H∞ Loop shaping control for PLL-based mechanical resonance tracking in NEMS resonant mass sensors

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***Abstract*—A simple dynamic detection of the resonance frequency shift in NEMS resonant mass sensors is described. This is done without the use of an external frequency sweep signal nor a frequency counter limiting the dynamic variation detection. Neither an amplitude control nor a phase switcher is required for maintaining the resonant oscillations. The sensor is driven directly by the VCO's output for which the control signal is calculated by a robust H∞ controller using loop-shaping method. Only the sensor and the VCO's signals signs are detected and compared so that the controller regulates the phase difference between them, maintaining it at $\pi/2$ which occurs on resonance frequency. The measurement issue is transformed to a novel control problem that rejects the disturbance described by the resonance frequency shift, attenuates the phase noise and guarantees good stability margins.**

I. INTRODUCTION

The use of vibrating cantilevers shows great promise for sensor applications. In resonance mode mass sensing applications, mass change causes the resonance frequency to shift. By tracking this resonance frequency shift information, mass change in the sensor system can be inferred. Due to the capability of achieving small mass and high resonance frequency, sensors with high sensitivity can be built using NEMS resonators which can be operated as a microbalance with femtogram mass resolution [1].

Changes in viscosity and density of the environment also influence the vibration characteristics of the transducer. When a cantilever is operated in liquid both the resonance frequency and the quality factor Q, shift towards lower values. This reduces the achievable resolution of frequency detection and therefore the mass resolution [2]. Thus, the minimal reachable frequency resolution and therefore the mass resolution are limited by the quality factor of the oscillating system. Traditional resonance frequency measurement uses a function generator in frequency sweep mode used in open-loop with a lock-in amplifier. However, the quality factor can be enhanced by an active feedback circuit with a variable amplifier and phase shifter [3, 4, 5] associated to a frequency counter limiting the dynamic detection. To overcome the disadvantage of an expensive and complicated measurement set-up with an external frequency generator and a lock-in amplifier the cantilever can be used as the frequency driving element in an oscillation circuit [6] and [7]. Some other feedback control methods have associated the oscillatory circuits to PLL techniques where the resonator's signal acts as an external frequency or phase reference signal to be tracked. However, this adds to the complexity of the control. Note that the PLL circuit may serve simultaneously as an accurate frequency detector because the control voltage applied to the VCO is an image of its frequency [8]. Another closed-loop strategy consists in compensating the parametric resonance frequency shift by the dc actuating offset signal applied to the sensor [9].

In this work, a dynamic measurement of the resonance frequency variation is described using a robust controlled PLL loop, transforming the measurement scheme to a disturbance rejection control issue which considers the resonance frequency variation as an added disturbance and gives its measure as a dynamic varied control signal rejecting the phase noise. In the first section, a brief theoretical background is presented. In the second, PLL technique for oscillating systems on resonance is described and the transformation to disturbance rejection control issue is explained to end up with the design of the robust H∞ controller based on loop-shaping method and the exposure of the simulation results.

II. THEORETICAL BACKGROUND

As already mentioned, microcantilevers are one of the best transducers, due to their simplicity, their mechanical characteristics and dynamic behavior. The working principle of resonating cantilevers as mass sensors comes from the dependence of the resonance frequency of the lever with the mass. Considering a mass-spring model for the cantilever movement, its transfer function is represented by:

$$G(s)=\frac{1/m}{s^2+2\xi\omega_0 s+\omega_0^2} \quad (1)$$

where ω_0 is the resonance pulsation, defined by :

$$\omega_0=\sqrt{\frac{k}{m_{eff}}} \quad (2)$$

with m_{eff} is the effective mass and k the spring constant. ξ is the constant damping factor. When the cantilever is driven by an oscillating signal on ω, the output's phase shift is defined by:

$$\varphi_{sen}=-\arctan\left(\frac{2\xi.\omega.\omega_0}{\omega_0^2-\omega^2}\right) \quad (3)$$

At the resonance frequency, the phase shift is equal to $-\pi/2$ and its variation according to the oscillating frequency can be linearized near resonance by the following equation:

$$\frac{d\varphi_{sen}}{d\omega}=k=-\frac{1}{\xi\omega_0} \quad (4)$$

For an additional mass δm, uniformly distributed over the surface, the resonance pulsation of the loaded beam becomes:

$$\omega_{res}=\sqrt{\frac{k}{m_{eff}+\delta m}}=\frac{\omega_0}{\sqrt{1+\delta m/m_{eff}}} \quad (5)$$

Using Taylor series and taking into account only the first order, one gets the expression of the mass responsivity, also called the mass sensitivity:

$$\Re=\frac{\delta\omega}{\delta m}=-\frac{\omega_0}{2.m_{eff}} \quad (6)$$

where $\delta\omega$ is the difference between ω_{res} and ω_0. For this derivation, we are assuming that the deposited mass will only produce a shift on the frequency and will not affect the spring constant. Fig.1 shows the Bode plots of a nanocantilever system with a resonance frequency $f_0=\omega_0/2\pi$ equal to 20 MHz and a damping coefficient $\xi=0.001$ in its initial state and after a resonance frequency shift due to an additional mass $\delta m=0,001.m_0$.

III. Phase-Locked Loop Technique and Measurement Control Formulation

A phase-locked loop (PLL) is a feedback system combining a voltage controlled oscillator (VCO) and a phase comparator connected so that the oscillator frequency (or

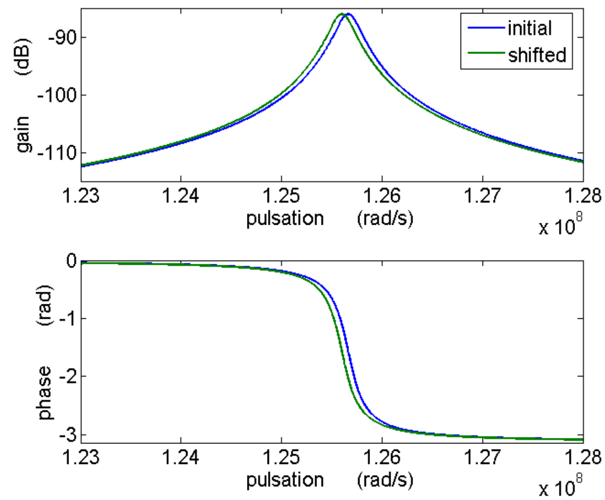

Figure 1. Bode plot of the system before (blue curve) and after (red curve) mass absorption.

phase) accurately tracks that of an applied frequency or phase modulated signal at the reference. The phase error between the reference and the VCO signals is detected by the phase comparator and is inserted into a controller that calculates the required VCO input u. The VCO output signal oscillates at $\omega_c+K_{VCO}.u$, where ω_c is the central pulsation and K_{VCO} is the sensitivity expressed in Hz/V. The phase comparator can be analogical (frequency mixer or analog multiplier associated to a low-pass filter) or digital using logic gates. In the first case, when the two compared signals have the same frequency and are 90° apart, the phase comparator outputs a constant level of zero ($\cos(\varphi_{ref}-\varphi_{VCO})=0$).

In this paper, we use a PLL technique applied to the resonant sensor, in which the VCO signal drives directly the sensor whose output accounts for the reference signal to be tracked by the VCO itself. The phase comparator is a digital one surpassing the need of the high resolution displacement measurement on the sensor's output. It consists of two relays detecting only the signs of each of the compared signals and a multiplier which outputs a constant level "1" when the two signals are in phase and a constant level "-1" when they are 180° out of phase as well as a square wave with a zero mean when they are 90° phase shifted, describing the same behavior of the analog comparator. As the VCO drives the sensor, the two signals have the same instantaneous frequency and thus, when multiplied by $\pi/2$, the phase comparator output represents exactly the difference between the phase shift and its reference value equal to $-\pi/2$ when resonance is sought, which is shown in the following equation:

$$\delta\phi_{comp}=(-\omega_{VCO}t+\omega_{sen}t+\varphi_{sen})+\pi/2=\varphi_{sen}+\pi/2 \quad (7)$$

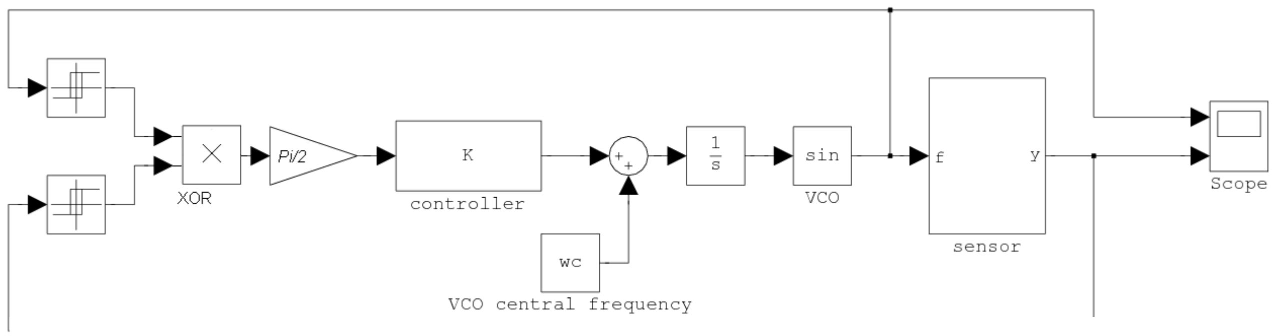

Figure 2. Closed-loop PLL with sensor driven by VCO and digital phase comparator.

Thus, the resonance tracking problem consists in regulating the VCO input control signal such that the output frequency induces $\delta\phi_{comp}$ equal to zero. Fig.2 shows the scheme that relates the different components of the loop. For simplification, K_{VCO} is assumed to be equal to one and the VCO is represented by the generator of a sinusoidal signal whose phase is obtained by the integration of the VCO output frequency $\omega_c + u$. Considering small frequency shifts, one can describe the phase shift evolution by the linear equation:

$$\varphi_{sen} = -\pi / 2 + k.(\omega_{VCO} - \omega_{res}) \qquad (8)$$

Thus, from a control point of view, the model of the system to be controlled is represented by the following:

$$\delta\phi_{comp} = k.(u + \omega_c - \omega_0 - \delta\omega) \qquad (9)$$

The pulsation shift $\delta\omega$ can be considered as a disturbance added on the input u that has to be rejected by control. When rejection is perfectly guaranteed, the constant steady state control signal u measures the exact value of the frequency shift (added to the constant $\omega_0 - \omega_c$). Furthermore, phase noise acts on the output and should be reduced. Also, k varies very slightly with the resonance frequency, so the designed controller must be robust to the parameter uncertainty and robust stability must be achieved on real nonlinear phase model. In this manner, the resonance measurement issue is reformulated by a robust control problem which ensures the required specifications mentioned above. The modified feedback control scheme is represented in fig.3.

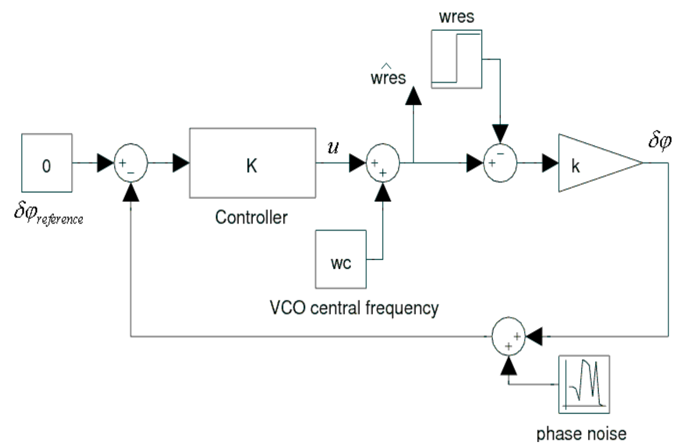

Figure 3. Resonance measurement problem described by phase shift control with disturbance rejection and phase noise reduction.

IV. H∞ LOOP-SHAPING CONTROLLER DESIGN

The controller K is designed using H∞ technique which is know to be commonly used to minimize the closed loop impact of a perturbation and to synthesize controllers achieving robust performance or stabilization by expressing the control problem as a mathematical optimization problem. Simultaneously optimizing robust performance and robust stabilization is difficult. One method that comes close to achieving this is H∞ loop-shaping, which allows the control designer to apply classical loop-shaping concepts to the frequency response to get good robust performance, and then optimizes the response near the system bandwidth to achieve maximum feedback bandwidth which respects the trade-off between output noise reduction and disturbance rejection. Using this approach and following the methodology described in [10], a pre-compensator W_c is designed to obtain a desired open-loop shape (of the extended system formed by $k.W_c$) with high gain in low frequencies to ensure good reference tracking and disturbance rejection and low gain in high frequencies for noise reduction and filtering of the high frequency square wave of the phase comparator in order to obtain its rms value describing the phase difference. In addition, the chosen pulsation of the open-loop response on unity gain corresponds almost to the bandwidth of the closed-loop system.

Once the pre-compensator is designed, the H∞ optimization problem is applied on the extended system and gives the minimum upper bound $\gamma_{\min} = \sqrt{1 + X.Y}$ for the H∞ norm of the stable achievable closed loop system and finds the controller F defined by the following state space model:

$$\begin{bmatrix} \dot{x}_c(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} A - BB^T X + \gamma^2 Z_\gamma Y C^T C & -\gamma^2 Z_\gamma Y C^T \\ B^T X & 0 \end{bmatrix} \begin{bmatrix} x_c(t) \\ y(t) \end{bmatrix} \qquad (10)$$

where γ is slightly higher then $\gamma_{\min}$ and *A*, *B* and *C* are the state-space model representation of the extended system:

$$\begin{cases} \dot{x} = A.x + B.u \\ y = C.x \end{cases} \tag{11}$$

$Z = \left(1 - \gamma^2 + Y.X\right)^{-1}$ where *X* and *Y* are the positive definite solutions of the two following Riccati equations:

$$\begin{aligned} XA + A^T X - XBB^T X + C^T C = 0 \\ YA^T + AY - YC^T CY + BB^T = 0 \end{aligned} \tag{12}$$

The final controller *K* which is implemented is composed of the pre-compensator W_c and the calculated controller *F*.

V. Simulation Results

The simulated nanocantilever has an initial mass of 633 *pg*, an initial resonance frequency of 20 MHz ($\omega_0 = 125{,}6.10^6$ rad/s) and a damping coefficient of 0.001 (quality factor of 500). The slope of the phase shift variation near resonance, *k*, is found to be equal to $7{,}9.10^{-6}$. The pre-compensator is designed as a PI regulator associated to a low-pass filter so that the open-loop shape has high gain at low frequencies and low gain on high frequencies with a cutoff frequency of 314 KHz, thus imposing a closed-loop bandwidth of 314 KHz. By applying the H∞ optimization problem on the extended system, one finds $\gamma_{\min} = 1.763$ and thus the calculated controller $K = F.W_c$ consists of an integrator with a first order filter of bandwidth equal to 314 KHz and a DC gain of 82 dB plus a second order filter with a natural frequency of 984 KHz and a damping coefficient of 0.892 having a zero on 1825 KHz and a DC gain of 92 dB. The open-loop shape of the controlled system is represented in fig.4 and shows that the controller is almost identical to the pre-compensator filter except on high frequencies. In addition, this 4th order controller shows very good stability margins attaining 68° for the phase and 20 dB for the gain. Simulation results applying a mass variation of $0.001\, m_0$ on the sensor are shown in fig.5. The resonance pulsation is measured on the control signal added to the central frequency of the VCO.

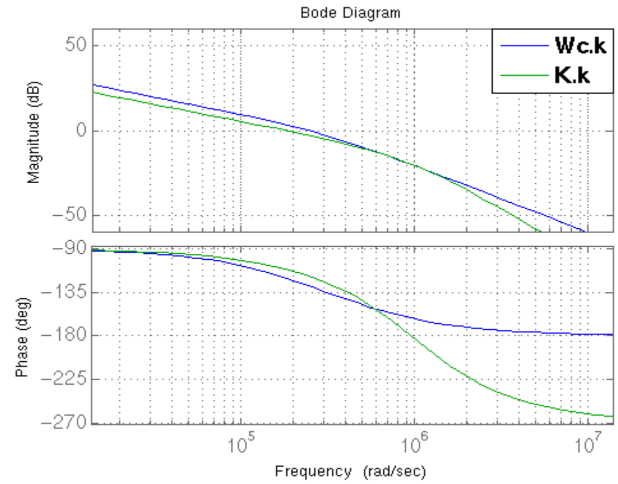

Figure 4. Bode diagram of the compensated system (blue) and the final open-loop transfer function (green).

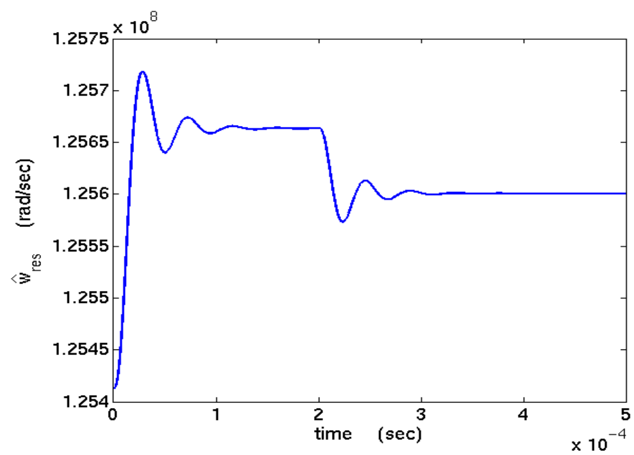

Figure 5. Resonance pulsation measure for mass variation = 0.001 m_0

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