

SIX-DIMENSIONAL BEAM MATCHING FOR  
AXIAL INJECTION INTO A CYCLOTRON

A.R. Beck, S. Chel, B. Bru and Ch. Ricaud  
GANIL, BP 5027, 14021 CAEN Cedex, France

ABSTRACT

The general optical structure of a beam line for axial injection into a cyclotron is proposed. It provides the beam matching in the six-dimensional phase space at the entrance of the cyclotron. As an illustration, the hyperboloid and the spiral inflectors are considered.

1. INTRODUCTION

The different kinds of existing inflectors for axial injection into cyclotrons generate couplings between the three phase planes. These couplings have to be controlled in order to get the best yield of injection in the six-dimensional phase space and therefore, to ensure the optimum transmission in the cyclotron.

The correlations required by the dynamics in the cyclotron can be obtained if the beam line associated with the inflector is designed in order to provide the corresponding correlations at entrance of the inflector.

After listing the required optical functions of the beam line, a general structure is proposed in view of easy optimization of its parameters.

The assumption is that the beam has no time structure and no betatron nor chromatic correlations at the beginning of the beam line.

As an illustration, we apply the described method to two types of inflectors, hyperboloid and spiral inflectors, that are studied in our laboratory.

Due to the low energy (a few keV/a.m.u.), space charge effect has often to be taken into account. A solution is proposed.

2. PRELIMINARY REMARKS

Any kind of axially injecting inflector always generates coupling between the vertical and the longitudinal phase planes. An example is the hyperboloidal inflector. In addition, some other types, like the spiral inflector, also generate a betatron and a chromatic coupling between the two transverse phase planes, thus leading to a coupling between the longitudinal and the

transverse phase planes.

The chromatic correlation generated by the inflector, or required by the dynamics in the cyclotron, have an opposed effect on the bunching efficiency. The influence depends on the transverse emittance value and on the phase acceptance in the cyclotron.

Now the questions are: 1) can the coupling effect generated by an inflector be controlled by classical optical elements (dipoles, quadrupoles, solenoids)? and 2) is a specific computer code necessary to optimize the parameters of the beam line including the inflector?

In the case of an ideal inflector (hard edge approximation), if the usual variables (canonically conjugated in the linear approximation of dipoles, quadrupoles and solenoids) are used, the 6 x 6 transfer matrix of the inflector is a canonical transformation. The symplectic conditions are verified. This does not necessarily imply that these variables are canonically conjugated inside the inflector.

However, a real axially injecting inflector has electric stray fields at entrance and exit. On the other hand the magnetic stray field in the hole of the yoke extends far upstream the inflector. The linear transformation of a real inflector is only canonical if its entrance is chosen in the region where the stray field is small and if its exit is chosen in the region where the velocity vector of the central particle is in the median plane of the cyclotron.

The fact that this transformation, including the inflector, exists, has the following consequence: a classical optical device consisting of dipoles, quadrupoles and solenoids, represented in the same variables by canonical transformations, can be associated with the inflector, in order to control its coupling effect.

If the transfer matrix of the inflector is known, the structure of the beam line can be defined and the parameters optimized with a classical beam optics code, in order to match the six-dimensional beam ellipsoid inside the cyclotron.

### 3. STRUCTURE OF THE BEAM LINE

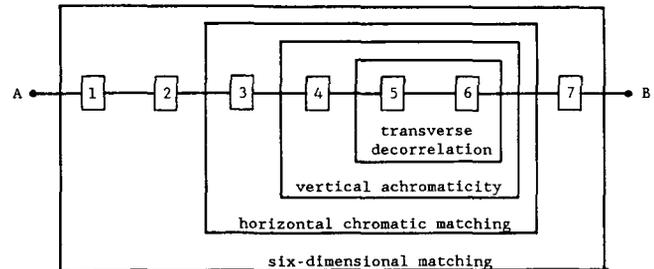
The optical functions of any kind of axially injecting beam line, the strictly necessary number of tuning parameters, and the required optical elements, are given in the following chart:

Optical function (matching inside cyclotron)	Number of parameters	Optical elements
Horizontal and vertical matching	4	4 quadrupoles
Horizontal chromatic matching	2	1 dipole 2 quadrupoles
Vertical chromatic matching	2	1 dipole 2 quadrupoles
Correlation matching between transverse phase planes	4	2 skew quadrupoles

This chart has to be completed with the following remarks:

- The buncher(s) must be located ahead of the chromatic matching structure that controls the phase terms depending on the betatronic conditions (even if the energy dispersion is small).
- It can be useful to add one quadrupole, especially in the chromatic sections, in order to reduce the beam dimensions.
- It can be useful to add a focusing element (for example an electrostatic quadrupole) between the inflector and the matching point inside the cyclotron, in order to reduce the beam dimensions at exit of the inflector. A rotated edge at the exit of the inflector can also be useful.
- If two skew quadrupoles are strictly needed to control the correlations between the two transverse phase planes, an additional skew quadrupole relaxes the constraint of the relative position of the quadrupoles and provides smaller beam dimensions.
- The chromatic planes of the two chromatic matching sections must be orthogonal. As a consequence, at least one section provides a 90° deviation.
- The angular orientation around the cyclotron axis should be chosen in the following way: if the transverse phase planes are uncorrelated at the matching point in the cyclotron, the chromatic plane of the cyclotron must correspond to the chromatic plane of the first chromatic matching section. If this orientation cannot be respected, a solenoid providing the required rotation can be put in the beam line.

If the number of tuning parameters is an important question, it is not less important to arrange the order of the optical section in view of getting easy calculation of the parameters and easy tuning of the beam line. The order of arrangement and the optical functions are given in the following diagram:



- A - entrance (transverse decorrelated beam, no time structure)
- B - matching point inside the cyclotron
- 1 - transverse betatron matching device
- 2 - buncher(s)
- 3 - horizontal chromatic matching device
- 4 - vertical chromatic matching device
- 5 - horizontal/vertical correlation device
- 6 - inflector
- 7 - inside cyclotron, from inflector exit to matching point B.

### 4. AXIAL INJECTION WITH THE HYPERBOLOID INFLECTOR

In the previous canonical conjugated variables, the 6 X 6 transfer matrix of the inflector in the hard-edge approximation can be obtained analytically (1, 2):

$$\begin{pmatrix}
 0 & 0 & 0 & r/2 & 0 & 0 \\
 0 & 0 & -2/r & 0 & 0 & 0 \\
 1 & 0 & 0 & 0 & 0 & -r/2 \\
 0 & 1 & 0 & 0 & 0 & 0 \\
 0 & -r/2 & 0 & 0 & 1 & \pi r/4 \\
 0 & 0 & 0 & 0 & 0 & 1
 \end{pmatrix}$$

where  $r = 2\sqrt{6} R$  and  $R =$  magnetic injection radius.

This matrix is symplectic. It expresses the transformation between the entry of the inflector, where the radial direction is colinear to the electric field and its exit, where the radial direction is in the median plane of the cyclotron.

The term  $t_{52}$  is not equal to zero: this means that the inflector generates coupling between the vertical phase plane and the longitudinal one. In order to get at the exit of the inflector an achromatic beam, i.e. betatronic isochronism, only sections 1, 2 and 4 are required. However, if the matching point is located downstream the inflector exit, or if horizontal chromatic correlations have to be matched, section 3 is also required.

5. AXIAL INJECTION WITH THE SPIRAL INFLECTOR

As an example the inflector studied in the frame of GANIL's intensity upgrade project (3), the spiral inflector (4, 5, 7) is considered. No analytical transfer matrix could be found. It has been calculated numerically with the previous canonically conjugate variables. The symplectic conditions are verified with a good approximation.

-0.379729	+0.045822	+0.295503	-0.047215	0.	+0.048322
-7.032999	-1.402020	+4.901125	-0.291224	0.	-0.716805
-0.308074	-0.091233	-1.217808	+0.029571	0.	-0.232523
-3.177977	-1.412930	-11.857491	-0.413866	0.	-1.858233
-0.445569	+0.124100	+0.942818	+0.103268	1.	+0.061937
0.	0.	0.	0.	0.	1.

The couplings between the three phase planes generated by the inflector can be controlled if the beam line comprises all the optical matching devices mentioned in paragraph 3. Figure 1 shows a general layout of these sections. The beam line parameters have been optimized with our GALOPR code (6). Figure 2 shows the corresponding beam envelopes in the case of a chromatic matching.

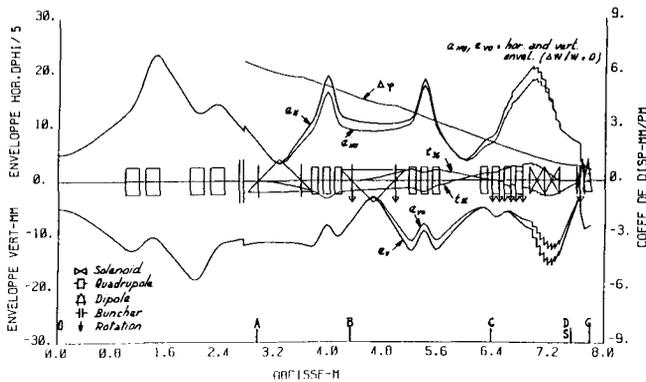


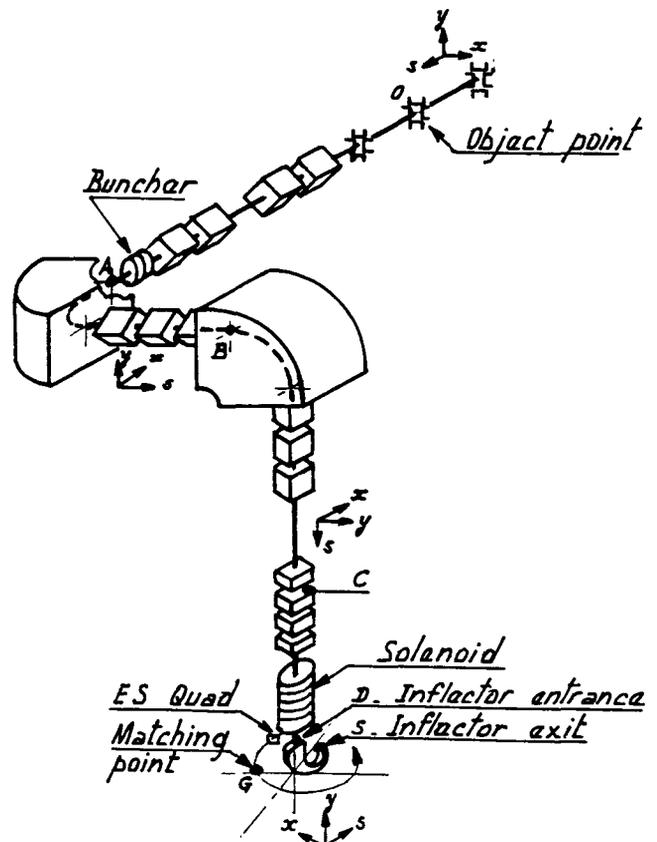
Figure 2. Beam envelopes.

6. SPACE CHARGE

With the beam line structure proposed in paragraph 3, the beam can still be matched in the six-dimensional phase space, if the space charge effects are taken into account. However, if the longitudinal space charge force becomes too important, the beam line has to be modified: the buncher(s) must be installed as downstream as possible i.e. at the entrance of the cyclotron yoke hole. Consequently, the energy width of the injected beam becomes larger and the chromatic matching sections are not required anymore. But a better solution consists in adding to the structure proposed in paragraph 3 a rebuncher, located at the entrance of the cyclotron yoke hole. The buncher(s) must then be tuned in such a way that the phase extension of the bucket coming in the rebuncher does not exceed  $\pm 45^\circ$ .

REFERENCES

- (1) Muller, R. W., "Novel inflectors for cyclic accelerators", NIM 54, (1967).
- (2) Beck, R., Bru, B. and Ricaud, Ch., "L'inflecteur hyperboloïdal de R.W.Muller," GANIL report 86N/TF01 and 86N/TF02, (1986).
- (3) Fermé, J., "Present and future development at GANIL," GANIL report A88/06, (1988).
- (4) Belmont, J. L. and Pabot, J. L., "Study of axial injection for the Grenoble Cyclotron," IEEE Trans. Nucl. Sc. NS13, 191, (1966).
- (5) Chel, S., "Etude de l'injection axiale d'un faisceau d'ions lourds dans un cyclotron compact," Thèse soutenue à Paris 7 (1988).
- (6) Bru, B., "GALOPR, a beam transport program with space charge and bunching," in Proceedings of the 1st EPAC, Rome, 1988.
- (7) Attal, P., Baron, E., Beck, R., Bony, J., Bourgarel, M. P., Bru, B., Chel, S. and Ricaud, Ch., "Design of a new high intensity injection system for GANIL", presented at this conference.



Caption: OA = transverse betatron matching  
 AB = horizontal chromatic matching  
 BC = vertical chromatic matching  
 CD = transverse correlation matching  
 DG = inflector + el.-stat. quadrupole

Figure 1. General layout of the beam line.