour does not exist. Indeed, the outer rim deformations are large and the outer rim properties change under deformations.

3 Improvement of the Small signal model using lumped parameters: experimental methodology

3.1 Introduction

This section presents a way of deriving the time dependence and the input current dependence of the Small signal parameters. For this purpose, an experimental way based on a measurement algorithm is described. The electrodynamic loudspeaker is characterized by the electrical impedance which, precisely measured, allows us to construct polynomial functions for each Small signal parameter. The knowledge of the Small signal parameter variations allows us to derive analytically the distortions created by the electrodynamic loudspeaker.

3.2 Principle of the measurement

In order to measure the electrical impedance of a loudspeaker, it must be placed in an anechoic chamber in a normalized plane. By varying the frequency and the input current, we can measure the electrical impedance. So as to increase the measurement precision when impedance variation is important, different measurement algorithms have been developed. Basically, the aim is to acquire more points when impedance variation is important and less information when impedance tends to be constant with frequency.

3.3 Measurement equipment and devices

The electrical impedance is measured by a Wayne Kerr wedge that has an excellent precision ($10^{-4}\Omega$). Different algorithms are used to determine at which frequencies impedance must be measured. Basically, points must be measured when electrical impedance reaches a maximum or when impedance variation with frequency is important. To do so, a dichotomic search of the maximum impedance is used first to measure accurately the impedance near the resonance frequency. The second algorithm is called in order to detect important variation of impedance while the first algorithm is called to refine measurement near impedance maxima.

3.4 Determination of the Small signal parameters

The Small signal parameters vary both in time and with the input current. As it is very difficult to find the two dependences for each parameter, the measurement algorithm is first used to derive the time dependence and afterwards to derive the input current dependence. On the one hand, the input current level is fixed and the electrical impedance is measured each time. On the other hand, Thiele and Small variations in time are neglected and the electrical impedance is measured for many input currents. In each case, we work with three degrees of freedom. These three degrees of freedom are the time t, the input current i and the frequency $f = \frac{w}{2\pi}$. The measured value is always the electrical impedance Z_e.

3.5 Nonlinear parameter variations

To determine the nonlinear parameter variations, two impedance layers are used. One of them can be called the experimental impedance layer $Z_e^{(exp)}$ and is determined by using the measurement algorithm described in section (3). The other one can be called the theoretical impedance layer $Z_e^{(theo)}$ and is determined as follows: the Small signal parameters are assumed to vary with either the input current or time. In a first approximation, a polynomial writing is used to represent the dependence on the parameters with either the input current or time. The expansion is truncated after the 2nd term. Therefore, in the case of the input current dependence, we assume the electrical resistance R_e and R_μ to be constant; the Small signal parameters are expressed as follows:

$$Bl(i) = Bl(1 + \mu_{Bl} i + \mu_{Bl}^2 i^2) \tag{11}$$

$$R_{ms}(i) = Rms(1 + \mu_{Rms} i + \mu_{Rms}^2 i^2) \tag{12}$$

$$k(i) = k(1 + \mu_k i + \mu_k^2 i^2) \tag{13}$$

$$M_{ms}(i) = M_{ms}(1 + \mu_{M_{ms}} i + \mu_{M_{ms}}^2 i^2) \tag{14}$$

$$L_e(i) = L_e(1 + \mu_{L_e} i + \mu_{L_e}^2 i^2) \tag{15}$$

and the electrical impedance becomes:

$$Z_{e1}^{(theo)}(i) = R_e + \frac{jR_\mu L_e(i)w}{jL_e(i)w + R_\mu} + \frac{Bl(i)^2}{Rms(i) + jM_{ms}(i)w + \frac{k(i)}{jw}} \tag{16}$$

Again, in the case of the time dependence, we assume that R_μ is constant. The Small signal parameters are expressed as follows:

$$R_e(t) = R_e(1 + \nu_{R_e} t + \nu_{R_e}^2 t^2) \tag{17}$$

$$Bl(t) = Bl(1 + \nu_{Bl} t + \nu_{Bl}^2 t^2) \tag{18}$$

$$R_{ms}(t) = Rms(1 + \nu_{Rms} t + \nu_{Rms}^2 t^2) \tag{19}$$

$$k(t) = k(1 + \nu_k t + \nu_k^2 t^2) \tag{20}$$

$$M_{ms}(t) = M_{ms}(1 + \nu_{M_{ms}} t + \nu_{M_{ms}}^2 t^2) \tag{21}$$

$$L_e(t) = L_e(1 + \nu_{L_e} t + \nu_{L_e}^2 t^2) \tag{22}$$

and the electrical impedance becomes:

$$Z_{e2}^{(theo)}(t) = R_e(t) + \frac{jR_\mu L_e(t)w}{jL_e(t)w + R_\mu} + \frac{Bl(t)}{Rms(t) + jM_{ms}(t)w + \frac{k(t)}{jw}} \tag{23}$$

A least square method is used to identify all the parameters in the both cases ; this method is based on the Symplex algorithm. The principle of this algorithm is to minimize the difference ΔZ_e between the experimental impedance and the theoretical impedance. In the case of the time dependence of the Small signal parameters, this difference is expressed as follows:

$$\Delta Z_e^1(t) = \sum_{n=0}^{n=2} \left|\left| Z_e^{(exp)}(t) - Z_{e1}^{(theo)}(t) \right|\right|^2 \tag{24}$$

In the case of the input current dependence of the Small signal parameters, this difference is expressed as follows:

$$\Delta Z_e^2(i) = \sum_{n=0}^{n=2} \left|\left| Z_e^{(exp)}(i) - Z_{e2}^{(theo)}(i) \right|\right|^2 \tag{25}$$

When the algorithm converges, all the values describing the nonlinear parameters are obtained and allow us to predict analytically the distortions created by the electrodynamic loudspeaker by solving the time-varying nonlinear differential equation.

4 Time dependence of the Small signal parameters

This section describes a temporal study of two electrodynamic loudspeakers. The electrodynamic loudspeakers used are two woofers (Eminence Alpha). One of them is run in and the other one is not. First, the measurement algorithm presented in the previous section is used in order to derive all the non-linear parameters. Then, time-varying effects experimentally observed are discussed and physically interpreted.

4.1 Obtaining the experimental impedance

The first step to derive the time dependence of the Small signal parameters is to use the experimental impedance layer. As explained previously, the current input current is fixed. A current which equals $i = 100mA$ is injected in the electrodynamic loudspeaker. The electrodynamic loudspeaker used is supposed to be run in. The lower measurement frequency equals $50Hz$ and the upper measurement frequency equals $250Hz$. The experimental impedance is measured for eight hours. Such an experimental impedance layer is represented in Fig.(1). It can be noted that the time-varying effects are not visible in this impedance layer but they are clearly shown in Figs.(2),(6) and (7).

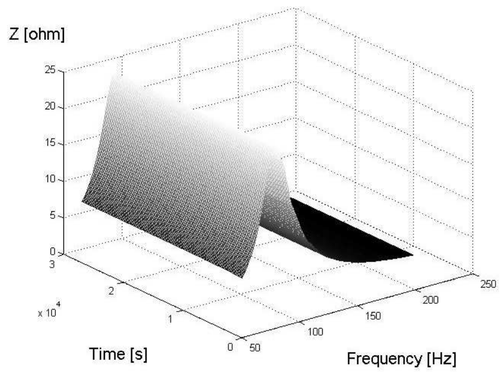

Fig. 1. Experimental three-dimensional representation of the electrical impedance modulus of the electrodynamic loudspeaker (x: time 0s to 3.10^4s) (y: 0Hz to 200Hz) (z: 0Ω to 25Ω)

4.2 Obtaining the parameters sensitive to time

4.2.1 Error sheet between the experimental impedance and the theoretical impedance

In the previous section, the experimental impedance layer is determined with the measurement algorithm presented in section (3). In this section, the experimental impedance is compared to the theoretical one calculated with the Small signal model using lumped parameters. For this purpose, the difference $\Delta Z_e^1(t)$ between the experimental impedance modulus and the theoretical impedance modulus is calculated for each frequency and at each time. This difference $\Delta Z_e^1(t)$ is represented in Fig.(2). The mean difference $\overline{\Delta Z_e}$ is defined as the difference $\Delta Z_e^1(t)$ divided by the number of points necessary to plot the experimental impedance layer. By using the Small signal model using lumped parameters with constant parameters, the mean difference $\overline{\Delta Z_e}$ equals $0,20\Omega$.

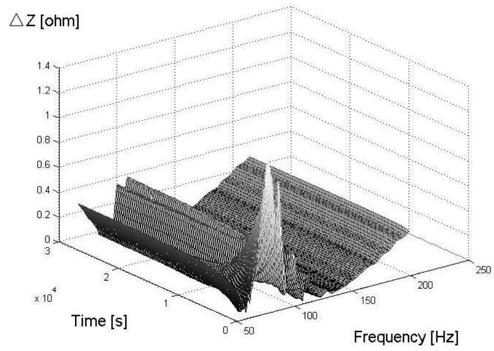

Fig. 2. Three-dimensional representation of the difference $\Delta Z_e^1(t)$ between the experimental impedance and the theoretical impedance ; the theoretical impedance is based on the Small signal model using lumped parameters with constant parameters (x: time 0s to 3.10^4s) (y: 0Hz to 200Hz) (z: 0Ω to 25Ω)

4.2.2 Parameter sensitive to time

To reduce $\overline{\Delta Z_e}$, we use the Symplex algorithm and the parameter which is the most sensitive to time is the equivalent mechanical stiffness k. As a remark, although the electrical resistance of the voice coil R_e increases in time, its time variation is less important than the mechanical stiffness one. Moreover, the variations of the other Small signal parameters are not so important as the mechanical stiffness variation. In Fig.(3), we represent the difference $\Delta Z_e^1(t)$ between the experimental impedance modulus and the theoretical impedance modulus which takes into account the time variation of the mechanical stiffness k. This difference is a function of both time and frequency. The impedance layer is zoomed for more legibility. The temporal axe varies from $0s$ to $200s$. The mean difference $\overline{\Delta Z_e}$ equals $0,19\Omega$. The figure (4) shows the relative mechanical stiffness as a function of time ($k_0 = 3714N/m$). The mechanical stiffness k decreases in time since heat produced by the electrical resistance passes through to the outer rim and modifies its properties. The increasing temperature is one factor contributing to the deformation of the outer rim,

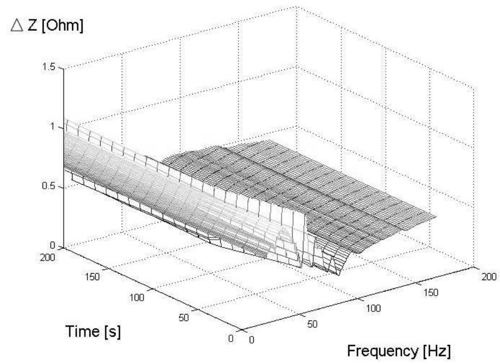

Fig. 3. Three-dimensional representation of the difference between the experimental impedance and the theoretical impedance ; the theoretical impedance is based on Small signal model using lumped parameters with variable mechanical stiffness (x: time 0s to 3.10^4s) (y: 0Hz to 200Hz) (z: 0Ω to 25Ω)

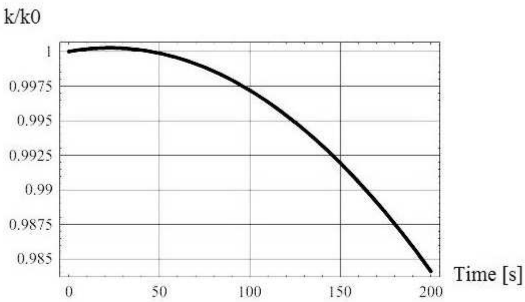

Fig. 4. The relative mechanical stiffness is a function of time [s]

and viscoelastic properties change with decreasing or increasing temperature.

4.3 Resonance frequency variation

Another interesting temporal effect is the resonance frequency variation. It is quite difficult to obtain the resonance frequency experimental measurement in time because its variation is very fast and the time necessary to get the measurement points by the algorithm is only about half a second. Fig.(5) shows the resonance frequency f_{res} as a function of time. We see in this figure

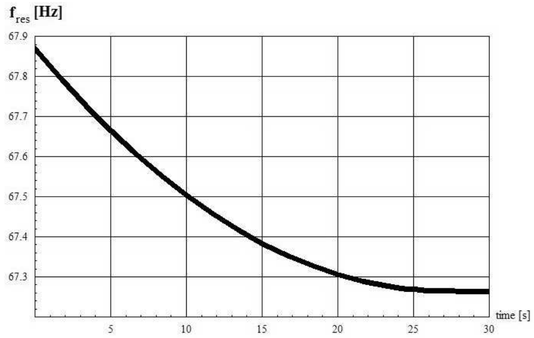

Fig. 5. The resonance frequency [Hz] is a function of time [s]

that the resonance frequency decreases in time. This effect can be explained since the mechanical stiffness of suspension (spider, outer rim) depends on time. In consequence, the resonance frequency is not constant and depends also on time. In short, the decrease in mechanical stiffness generates the decrease in the resonance frequency.

$$f_{res}(t) = \frac{1}{2\pi}\sqrt{\frac{k - k_3 t - k_4 t^2}{Mms}} \tag{26}$$

4.4 Comparison between two loudspeakers: one of them is not run in and the other one is

This section presents an experimental comparison between two electrodynamic loudspeakers. One of them is supposed to be run in and the other one is not. The electrodynamic loudspeaker which is run in has been used for one year. In consequence, its mechanical properties have changed, particularly for the outer rim and the spider which have become both more elastic and worn. For five hours, we measured continually the electrical impedance of the two electrodynamic loudspeakers. The experimental electrical impedance modulus $Z_e(t)$ of the electrodynamic loudspeaker which is not run in is represented in Fig.(6). As said previously, $Z_e(t)$ is plotted at different instants and is a func-

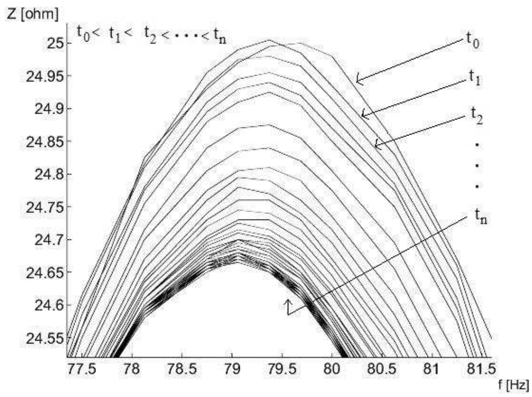

Fig. 6. Electrical impedance modulus of the woofer which is not run in. The electrical impedance modulus [Ω] is a function of frequency [Hz] and is plotted at different instants around the resonance frequency.

tion of frequency. In this figure, we see that the electrical impedance decreases in time and it is mainly due to the change of the mechanical properties. Another interesting point is that the resonance frequency varies quickly in time between t_0 and t_1 which corresponds to 8 seconds. This variation is probably due to the dry friction behaviour of the outer rim.

Fig. (7) represents the electrical impedance modulus of the electrodynamic loudspeaker which is supposed to be run in. As in the previous case, $Z_e(t)$ is plotted at different instants and is a function of frequency. This figure shows that the decrease in electrical impedance modulus is less important for the woofer which is run in than the one which is not. This diminution is about $0,4\Omega$ for the woofer which is not run in, whereas this diminution is $0,05\Omega$ for the woofer which is run in. Moreover, the resonance frequency variation is less important for the woofer which is run in than the one which is not. This resonance frequency variation is about $1Hz$ for the woofer which is not, whereas this variation is $0,4Hz$ for the woofer which is run in. Furthermore, the resonance frequency is very different between the two loudspeakers although they

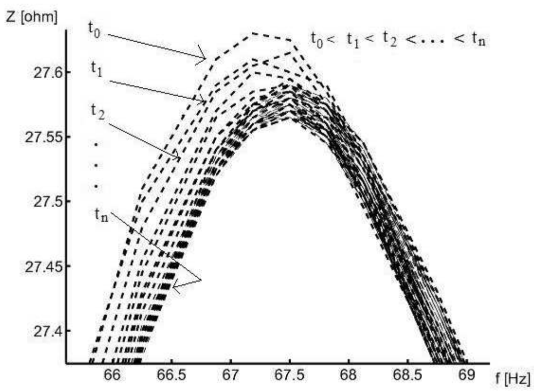

Fig. 7. Electrical impedance modulus of the woofer which is run in. The electrical impedance modulus [Ω] is a function of frequency [Hz] and is plotted at different instants around the resonance frequency.

are both the same. The resonance frequency of the woofer which is run in is about $67Hz$ whereas the resonance frequency of the woofer which is not run in is about $79Hz$. This resonance frequency discrepancy is probably due to the fabrication scattering and the change in time of the membrane mechanical properties.

4.5 Electrical impedance variation in time

The previous section shows that the electrical impedance varies in time. The aim of this section is to show that the electrical impedance does not vary in the same way according to the frequency measurement. For this purpose, we plot the electrical impedance for the two loudspeakers at two different fixed frequencies. One of them is at the resonance frequency and the other one is at 200 Hz. In Fig. (8), the electrical impedance modulus of the woofer which is run in is a function of time. The fixed frequency equals $200Hz$ and the input current equals $100mA$. This figure shows that the electrical impedance modulus increases in time. In Figure (9), we still plot the electrical impedance modulus

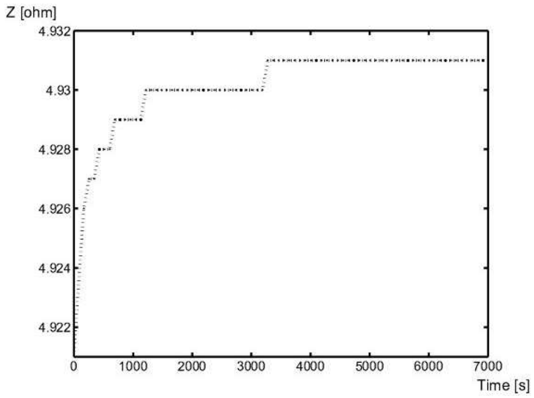

Fig. 8. Electrical impedance modulus of the woofer which is run in. The frequency equals $200Hz$ and the input current equals $100mA$. The electrical impedance modulus is a function of time.

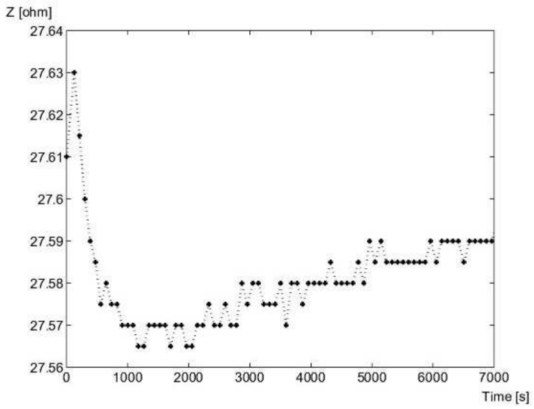

Fig. 9. Electrical impedance modulus of the boomer which is run in. The frequency equals the resonance frequency and the input current equals $100mA$. The electrical impedance modulus is a function of time.

of the woofer which is run in, but the fixed frequency equals the resonance frequency. The temporal behaviour of the electrical impedance is very different according to the frequency measurement. Actually, the electrical impedance of the woofer which is run in decreases a lot at the beginning and increases only after three hour measurement. Moreover, the electrical impedance modulus varies more in time at the resonance frequency than another frequency (here: $200Hz$).

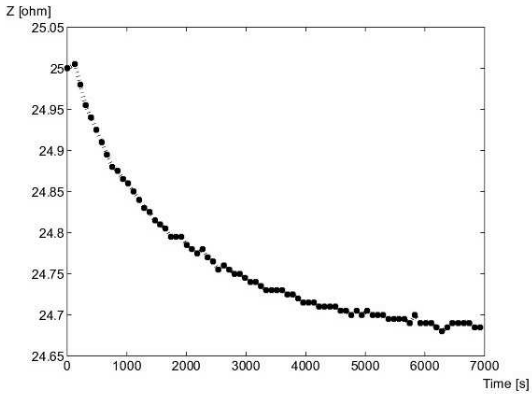

Fig. 10. Electrical impedance modulus of the woofer which is not run in. The frequency equals the resonance frequency and the input current equals $100mA$. The electrical impedance modulus is a function of time

The same experimental measurements are done with the electrodynamic loudspeaker which is not run in. Again, an experimental measurement is realized with a fixed frequency which equals the resonance frequency. Such an experimental measurement is represented in Fig.10 This figure shows that the electrical impedance decreases in time. The behavior of the electrical impedance is very different according to the electrodynamic loudspeaker used at the resonance frequency. The figure (10) shows the electrical impedance modulus of the woofer which is not run in as a function of time. The fixed frequency equals $200Hz$ and the input current equals $100mA$.

As seen previously in the case of the run in electrodynamic loudspeaker, the electrical impedance modulus increases in time. In Fig.(11), the electrical impedance modulus decreases in time. Moreover, we see that the electrical impedance modulus varies more at the resonance frequency than another frequency (here: $200Hz$).

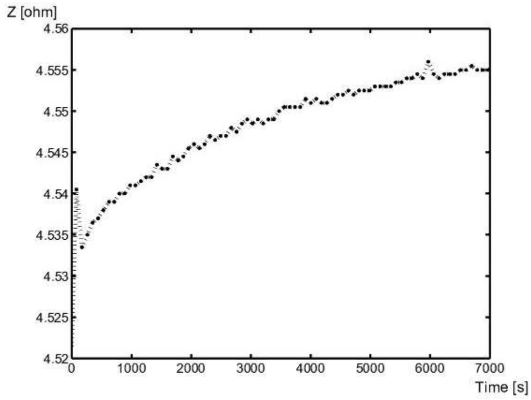

Fig. 11. Electrical impedance modulus of the woofer which is not run in. The frequency equals $200Hz$ and the input current equals $100mA$. The electrical impedance modulus is a function of time.

4.6 Running in an electrodynamic loudspeaker

The aim of this section is to show the time necessary to consider that an electrodynamic loudspeaker is run in. For this purpose, we use the electrical impedance modulus of the electrodynamic loudspeaker. We take a frequency which equals the resonance frequency, an input current which equals $100mA$ and we plot the electrical impedance modulus at each instant. Such an electrical impedance modulus is plotted in Fig.(12). This figure shows that the electrical impedance modulus does not vary after $10^4 s$, which corresponds to about three hours. It can be concluded that this is the time necessary for breaking in this electrodynamic loudspeaker.

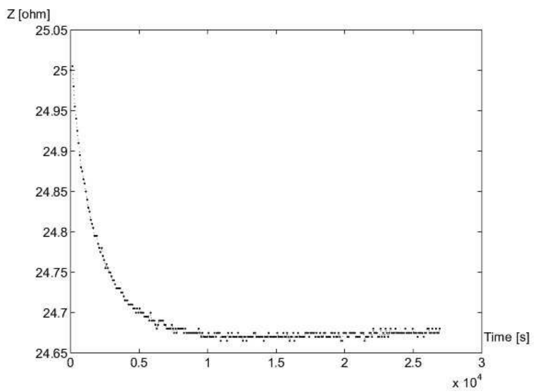

Fig. 12. Electrical impedance modulus of the woofer which is not run in. The frequency equals the resonance frequency and the input current equals $100mA$. The electrical impedance modulus is a function of time.

5 Analytical study of the distortions created by an electrodynamic loudspeaker

5.1 Obtaining the experimental impedance

The way of obtaining the experimental impedance is similar to the one described previously. In order to derive the input current dependence of Small signal parameters, the first step is to use the experimental impedance layer. The time dependence of Small signal parameters is neglected and the input current varies from $20mA$ to $200mA$. The Wayne Kerr wedge cannot deliver currents higher than $200mA$. The electrodynamic loudspeaker used is supposed to be run in. The lower measurement frequency equals $50Hz$ and the upper measurement frequency equals $650Hz$. The experimental impedance layer is represented in Fig.(13).

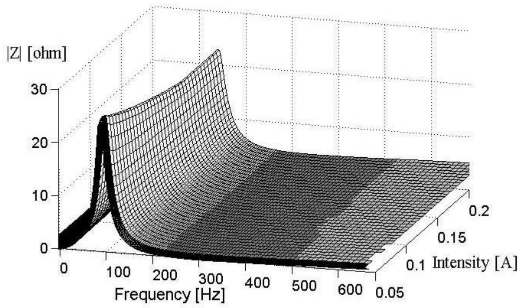

Fig. 13. Experimental three-dimensional representation of the electrical impedance modulus of the electrodynamic loudspeaker (x:$0.05A$ to $0,2A$) (y: $0Hz$ to $650Hz$) (z: 0Ω to 25Ω)

5.2 *Obtaining the parameters sensitive to the input current*

5.2.1 *Error sheet between the experimental impedance and the theoretical impedance*

In the previous section, the experimental impedance layer is determined with the measurement algorithm presented in section (3). In this section, the experimental impedance is compared to the theoretical one calculated with the Small signal model using lumped parameters. For this purpose, the difference $\Delta Z_e^2(i)$ between the experimental impedance modulus and the theoretical impedance modulus is calculated for each frequency and at each intensity. The intensity step is $10mA$. This difference $\Delta Z_e^2(i)$ is represented in Fig.(14). We define the mean difference $\overline{\Delta Z_e}$ as the difference $\Delta Z_e^2(i)$ divided by the number of points necessary to plot the experimental impedance layer. By using the Small signal model using lumped parameters with constant parameters, the mean difference $\overline{\Delta Z_e}$ equals $2,04\Omega$.

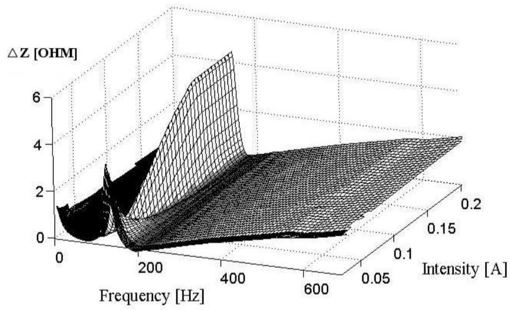

Fig. 14. Three-dimensional representation of the difference $\Delta Z_e^2(i)$ between the experimental impedance and the theoretical impedance ; the theoretical impedance is based on the Small signal model using lumped parameters with constant parameters (x:$0,05A$ to $0,2A$) (y: $0Hz$ to $200Hz$) (z: 0Ω to 6Ω)

5.2.2 Parameters sensitive to the input current

To reduce $\overline{\Delta Z_e}$, the Symplex algorithm is used and five nonlinear parameters are taken into account to reduce $\overline{\Delta Z_e}$. In Fig.(15), we represent the difference $\Delta Z_e^2(i)$ between the experimental impedance modulus and the theoretical impedance modulus which takes into account the Small signal parameter variations. This difference is a function of both the input current and frequency. The mean difference $\overline{\Delta Z_e}$ equals $0,39\Omega$.

In table (1), all the parameters and their expansions are described and the sensitivity to the least square is precise. This table shows that the parameter which is the more sensitive to the input current is the equivalent damping parameter R_{ms}.

5.3 Obtaining the time-varying nonlinear differential equation

This section presents the time-varying nonlinear differential equation of the electrodynamic loudspeaker which is run in. For this purpose, we take into

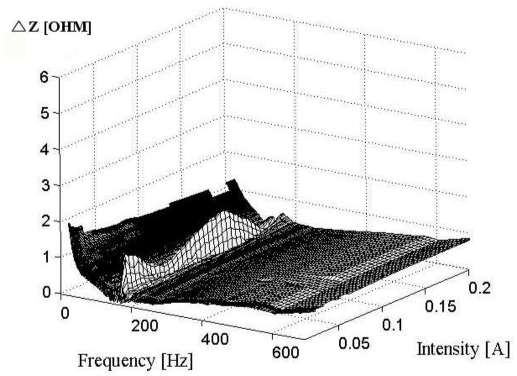

Fig. 15. Three-dimensional representation of the difference between the experimental impedance and the theoretical impedance ; the theoretical impedance is based on the Small signal model using lumped parameters with variable parameters (x: $0A$ to $0,2A$) (y: $0Hz$ to $200Hz$) (z: 0Ω to 25Ω)

Ranking	**Parameter**	**Law of variation**	$\overline{\Delta Z_e}[\Omega]$	**Sensitivity**
1	R_{ms}	$1.1(1+4.09i-8.36i^2)$	1.24	33%
2	Bl	$5.5(1+0.33i-1.02i^2)$	1.67	18%
3	M_{ms}	$0.009(1+0.56i-0.22i^2)$	1.74	14%
4	k	$7440(1-0.2i+0.9i^2)$	1.86	8%
5	L_e	$0.0017(1-1.68i+7.58i^2)$	1.98	3%
6	R_{μ}	$2,28$	2.04	0%
7	R_e	$3,17$	2.04	0%

Table 1

Ranking of the parameters according to their sensitivity to the least square algorithm account the nonlinear parameters defined in the previous section and we also take into account the time variation of the mechanical stiffness k. The time-varying nonlinear differential equation is defined by Eq.(27) in the case when we also take into account the variation of the electrical resistance R_e in time.

$$a(i)\frac{d^3x(t)}{dt^3} + b(i,t)\frac{d^2x(t)}{dt^2} + c(i,t)\frac{dx(t)}{dt} + d(i,t)x(t) = u(t) \tag{27}$$

with

$$a(i) = \frac{(Mms(1+\mu_{Mms}i+\mu_{Mms}^2 i^2))\,(Le(1+\mu_{Le}i+\mu_{Le}^2 i^2))}{Bl(1+\mu_{Bl}i+\mu_{Bl}^2 i^2)} \tag{28}$$

$$\begin{aligned} b(i,t) =& \frac{\Big((Mms(1+\mu_{Mms}i+\mu_{Mms}^2 i^2)R_e(1+\nu_{R_e}t+\nu_{R_e}^2 t^2\Big)}{Bl(1+\mu_{Bl}i+\mu_{Bl}^2 i^2)} \\ &+\frac{R_{ms}(1+\mu_{R_{ms}}i+\mu_{R_{ms}}^2 i^2)L_e(1+\mu_{L_e}i+\mu_{L_e}^2 i^2)}{Bl(1+\mu_{Bl}i+\mu_{Bl}^2 i^2)} \end{aligned} \tag{29}$$

$$\begin{aligned} c(i,t) =& \frac{R_e(1+\nu_{R_e}t+\nu_{R_e}^2 t^2)\,(Rms(1+\mu_{Rms}i+\mu_{Rms}^2 i^2))}{Bl(1+\mu_{Bl}i+\mu_{Bl}^2 i^2)} \\ &+\frac{(Le(1+\mu_{Le}i+\mu_{Le}^2 i^2))\,k(1+\nu_k t+\nu_k^2 t^2)(1+\mu_k i+\mu_k^2 i^2)}{Bl(1+\mu_{Bl}i+\mu_{Bl}^2 i^2)} \\ &+\frac{\big(Bl(1+\mu_{Bl}i+\mu_{Bl}^2 i^2)\big)^2}{Bl(1+\mu_{Bl}i+\mu_{Bl}^2 i^2)} \end{aligned} \tag{30}$$

$$d(i,t) = \frac{R_e(1+\nu_{R_e}t+\nu_{R_e}^2 t^2)\,(k(1+\nu_k t+\nu_k^2 t^2)(1+\mu_k i+\mu_k^2 i^2))}{Bl(1+\mu_{Bl}i+\mu_{Bl}^2 i^2)} \tag{31}$$

5.4 Solving the time-varying nonlinear differential equation

We explain here how to solve the equation defined in the previous section. We can point out that the coefficient $a(i)$ defined in Eq.(28) is the only coefficient which is constant in time. We use the notation Re_t and k_t to indicate that these parameters depend on time. To solve the time-varying nonlinear differential equation, a Taylor series expansion is used.

5.4.1 Discussion about the time-varying differential equation

It is noticeable that the temporal variations of the Small signal parameters do not create any important distortions. Indeed, if we assume all the Small signal parameters to be constant with the input current, the general differential equation of the electrodynamic loudspeaker is written:

$$\tilde{a}\frac{d^3x(t)}{dt^3} + \tilde{b}(t)\frac{d^2x(t)}{dt^2} + \tilde{c}(t)\frac{dx(t)}{dt} + \tilde{d}(t)x(t) = u(t) \tag{32}$$

with

$$\tilde{a} = \frac{M_{ms}L_e}{Bl} \tag{33}$$

$$\tilde{b}(t) = \frac{M_{ms}R_e(1+\nu_{R_e}t+\nu_{R_e}^2t^2)}{Bl} + \frac{R_{ms}L_e}{Bl} \tag{34}$$

$$\tilde{c}(t) = \frac{R_e(1+\nu_{R_e}t+\nu_{R_e}^2t^2)R_{ms} + Bl^2 + k(1+\nu_k t+\nu_k^2t^2)L_e}{Bl} \tag{35}$$

$$\tilde{d}(t) = \frac{k(1+\nu_k t+\nu_k^2t^2)R_e(1+\nu_{R_e}t+\nu_{R_e}^2t^2)}{Bl} \tag{36}$$

The time-varying differential equation defined in Eq.(32) is a hypergeometric equation and can be solved in the general case by using the theory of the Power Series Method [12]. However, if we take $u(t) = Ae^{(jwt)}$ where A is a term of amplitude, the response does not contain terms in $e^{(j2wt)}$,$e^{(j3wt)}$,etc... In consequence, we deduct that the time dependence of the Small signal parameters does not generate any distortions.

5.5 Solving the nonlinear differential equation

The nonlinear differential equation can be solved at each time. By assuming the electrical resistance to be constant in time, the only parameter sensitive to time is the mechanical stiffness. To simplify the resolution of the time-varying nonlinear differential equation, we can write that at each time, the nonlinear differential equation is stationary. The distortions predicted by the nonlinear differential equation depend on time but can be solved at each time. The study of the nonlinear small signal parameters can be done with either the input current or with the position of voice coil. In fact, the relation between the input current i and the position $x(t)$ of the voice coil is linear. Indeed, by using the classical approach, Laplace Law describes the movement of the voice coil at first order.

$$M_{ms}\frac{d^2x(t)}{dt^2} = Bli(t) \tag{37}$$

If we consider that the current is varying sinusoidally in time, above the frequency resonance, the displacement of the voice coil is proportional to the Laplace force and in opposed directions. The displacement of the voice coil can be described by:

$$x = -\frac{Bli}{M_{ms}w^2} \tag{38}$$

where w is the radian frequency of the input current. In consequence, it exists a parameter α which verifies:

$$x = \alpha i \tag{39}$$

where $\alpha = -\frac{Bl}{M_{ms}w^2}$. All the Small signal parameters can be expressed as a Taylor series expansion. By inserting all these expansion series in Eq.(28), we obtain a classical nonlinear differential equation. Its solution is given by Eq.(40). The solution is developed until the order 2 (μ_2).

$$x(t) = x_0(t) + \mu x_1(t) + \mu^2 x_2(t) + ... \tag{40}$$

where $x_0(t)$ is the solution of the nonlinear differential equation of the electrodynamic loudspeaker when the terms with orders higher than zero are neglected, $x_1(t)$ is the solution of the nonlinear differential equation when the terms with orders higher than one and smaller than one are neglected, $x_2(t)$ is the solution of the nonlinear differential equation when the terms with orders higher than two and smaller than two are neglected. In short, the solution of the nonlinear differential equation of the electrodynamic loudspeaker is given by:

$$x(t) = A\cos(wt) + B\sin(wt) + C\cos(2wt) + D\sin(2wt) + ... \tag{41}$$

The terms A and B can be found by inserting $A\cos(wt)+B\sin(wt)$ in Eq.(27) with an excitation $u(t)$ which equals $P\sin(wt)$ where P is an amplitude. The terms C and D can be found by taking the terms with orders higher than one and smaller than one into account,etc...

5.6 *Experimental and theoretical displacement spectrums*

This section presents the experimental and the theoretical displacement spectrums of the electrodynamic loudspeaker which is run in. The theoretical displacement spectrum is obtained by calculating the Fourier transform of the

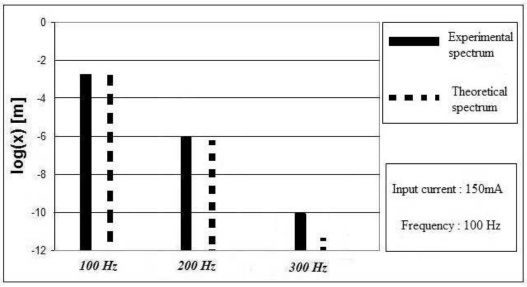

Fig. 16. Experimental and theoretical spectrums of the electrodynamic loudspeaker which is run in. The input current equals $100mA$ and the input frequency equals $100Hz$.

solution given in Eq.(41). The experimental displacement spectrum is obtained by using a laser Doppler velocimeter. The theoretical displacement spectrum is consistent with the experimental displacement spectrum. The theoretical and experimental first-harmonic and second-harmonic shows a very good agreement. However, the theoretical third-harmonic is lower than the experimental one. This discrepancy between the theoretical third-harmonic and the experimental one shows the limit of the use of a series Taylor expansion. It can be noted that the experimental spectrums have been measured at low frequencies. For higher frequencies, the theoretical model should take into account membrane modes.

6 Conclusion

The aim of this paper is the study of the time-varying effects and nonlinear effects of electrodynamic loudspeakers. A temporal study based on a very precise measurement shows the time dependence of the membrane mechanical stiffness k. However, this time dependence does not create any distortions. Moreover, two identical electrodynamic loudspeakers are compared and important time discrepancies are discussed. The resonance frequency between an electrodynamic loudspeaker which is run in and one which is not is extremely different and does not vary in time in the same way. Then, the time-varying nonlinear differential equation of the electrodynamic loudspeaker is solved by using a series Taylor expansion. For this purpose, the time-varying effects are neglected but can be taken into account by solving the nonlinear differential equation at different instants. The theoretical displacement spectrum is consistent with the experimental displacement spectrum.

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