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Mohamed Mahjoub, Ahmed Rouabhi, Michel Tijani, Sylvie Granet

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M. MAHJOUB[1], A. ROUABHI[1], M. TIJANI[1] and S. GRANET[2]

[1]Mines ParisTech, PSL Research University, Centre de géosciences. Fontainebleau, France
[2]EDF - R&D. AMA - Groupe mécanique non linéaire. Clamart, France

Abstract

In a nuclear disposal project, the damage of the hosting rock is a capital issue that should be studied and understood. This problem is even more complex when the studied rock is anisotropic. In the present paper, a damage model that takes into account both initial and induced anisotropy is introduced using equivalence relations between the real material and a fictitious isotropic one on which we can take all the advantages of the well established isotropic theory. Numerical simulations using a Finite Element Method (FEM) code shows an agreement between the theoretical predictions and the experimental data of Brazilian tests with different orientation angles.

1 Introduction

In rock engineering, anisotropy is an important aspect that has to be taken into account. This material feature has been a long-standing issue in geomechanics. Multiple studies are interested in this problem since the early developmental stages of rock mechanics [1–10]. Recently, new applications such as nuclear waste disposal in argillaceous formations and shale gas exploitation increased the interest to characterize these materials [11–27]. Rocks anisotropy may be caused by mineral foliation, stratification or discontinuities. For the sake of simplification, it is generally admitted that the elastic behavior of sedimentary rocks like shales and schists can be reasonably described by transverse isotropy characterized by five independent parameters. It has been proven that the assumption of complete symmetry (isotropy) for this kind of materials may induce large errors in the measured in situ stress field [2].

Numerous studies attempted to model the mechanical behavior of transversely isotropic materials. Multiple failure criteria were developed and compared to experimental results of triaxial compression tests [11–15]. Rouabhi et al. [13] proposed a viscoplastic constitutive model in order to characterize triaxial creep tests. On the other hand, a crack propagation method was employed to define anisotropic damage behavior laws for transversely isotropic materials (*cf.* [28, chapter 4 and 5] and [29, chapter 5 and 6]). However, a general methodology to introduce such a model is not yet available.

In order to define a general procedure to produce mechanical behavior models for transversely isotropic materials, the present work proposes an approach to adapt existing isotropic models. This method consists in a three steps process. First, we introduce a fictive equivalent isotropic material. Second, an isotropic model is chosen to characterize the new material. Finally, equivalence relations are used to adapt this model to the real material. A transformation between the behavior of the real anisotropic material and that of an isotropic fictitious one is introduced. This idea is proposed by [30] to simulate the non-linear constitutive behavior of composite materials and used in [13] to define a viscoplastic behavior law and two failure criteria. The respect of the material symmetry is insured by the application of the representation theorem (*cf.* Truesdell and Noll [31, part B]).

As an application of this approach, a damage constitutive model taking into account both initial and induced anisotropy is introduced. This model is based on the works [32–34] and [35, chapter 2] where a damage second order tensor is considered. The definition and evolution of the damage is established for the fictive isotropic material and afterward transferred to the real material. Numerical simulations of the Brazilian test for two rocks using this behavior law are presented and compared to experimental data from [16].

The constitutive law proposed in this paper is the cornerstone of a more complete model coupling the mechanical damage with the fluids flow and the heat transfer in the material. Nevertheless, for the sake of clarity, in this paper, the effective stress in the poromechanics sens and the mechanical strain tensors will be simply called stress and strain tensors. Moreover, the transformation from the real material to the fictitious isotropic one concerns only the mechanical behavior and not the other material properties such as the permeability $\underline{\underline{K}}$ and the coefficient of thermal expansion $\underline{\underline{\alpha}}$...

This paper is organized as follows. Section 2 presents the approach of equivalent isotropic material and the necessary relations. In section 3, this procedure is applied to define an anisotropic damage behavior law for transversely isotropic materials. The model parameters are fitted using measures of the tensile strength.

Afterward, the behavior law is used to simulate 3D Brazilian tests with different orientation angles and the results are compared to experimental data.

Notations. *Throughout this paper, a first order tensor (vector) is designed by an arrow over an undercase letter* $(\vec{a})$*, a second order one is denoted by two lines under a letter* $(\underline{\underline{a}})$ *and a fourth order tensor is designed by a tilda beneath a capital letter* $(\underset{\sim}{A})$*. Superscript '* ^{t} *' indicates the transpose operation, while 'tr' is the trace operator. Symbol ':' denotes the product with double contraction, e.g.* $\underline{\underline{a}} : \underline{\underline{b}} = a_{ij}b_{ji}$*, where the index denotes cartesian components and repeated subscript imply summation unless otherwise indicated. The product* $\underline{\underline{a}}\,\underline{\underline{b}}$ *denotes a single contraction, i.e.* $\left(\underline{\underline{a}}\,\underline{\underline{b}}\right)_{ij} = a_{ik}b_{kj}$*. In the same way, we have* $\left(\underset{\sim}{A}\underset{\sim}{B}\right)_{ijkl} = A_{ijmn}B_{nmkl}$*. The dyadic or the tensor product is so* $\left(\underline{\underline{a}} \otimes \underline{\underline{b}}\right)_{ijkl} = a_{ij}b_{kl}$ *and* $(\vec{a} \otimes \vec{b})_{ij} = a_i b_j$*, whereas '* $\overline{\otimes}$ *' denotes the symmetrized dyadic product defined as* $\left(\underline{\underline{a}}\,\overline{\otimes}\,\underline{\underline{b}}\right)_{ijkl} = \frac{1}{2}\left(a_{ik}b_{jl} + a_{il}b_{jk}\right)$*. The second-order* $\underline{\underline{1}}$ *and the fourth-order* $\underset{\sim}{1}$ *are identity tensors.*

2 Equivalent fictive isotropic material approach

Given a transversely isotropic material where the stress field $\underline{\underline{\sigma}}$ and strain field $\underline{\underline{\varepsilon}}$ are linked by the transversely isotropic Hooke tensor $\underset{\sim}{H}(E, E', \nu, \nu', G)$, we define a fictive isotropic material where the stress $\underline{\underline{\bar{\sigma}}}$ and strain $\underline{\underline{\bar{\varepsilon}}}$ are linked by an isotropic tensor $\underset{\sim}{\bar{H}}$ associated to the fictive elastic parameters $(\bar{E}, \bar{\nu})$. The real and the fictitious stress are related by a second order tensorial transformation function $\underline{\underline{\ell}}$:

$$\underline{\underline{\bar{\sigma}}} = \underline{\underline{\ell}}(\underline{\underline{\sigma}}) \tag{1}$$

This function must respect material symmetry. In this work and in order to go further and allow analytical developments, $\underline{\underline{\ell}}$ is assumed to be linear. Therefore, this transformation may be represented by a fourth order transversely isotropic tensor $\underset{\sim}{L}$:

$$\underline{\underline{\bar{\sigma}}} = \underset{\sim}{L} : \underline{\underline{\sigma}} \tag{2}$$

In order to relate the real and fictive strain, we use the restrictive relation $\underline{\underline{\bar{\sigma}}} : \underline{\underline{\bar{\varepsilon}}} = \underline{\underline{\sigma}} : \underline{\underline{\varepsilon}}$ which leads to:

$$\underline{\underline{\varepsilon}} = \underset{\sim}{L} : \underline{\underline{\bar{\varepsilon}}} \tag{3}$$

Combining equation 2 and 3 with the fact that $\underline{\underline{\bar{\sigma}}} = \underset{\sim}{\bar{H}} : \underline{\underline{\bar{\varepsilon}}}$, we deduce that:

$$\underset{\sim}{\bar{H}} = \underset{\sim}{L}\underset{\sim}{H}\underset{\sim}{L} \tag{4}$$

In order to drive the expression of $\underset{\sim}{L}$ as a function of $\underset{\sim}{H}$ and $\underset{\sim}{\bar{H}}$, let's start by solving the general problem $\underset{\sim}{A} = \underset{\sim}{L}\underset{\sim}{B}\underset{\sim}{L}$ for any positive definite tensors $\underset{\sim}{A}$, $\underset{\sim}{B}$ and $\underset{\sim}{L}$. Multiplying this equation by $\underset{\sim}{B}^{\frac{1}{2}}$ on the left and on the right, one gets $\underset{\sim}{B}^{\frac{1}{2}}\underset{\sim}{A}\underset{\sim}{B}^{\frac{1}{2}} = \underset{\sim}{B}^{\frac{1}{2}}\underset{\sim}{L}\underset{\sim}{B}\underset{\sim}{L}\underset{\sim}{B}^{\frac{1}{2}}$. By writing $\underset{\sim}{B} = \underset{\sim}{B}^{\frac{1}{2}}\underset{\sim}{B}^{\frac{1}{2}}$, we obtain $\left(\underset{\sim}{B}^{\frac{1}{2}}\underset{\sim}{L}\underset{\sim}{B}^{\frac{1}{2}}\right)^2 = \underset{\sim}{B}^{\frac{1}{2}}\underset{\sim}{A}\underset{\sim}{B}^{\frac{1}{2}}$. Finally, we find:

$$\underset{\sim}{L} = \underset{\sim}{B}^{-\frac{1}{2}}\left(\underset{\sim}{B}^{\frac{1}{2}}\underset{\sim}{A}\underset{\sim}{B}^{\frac{1}{2}}\right)^{\frac{1}{2}}\underset{\sim}{B}^{-\frac{1}{2}} \tag{5}$$

Writing the equation 4 in the form $\underset{\sim}{H}^{-1} = \underset{\sim}{L}\underset{\sim}{\bar{H}}^{-1}\underset{\sim}{L}$, replacing $\underset{\sim}{A}$ by the transversely isotropic tensor $\underset{\sim}{H}^{-1}$ and $\underset{\sim}{B}$ by the isotropic tensor $\underset{\sim}{\bar{H}}^{-1}$ and using the basis $(\underset{\sim}{I}, \underset{\sim}{J}_1, \underset{\sim}{J}_2, \underset{\sim}{J}_3, \underset{\sim}{J}_4, \underset{\sim}{J}_5)$, we obtain:

$$[L] = [\bar{H}^{\frac{1}{2}}]\left([\bar{H}^{-\frac{1}{2}}][H^{-1}][\bar{H}^{-\frac{1}{2}}]\right)^{\frac{1}{2}}[\bar{H}^{\frac{1}{2}}], \quad \hat{\ell}_4 = \sqrt{\hat{h}_4/\hat{\bar{h}}_4}, \quad \hat{\ell}_5 = \sqrt{\hat{h}_5/\hat{\bar{h}}_5} \tag{6}$$

We finally get the following expressions of $\hat{\ell}_i$ as function of (E, E', ν, ν', G) and $(\bar{E}, \bar{\nu})$:

$$\hat{\ell}_1 = \sqrt{\bar{E}}.\frac{\hat{h}_1 + \sqrt{(\bar{b}/\bar{a})}\gamma}{\Delta}, \quad \hat{\ell}_2 = \sqrt{\bar{E}}.\frac{\hat{h}_2}{\Delta}, \quad \hat{\ell}_3 = \sqrt{\bar{E}}.\frac{\hat{h}_3 + \sqrt{(\bar{a}/\bar{b})}\gamma}{\Delta}, \quad \hat{\ell}_4 = \sqrt{\bar{E}}.\sqrt{\frac{\hat{h}_4}{\bar{b}}}, \quad \hat{\ell}_5 = \sqrt{\bar{E}}.\sqrt{\frac{\hat{h}_5}{\bar{b}}} \tag{7}$$

where $\bar{a} = 1 - 2\bar{\nu}$, $\quad \bar{b} = 1 + \bar{\nu}$, $\quad \gamma = \sqrt{\hat{h}_1\hat{h}_3 - \hat{h}_2^2}$, $\Delta = \sqrt{\bar{a}\hat{h}_1 + \bar{b}\hat{h}_3 + 2\sqrt{\bar{a}\bar{b}}\gamma}$ and

$$\begin{aligned} \hat{h}_1 &= \frac{1}{3}\left[2(1-\nu)/E + (1-4\nu')/E'\right], \quad \hat{h}_2 = \frac{\sqrt{2}}{3}\left[(1-\nu)/E - (1-\nu')/E'\right], \\ \hat{h}_3 &= \frac{1}{3}\left[(1-\nu)/E + 2(1+2\nu')/E'\right], \qquad \hat{h}_4 = (1+\nu)/E \quad \hat{h}_5 = 1/2G \end{aligned} \tag{8}$$

$$\hat{\bar{h}}_1 = (1-2\bar{\nu})/\bar{E}, \quad \hat{\bar{h}}_2 = 0, \quad \hat{\bar{h}}_3 = \hat{\bar{h}}_4 = \hat{\bar{h}}_5 = (1+\bar{\nu})/\bar{E} \tag{9}$$

For a given elastic tensor $\underset{\sim}{H}$, we construct an infinity of isotropic equivalent materials each one is defined by the couple $(\bar{E}, \bar{\nu})$. These parameters may be fixed for instance in such a way that a chosen distance between $\underset{\sim}{H}$ and $\underset{\sim}{\bar{H}}$ be minimized but we prefer to keep free $(\bar{E}, \bar{\nu})$ to take into account, as best as possible, the observed independence between the elastic behavior and damage for instance.

3 *Anisotropic damage behavior law for transversely isotropic materials*

J. Lemaitre [36, chapter 1] defined the mechanical damage as the creation and growth of discontinuities, microvoids and microcracks, in a medium considered as continuous at a large scale. In engineering, the damage is represented by continuous variables in the space. In fact, the mechanics of continuous media introduces a Representative Volume Element (RVE) on which all properties are represented by homogenized variables. Mathematically, the damage variable may be represented either by a scalar, a higher order tensor or a combination of two or more scalar and tensorial variables if several mechanisms of damage occur. In order to take into account the directional bias introduced by the formation of cracks, an anisotropic damage model is to be considered *i.e.* the damage is represented by a higher order tensor variable. In this case, the largest generality for a damage variable is a representation by a fourth order tensor. However, such a representation is too difficult to use. Therefore, second order symmetric tensors are commonly employed since they are mathematically simpler and yet can describe most essential features of anisotropic damage [32–38]. In general numerical codes, what is called "damage-model" consists principally on a linear relationship between stress and strain with an elastic tensor $\underset{\sim}{H}$ which varies as a function of a damage variable. One can notice that this formalism is commonly used to describe the damage of initially isotropic solids. To the best of our knowledge, an anisotropic damage model for transversely isotropic materials using the mechanics of continuous media is not available.

In order to define an anisotropic damage behavior law for the transversely isotropic material, we will use the equivalent isotropic material approach defined in the previous section. Further, we assume that the relationship between the real and fictive materials $(\underset{\sim}{\bar{H}} = \underset{\sim}{L}\underset{\sim}{H}\underset{\sim}{L})$ remains valid at every stage of the damage but with a fixed $\underset{\sim}{L}$. This assumption means that we consider the following relationship between the real and the fictive materials:

$$\underset{\sim}{H}^{-1}(\underline{\underline{\omega}}) = \underset{\sim}{L}\underset{\sim}{\bar{H}}^{-1}(\underline{\underline{\omega}})\underset{\sim}{L} \tag{10}$$

where $\underline{\underline{\omega}}$ is the damage second order tensor. This may be interpreted as a separation between the effect of natural anisotropy represented by the tensor $\underset{\sim}{L}$ and the induced anisotropy represented by the damage tensor $\underline{\underline{\omega}}$.

It remains now to define a damage behavior law for the fictitious isotropic material $\left(\underset{\sim}{\bar{H}}(\underline{\underline{\omega}})\right)$. An established constitutive model for isotropic materials may be employed. We select here the damage behavior law introduced by Rouabhi et al. [32, 35]. The developed model is based on the so called, in damage mechanics, effective stress $\underline{\underline{\tilde{\sigma}}} = \underline{\underline{\omega}}^{\frac{1}{2}}\,\underline{\underline{\bar{\sigma}}}\,\underline{\underline{\omega}}^{\frac{1}{2}}$ and strain $\underline{\underline{\tilde{\varepsilon}}} = \underline{\underline{\omega}}^{-\frac{1}{2}}\,\underline{\underline{\bar{\varepsilon}}}\,\underline{\underline{\omega}}^{-\frac{1}{2}}$ tensors where $\underline{\underline{\bar{\sigma}}}$ and $\underline{\underline{\bar{\varepsilon}}}$ denote the stress and strain tensors in the damaged isotropic material. The compliance tensor of the fictitious damaged material is given by the following expression:

$$\underset{\sim}{\bar{H}}^{-1}(\underline{\underline{\omega}}) = -\frac{\bar{\nu}}{\bar{E}}\underline{\underline{\omega}} \otimes \underline{\underline{\omega}} + \frac{1+\bar{\nu}}{\bar{E}}\underline{\underline{\omega}} \mathbin{\overline{\underline{\otimes}}} \underline{\underline{\omega}} \tag{11}$$

The rate-dependent evolution law of the damage variable $\underline{\underline{\omega}}$ is given by:

$$\underline{\underline{\dot{\Lambda}}} = 2\underline{\underline{\omega}}^{-\frac{1}{2}}\underline{\underline{\dot{\omega}}}\,\underline{\underline{\omega}}^{-\frac{1}{2}} = \sum_{k=1}^{3}\dot{\lambda}(a_k)\tilde{\vec{n}}_k \otimes \tilde{\vec{n}}_k, \quad \dot{\lambda}(x) = v_d\left[1 - exp\left(-\left\langle\frac{x}{R_d} - 1\right\rangle^{n_d}\right)\right] \tag{12}$$

where a_k and $\tilde{\vec{n}}_k$ are respectively the eigenvalues and the eigenvectors of the tensor $\underline{\underline{a}} = \dfrac{1}{2}\left\langle\underline{\underline{\tilde{\sigma}}}\right\rangle\left\langle\underline{\underline{\tilde{\varepsilon}}}\right\rangle$, v_d, R_d and n_d are material parameters and $\left\langle\underline{\underline{x}}\right\rangle$ denotes the positive part of the tensor $\underline{\underline{x}}$. It's the unique tensor having the same eigenvectors as $\underline{\underline{x}}$ and whose eigenvalues are deduced by the operator $\langle x_i\rangle = (x_i + |x_i|)/2$.

Finally, in order to distinguish the different response in tension and compression, an active damage tensor was introduced as follows:

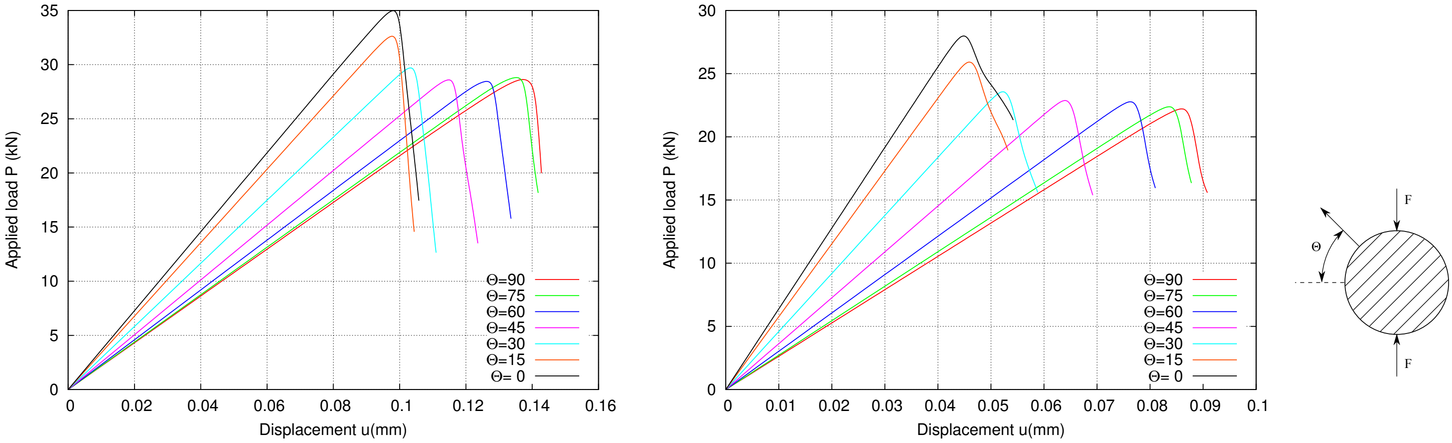
$$\underline{\underline{\omega}}^{ac} = \underline{\underline{1}} + \underline{\underline{P}}(\underline{\underline{\omega}} - \underline{\underline{1}})\underline{\underline{P}}, \quad \underline{\underline{P}} = \sum_{k=1}^{3}\mathcal{H}(\bar{\varepsilon}_k)\vec{n}_k \otimes \vec{n}_k \tag{13}$$

where $\bar{\varepsilon}_k$ and $\vec{n}_k$ are the eigenvalues and eigenvectors of $\underline{\underline{\bar{\varepsilon}}}$ respectively and $\mathcal{H}$ is the Heaviside function.

3.1 Simulation of Brazilian tests

Indirect tensile tests such as Brazilian tests are very useful in rock mechanics since it is too difficult to carry out direct tensile tests. It is generally used to find the tensile strength for isotropic materials since its value could be directly estimated knowing the ultimate applied load (P) and the sample dimensions (diameter D and thickness t):

$$R_t = \frac{2P}{\pi D t} \tag{14}$$



(a) Boryeong shale (b) Yeocheon schist

Figure 1: Brazilian test simulation for different orientation angles

Unfortunately, this result is not valid in the case of transversely isotropic materials and that is why some authors have attempted to develop more complicated solutions which take into account not only the applied load and the sample dimensions but also the elastic parameters and the orientation angle [20–22, 26]. J. Claesson [19] gave the following approximate formula:

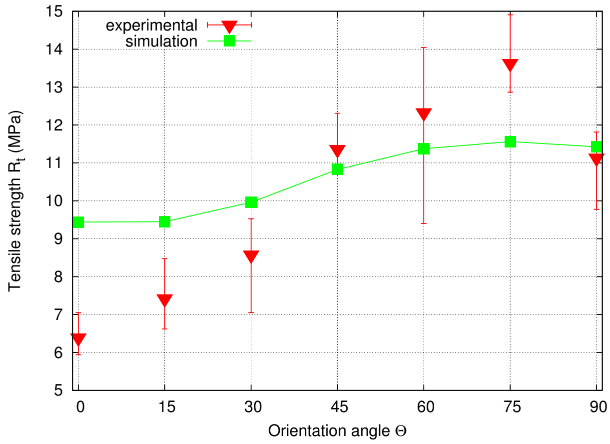
$$R_t = \frac{2P}{\pi D t}\left[\left(\frac{E}{E'}\right)^{\frac{\cos(2\Theta)}{4}} - \frac{\cos(2\Theta)}{4}(b-1)\right], \quad b = \frac{\sqrt{EE'}}{2}\left(\frac{1}{G} - \frac{2\nu'}{E'}\right) \tag{15}$$

In order to show the capabilities of the developed oriented damage model for transversely isotropic materials, in what follows, we will simulate numerically the Brazilian test in 3D configuration ($t = D/2 = 30\,mm$) for different orientation angles and for two rocks [16] using the parameters in table 1 (the parameter $\bar{E}$ is not involved in this case). In these numerical simulations, the rock sample is supposed to be totally adhering the platens of the press therefore we have a totally 3D problem. Figure 1 shows the whole simulation plot (the applied load versus the axial displacement). The peak of these curves is recorded and used in the expression 15 in order to estimate the tensile strength of the rock for the corresponding orientation angle. Figure 2 shows the obtained tensile strength values compared to experimental data. In order to visualize the sample's damage state at the end of the test, the horizontal damage for the two rocks for orientations 0, 45 and 90 is presented in figure 3. It is visible that the damage develops mainly along the centerline of the sample.

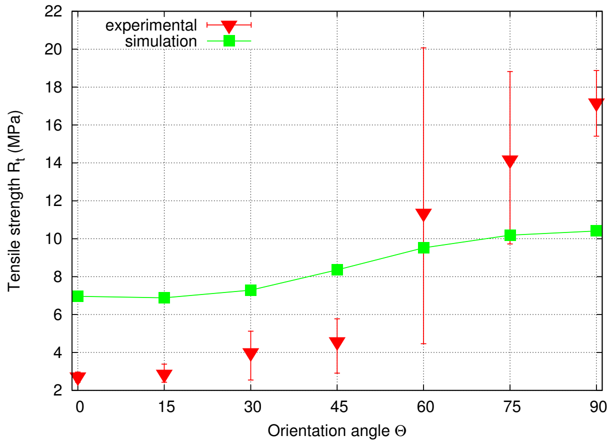
Table 1: Elastic and model parameters for the studied rocks

Rock	$E(GPa)$	$E'(GPa)$	ν	ν'	$G(GPa)$	$\bar{\nu}$	R_d
Boryeon shale	39.3	19.0	0.18	0.20	8.7	0.1242	4.4325
Yeoncheon schist	72.1	21.2	0.25	0.16	13.7	0.2586	2.2383

As can be seen, the proposed model furnishes satisfactory results. However, the obtained compression strength anisotropy is of same range as the Young moduli anisotropy and can not reach the observed values for rocks where a big difference between the two anisotropy values is measured. A further study on the used yield function has to be done in order to overcome this problem. Moreover, some authors such as [3, 4, 6, 8, 22] proved that the Brazilian test is not very convenient to determine the tensile strength especially for laminated materials. In fact, in the most of cases, the crack does not happen within the loading plane but within an inclined one which means that the failure is of a shear nature and not tensile one. This observation could be explained by the fact that the compression stress next to the platens of the press reaches the shear limit before that the tensile stress in the sample's center reaches the tensile strength. For that reason, an other test was proposed and consists on making a hole in the sample's center (This test is called the ring test). This way the tensile stress next to the hole is increased and will probably reach the stress strength before that the compression stress reaches the shear strength. As a future work, it would be of a great interest to simulate the ring test and compare the results to experimental data.



(a) Boryeong shale



(b) Yeocheon schist

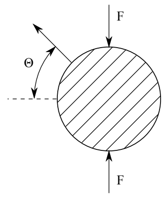
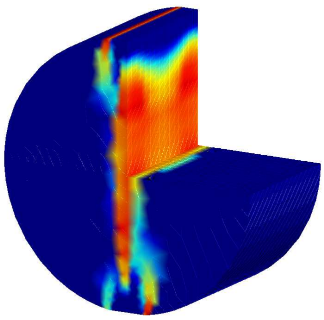
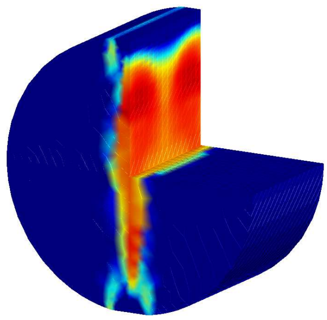


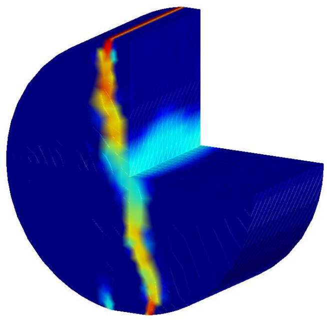
Figure 2: Tensile strength with simulation



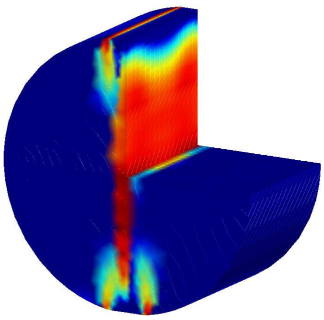
(a) Boryeong shale $\Theta = 0$



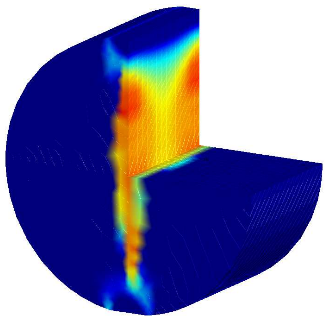
(b) Boryeong shale $\Theta = 45$



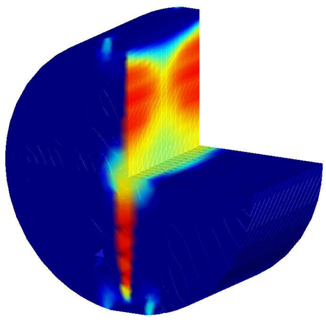
(c) Boryeong shale $\Theta = 90$



(d) Yeocheon schist $\Theta = 0$



(e) Yeocheon schist $\Theta = 45$



(f) Yeocheon schist $\Theta = 90$

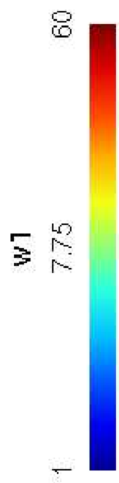


Figure 3: Horizontal damage state at the end of the Brazilian test

4 Conclusions

In this paper, an approach to describe the behavior of transversely isotropic materials is introduced. The method consists in introducing a fictive isotropic equivalent material, choosing an established isotropic model and transferring it to the real material. The elastic moduli of the fictitious material are the only two supplementary parameters introduced by this procedure. This method is used here to define an anisotropic damage behavior law in tensile regime. The model parameters are adjusted to experimental results and used in an FEM code (VIPLEF 3D and Code-Aster) to simulate 3D Brazilian tests for two rocks with different orientation angles. Compared to experimental data, the furnished results are rather satisfactory. However, the obtained tensile strength anisotropy is of the same range as the Young moduli anisotropy and can not reach the observed values for rocks where a big difference between the two anisotropy values is measured. A further work must be done to overcome this issue. Moreover, since the failure in a Brazilian test might be of a shear nature and not only a pure tensile nature, other simulations of the ring test for instance have to be compared to experimental data.

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