Topology Preserving Simplification of 2D Non-Manifold Meshes with Embedded Structures

Fabien Vivodtzev, Georges-Pierre Bonneau, Paul Le Texier

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Fabien Vivodtzev · Georges-Pierre Bonneau · Paul Le Texier

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Abstract Mesh simplification has received tremendous attention over the past years. Most of the previous works deal with a proper choice of error measures to guide the simplification. Preserving the topological characteristics of the mesh and possibly of data attached to the mesh is a more recent topic, the present paper is about. We introduce a new topology preserving simplification algorithm for triangular meshes, possibly non-manifold, with embedded polylines. In this context embedded means that the edges of the polylines are also edges of the mesh. The paper introduces a robust test to detect if the collapse of an edge in the mesh modifies either the topology of the mesh or the topology of the embedded polylines. This validity test is derived using combinatorial topology results. More precisely we define a so-called *extended complex* from the input mesh and the embedded polylines. We show that if an edge collapse of the mesh preserves the topology of this extended complex, then it also preserves both the topology of the mesh and the embedded polylines. Our validity test can be used for any 2-complex mesh, including non-manifold triangular meshes. It can be combined with any previously introduced error measure. Implementation of this validity test is described. We demonstrate the power and versatility of our method with scientific data sets from neuroscience, geology and CAD/CAM models from mechanical engineering.

Keywords Computational Geometry and its Applications · LOD Techniques · Multi-resolution

F. Vivodtzev · G.-P. Bonneau
Laboratoire GRAVIR (UJF, CNRS, INP Grenoble, INRIA)
INRIA Rhone-Alpes, 655 avenue de l'Europe, 38334 Saint Ismier Cedex, France. E-mail: fabien.vivodtzev@imag.fr, georges-pierre.bonneau@imag.fr

F. Vivodtzev · P. Le Texier
CEA/CESTA (French Atomic Energy Commission)
Route des Gargails, BP 2, Le Barp Cedex, France. E-mail: fabien.vivodtzev@cea.fr,paul.letexier@cea.fr

1 Introduction

We assume the reader is familiar with mesh simplification techniques in general and their importance in Scientific Visualization. Most of the previous works on mesh simplification are devoted to the development of a specific scalar error measure, taking into account the geometry of the mesh and possibly the values of data attached to the mesh. A more recent topic in mesh simplification is the preservation of topological characteristics of the mesh and of data attached to the mesh. In this paper a new topology preserving simplification algorithm is introduced for triangular meshes, possibly non-manifold, in which polylines are embedded. In this context embedded means that the edges of the polylines are also edges of the mesh. The mesh is simplified by repeated edge collapses, following the classical scheme introduced in previous works. For each edge of the mesh, the cost of its collapse is computed and inserted in an ordered heap of edges. The algorithm iteratively pops the edge collapse introducing the lowest error in the simplified mesh, checks for the validity of the collapse (i.e. geometry, topology or attribute consistency) and if this operation is accepted, update both the mesh and the heap. Since the polylines form a subset of the edges of the mesh, edge collapses modify both the mesh and the set of polylines embedded in the mesh. The main contribution of this paper is a robust validity algorithm, that detects whether or not an edge collapse preserves the topology of the mesh *as well as* the topology of the embedded polylines. To this end we define a so-called *extended complex* that implicitly encodes both topologies. Thereafter edge collapses that preserve the topology of this extended complex are detected. We show that these edge collapses preserve also both the topologies of the mesh and the embedded polylines. As we will see in this introduction, the preservation of these topological characteristics is crucial in several applications.

Many application areas may produce meshes with embedded polylines. This work has been motivated by three applications, for which the topology of the mesh and the topology of the embedded polylines must be preserved throughout the simplification.

Neuroscience

One of the great challenges of these years, in neuroscience, is to automate the *brain mapping*. As annotating brains is complicated and time consuming, neuroscientists are working on a unified atlas created from a collection of different brains which will later be use to directly transfer information to the patient. One method to map brains is to segment the cortical surface according to the main features (i.e. the gyri and the sulci) and to match these features. This segmentation produces polylines surrounding these features on the cortical surface. The topology of the polylines is simple: they form closed curves without self-intersections. Due to the high complexity of the surface, pre-processing needs to be done. Simplifiying the surface while preserving the main folds is one possible method. In this context it is critical to preserve the topology of the polylines and the mesh: the mesh should remain a manifold surface, and the polylines should remain closed curves without self-intersections.

Geology

Understanding the soil quality is fundamental for the agriculture or to estimate the health of the environment. Soil surveys are presented as maps encoding the proportion of each soil within a specific area. The boundaries of these domains are really important because they will be used to drive a specific land use (e.g. oil exploitation). In terms of visualization, data reduction is often used because of the difference between the area of interest (e.g. country) and the size of one soil type. While these data are characterized by a fairly simple surface representing a height field, embedded structures such as the boundaries among soil types can be really complex. It is crucial to preserve the topology formed by these boundaries in order to properly recover the different soil types in the simplified model. Also in a volume these layers of soils are creating multiple surfaces intersecting each oters which can be seen as a complex non-manifold surface.

CAD/CAM

CAD/CAM models are based on a geometry produced by a modeler and adapted to the design of a mechanical model. The hand-made geometry can be highly complex and features, such as materials, can be added to the mesh. In addition there is a coherence between the material interfaces and the geometric features that is important for the consistency of the whole CAD/CAM model. 2D Numerical simulations based on FEM (Finite Element Methods) on a CAD/CAM model produce a triangular mesh, possibly non-manifold, in which the edges follow the geometric features of the CAD/CAM model, as well as the interfaces separating the different materials. Preserving the topology of the polylines formed by these characteristic edges is essential to maintain the consistency of the CAD/CAM model throughout the simplification.

This paper is structured as follows. Section 2 reviews related works, and points out differences with the present paper. Section 3 recalls the previously introduced topological tests for edge collapses in 2-complexes, as these results are used to detect valid edges in the extended complex. Section 4 explains how the extended complex is defined, and describes the actual implementation of the validity algorithm. Finally section 5 presents results in the different application areas cited above, and section 6 gives a conclusion with future works.

2 Related Work

Research on surface simplification in Computer Graphics and Scientific Visualization has led to a substantial number of methods within the last twelve years. An exhaustive description of this field is beyond the scope of this paper and one can refer to the many surveys [2,24, 10] available. In the remainder of this section we only review the methods close to our work.

2.1 Surface Simplification

In order to decimate a discrete surface, different operators can be applied on its elements (i.e vertex, edges, faces). Historically started with region merging [17], numerous methods followed such as surface re-tiling [30], vertex decimation [29], vertex clustering [27], subdivision meshes [7] or wavelet decomposition [15]. Amongst these decimation methods the local iterative edge collapse operator has been widely used in order to control precisely the simplification error. These methods highly depend on the metric used to determine the order of the modifications and many heuristics have been proposed depending on the application. Particularly relevant algorithms used progressive meshes [18], tolerance volumes [16], plane deviations [26], quadric-based metrics [11] or the memoryless simplification [22]. While this wide range of methods deal well this the geometry, models used in Computer Graphics or in Solid Modeling may have different non-geometric attributes defined on the surface which need to be preserved through the simplification process.

2.2 Attribute Preserving Simplification

Discontinuities are often observed in the non-geometric attribute field of a surface such as high color varitation due to shadows projected on a surface or texture coordinates defined on vertices. Attribute errors can be estimated on each face to compute an error-bounded simplification [1], treated with dedicated datastructure such as *wedges* [19] or integrated into the computation of a quadric error metric [12]. [8] tracks attribute variations by minimizing an error computed on point clouds defined by these field values. A Resampling of the attributes can also be used as in [3], where a displacement map is computed on high resolution and applied as a texture on the simplifed mesh. [4] dynamically computes a *texture deviation metric* used to decouple the sampling of the attributes and to ensure an error bound of the simplification. In some cases, attributes can describe geometric features of a surface but usually specialized structures and algorithms are required, especially when these features are defined in different dimensions.

2.3 Application Driven Simplification

Feature-guided simplification provides dedicated error metrics to integrate strong constrains on subsets of the original model such as intersection of roads on a terrain data. These metrics highly depend on the application because the definition itself of a feature is based on the field of study. Many methods have been proposed to extract a collection of piecewise linear curves on the surface [25, 28, 21]. Once such features have been extracted, associated weights can be used to integrate them in any kind of error mesure previously presented like [28] which extends the quadric-based error metric of [11]. User-guided simplification also provides specific error metrics to simplify chosen features.

Terrain Simplification is a stand-alone branch of the mesh simplification field as it has been studied for almost 30 years and has led to an incredible number of methods. Among others, [9, 23] provide survey of this wide field. Several papers deal with the preservation of the topology of polylines on these terrains. However, due the nature of terrain data, they are restricted to height field meshes having a very simple topology. All these methods are either extension of the Douglas-Peucker [6] algorithm (which does not deal with the topology) for curve simplification or based on heuristics defined by the local topology around the polyline vertices.

Material boundaries are special properties of CAD-/CAM models. Preservation of the material interface through simplification in CAD/CAM applications has been mentioned in some previous works, including [14] for volume data. In these cases, the interface is integrated in a scalar attribute such as a color and the global geometric appearance is maintained through simplifcation. Preservation of the topology of a scalar field and isosurfaces defined on this field, has been considered and guaranteed in several methods like [13]. However, in our framework we are interested in preserving the topology of such properties defined for the model (i.e. patches of surfaces), for the material boundaries (i.e. subsets of the mesh edges) and for the intersections amongst boundaries (i.e. specific vertices of these boundaries). Existing methods lack a unique combinatorial validity test preserving the topology of all the elements defined in all dimensions lower or equal to 3.

2.4 Topology Preserving Simplification

Surface mesh simplification algorithms transform a general simplicial complex in another one. Edge collapse is the most used operator to remove sub-simplicies from the input because it allows to control efficiently and accurately the deformation introduced in the initial mesh. Some of these algorithms preserve the topology of the simplified mesh by investigating the organisation of the triangles around an edge leading to a certain number of cases. In some situations the edge collapse is rejected as it would introduce topological error [20]. A similar treatement can be done on feature lines based on local tests in between the edges around a collapse and the triangulation. However non-manifold surfaces can not be treated with this method and there is not an obvious extension to higher dimensions in contrast with our results, as pointed out in Section 6.

More recently, [5] has proven that the complex obtained after an edge collapse is homeomorphic to the first one if the neighborhood of the edge collapse satisfies the *link condition*. This result based on concepts of Computational Topology has been more and more used, even in volume simplification as it is not restrictived to 2-complexes. Especially, non-manifold surfaces can be treated with this method using the link condition.

3 Preserving the Topology of 2-Complexes

Appendix A recalls the basic definitions from combinatorial topology needed in the following sections. Given a simplicial complex, and an edge (a 2-simplex) in this complex, the problem of detecting if the collapse of this edge modifies the topology of the complex has been extensively studied in [5]. For 2- and 3-complexes, necessary and sufficient conditions have been developed, ensuring that a continuous one-to-one mapping between the complex before edge collapse and the complex after edge collapse exists. These conditions are slightly more restrictive than topology preservation since the mapping is required to be the identity outside a neighbourhood of the edge being contracted. In this section we state the results developed in [5] for 2-complexes, as they will be used later in this paper. The reader is referred to the original paper for details.

3.1 Order of a Simplex in a 2-Complex

The order of a simplex τ in a complex K measures the topological complexity of the neighbourhood of τ in K. It is denoted by ord τ. For a 2-complex, the order of simplices may be 0, 1 or 2. A simplex τ has an order 0 if St τ is homeomorphic to an open disc. This is the simplest case, where locally the complex is a 2-manifold. The order is 1 if St τ is homeomorpic to p triangles sharing a common edge with $p \neq 2$. This is for example the case for non-manifold edges and boundary edges. In all other cases the order of the simplex is 2. Note that all triangles have order 0, and that edges may have order 0 or 1 only. Figure 1 (a) illustrates the order of vertices and edges in a 2-complex.

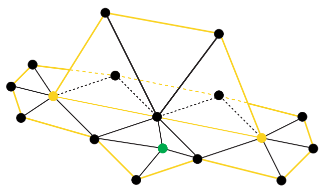


Fig. 1 (a) Order of simplices in a 2-complex: the green vertex has order 0, black vertices have order 1, yellow vertices order 2, black edges order 0, yellow edges order 1, all triangles have order 0. (b) shows the edges and vertices in the first boundary of the 2-complex shown in (a). The two yellow vertices form the second boundary of this complex.

Boundaries of a 2-Complex
The topological tests rely on a generalized notion of a boundary. The j-th boundary in a 2-complex K is defined as the set of all simplices with order j or higher:

$$\mathrm{Bd}_j\ K = \{\tau \in K | \mathrm{ord}\ \tau \geq j\}.$$

The 0-th boundary of K is K. Figure 1 (b) illustrates the first and second boundaries of a 2-complex. Note that the first boundary contains not only the usual boundary edges, but also non-manifold edges adjacent to three triangles or more. Since only vertices may have order 2, the second boundary of a 2-complex is always a set of vertices.

Topological test
Let ω be a dummy vertex, define $K^\omega = K \cup (\omega \cdot \mathrm{Bd}_1\ K)$ and $G^\omega = \mathrm{Bd}_1\ K \cup (\omega \cdot \mathrm{Bd}_2\ K)$. See Appendix A for the definition of the cone $\omega \cdot T$, where T is a set of simplices. Let $\mathrm{Lk}_0^\omega\ \tau$ and $\mathrm{Lk}_1^\omega\ \tau$ denote the link of τ in K^ω and G^ω respectively. With all these definitions at hand, one can now state the main result of [5] for 2-complexes.
Topological Test:

Theorem 1 *Let K be a complex and L be the complex obtained by contracting the edge uv in K. The following two conditions ensure that L has the same topology as K:*

(i) $Lk_0^\omega\ u \cap Lk_0^\omega\ v = Lk_0^\omega\ uv$
(ii) $Lk_1^\omega\ u \cap Lk_1^\omega\ v = \emptyset$.

Conditions (i) and (ii) are illustrated in Figure 2 for a non-manifold edge.

4 Topology Preserving Edge Collapse With Embedded Structures

Let K be a 2-complex and E be a set of edges in K. The closure $\overline{E}$ of E is a collection of edges and vertices

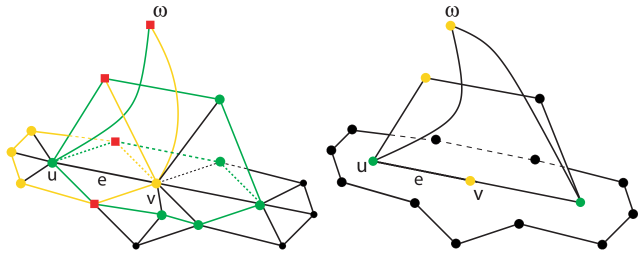


Fig. 2 Topological test for an edge e=(u,v): the left and right parts illustrate respectively the first and second conditions. Left part: $\mathrm{Lk}_0^\omega\ u$ contains the yellow and red vertices and the yellow edges, $\mathrm{Lk}_0^\omega\ v$ contains the green and red vertices and the green edges, $\mathrm{Lk}_0^\omega\ e$ contains the red vertices: the first condition is fulfilled. Right part: $\mathrm{Lk}_1^\omega\ u$ contains the three yellow vertices, $\mathrm{Lk}_1^\omega\ v$ contains the two green vertices, the intersection is empty: the second condition is fulfilled.

that may be viewed as a set of several polylines embedded in K, with possible self intersections. These polylines together with the polylines formed by all boundary and non-manifold edges of K define a 1-complex $F = \overline{E} \cup Bd_1K$, whose topology is an important characteristic of the data, and should be preserved. Contracting edges in K possibly modifies both the topology of K and the topology of $\overline{F}$. Our goal is to develop a *robust* test to detect which edge collapse preserves the topology of the 2-complex K *together* with the topology of the polylines embedded in K. The key idea of the paper is to implicitly encode the topology of the polylines $\overline{F}$ and of the embedding complex K in a single extended 2-complex $\widetilde{K}$. Thereafter the topological test developed in [5] (see Section 3) can be used to select the valid edge collapses in the extended 2-complex $\widetilde{K}$. $\widetilde{K}$ is built in such a way that valid edge collapses in $\widetilde{K}$ preserve the topology of both K and the embedded polylines $\overline{F}$.

4.1 Implicitly Encoding the Topology of the Embedded Structures

We can assume that the collection of edges E contains only order 0 edges, without changing F: if there is an order 1 edge in E, it is also in Bd_1K. The extended complex $\widetilde{K}$ is built by adding to K the cones from the dummy vertex ω to each edge in E. More precisely:

$$\tilde{K} = K\ \cup\ w.\overline{E}$$

In other words, the embedded edges are extended to a 2D sub-complex of the extended complex $\tilde{K}$. Figure 3 illustrates an extended complex with two intersecting embedded polylines. The extended complex $\tilde{K}$ is defined such that its first boundary equals F.

Lemma 1 $Bd_1\tilde{K} = F$

Proof An edge in E has an order 0, i.e. it has exactly 2 adjacent triangles in K, and three adjacent triangles in $\tilde{K}$. Thus it has an order 1 in $\tilde{K}$. An edge of order 1 in K

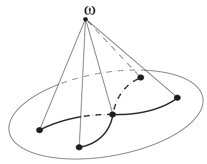


Fig. 3 Extended complex: the extended complex is defined by adding to the mesh all edges and faces connecting a dummy vertex ω with the vertices and edges of the embedded polylines.

is not in E, therefore it has the same star in K and $\tilde{K}$, and is still of order 1 in $\tilde{K}$. Thus we have $F \subset Bd_1\tilde{K}$. Now let e be an edge of order 1 in $\tilde{K}$. e has one or three or more triangles in its star. If e has only one triangle in its star, it is in the boundary of K. If it has three or more triangles in its star, it is either a non-manifold edge of K, or an edge in E. Thus we have also $Bd_1\tilde{K} \subset F$. □

Lemma 1 means that the topology of the embedded structure F is encoded implicitly in the extended complex $\tilde{K}$. An edge contraction in K corresponds to an edge contraction in $\tilde{K}$. After an edge contraction, the modified set of polylines F can be retrieved as the set of edges facing the dummy vertex ω in a triangle. The following Lemma applied to the extended complex $\tilde{K}$, proves that if an edge contraction preserves the topology of $\tilde{K}$, it also preserves the topology $Bd_1\tilde{K}$, i.e. it also preserves the topology of the modified set of polylines F, since $Bd_1\tilde{K} = F$ from Lemma 1.

Lemma 2 *Let K be a 2-complex and (uv) be an edge in K such that (i) and (ii) are satisfied. Then the contraction of (uv) preserves the topology of Bd_1K.*

Proof Let $M = Bd_1K$, and $M^w = Bd_0(M) \cup \omega \cdot Bd_1(M)$. M is a 1-complex: it is the closure of the set of order 1 edges in K. Since M is a 1-complex, it is sufficient to show that there is no vertices in the intersection of the links of u and v in M^w. Since $Bd_1M = Bd_1(Bd_1K) \subset Bd_2K$, any vertex in the link of u (resp. v) in M^w, is also in the link of u (resp. v) in G^ω. Therefore condition (ii) implies the lemma. □

4.2 Implementation of the Topological Preservation Test

In this section we describe the actual implementation of the algorithm that detects if an edge $e = (u, v)$ can be collapsed without changing the topology of the surface nor the topology of any polyline defined on the triangle edges.

1. **Initialisation:** Set up local variables in the neighborhood of the 2 vertices u and v of the edge e. For each edge e_i adjacent to u or v, add dummy triangles $(\omega \cdot e_i)$ if e_i is either on a polyline or in the first boundary of the mesh. This amounts to locally computing the extended complex.
2. **Compute the vertices in $\mathbf{Lk}_0^\omega$ u:** Loop through the edges e_i adjacent to u in the extended complex. For each edge e_i, store the vertex facing u in Lk_0^ω u. In some cases the opposite vertex is the dummy vertex ω if the edge is in a polyline, or in the first boundary of the mesh. This case is implicit and it is not treated explicitly as the neighborhoods of u and v have been triangulated by adding dummy cells using the dummy vertex ω.
3. **Compute the edges in $\mathbf{Lk}_0^\omega$ u:** Loop through the faces f_i adjacent to u in the extended complex. For each face f_i, store the edge facing u in Lk_0^ω u. Figure 4 illustrates the computation of the link around a non-manifold vertex.

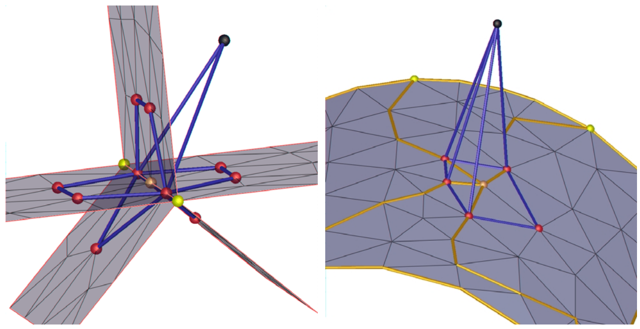


Fig. 4 Link of a non-manifold vertex on the intersection of several surfaces (*left*). Link of a vertex on the intersection of several polylines. In the extended complex, dummy triangles are added from the segment polylines modifying the link of the vertex as shown (*right*).

4. **Compute $\mathbf{Lk}_0^\omega$ v:** The exact same process is applied around v in order to compute Lk_0^ω v.
5. **Compute $\mathbf{Lk}_0^\omega$ e:** Loop through the faces f_i adjacent to e in the extended complex. For each face f_i, store the vertex facing e in Lk_0^ω e.
6. **Compare $\mathbf{Lk}_0^\omega$ $u \cap \mathbf{Lk}_0^\omega$ v and $\mathbf{Lk}_0^\omega$ e:** The intersection of Lk_0^ω u and Lk_0^ω v is computed. If this intersection contains an edge, the edge collapse is rejected, as Lk_0^ω e contains only vertices. Compare the number of vertices in Lk_0^ω $u \cap \mathrm{Lk}_0^\omega$ v with the number of vertices in Lk_0^ω e. The edge collapse is rejected if these numbers are not equal. It is sufficient to compare the number of vertices, and not the actual indices of the vertices, since in any case the vertices in Lk_0^ω e form a subset of the vertices in Lk_0^ω $u \cap \mathrm{Lk}_0^\omega$ v.
7. **Compute $\mathbf{Lk}_1^\omega$ u:** Loop through the edges e_i adjacent to u and in the first boundary of the extended complex. For each edge e_i store the vertex facing u in Lk_1^ω u. Note that if u is in the second boundary of the mesh, then the edge (ω, u) is in G^ω (more precisely in $(\omega \cdot \mathrm{Bd}_2\ K)$) and thus the dummy vertex ω is in Lk_1^ω u.
8. **Compute $\mathbf{Lk}_1^\omega$ v:** The exact same process is applied around v computing the Lk_1^ω v.

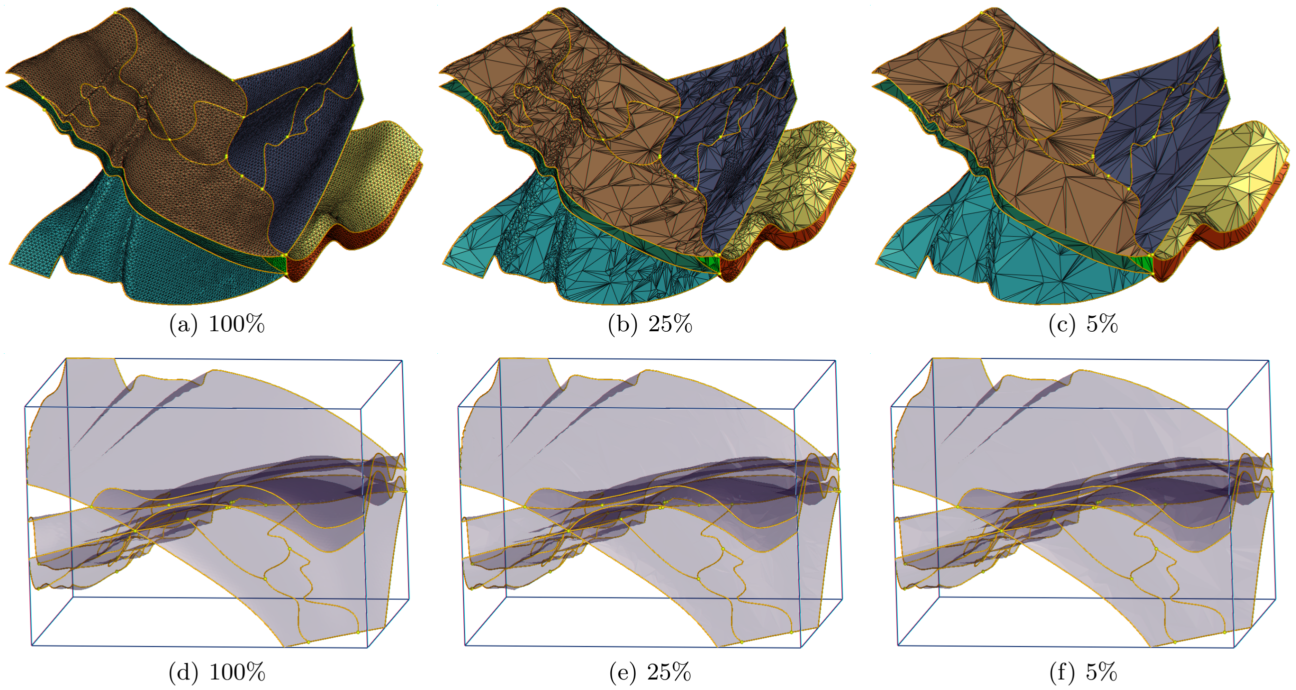


Fig. 5 Geological data showing several surfaces separating different soil types. Linear features are present in the data for cetain layers. This set of interfaces among soil type creates non-manifold surfaces.

9. **Compute $\mathbf{Lk}_1^\omega\ u \cap \mathbf{Lk}_1^\omega\ v$:** The intersection of $\mathrm{Lk}_1^\omega\ u$ and $\mathrm{Lk}_1^\omega\ v$ is computed. If this intersection is not empty then the edge e is rejected for collapse as the second link condition breaks.
10. **Accept collapse of e:** If the edge e has not been rejected in previous steps, it can now be contracted. The dummy cells introduced in the first step are removed.

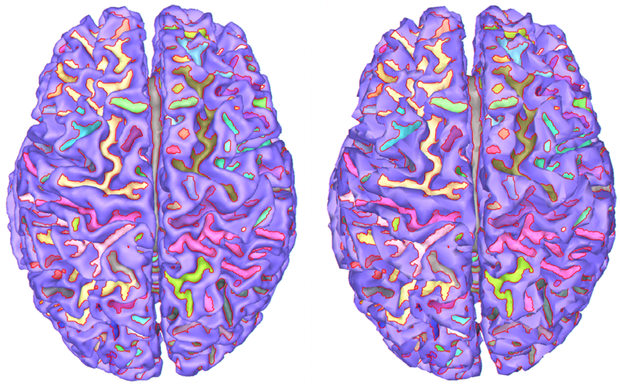
5 Applications and Results

We have applied our topology preserving simplification algorithm on a wide range of datasets. The error measure used in all examples to sort the edges in the heap is a simple weighted combination taking into account the maximal normal deviation of the faces and the polyline's edges, before and after collapse. This simple error measure gives already satisfactory results, but we insist on the fact that our algorithm can be combined with any error measure. All simplifications run at about 20000 removed edges per second on a 1.7Ghz Pentium M laptop. In the following section, all decimation percentages are taken from the original number of triangles in the mesh. The first paragraph shows an neuroscience application involving a complicated surface geometry but with a really simple feature line topology. Then a CAD/CAM application is presented illustrating both a complicated geometry and feature line topology. Finally, geological data allow us to validate our method with a simple geometry but an extremely complex polyline topology.

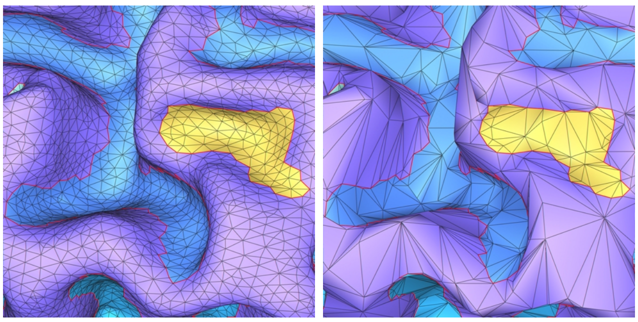
Geology Data with Non-manifold Models As our edge collapse criteria for preserving topology is valid for 2-complexes, non-manifold models can directly be simplified with the same topological test. Figure 5 shows the simplification of geological data. Several layers separating different soil type create a non manifold surface. Linear features are present in the model which give a specific information on a subset of the data and thus need to be handle in the simplification process. Even with a high simplification ratio of 95%, the topology of the non-manifold surface is preserved as well as the linear features embedded on the triangle edges. Note that several polylines are crossing a non-manifold region which leads to a vertex of order two which is successfully preserved after the collapses.

Neuroscience
We have applied our simplification algorithm on a triangle mesh extracted from MRI data and segmented according to the sulci and the gyri (Figure 6). Thus we were able to generate a low resolution mesh while preserving its main features: the mesh remains a manifold surface, and the polylines remain closed curves without self-intersections. The close-up view shows the effect of the simplification on a feature. Note that the boundaries are preserved despite the reduction of vertices along them. As many features can be small and close to each other, allowing the collapse of edges around and along

the boundaries is essential to keep the simplification rate low.



(a) 100% of original data (b) 25% of original data

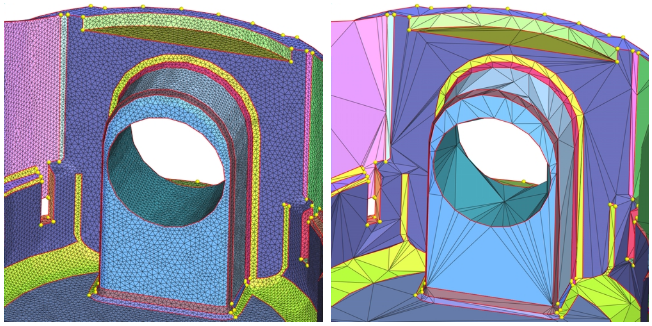


(c) 100% of original data (d) 25% of original data

Fig. 6 Segmented surface of a human Cortex. The sulci and gyri of the cortex (i.e. main features) are preserved by the simplification.

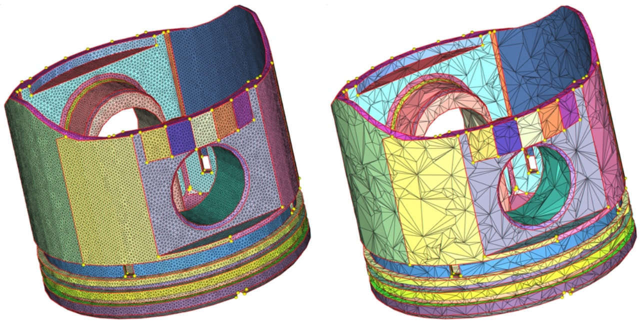
CAD/CAM model with complex polylines Figures 7 and 8 show the results on a CAD/CAM model of a piston after a simplification ratio up to 98%. The solid views show the high simplification ratio applied to the triangles, while the polyline views exhibit the preservation of both the geometry and the topology of the polylines. In this particular exemple polylines are extracted as interfaces in between materials defined on the surface. Even with a high simplification ratio of 98%, the interfaces are not altered as illustrated in the close-up views.

Terrain Data Figure 9 shows a subset of the surficial materials of Canada. Boundaries among soil types are represented by red polylines. By applying our algorithm we successfully simplify the surface down to $1/5^{th}$ of original data without affecting the topology of the polylines. The simplification process is stopped when no edges can be removed without introducing a certain deviation into the polylines. The simplification process stops early because of the complexity of the polyligne and not because of changes in the surface topology. The close-up view shows the simplification emphasizing the complexity of the soil structure. Note that intersections among polylines are represented by yellow spheres and that no such features are removed through the simplification (i.e it would introduced a topological modification of the polylines).

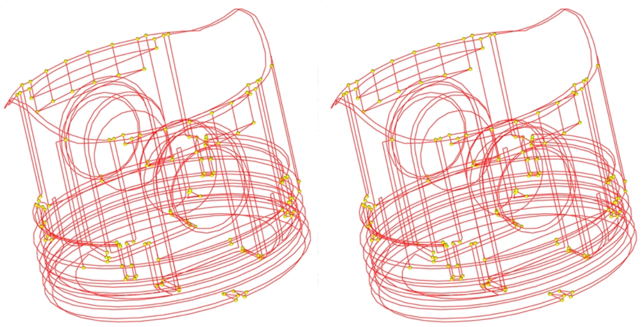


(a) 100% of original data (b) 2% of original data

Fig. 7 CAD/CAM model of a piston simplified by preserving its material properties.



(a) 100% (b) 5%

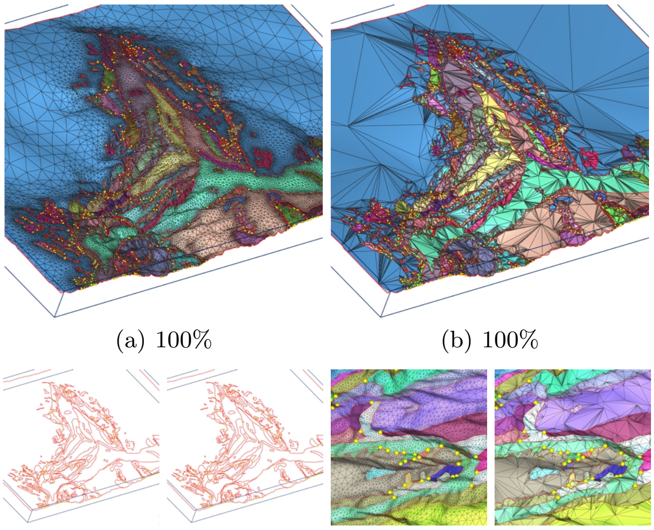


(c) 100% (d) 5%

Fig. 8 CAD/CAM model of a piston simplified by preserving its material properties. The percentages are ratios of original data. Top row shows solid faces of the model with the material boundaries defined as polylines. We ensure that the topology of these polylignes remains unchanged. The bottow row shows that the main features of the model are preserved even after removing 95% of the vertices.

6 Conclusion

This paper has introduced a new topology preserving simplification algorithm, based on edge collapse, for non-manifold triangular meshes with embedded polylines. The



(a) 100% (b) 100%

(c) 100% (d) 20% (e) 20% (f) 20%

Fig. 9 Subset of surficial materials of Canada. The percentages are ratios of original data. Each soil type is color-coded on the left views. Interfaces among them are shown by red polylines. Note the complexity of the polyline topology emphasized by yellow spheres showing all polyline intersections.

topology of the polylines as well as the mesh is preserved throughout the simplification using a *unified and robust validity algorithm* to select edges that can be collapsed. Our algorithm is based on powerful combinatorial topology results, and can be applied to general triangular meshes, including non-manifold meshes. The idea of the *extended complex*, encoding implicitly the topologies of the mesh and the polylines in a unified way, is at the heart of our results. One of the main appeal of this idea is its possible extension to 3-complexes. We are currently working on it, in order to generalize this concept to tetrahedral meshes with embedded 2D and 1D structures.

Appendix A

A $n-$simplex τ is the set of convex combinations of $n+1$ affinely independent points called the vertices of τ. A simplex η whose vertices are a subset of the vertices of τ is called a face of τ. This is denoted $\eta \leq \tau$ or $\tau \geq \eta$. If η is a face of τ then τ is called a coface of η. A complex K is a collection of simplices such that:

(i) if $\tau \in K$ then all faces of τ are also in K.

(ii) if $\eta, \tau \in K$ then $\eta \cap \tau$ is empty or a face of η and τ.

The star, the closure and the link of a set of simplices T in a complex K are denoted respectively St (T), $\overline{T}$, Lk (T), and defined by:

St $(T) = \{\eta \in K | \eta \geq \tau \in T\}$.

$\overline{T} = \{\eta \in K | \eta \leq \tau \in T\}$.

Lk $(T) = \overline{\text{St}T}$ - St$\overline{T}$.

If τ is a simplex and ω is a point affinely independant of the vertices of $v_1, \cdots, v_n$ of τ then the cone from ω to τ is denoted by $\omega \cdot \tau$ and defined as the simplex with vertices $\omega, v_1, \cdots, v_n$. If T is a set of simplices, then the cone from ω to T is the union of the cones from ω to τ, with $\tau \in T$. It is denoted by $\omega \cdot T$.

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