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LIMITED ENERGY SPREAD IN AN SSC

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ABSTRACT

After having recalled the tolerances on the adjustments which mainly affect the energy spread out of an SSC either with or without RF flat-topping cavities, the special case of no flat-topping is considered with more detail.

A method for cutting the bunches in phase in an SSC to a very short length while minimizing the loss is described. This length is then kept approximately constant in the isochronous cyclotrons themselves, since it is only affected by the radial acceleration gradient. In between, on the contrary, rebunchers are needed to counteract drift effects. The cases of curved paths and of the entrance or exit of a cyclotron, where the definition of an equivalent drift length requires special attention, is discussed. The importance at input and output of an SSC of having the particles transfered about their own energy dependent equilibrium orbit is emphasized.

1) INTRODUCTION

GANIL (Grand Accélérateur National d'Ions Lourds) is a complex cyclotron system made of two SSC's in cascade, following a small injector cyclotron, intended to accelerate all ions to an energy of 10 MeV/nucleon for very heavy ions up to 100 MeV/nucleon for light ones [1].

Emphasis is put on the beam quality to be delivered from that installation. Here, the problems related to energy spread are especially considered with some of the most stringent tolerances to satisfy.

2) ENERGY SPREAD PRODUCED DURING ACCELERATION

2.1 Energy spread expressions

In an SSC where the field is close to isochronism, the particles of the bunches receive a slightly different energy according to their phase. For one particle, it is possible to represent acceleration in each gap by a vector. The total energy received depends almost only on the phase of the sum of these vectors. The energy spread produced is then related to the distribution in phase of the particles in the bunches.

The use of r.m.s. values for phase and energy spread *(a)*,

$$\Delta\varphi = 2\sqrt{\overline{\varphi^2}} \qquad (1)$$

$$\Delta W = 2\sqrt{\overline{W^2} - \overline{W}^2}\ , \qquad (2)$$

leads to expressions where the exact distribution has a reduced influence. It is shown in Appendix I that one has, to a good approximation in the case of a classical cyclotron without flat topping,

$$\left|\frac{\Delta W}{W}\right|^2 \approx \left|\frac{\Delta W_0}{W}\right|^2 + \frac{\Delta\varphi^4}{16} + \Delta\varphi^2 . \delta\varphi_0^2\ , \qquad (3)$$

where $\delta\varphi_0$ is the phase adjustment of the bunch centre and ΔW_0 represents the contribution of phase independent sources of spread, like the injected beam energy spread. Neglecting this last term and assuming the first term as predominant, one can write :

$$\Delta W/W \approx \Delta\varphi^2/4 + 2\,\delta\varphi_0^2\ . \qquad (4)$$

With flat topping, on the contrary,

$$\Delta W/W \approx \Delta\varphi . \delta\varphi_{f.t} \qquad (5)$$

where $\delta\varphi_{f.t}$ is the error in phase of the f.t. voltage with respect to the fundamental, expressed in phase of the fundamental frequency. Initial energy spread would again add quadratically.

2.2. Discussion

A look at relations (4) and (5) immediately shows that if one likes to be able to accelerate long bunches (e.g., for maximum intensities, when space charge effects become appreciable *(b)*), in order to keep small energy spread ($\lesssim 10^{-3}$), flat topping is necessary. The most stringent tolerance is then put on the relative phase between fundamental and f.t. frequencies (of the order of .1° or .2°) inasmuch as the bunches are longer.

If, on the contrary, for experimental requirements, bunches have to be kept short, there is no need for flat topping. In this case, however, the relative phase of the bunches with respect to RF has to be carefully stabilized (of the order of .5°).
One should, in fixing tolerances, not forget the effect of other sources of energy spread, like that coming from the injected beam or the error in mean energy which is almost directly proportional to the RF voltage (but also depends slightly on bunch centre phase in the case of f.t.).
In addition, one must keep in mind that the particle phases introduced here are average values for which the particular phase at injection or ejection may not be exactly representative when variations can take place during acceleration, as will now appear. As soon as an exact adjustment has been found, however, the required tolerance in phase can be kept satisfied only from the measurements of input and output values.

3) LIMITING BUNCH LENGTH [2]

The short length of bunches required may not be obtained from the injector of an SSC, nor reached for all types of operation from the rebuncher system used between it and the SSC itself. It can then be necessary to still reduce that length inside the SSC.

Provided one accepts loss of those particles which are outside the necessary $\pm\Delta\varphi$, it is relatively simple to make the selection in an SSC as follows.

A small change is applied to the value of magnetic induction of the sectors and the injection phase is displaced in such a way as to have the central phase of the bunches shifting during acceleration from say $-\varphi_0$ to $+\varphi_0$, keeping the average $\delta\varphi_0$ close to zero. For φ_0 of the order of 30°, after 5 to 10 turns, particles with different phase have received different energies and are separated radially without successive turns being yet mixed.

(a) The factor 2 gives, in case of betatron motion with uniform distribution in phase space (Kapchinsky Vladimirsky), the true value of half beam size.

(b) The linear part of space charge can be compensated for by a small slope of the RF voltage along the bunch length. This is more easily produced with f.t. than without.

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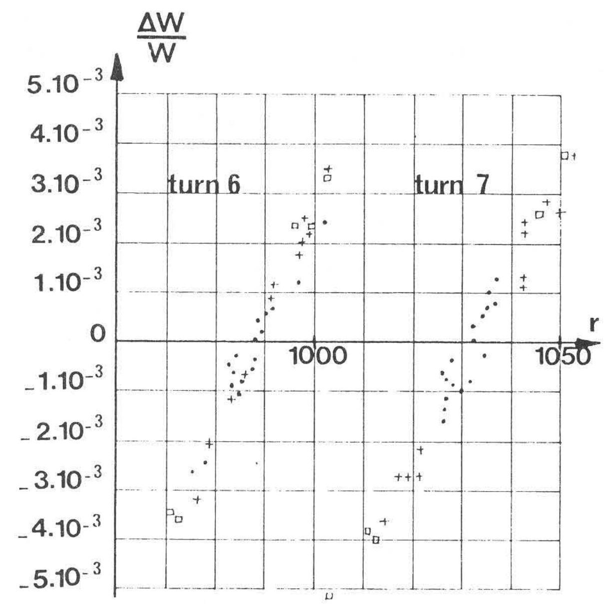

Figure 1 : Simulated r-ΔW distribution of the particles after 6 and 7 turns. At such a place, it is possible to cut down the bunch length from ± 7.5° to ± 3° with a slit or a target between two turns

.	$0 < \Delta\varphi < 2°5$
+	$2°5 < \Delta\varphi < 5°$
□	$5° < \Delta\varphi$

Numerical simulation has shown (see fig.1) that it is then possible with a single slit or target, properly placed, to cut bunches of ±7.5° down to ±3°, at least for cases where turn to turn separation is large. The value of transverse emittance is not an important parameter, provided injection is correctly adjusted with a proper correlation of radius with energy and corresponding phase with angle.

For smaller turn to turn separation or to reach shorter lengths, a second target can be used : it can either be put on the same turn at an opposite azimuth or on the same azimuth a few turns later after half a radial betatron oscillation.

The intensity loss to expect in the case mentioned remains of the order of 50 %. It would increase for a larger reduction in bunch length.

The energy spread obtained at ejection corresponds to (3) or (4).

4) PHASE CONSERVATION IN AN SSC

For a correct isochronism adjustment and orbits well centered (no radial oscillation), the central phase of a bunch is supposed to change regularly with azimuth and the phase width to remain constant.

This is true only for a very small acceleration rate.

4.1. Central phase change during one turn resulting from acceleration

Acceleration being not continuous, the accelerated orbit is not a spiral. Let us consider the case of two cavities per turn and replace the effect of each cavity by a single kick given in the middle. There are several ways to approach such an effect.

In a compact classical cyclotron, it is seen to produce the displacement of the centre of the orbits which twice a turn jumps alternatively from one to the other of two positions.

It can also be described in terms of orbit oscillations: Fig.2 shows how, from one energy kick to the next, the orbit can oscillate around an orbit whose radius is just the average of those at successive gap crossings.

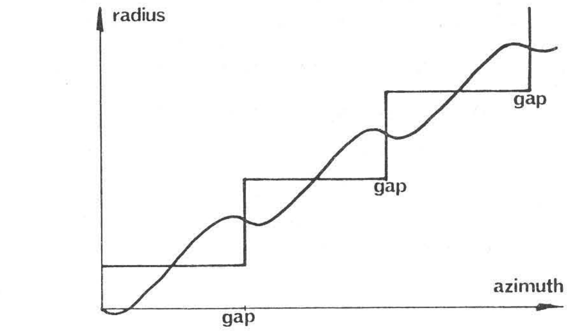

Figure 2 : Accelerated orbit with minimum oscillation in a two gap or two dee geometry.

In a 4 SSC, the unaccelerated orbit has a squared shape ; the effect described is changing it to a lozenge (see fig. 3) (c). The difference in length of the orbits followed in the two sectors crossed between accelerating cavities entails a shift in phase in the middle of the free valley with respect to the average between cavities. Such an effect may greatly affect beam phase measurements, particularly when turn spacing, i.e., relative energy gain, and RF harmonic number are large.

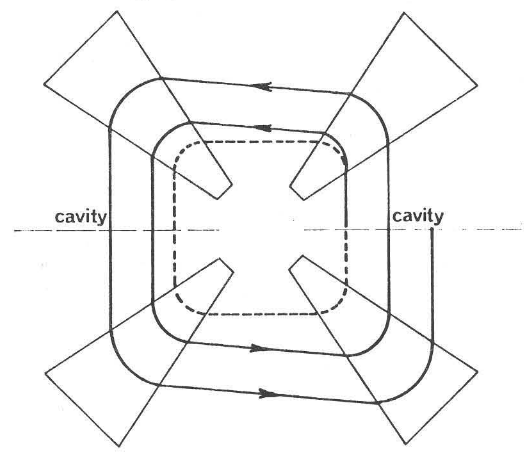

Figure 3 : Lozenge shape of the accelerated orbit in a two dee four sector SSC.

4.2. Bunch length change during one turn

Apart from oscillations resulting from unequal lengths of the bunches in radial and azimuthal directions, other variations are due to acceleration.

Accelerating cavities used in SSC are usually double gap and generally of radial structure. As already mentioned, it is possible to replace the effect of two gaps by a single kick ; this is obtained from a Fourier analysis of the field distribution along the orbit.

The treatment of one complete cavity for a single crossing leads to a Fourier integral in the spectrum of which only the component corresponding to a periodic orbit acts on the particles. A proper choice of the azimuth (the axis of symmetry, if it exists) or a kind of centre of gravity leads to a single term equivalent to a single kick (See note of appendix II).

This kick however is both function of W (or rather wave vector $k = \omega/v$)and radius r. It can be put in the form :

$$\delta W = q\, V_o(r)\, S(r,k)\, \sin\varphi \qquad (6)$$

with

$$S(r,k) = 2 \sin(k\, r\, \mathrm{tg}\, \theta) \qquad (7)$$

(c) This effect is described by S. ADAM as GABA effects (15th European Cyclotron Progress Meeting. Berlin 1978 The Injector II for SIN).

if cavities are radial sectors with a half angle θ and r is the radial coordinate at the mid-valley azimuth. In (6), $qV_o(r)$ is the energy gained at one accelerating gap by a particle of coordinate r as defined above. (The dependance of V_o with k is assumed negligible as well as the coupling with the axial direction z).

With such a field where only the space harmonic Fourier component S is acting on the particles, one has [3], during the crossing of a full cavity assumed concentrated at its centre, a slight phase displacement and a small radial kick given by (*d*) :

$$\delta\varphi_1 = \varphi_o - \varphi_i = -\frac{qV_o}{2W}\, k\, \frac{\partial S}{\partial k} \cos\varphi \qquad (8)$$

$$\frac{\delta(mv_r)}{mv} = \frac{q}{2W}\cdot\frac{1}{k}\cdot\frac{\partial(V_o S)}{\partial r}\cos\varphi\,. \qquad (9)$$

This radial kick moves the orbit from that cavity to the next and changes its length and therefore input phase at this next cavity by an amount (see appendix II).

$$\delta\varphi_2 = kr\,\delta(mv_r)/mv \qquad (10)$$

With the expression (7) for S, one gets:

$$\delta\varphi_1 = -A\,\mathrm{cotg}\,\varphi \qquad (11)$$

with

$$A = \delta W/2W\cdot kr\,\mathrm{tg}\,\theta\,\mathrm{cotg}(kr\,\mathrm{tg}\,\theta)\,. \qquad (12)$$

Adding $\delta\varphi_2$ partly cancels out this term and one only has

$$\delta\varphi_1 + \delta\varphi_2 = \frac{\delta W}{W}\cdot\frac{r}{V_o}\cdot\frac{dV_o}{dr}\cdot\mathrm{cotg}\,\varphi\,. \qquad (13)$$

For a constant voltage along the accelerating gaps, there is a complete cancellation effect such that twice a turn, from cavity centre to cavity centre, isochronism is conserved. In between, however, besides a betatron oscillation which may be due to a mismatch between azimuthal and radial dimensions, the bunch length has a variation of the type shown on figure 4.

The various aspects of this effect have to be taken into account for bunching adjustments. According to (11) it is particularly important when A (defined by (12)) is not small.

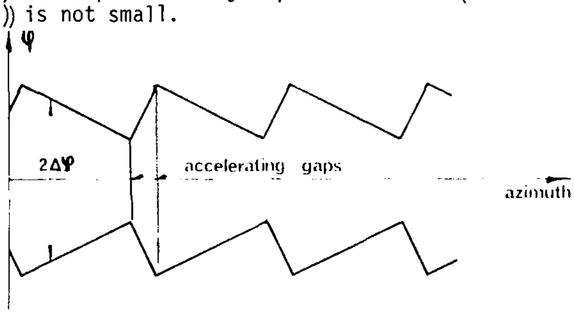

Figure 4 : Bunch length variation around a double gap two dee SSC. The amplitude of variation is proportional to the parameter A (here A = 0.4). The bunches are assumed matched in r,r',φ–space at mid-valley azimuth.

In practice, the situation is still more complicated and even if the previous cancellation is perfect, one should use cavity geometries and harmonic numbers minimizing this term. Besides equations (8) and (9) there are indeed other relations describing all the possible couplings between the four coordinates of radial and azimuthal motion (assuming axial not coupled). In particular, associated with (9), through Maxwell's equations, one has

$$\delta W/W = \delta W_o/W + 2Ak\Delta r \qquad (14)$$

expressing the fact that, for a fixed energy, a cavity of radial shape delivers an acceleration which depends on the radial coordinate (even for a zero voltage gradient), which periodically moves the equivalent orbit.

The motion of a particle in a Δr-Δφ diagram , seen at each turn in a stroboscopic view [3], appears then as shown on fig. 5 on dashed or dotted curves according to the sign of A, the A ≠ 0 case being shown by the solid curve. Final energy spread may be appreciably affected by such an effect, which leads to a growth of phase spread when A is too large (*e*).
In order to avoid any degradation due to this, A should be kept << 1.

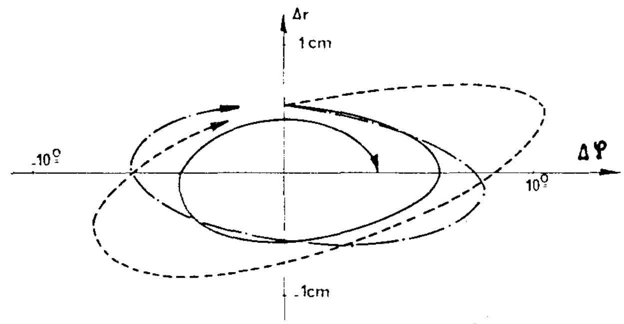

Figure 5 : Motion of a particle in a Δφ-Δr plane obtained from a stroboscopic view at periodic points showing the effect of the parameter A. Here the initial values of A are :
—— A = – 0.03   —.— A = – 0.3   - - - A = 0.5

When the gradient dV/dr is not zero, the term (13) can lead to a phase compression or dilatation.(13) can be written [3]

$$\delta\varphi_1 + \delta\varphi_2 = \frac{r}{2W}\cdot\frac{\partial(\delta W)}{\partial r}\cdot\mathrm{cotg}\,\varphi \qquad (15)$$

from which one gets the invariant initially given by R.W. MULLER, et al. [4]

$$\delta W.\,\sin\Delta\varphi = \mathrm{const}$$

or, coming back to the voltage on the gap and applying the invariant from injection to ejection,

$$V_{o.inj}\,\sin\Delta\varphi_{inj} = V_{o.ej}\,\sin\Delta\varphi_{ej}\,. \qquad (16)$$

Such an effect, which has also been confirmed on numerical simulation, can be used as a very elegant way for phase compression ; practical considerations, however, reduce the extent of what can be obtained from it.

5) DRIFT EFFECTS AND REBUNCHING

In between SSC's, over a rectilinear path of length L, two particles of slightly different velocities ($v \pm \Delta v/2$) slip in phase, one with respect to the other, by an amount :

$$\Delta\varphi = kL\,\Delta v/v\,. \qquad (17)$$

Such a situation can introduce a deterioration of the bunching ; it may also be used, by the introduction of cavities properly placed along the transfer lines, to obtain rebunching, virtually even on a shorter length [5].

One must be aware, however, that relation (17) only holds for a straight transfer. Over a curved path particles of different momentum have different trajec-

d) In case of unequal gaps, the equivalent centre can move in azimuth with r and this results in an additional radial kick term which, as for spiral sector machines, entails a phase delay just equal, in this linear approximation, to the spiralization.

e) This has been described by W.M. SCHULTE with a completely different formalism (15th European Cyclotron Progress Meeting. Berlin 1978 : Radial Beam Behaviour in 2 dee cyclotrons).

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tories ; this fact can appreciably change the phase slip, making it nil, as in an isochronous cyclotron, or, on the contrary, very large, as for some adjustments of high resolution spectrometers.

In order to study phase slip effects in beam transfer systems, one may use matrix formalism which, limited to the coordinates in the plane of the beam (transverse and longitudinal) and assuming a constant velocity transfer, can be written

$$\begin{vmatrix} x \\ x' \\ \Delta\varphi/k \\ \Delta v/v \end{vmatrix}_2 = \begin{vmatrix} a_{11} & a_{12} & 0 & a_{14} \\ a_{21} & a_{22} & 0 & a_{24} \\ a_{31} & a_{32} & 1 & a_{34} \\ 0 & 0 & 0 & 1 \end{vmatrix} * \begin{vmatrix} x \\ x' \\ \Delta\varphi/k \\ \Delta v/v \end{vmatrix}_1 \qquad (18)$$

with the simplectic conditions

$$\left. \begin{aligned} a_{14} &= a_{32} \cdot a_{11} - a_{31} \cdot a_{12} \\ a_{24} &= a_{32} \cdot a_{21} - a_{31} \cdot a_{22} \end{aligned} \right| \qquad (19)$$

and unity determinant

$$a_{11} \cdot a_{22} - a_{12} \cdot a_{21} = 1 \ . \qquad (20)$$

A particularly interesting class of transfer systems is the achromatic case for which

$$a_{14} = a_{24} = 0 \ , \qquad (21)$$

entailing, from (19) and (20),

$$a_{31} = a_{32} = 0 \qquad (22)$$

In this case, it is easy to see that

$$\Delta\varphi_2 = k \, a_{34} \, \Delta v/v \ . \qquad (23)$$

Comparison with (17) shows that a_{34} can then be called the equivalent length of the system.

Achromatic systems possess several other interesting properties. Provided they do not include more than two curved components, their equivalent length is simply the sum of equivalent elements [6]. This is not generally the case. When entering or exiting an SSC, a choice must be made of what kind of matching to achieve in the above coordinates.

Achromatic matching can be made onto an orbit inside the machine. This choice, however, only yields correct results at a given azimuth and with oscillations induced along this orbit for particles of different velocities ; in addition, the length of the transfer depends upon the azimuth.

It is much better, then, to try to ensure chromatic isochronism in the transfer. This is achieved when each particle has a displacement proportional to its momentum or velocity with respect to the reference orbit:

$$\Delta r = r \, \Delta v/v. \qquad (24)$$

In order to make a correct numerical study of this case, one should not forget, however, that such a chromatic transfer should satisfy the simplectic conditions (19). It is then seen [6] that, in addition to (24), there must exist another correlation (*f*),

$$\Delta\varphi = k \, r \, \Delta r' \qquad (25)$$

which is automatically introduced by any physical system.

(f) The necessity of this term was pointed to us by P. YVON.

Numerical simulation has demonstrated a very noticeable improvement on output energy spread when injection is made in this way ; it is then possible to reach the limits given by (3) or (4) even for large emittance beams with high harmonic numbers. One must insist on the real importance of contribution (25) which, especially for high harmonic numbers, is responsible for the improvements obtained, through a better matching.

6) CONCLUSION

In order to reach given energy spread specifications in an SSC complex like GANIL, strict tolerances must be satisfied. In particular, special attention has to be given to phase and bunch length, to the method used to obtain and keep the necessary value, and to procedures for checking the quality of adjustments. Several aspects of these problems have been considered here: the methods proposed, as confirmed by numerical simulation, show very promising prospects provided proper precautions are taken.

The complexity of some effects emphasize nevertheless the importance of finding accurate adjustments from the measurement of the energy spread itself.

7) ACKNOWLEDGEMENTS

Mrs. CARDIN and SAURET have greatly helped in the numerical simulation work ; a positive contribution also came from very fruitful discussions with P. YVON. They are all acknowledged here.

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APPENDIX I

Let us assume a density distribution in phase defined by a truncated gaussian law :

$$d = d_o\, e^{-\varphi^2/a^2} \quad \text{for} \quad -b < \varphi < +b$$

According to (1), one has

$$\Delta\varphi^2 = 4\int_{-b}^{+b} \varphi^2 e^{-\varphi^2/a^2} d\varphi \Big/ \int_{-b}^{+b} e^{-\varphi^2/a^2} d\varphi$$

If the relative energy gain of the particles is given by the cosine of their phase with respect to the RF phase $-\delta\varphi_o$, one has for the mean energy in the bunch, with respect to the peak,

$$\frac{\overline{\delta W}}{W} = \frac{1}{2}\, \overline{(\varphi - \delta\varphi_o)^2}$$

and according to (2)

$$\left(\frac{\Delta W}{W}\right)^2 = \left\{ \overline{(\varphi - \delta\varphi_o)^4} - \overline{(\varphi - \delta\varphi_o)^2}^{\,2} \right\}.$$

Straightforward but tedious computations give

$$\frac{\overline{\delta W}}{W} = \frac{1}{2}\left\{ \frac{\Delta\varphi^2}{4} + \delta\varphi_o^2 \right\}$$

and

$$\left(\frac{\Delta W}{W}\right)^2 = \frac{\Delta\varphi^4}{16} f^2\left(\frac{b}{a}\right) + \Delta\varphi^2\, \delta\varphi_o^2$$

with

$$f^2\left(\frac{b}{a}\right) = \frac{4b^2}{\Delta\varphi^2}\left\{1 - \frac{2a^2}{\Delta\varphi^2}\right\} + \frac{6a^2}{\Delta\varphi^2} - 1 .$$

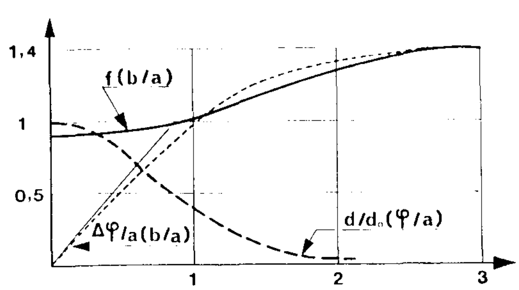
Assuming $\delta\varphi_o \ll \Delta\varphi$ the following expression can be used :

$$\frac{\Delta W}{W} = \frac{\Delta\varphi^2}{4} f\left(\frac{b}{a}\right) + 2\,\delta\varphi_o^2 \Big/ f\left(\frac{b}{a}\right).$$

Numerical values obtained from these various relations are given on the figure as functions of b/a or φ/a.

One sees that for b/a of the order or 1 or 1.5, $f(b/a)$ is not far from 1.
One may add that a similar computation for a parabolic distribution gives an expression equivalent to $f(b/a) = \sqrt{8/7}$, while a uniform distribution corresponds to b/a = 0, i.e., $f(b/a) = \sqrt{4/5}$
For f = 1 one has

$$\Delta W / W = \Delta\varphi^2/4 + 2\,\delta\varphi_o^2$$



APPENDIX II

Reference [3] describes the formalism *(a)* used to represent the action of a two gap radial dee by a single kick applied at its centre.

The derivation of the phase slip from one cavity to the next resulting from the radial kick received at dee crossing is, however, slightly incorrect because an average expression has been used for the betatron function.

Let us consider accurately a hard edge SSC configuration with half valley angle α (sector angle $\pi/2 - 2\alpha$); for a mid-valley radial coordinate r, a radius of curvature ρ in the sector, with

$$\rho = r\,(1 - tg\,\alpha)$$

two half trajectories having a radial displacement δ at mid-free-valleys cross in accelerating valleys with an angle

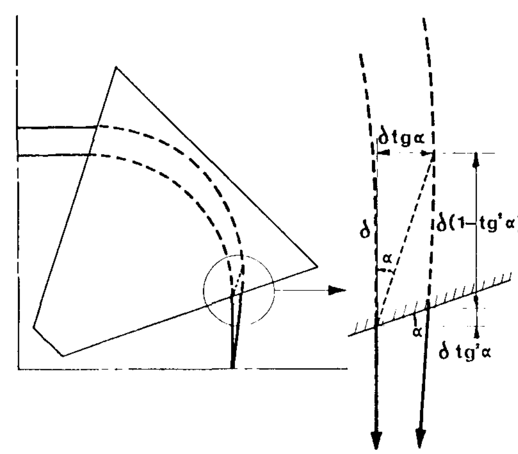
$$2\,\frac{\delta}{\rho}\,(1 - tg^2\alpha)$$

and differ in length from the closed orbit by (see the figure)

$$2\,\delta\,(1 + tg\,\alpha)$$

A simple check on relations (8),(9) and (10) shows that, provided V_o is constant in r,

$$\delta\varphi_1 + \delta\varphi_2 = 0 .$$



This has been confirmed on the simulation computations for the soft edge case of GANIL.

Such a property is then valid for an SSC ; it is also true for a classical cyclotron and, as already mentioned in the note of section 4.2., for a spiral gap shape. It is probably true for any shape either outside or inside a magnetic field as long as linear approximation is valid and curvature in the gaps may be neglected ; no general proof of this property is known, however.

(a) The principles leading to it can be found in : Accurate beam dynamics equations in proton linear accelerators, by A. Carne, P. Lapostolle, M. Prome, 5th Int. Conf. on High Energy Accelerators FRASCATI 1965 pp 656. 662.

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** DISCUSSION **

W. JOHO: In the new injector for SIN we have a very large energy gain per turn compared with the initial injection energy of only 800 kV. Therefore, this effect of diamond-shaped orbits is even more pronounced in our case. Since the beam cuts the edge angle of the magnet differently in the even and odd sectors, the focusing does not have four-fold symmetry. Therefore, we plan to make the even sectors different from the odd sectors, so that the orbit and not the magnets has four-fold symmetry. Do you plan something similar for GANIL?

P. LAPOSTOLLE: No, we don't. In our case, having different acceleration harmonics with different turn separation, it would be difficult to make a correction good for all cases. We have managed to choose parameters so that the effect is reasonably small. So, apart from complications in measurements, this particular effect does no harm to the beam quality in our case.

W. SCHULTE: The correlation between high frequency phase and radial momentum at the injection point in the cyclotron, as you point out, is equivalent to dispersion matching at that point. This point was discussed by Hinderer two years ago at the ECPM. Secondly, there is a definition of central position phase which will facilitate the description of the accelerated particles. A poster contribution deals with this definition and with the injection problem.

P. LAPOSTOLLE: Thank you for your comments. Though using very different approaches, our conclusions agree. What you find from your Hamiltonian reduced variables I have obtained in actual laboratory coordinates.

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