Sagnac effect in superfluid liquids

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Abstract. **The interpretation of the Sagnac effect is re-examined in the context of recent cold atomic beam and superfluid experiments. A widespread misconception concerning the understanding of this effect in a superfluid liquid is discussed.**

In this letter, we wish to consider some problems of interpretation of Sagnac-type experiments with beams of cold atoms and with superfluids. In particular, we intend to disprove Malykin's following comment on the latter system made in his otherwise well-documented and comprehensive review article on the Sagnac effect [1]:

"It should be noted that the inertial properties of waves (or wave packets, for that matter) are made use of in such gyroscopic instruments as solid-state wave gyroscopes and also gyroscopes whose principle of action is grounded on the macroscopic quantum properties of superfluid helium. These instruments along with the Foucault pendulum and mechanical gyroscopes are applied to determine the angular position in space. In contrast, devices in which the Sagnac effect provides the working principle... serve as angular velocity pickups. This makes the fundamental distinction between instruments based on the Sagnac effect and those in which the property of physical bodies or wave packets to maintain orientation in space is employed."

In spite of the maturity of its subject matter, Malykin's review stirred the need for further clarification and comments [2, 3]. Here, we want to point out that contrary to the statement above, superfluid interferometers do measure the absolute angular velocities of the platforms on which they are mounted. But, more importantly, we also attempt to address the somewhat widespread (see, e.g., Refs [1, 2, 4]) misconception that superfluid rotation sensors, unlike atomic beam gyros, do not belong to the same class of quantum interference effects as Sagnac light-wave experiments.

The Sagnac effect is no longer an object of sole academic curiosity studied to prove (or disprove, in the eyes of some, Sagnac being one) the foundations of the theory of relativity; it has spread to applications of daily usefulness such as the ring laser gyros in inertial guidance devices and the Global Positioning System.

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For these purposes, the effect is well understood [5 – 8]. In the classic textbook by Landau and Lifschitz [5], the rotating frame of reference, embodied by orbiting satellites carrying atomic clocks, our Earth, or turntables supporting interferometers, is treated as an accelerated frame from the standpoint of general relativity. In such frames, characterized by a space – time metric

$$-\mathrm{d}s^2 = g_{00}\,\mathrm{d}(x^0)^2 + 2g_{0i}\,\mathrm{d}x^0\,\mathrm{d}x^i + g_{ii}\,\mathrm{d}(x^i)^2\,,$$

clocks can be synchronized for infinitely close points by the time shift $\mathrm{d}t = -g_{0i}\,\mathrm{d}x^i/g_{00}$. If a clock is transported around a finite path Γ in the frame rotating with a velocity $\mathbf{\Omega}$, the resulting total time shift is ([5, § 89])

$$\Delta t = \frac{1}{c}\oint_\Gamma \frac{g_{0i}\,\mathrm{d}x^i}{-g_{00}} = \oint_\Gamma \frac{(\mathbf{\Omega}\times\mathbf{r})\,\mathrm{d}\mathbf{r}}{c^2 - (\mathbf{\Omega}\times\mathbf{r})^2} \approx \frac{2}{c^2}\,\mathbf{\Omega}\mathbf{S}\,, \qquad (1)$$

$\mathbf{S}$ being the vector area subtended by Γ. Time delay (1) between the reading of the transported clock and that of the clock standing still on the rotating platform lies at the root of the Sagnac effect. Such a standpoint was held long ago by Langevin [9] and others [1].

For light waves with angular frequency ω, the corresponding phase shift is given by

$$\Delta\varphi = \omega\,\Delta t = \frac{4\pi\,\mathbf{\Omega}\mathbf{S}}{\lambda c}\,, \qquad (2)$$

where λ is the wavelength in the vacuum, $\lambda = 2\pi c/\omega$.

Formulas (1) and (2) are usually derived for optical interferometric experiments in the framework of the special theory of relativity, using Lorentz boosts to calculate the velocity of the moving clock or wave (see, e.g., Refs [1, 2]). Since Sagnac's early experiments in 1913, their validity has been confirmed in detail with optical interferometers and by atomic clock transportation, as reviewed, for instance, in Refs [10, 11].

New physical systems to which the same conceptual framework as for the original Sagnac experiment can be applied have been studied in the past twenty years or so, when it became possible to split beams of particles and to have them recombine and interfere. Interferometers were built using neutrons and electrons, and, more recently, atomic beams and superfluids. Together with these experimental advances came alternative interpretations of the effect.

We first consider particles — electrons, neutrons, or atoms — represented by localized wave packets with a slowly varying overall phase φ. These wave packets can be treated in the semiclassical approach: the phase is related to the classical action $\varphi = \mathcal{S}/\hbar$. This action can be computed in a rotating frame following, for instance, [12, 13]. The Lagrangian for a free particle with mass m located at a position $\mathbf{r}$ and moving with a velocity $\mathbf{v}$ in the reference frame rotating with an angular velocity $\mathbf{\Omega}$ is expressed by

$$\mathcal{L}(\mathbf{r}, \mathbf{v}) = \frac{m}{2}\, v^2 + m\,\mathbf{\Omega}\,(\mathbf{r} \times \mathbf{v}) + \frac{m}{2}\,(\mathbf{\Omega} \times \mathbf{r})^2 \,. \tag{3}$$

The discussion is restricted to the case of slow rotations, which are treated as a small perturbation. The action is then obtained as the integral of the Lagrangian, Eqn (3), over the unperturbed path of the particle, along which its velocity $\mathbf{v}$ is constant. In the first order in $\Omega r/c$, the last term in Eqn (3) can be neglected and the expression for the action reduces to

$$\mathcal{S} = \int_{\Gamma} \mathrm{d}t\; \mathcal{L}\big(\mathbf{r}(t), \mathbf{v}(t)\big) = m\mathbf{\Omega} \int_{\Gamma} \mathrm{d}t\; \big[\mathbf{r}(t) \times \mathbf{v}(t)\big] \,. \tag{4}$$

Since $\mathbf{v}(t) = \mathrm{d}\mathbf{r}(t)/\mathrm{d}t$, the last integral in Eqn (4) is twice the area swept along Γ. For a closed path, the change of the phase of a wave packet upon completing a round trip involves the area $\mathbf{S}$ subtended by Γ:

$$\Delta\varphi = \frac{m}{\hbar}\, \mathbf{\Omega} \oint_{\Gamma} \mathbf{r} \times \mathrm{d}\mathbf{r} = \frac{m}{\hbar}\, 2\,\mathbf{\Omega}\,\mathbf{S} \,. \tag{5}$$

Equation (5) expresses the Sagnac phase shift for massive particles as obtained from a purely nonrelativistic kinematical approach.

We now turn to helium liquids. The inertial properties of superfluids have been the subject of numerous studies [14]. These properties are governed by the existence of an order parameter that acts as a macroscopic wave function with a well-defined overall phase φ. The superflow velocity is proportional to the gradient of this phase,

$$\mathbf{v}_{\mathrm{s}} = \frac{\hbar}{m}\, \nabla\varphi \,, \tag{6}$$

where m is the atomic mass, m_4, for ^{4}He and the Cooper-pair mass,[1] $2m_3$ for ^{3}He-B. No gauge field added to φ can allow this expression to transform through rotation of the reference frame; it only holds in inertial reference frames.

For a pool of superfluid in the shape of a torus, the continuity of the phase requires the circulation of the velocity along a closed contour Γ threading the torus to be quantized *in the inertial frame*:[2]

$$\oint_{\Gamma} \mathbf{v}_{\mathrm{s}}\, \mathrm{d}\mathbf{r} = \frac{\hbar}{m} \oint_{\Gamma} \nabla\varphi\, \mathrm{d}\mathbf{r} = n\kappa \,, \tag{7}$$

where $\kappa = 2\pi\hbar/m$ is the quantum of circulation and n is an integer.

This quantum feature of superfluids has been demonstrated experimentally by setting the toroidal vessel into rotation. As shown in [16], states of circulation quantized in the inertial frame spontaneously appear at the superfluid transition. In particular, a state of zero circulation, $n = 0$, the so-called Landau state, can exist. The superfluid fully decouples from its container: it settles at rest with respect to the distant stars, that is, in motion with respect to the container walls.

At finite temperature, a nonsuperfluid fraction appears in the fluid, formed by the thermally excited elementary excitations in the superfluid, the phonons and rotons for ^{4}He, and thermal quasi-particles and quasi-holes for ^{3}He. As shown in [17], not the angular momentum associated with the motion of the superfluid component but the superfluid velocity circulation defined by Eqn (7) is a conserved quantity as the temperature, and hence the superfluid fraction changes.

A rotating superfluid is not simply a classical inviscid fluid with angular momentum; circulation quantization constitutes a stricter constraint, immune to perturbations by moving boundaries and to temperature changes, as is illustrated by the experiments mentioned above and many others. These properties fundamentally follow from Eqn (6) and the continuity of the quantum phase throughout the superfluid. They entail the existence of a Sagnac effect.

In the frame rotating with an absolute rotation vector $\mathbf{\Omega}$, the superfluid velocity transforms as $\mathbf{v}_{\mathrm{s}}' = \mathbf{v}_{\mathrm{s}} - \mathbf{\Omega} \times \mathbf{r}$ and the quantization of circulation condition (7) is given by

$$\oint_{\Gamma} \mathbf{v}_{\mathrm{s}}'\, \mathrm{d}\mathbf{r} = \oint_{\Gamma} (\mathbf{v}_{\mathrm{s}} - \mathbf{\Omega} \times \mathbf{r})\, \mathrm{d}\mathbf{r} = n\kappa - 2\,\mathbf{\Omega}\,\mathbf{S} \,. \tag{8}$$

The last term in the right-hand side of Eqn (8) amounts to a nonquantized contribution to the circulation in the rotating frame that varies with the rotation vector $\mathbf{\Omega}$. This circulation gives rise to a phase change $\Delta\varphi = (m/\hbar)\, 2\,\mathbf{\Omega}\,\mathbf{S}$ that, measured by means of Josephson-type devices [15], gives access to the rotation vector $\mathbf{\Omega}$, contrarily to the statement in Ref. [1] quoted above. The superfluid gyros in Ref. [15] are gyrometers, not gyroscopes.

The phase difference stemming from Eqn (8) is precisely that arising from the Sagnac effect for particles with mass m, Eqn (5). This coincidence is not simply formal: an applied rotation has the same effect on the phase of an atomic wave packet in an atom-interferometric experiment as on the phase of the superfluid macroscopic wave function in a toroidal vessel.

If we now invoke the wave – particle duality and introduce the de Broglie wavelength $\lambda_{\mathrm{B}} = 2\pi\hbar/(mv)$ of the particle of mass m and velocity v in Eqn (5), we find

$$\Delta\varphi = \frac{4\pi\,\mathbf{\Omega}\,\mathbf{S}}{\lambda_{\mathrm{B}} v} \,. \tag{9}$$

For photons in the vacuum, $v = c$, and we recover Eqn (2).

In a rotating material medium such as a glass fiber ring gyro, the simple Eqn (2) does not hold. It is necessary to consider both the wave propagating in the corotating direction and that in the counterrotating direction to eliminate the refraction properties of the medium (see, e.g., Ref. [18] for a discussion). This circumstance takes advantage of the reciprocity principle to cancel the retarded propagation of light signals in opposite directions along precisely the same travel path. What is left is the difference in clock readings, Eqn (1).

Other examples of the same kind of cancellation between counterrotating waves are discussed by Malykin [1] (see also Ref. [2]). For interferometry with massive particles, the beam-

[1] The case of the anisotropic phase ^{3}He-A is more complicated because gradients of $\mathbf{l}$, the orbital anisotropy axis, also contribute to the right-hand side of Eqn (6).

[2] For an actual conduit with a finite cross section, the circulation of the mean velocity results from a suitable *average* over the various distinct superfluid contours threading the conduit, which may contain pinned vortices and textures [15]. Taking this average may lead to a constant, nonquantized, contribution to the overall phase. This phase bias is unimportant for the present discussion.

deflecting devices acting as 'mirrors' introduce additional phase shifts that must be taken into account. So do gravity and electromagnetic fields. Each separate experiment requires special considerations (see [19] for electrons and [20] for atoms). In most instances, Eqns (1) and (9) for the Sagnac effect are found to be obeyed.

We emphasize that all massive-particle interferometric experiments obey Eqn (9) and belong to the same class. The superfluid is not the odd man out. It offers so far the only experimental situation in which a matter-wave field, coherent over the full length of a pickup loop, is involved, but it is quite conceivable that in the near future, Sagnac-type experiments will be conducted with Bose–Einstein condensates of ultra-cold atoms. The required techniques are on the verge of becoming available [21]. An atom of a given atomic species could be made to interfere with itself or collectively. In the first case (atomic beam experiments), each single atom interferes with itself after having traveled along either arm of the rotating interferometer. In the second (Bose-condensed) case, the condensate sits nearly idle between the two banks of a Josephson junction, where it interferes with itself. The interference pattern arises from the overlap of the macroscopic wave function on one side of the junction with the weakly coupled part that leaks out from the other side. There is no breach of conceptual continuity between superfluid and particle Sagnac experiments: we have, on the one hand, massive particles and matter waves, and on the other hand, light signals, clocks, and photons.

The Sagnac phase shift for massive particles, Eqn (9), has been amply verified by classic experiments on electronic Cooper pairs [22], neutrons [23, 24], electrons in the vacuum [12, 19], and atom beams [25–37]. For superfluid helium, the same equation underpins the experiments reported in Refs [15, 28, 29]. Yet, this equation differs markedly from Eqn (2), quantitatively by a factor $mc^2/\hbar\omega \sim 10^{10}$ to 10^{11}, and qualitatively because Einstein's relativity does not enter its derivation.

There are several, equivalent, ways to restore explicit relativistic invariance for massive particles and superfluids.

It is possible [30, 31] to a priori derive the rotation terms appearing in Lagrangian (3) from a fully general-relativistic description of the matter-wave field. The particle quantum field is the solution of a Dirac-like equation (or the Proca, or a higher-order equation). In the rotating frame, the curved metric appears through the Dirac gamma matrices, and their low-velocity expansion yields a Hamiltonian and, correspondingly, a Lagrangian that generalizes (3). Rotation terms in this Lagrangian are a direct expression of the effects of the local space–time curvature on the phase of the quantum field; the Sagnac term for light waves has the same physical origin.

In the relativistic form of the Lagrangian for weakly interacting particles, the kinetic energy term in Eqn (3) is replaced by $-mc^2(1 - v^2/c^2)^{1/2}$ (see Refs [19, 25, 32]). A frequency such that $\hbar\omega = mc^2$ appears that turns Eqn (2) formally into Eqn (9). Massive and massless particles are thus put on the same footing. This prescription was re-examined recently on different grounds by a number of authors for massive particles [2, 4, 33] and by Volovik for superfluid helium [34].

For superfluids, we can take a more direct approach. A relativistic two-fluid model can be built over the usual Landau superfluid hydrodynamics by imposing Lorentz invariance, as is done in Ref. [35]. The invariant velocity circulation, the generalization of Eqn (7), is given by

$$\int_{\Xi} \{ v_0' \, \mathrm{d}x^0 + v_i' \, \mathrm{d}x^i \} = n\kappa \,, \tag{10}$$

where (v_0', v_i') is the four-velocity in the rotating frame $(c^2 + \mathbf{v}_\mathrm{n}' \mathbf{v}_\mathrm{s}', -\mathbf{v}_\mathrm{s}')$. Both the normal fluid velocity v_n' and the superfluid velocity v_s' are small compared to c, and therefore the time-like component of the four-velocity reduces to c^2. The integration over Ξ is an actual loop integral only for the space-like components. The corresponding world line is not closed because the time for synchronized clocks varies as $\mathrm{d}x^0 = -g_{0i}\,\mathrm{d}x^i/g_{00}$. Using Eqn (1), we recover Eqn (8),

$$\oint_{\Gamma} v_i' \, \mathrm{d}x^i = n\kappa + \int \frac{c^2 g_{0i}}{g_{00}} \, \mathrm{d}x^i = n\kappa - \frac{2}{c^2} \, \mathbf{\Omega} \mathbf{S} \,, \tag{11}$$

which establishes a unifying link between superfluid physics and the relativistic particle approach. It shows that the effect described by Eqns (2) and (9) is one and the same in spite of the quantitative and qualitative differences stated above.

Thus, Einstein-synchronized clocks provide the time standard by which phase differences can be kept track of in all the studied physical systems. As appropriately summarized by Greenberger [36, Sec. IX], for neutron interferometry experiments: "the phase shift (in the rotating interferometer) is seen to be caused by the different rates at which a clock ticks along each of the two beams."

Needless to say, low-temperature experiments, and even those in cold-atom or neutron physics, in no way measure relativistic corrections to Eqn (9) derived for massive particles. The experimental implications of the observation of the Sagnac phase shifts are that no reference to special or general relativity need be made. In fact, the derivation of Eqn (9) makes no explicit reference to Einstein's relativity. The nonrelativistic limit, obtained by letting $c \to \infty$, leaves Eqn (2) for the phase shift unchanged. Clocks and light-wave experiments, which involve no rest-mass energy, are, for their part, fully relativistic. The reference to clocks tied to a particle rest-energy provides a fully covariant formalism to describe the Sagnac effect; it bears no direct relevance to laboratory observations but provides a common viewpoint on the various physical systems.

We hope to have clarified the case for Sagnac experiments in superfluids. As those with atoms, neutrons, and electrons, they do obey Eqn (1) when the proper transcription to the time domain is effected. With clock transportation, they share the feature that the relevant variables, superfluid phase or clock time, are defined and obey Eqns (1) and (9) along any given path, irrespective of the details of the paths of well-balanced interferometric devices. Also, they demonstrate a notably extreme case of 'giant matter waves,' close to the borderline between quantum systems and classical ideal fluids, but resting on the existence of a quantum phase, which is a prerequisite for the appearance of phase shifts, circulation quantization, and Josephson interference patterns.

Thus, to summarize: (1) The Sagnac effect takes a particularly simple form in superfluids because the order parameter phase is a macroscopically defined and directly measurable quantity [15, 28, 29]; (2) Its experimental implementation varies considerably between different physical systems, but a unifying relativistic formalism is offered by clock transportation — massive quantum particles, superfluids, waves, and actual clocks all carrying their own time reference, as implied previously by a number of authors (e.g., [32, 36, 37]).

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