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Citation: Physics of Plasmas **20**, 122901 (2013); doi: 10.1063/1.4836595
View online: http://dx.doi.org/10.1063/1.4836595
View Table of Contents: http://scitation.aip.org/content/aip/journal/pop/20/12?ver=pdfcov
Published by the AIP Publishing

PHYSICS OF PLASMAS 20, 122901 (2013)

Nonlinear electron acceleration by oblique whistler waves: Landau resonance vs. cyclotron resonance

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(Received 2 September 2013; accepted 14 November 2013; published online 3 December 2013)

This paper is devoted to the study of the nonlinear interaction of relativistic electrons and high amplitude strongly oblique whistler waves in the Earth's radiation belts. We consider electron trapping into Landau and fundamental cyclotron resonances in a simplified model of dipolar magnetic field. Trapping into the Landau resonance corresponds to a decrease of electron equatorial pitch-angles, while trapping into the first cyclotron resonance increases electron equatorial pitch-angles. For 100 keV electrons, the energy gained due to trapping is similar for both resonances. For electrons with smaller energy, acceleration is more effective when considering the Landau resonance. Moreover, trapping into the Landau resonance is accessible for a wider range of initial pitch-angles and initial energies in comparison with the fundamental resonance. Thus, we can conclude that for intense and strongly oblique waves propagating in the quasi-electrostatic mode, the Landau resonance is generally more important than the fundamental one. © *2013 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4836595]

I. INTRODUCTION

Planetary radiation belts are effective accelerators for relativistic electrons.[1] In turn, electron acceleration and transport determine the dynamics of entire radiation belts.[2] Moreover, high-energy electrons represent a hazard to satellite electronics.[3,4] Therefore, the investigation of electron acceleration in the radiation belts is an important problem of modern plasma physics.

One of the most significant processes responsible for electron acceleration is the wave-particle resonant interaction.[5,6] Wave amplitudes determine the character of such an interaction: scattering of particles on small-amplitude waves can be described in the frame of the quasi-linear theory, while large wave-amplitudes correspond to nonlinear effects of particle trapping.[7] The basis of the quasi-linear description of wave-particle interaction was given in Refs. 8 and 9. A comprehensive scheme of calculation of effective diffusion coefficients corresponding to particle pitch-angle and energy diffusion was introduced in Refs. 10 and 11. There are several modern numerical[12,13] and semi-analytical[14–16] models of these diffusion coefficients, which are used in 3D codes of the radiation belts to provide a solution of the Fokker-Plank diffusion equation.[17–20]

Although, the major part of wave statistics corresponds to small enough wave amplitude values allowing the application of quasi-adiabatic theory,[13,21–23] modern spacecraft observations have revealed the presence of whistler waves with extremely high amplitudes.[24–26] These waves form short intense wave-packets with 10–10^4 wave-periods inside each packet.[27] Spatial scales along the background magnetic field allow one single packet to be present over most of a field line starting from the equatorial plane and up to 45° of latitude.[28] Transverse scales of wave-packets $\sim$500 km are substantially larger than wavelengths.[29]

High-amplitude whistler waves can interact with relativistic electrons in the nonlinear regime when particle trapping is possible.[30,31] Previously, this regime of interaction was mainly considered for parallel whistler waves. In this case, the fundamental cyclotron resonance is the most accessible. In this case, particles must propagate in the direction opposite to the waves to be trapped in the regime of nonlinear acceleration (see review Ref. 32 and references therein). There are several generalizations of this mechanism of acceleration, together with interesting applications. It was shown that trapping of particles into cyclotron resonance results in pitch-angle increase, while transient particles decrease their pitch-angles due to resonant interaction.[33] Interaction of electrons with lightning-generated waves at small *L*-shell was described in Refs. 34 and 35. The influence of the variation of the mean frequency of the wave-packet on nonlinear acceleration was described in Refs. 36 and 37. An important effect at large electron energy was found in Ref. 38, when the decrease of the effective gyrofrequency due to the increase of the relativistic gamma factor may result in a turning of resonantly trapped particles in the course of acceleration (so-called turning acceleration). If the initial energy of electrons is large enough ($\geq$1 MeV), resonant interaction with parallel waves becomes possible for electrons moving with the wave (so-called ultra-relativistic acceleration, see Ref. 39).

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The important influence of wave-frequency variation with time on the turning acceleration mechanism was investigated in Ref. 40. There is also a series of important publications devoted to the study of a realistic self-consistent acceleration of $\sim$100 keV–1 MeV electrons by parallel waves. In this case, the strong temperature anisotropy of low-energy electrons generates whistler waves, while resonant interaction of these waves with high energy electrons results in electron acceleration.[41] In the frame of this self-consistent model, the effect of wave-frequency variation with time and the turning acceleration mechanism were also investigated.[42,43]

The latest spacecraft observations show that chorus waves can propagate with a large normal angle θ relative to the background magnetic field.[44,45] Moreover, oblique waves represent a statistically significant population.[23,46] Thus, some portion of high-amplitude chorus waves can be strongly oblique. Indeed, an analysis of THEMIS (Time History of Events and Macroscale Interactions during Substorms) data[47] has shown that a number of high-amplitude whistler waves are observed with θ within the range $\theta \in [\theta_g, \theta_r]$, where θ_g is Gendring angle[48] and θ_r is the resonance cone angle (for a simplified whistler-mode dispersion in the cold plasma-high density limit, one has $\cos\theta_g = 2\omega/\Omega_c$, $\cos\theta_r = \omega/\Omega_c$ where ω is a wave frequency and Ω_c is a electron local gyrofrequency). These strongly oblique whistler waves propagate in the electrostatic mode: the main part of the wave energy then resides in the wave electric field (see Ref. 49). Thus, amplitudes of wave electric field reach 100–300 mV/m (see Refs. 24, 26, and 47).

Nonlinear interaction of oblique high-amplitude whistler waves with relativistic electrons involves the Landau resonance besides the fundamental cyclotron one (see review Ref. 6). Even for waves with $\theta < \theta_r$, it has been shown that the effect of the parallel electric field can become dominant in the nonlinear interaction for $\theta \geq \theta_g$ and $\omega/\Omega_c > 0.3$ (see Ref. 50). For such waves, Landau resonance can therefore be described within the electrostatic approximation. Electrons trapped into the Landau resonance can effectively increase their energy and decrease their equatorial pitch-angles in the course of their propagation along with the waves from the equatorial plane up to high latitudes.[28] Electron acceleration via Landau resonance has already been studied in several papers (e.g., Refs. 51–53), while a comparison of pitch-angle diffusion rates by Landau and cyclotron resonances with very oblique low-amplitude waves was achieved in Ref. 54. To the best of our knowledge, however, no statistical comparison of the respective effectiveness of Landau and fundamental cyclotron resonances with intense very oblique waves was undertaken before. Such a comparison requires detailed informations concerning the actual distribution of wave-amplitudes along the field lines, as well as a realistic model of wave-normal angle evolution during wave propagation, both of which became available only very recently from modern spacecraft missions.

According to wave statistics collected by CRRES (Combined Release and Radiation Effects Satellite)[21] and Cluster,[23] the intensity of the wave electric field on the dayside of the Earth's radiation belts has a local minimum in the vicinity of the equator. At latitudes $\lambda \sim 15^\circ - 20^\circ$, the intensity of the wave electric field reaches a maximum value and does not vary substantially at $\lambda > 20^\circ$. Such a behavior can be explained by a combination of two processes: (1) with propagation away from the equator, waves become more intense due to a local instability of the background plasma medium, (2) wave propagation in the inhomogeneous magnetic field results in an increase of electric field amplitude due to the transformation of the wave-mode into an electrostatic one (e.g., Refs. 55–57). Thus, one can introduce an effective wave amplitude as a function of latitude $u(\lambda)$. Below, we use $u(\lambda)$ derived previously in the work (Ref. 28).

In this paper, we compare the effectiveness of two mechanisms of resonant accelerations (via Landau and fundamental cyclotron resonances). To this aim, we first obtain analytical estimates of the energy gained by electrons trapped into both resonances. We also derive expressions for the probability of trapping in both cases. On this basis, the relative importance of each resonance in electron energization can be assessed statistically.

II. GENERAL EQUATIONS

We use a simplified model of the background magnetic field with two components $B_z = B(z)$ and $B_x = -x dB(z)/dz$ (see Ref. 50), where z is the parallel coordinate, while x is the perpendicular coordinate. This model can mimic any variation of the magnetic field amplitude with z (e.g., for the dipole field, we have $B(z) = B_0\sqrt{1+3\sin^2\lambda}/\cos^6\lambda$, where λ is magnetic latitude $dz/d\lambda = R_0\sqrt{1+3\sin^2\lambda}\cos\lambda$, R_0 is the radial distance from the planet to the point in the equator, and $B_0 \sim R_0^{-3}$ is the magnetic field amplitude). However, this magnetic field model cannot reproduce arbitrary curvatures of the magnetic field lines.

To derive analytical estimates of electron acceleration due to wave-particle resonant interaction, we consider a simplified model of the high-amplitude whistler wave. We only take into account the electrostatic component of obliquely propagating whistler waves generated in the vicinity of the equatorial region. This approximation is valid for waves propagating near the resonance cone angle (see Ref. 50). Such waves are observed in the Earth magnetosphere (see statistics presented in Refs. 23 and 58). Moreover, as shown below, resonant interactions of particles with these waves only occur at medium to high latitudes, where the Gendrin angle is merely 5°–10° below the resonance cone angle. Thus, we hereafter assume that the electrostatic approximation can be used to derive estimates of electron acceleration for waves between the Gendrin and resonance cone angles. In this case, the wave can be described by the scalar potential $\hat{\Phi}(z,x,t) = \Phi_0 u(z)\cos\phi$, where the dimensionless function $u(z)$ corresponds to the distribution of wave-field amplitude along the magnetic field lines and ϕ is the wave phase. We further assume that the wave-phase can be written as

$$\phi = \phi_0 + \int^{z} k_\parallel(z')dz' + k_\perp x - \omega t,$$

where the perpendicular component of wavenumber $k_\perp$ is a constant. Wave amplitude Φ_0 is defined as $\Phi_0 = E_{0\parallel}/k_\parallel(0)$, where $E_{0\|}$ is the amplitude of the parallel electric field. We also assume that the wave-frequency ω is constant. We can derive analytical equations for the general dependence of $k_\|$ on z. However, for numerical estimates, we use expressions for waves propagating with the Gendrin angle:[48] $k_\| = k_0\omega/\Omega_c$ and $k_\perp \approx k_0$, where $k_0 = const$ and $\Omega_c = eB(z)/m_e c$ (see Ref. 47).

The Hamiltonian of a relativistic electron with charge $-e$ and rest mass m_e can be written as

$$H = m_e c^2\gamma - e\Phi_0 u(z)\sin\phi,$$
$$\gamma = \sqrt{1 + \frac{p_x^2 + p_z^2}{(m_e c)^2} + \left(\frac{e}{c^2 m} xB(z)\right)^2},$$

where (p_x, p_z) are components of electron momentum. Here, we introduce dimensionless variables and parameters $(p_x,p_z) \to (p_x,p_z)/m_e c$, $(x,z) \to (x,z)/R_0$, $t \to tc/R_0$, $(k_\|,k_\perp) \to (k_\|,k_\perp)R_0$, $\omega \to \omega R_0/c$, $\varepsilon = e\Phi_0/m_e c^2$, $H \to H/m_e c^2$, $b = B(z)/B_0\,\chi = \Omega_{c0}R_0/c$, where $B_0 = B(0)$, $\Omega_{c0} = eB_0/m_e c$. We also introduce the parameter $\omega_m = \omega/\Omega_{c0}$ (in normalized variables $\omega = \omega_m\chi$). The radius $R_0 = R_E L$ is defined by L ($R_E \approx 6400\,\mathrm{km}$). The dimensionless Hamiltonian now takes the form

$$H = \gamma - \varepsilon u(z)\sin\phi,$$
$$\gamma = \sqrt{1 + p_x^2 + p_z^2 + (\chi x b(z))^2}. \tag{1}$$

We consider systems where the background magnetic field is strong enough to make gyrorotation the fastest type of motion. Thus, we can introduce the adiabatic invariant I_x (see Ref. 59) for a non-perturbed ($\varepsilon = 0$) system,

$$I_x = \frac{1}{2\pi}\oint p_x dx = \frac{1}{\pi}\int \sqrt{\gamma^2 - 1 - p_z^2 - (\chi x b(z))^2}\,dx,$$
$$= \frac{1}{2}\left(\gamma^2 - 1 - p_z^2\right)/(\chi b(z)).$$

This is the classical magnetic moment and it can be introduced as a new canonical variable with the conjugate variable θ defined with $\dot\theta = -\chi b(z)/\gamma$. Corresponding expressions for (x, p_x) in terms of (I_x, θ) are

$$x = \sqrt{2I_x/\chi b(z)}\sin\theta,$$
$$p_x = \sqrt{2I_x\chi b(z)}\cos\theta,$$

with these new variables, the Hamiltonian (1) can be rewritten as

$$\begin{aligned} H &= \gamma - \varepsilon u(z)\sum_n J_n(\eta)\sin\phi_n,\\ \gamma &= \sqrt{1 + p_z^2 + 2I_x\chi b(z)},\\ \phi_n &= \phi_{0n} + \int^z k_\|(z')dz' + n\theta - \omega t,\\ \eta &= k_\perp\sqrt{2I_x/\chi b(z)}, \end{aligned} \tag{2}$$

where J_n is the Bessel function of the n-th order.

The resonant condition $\dot\phi_n = 0$ for the system (2) can be written as

$$\omega - k_\|(z)v_z = -n\frac{\chi b(z)}{\gamma},$$

where $v_z = p_z/\gamma$. The distance between resonances is $\Delta p_z = \chi b(z)/k_\|(z)$. If this distance is substantially larger than the width of the resonances, then we can consider each resonance separately (see Ref. 51 and Sec. VII). In this paper, we study two main resonances: the Landau resonance with $n = 0$ and the fundamental cyclotron resonance with $n = -1$.

For Landau and cyclotron resonances, we estimate the energy gained by particles due to trapping by a wave. Trapping itself corresponds to a change in the type of particle motion: before trapping, a particle oscillates along the field lines (bounce motion) with parallel velocity $p_z/\gamma = \sqrt{1 - \gamma^{-2} - 2I_x b(z)/\gamma^2}$ and constant energy $\gamma = const$, while during trapping this particle starts moving at the resonance velocity v_R. Trapped motion is therefore characterized by energy evolution. Trapping is possible if the wave electric field is strong enough to compensate the force due to magnetic field inhomogeneity (some analog of the mirror force). The magnetic field inhomogeneity varies along the field lines. Thus, for each particle with a given energy and pitch-angle, there is a point where the particle can get trapped (at this point the resonance condition is satisfied and the magnetic field inhomogeneity is weak enough) and a point where the particle should escape from the resonance. The variation of particle energy induced by trapping is simply calculated as the energy difference between these two points. Comparison of such gains of energy for the two considered resonances should allow us to determine which resonance is statistically more effective for particle acceleration.

For fixed particle energy and equatorial pitch-angle, the possibility of trapping depends on the initial position of the particle relative to the wave (i.e., on the value of ϕ_{0n}). Initial coordinates of potentially trapped and transient particles are mixed in phase space. Due to the fast variation of the wave phase ϕ, even a small variation of the initial coordinates of the particle can result in a significant change of the following scenario of wave-particle interaction. Potentially trapped particles can become transient, while potentially transient particles can become trapped. Moreover, already after one bounce-period (or a single passage through the resonance), a potentially transient particle can become potentially trapped and vice versa. Therefore, it is not sufficient to consider simply all the possible ranges of initial coordinates where particles could be trapped during their first resonant interaction with one wave. A great many bounce periods as well as the full ranges of initial coordinates should be investigated, which represents an enormous amount of simulation runs. An alternative, probabilistic approach is to consider the phase volume of the initial coordinates of particles which should be trapped. The probability of particle trapping $\Pi \le 1$ can be defined as the ratio of this *trapped* volume to the whole phase volume of the initial coordinates. For given energy and equatorial pitch-angle, this probability is the percentage of particles that are to be trapped in the course of the next passage through the resonance.[60] The comparison of such probabilities calculated for the Landau and cyclotron resonances can be used as an indicator of the potential effectiveness of these resonances.

III. LANDAU RESONANCE

In this section, we consider the Landau resonance ($n=0$). The wave phase ϕ_n (2) does not depend on θ and, as a result, the invariant I_x is conserved even in the presence of the wave. The argument of the Bessel function $\eta \sim \sqrt{I_x} \sim \sin\alpha_0$ strongly depends on the equatorial pitch-angle α_0 of particles and can vary in wide range. So, we introduce the function $u_0(z) = u(z)J_0(\eta)$ and rewrite the Hamiltonian (2) as

$$\begin{aligned} H &= \gamma - \varepsilon u_0(z)\sin\phi, \\ \gamma &= \sqrt{1 + p_z^2 + 2I_x\chi b(z)}, \\ \phi &= \phi_0 + \int^z k_\parallel(z')dz' - \omega t. \end{aligned} \tag{3}$$

Corresponding equations of motion are

$$\begin{cases} \dot{z} = p_z/\gamma \\ \dot{p}_z = -I_x\chi b'/\gamma + \varepsilon k_\parallel u_0 \cos\phi, \end{cases}$$

where $' = d/dz$ and we assume $u_0' \ll k_\parallel u_0$.

In the vicinity of the resonance $\dot{\phi} = 0$, we can introduce the resonant velocity $v_R = \omega/k_\parallel$ and resonant gamma factor $\gamma_R = 1/\sqrt{1 - v_R^2}$. In this case, the equation for the wave-phase has the form

$$\begin{cases} (\gamma/k_\parallel)\ddot{\phi} = -\mathrm{A} + \mathrm{B}\cos\phi \\ \mathrm{A} = \dfrac{\gamma_R^2}{\gamma}(-v_R^2\gamma^2(k'_\parallel/k_\parallel) + I_x\chi b') \\ \mathrm{B} = \varepsilon k_\parallel u_0 \\ \gamma = \gamma_R\sqrt{1 + 2I_x\chi b(z)}. \end{cases}$$

This is the classical equation of the nonlinear pendulum with a constant torque: if B > A there is a region with trapped trajectories in the phase plane and for A > B all particles are transient (e.g., Ref. 6). Here, we consider the wave propagating with Gendrin angle $k_\parallel = k_0/b(z) = 2\omega_{pe}\omega/b(z)$ (see Ref. 47) to show variations of A, B along field lines ($\omega_{pe} = \Omega_{pe}/\Omega_{c0}$, where $\Omega_{pe} =$ const is the plasma frequency, see approximate dependence of Ω_{pe} on L in Ref. 61). In this case, $v_R = \omega/k_\parallel = b(z)/(2\omega_{pe})$ and

$$\mathrm{A} = \frac{\gamma_R^2}{\gamma}\left(\gamma^2\frac{b}{(2\omega_{pe})^2} + I_x\chi\right)b',$$

$$\mathrm{B} = \frac{\varepsilon k_0 u_0}{b}.$$

Corresponding profiles of A and B are shown in Fig. 1 for various particle energies and equatorial pitch-angles. One can see that B > A at the equator and only above a certain latitude λ^* we get B < A. Therefore, there is some range of latitudes where trapped trajectories exist in the phase space.[28]

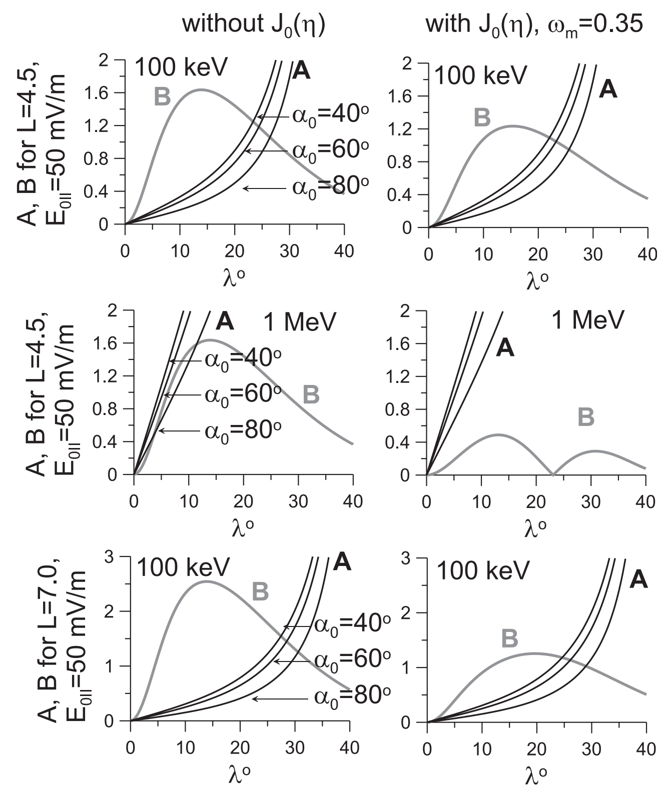

FIG. 1. Profiles of A and B for various system parameters. We show B coefficient calculated with $J_0(\eta)$ and without this factor.

In absence of the wave, particles oscillate along the bounce trajectories and their parallel velocities can be described by the adiabatic relation

$$p_z = \sqrt{\gamma^2 - 1 - 2I_x\chi b(z)}.$$

Thus, for any given value of the equatorial pitch-angle α_0 ($2\chi I_x = (\gamma^2 - 1)\sin^2\alpha_0$), one can determine the coordinate of the resonance z_R where $\gamma v_R = p_z$. Particles can be trapped by the wave if in the region $z \approx z_R$ we have A < B and $d\mathrm{B}/dt > 0$ (here the derivative is taken along the particle trajectory).[60] Trapped particles should be transported by the wave to higher latitudes. Near the point z^* where A = B, particles finally escape from the resonance. The corresponding energy gain is $\Delta\gamma \approx \gamma(z^*) - \gamma(z_R)$.

Using the magnetic field model $b(z)$, one can further analytically obtain 2D maps in the (γ, α_0) space of the distribution of gained energy $\Delta\gamma$ for various system parameters (see Fig. 2). Moreover, trapping is a probabilistic process with a certain probability Π (see Ref. 60). Thus, only some subpopulation of resonant particles can be trapped and accelerated. We derive the corresponding analytical expressions for the probability Π in Appendix A (see Eq. (A20)) and plot 2D maps of Π in Fig. 2.

In both Figures 1 and 2, we show the effect of the factor $\sim J_0(\eta)$. This factor is important for high energy electrons (>100 keV) for which the decrease of the effective wave amplitude $u_0 \sim J_0$ can result in a disappearance of trapping.

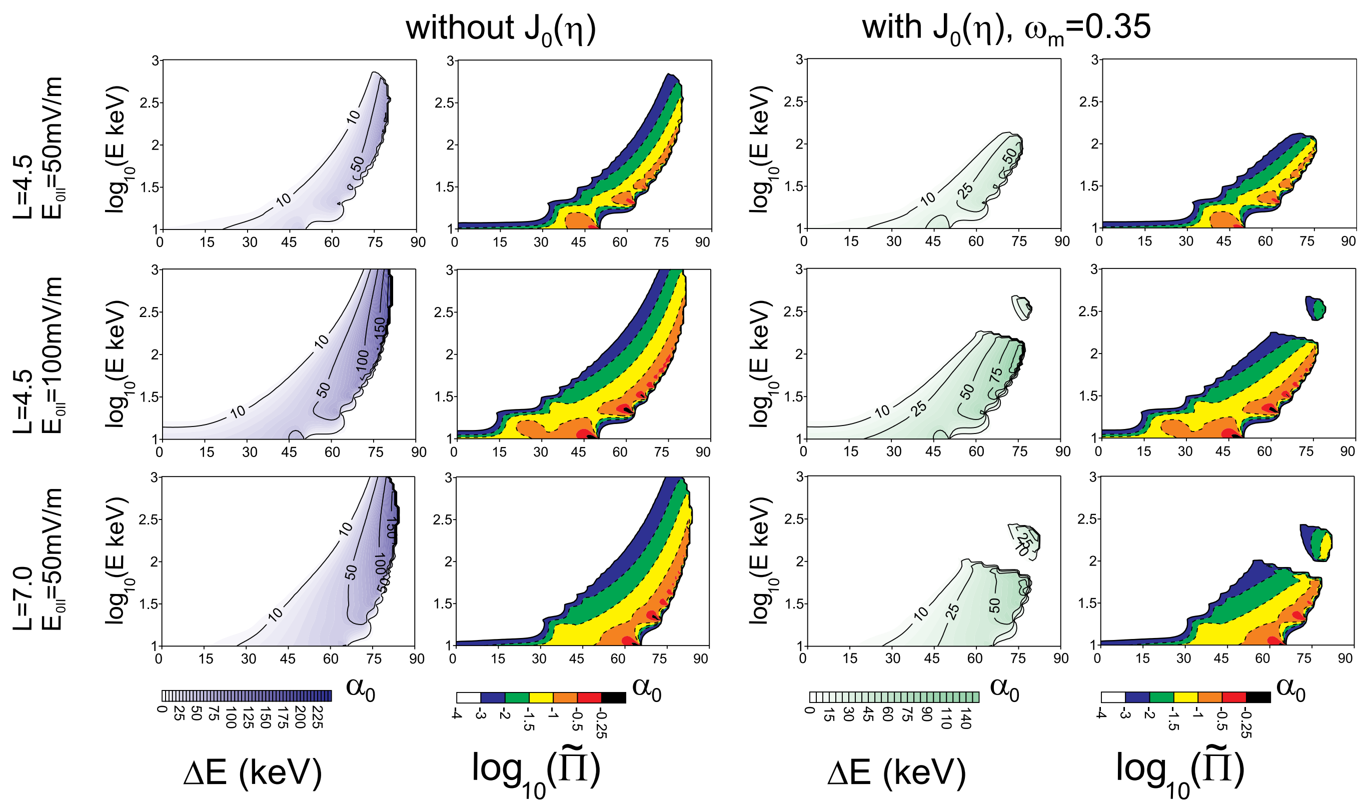

FIG. 2. 2D maps of energy gain and probability of trapping $\tilde{\Pi} = \Pi\sqrt{\omega_m}$ for various system parameters. The gain of energy is obtained as the difference between energies of trapped particles at points of trapping and escape. Probability Π is given in Appendix A.

To further illustrate the effects of trapping and acceleration, we solve the system (3) numerically and plot the particle trajectory in Fig. 3. Initially, the particle oscillates along the bounce trajectory (closed trajectory in the plane (z, p_z)). Then, after a certain time the particle becomes trapped by the wave and is transported to higher latitudes. This trapping motion corresponds to energy gain. After escape from the resonance, the particle returns to bounce oscillations with larger amplitudes. Thus, trapping into the Landau resonance results in a decrease of the equatorial value of the particle pitch-angle.

IV. FUNDAMENTAL CYCLOTRON RESONANCE

In this section, we consider the fundamental cyclotron resonance ($n=-1$). The wave phase ϕ_n given in Eq. (2) is a function of θ and, as a result, the invariant I_x is not conserved in the presence of waves. Moreover, the Bessel function $J_1(\eta)$ is small for small values of its argument. We introduce the function $u_1(z) = u(z)J_1(\eta)$ and write the Hamiltonian (2) as

$$
\begin{aligned}
H &= \gamma - \varepsilon u_1(z)\sin\phi,\\
\gamma &= \sqrt{1 + p_z^2 + 2I_x\chi b(z)},\\
\phi &= \phi_0 + \int^z k_\parallel(z')dz' - \theta - \omega t.
\end{aligned}
\tag{4}
$$

Corresponding equations of motion are

$$
\begin{cases}
\dot{z} = p_z/\gamma\\
\dot{p}_z = -I_x\chi b'/\gamma + \varepsilon k_\parallel u_1 \cos\phi\\
\dot{I}_x = \varepsilon u_1 \cos\phi\\
\dot{\theta} = -\chi b(z)/\gamma,
\end{cases}
\tag{5}
$$

where we assume $u_1' \ll k_\parallel u_1$.

In the vicinity of the resonance $\dot{\phi} = 0$, we can introduce the resonant velocity v_R as

$$
v_R = -\frac{\chi b(z) - \omega\gamma}{\gamma k_\parallel} = -\frac{w_R}{k_\parallel}
$$

and the resonant gamma factor is $\gamma_R = 1/\sqrt{1 - v_R^2}$. One can see that the resonant velocity is negative for small wave frequency, i.e., resonant particles move in a direction opposite to the waves.[62] We introduce several useful expressions valid in the vicinity of the resonance

$$
\begin{aligned}
\frac{k_\parallel'}{k_\parallel} &= -\frac{1}{v_R}\left(v_R' + \frac{w_R'}{k_\parallel}\right),\\
\ddot{z} &= \frac{\ddot{\phi}}{k_\parallel} - \frac{k_\parallel'}{k_\parallel}v_R^2 - \frac{w_R' v_R}{k_\parallel} = \frac{\ddot{\phi}}{k_\parallel} + v_R v_R',\\
\dot{\gamma} &= \gamma_R^2 \gamma v_R' v_R^2 + \gamma_R^2 \frac{I_x \chi b' v_R}{\gamma} + \gamma_R^2 \frac{\dot{I}_x \chi b}{\gamma}.
\end{aligned}
$$

In this case, the equation for the wave-phase has the form

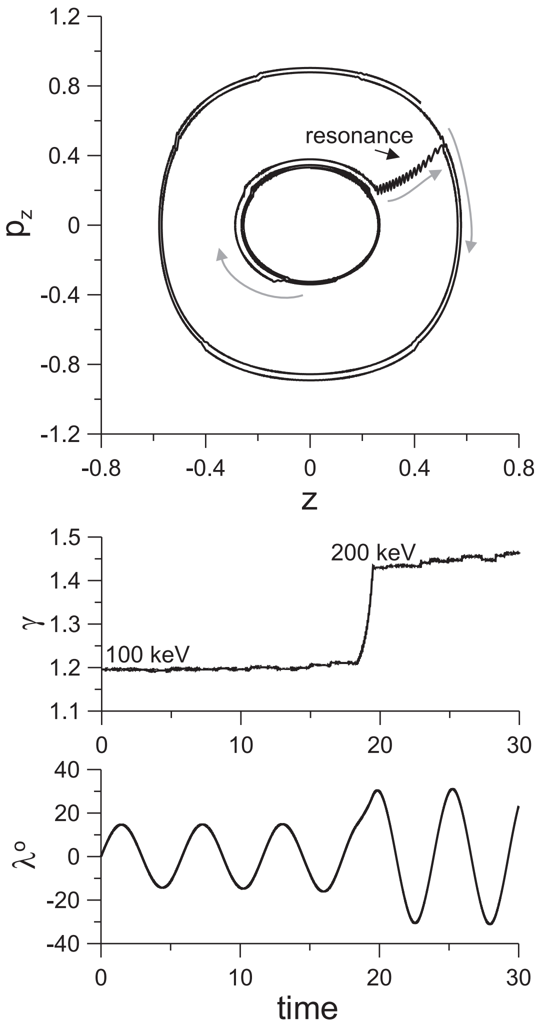

FIG. 3. Example of particle trajectory, energy and latitude λ as functions of time. The initial energy of the particle is 100 keV, initial pitch-angle is 60°, system parameters are: $E_{0\|} = 100\,\mathrm{mV/m}$, $L = 4.5$, $\omega_m = 0.35$. The factor $\sim J_0(\eta)$ is not taken into account here.

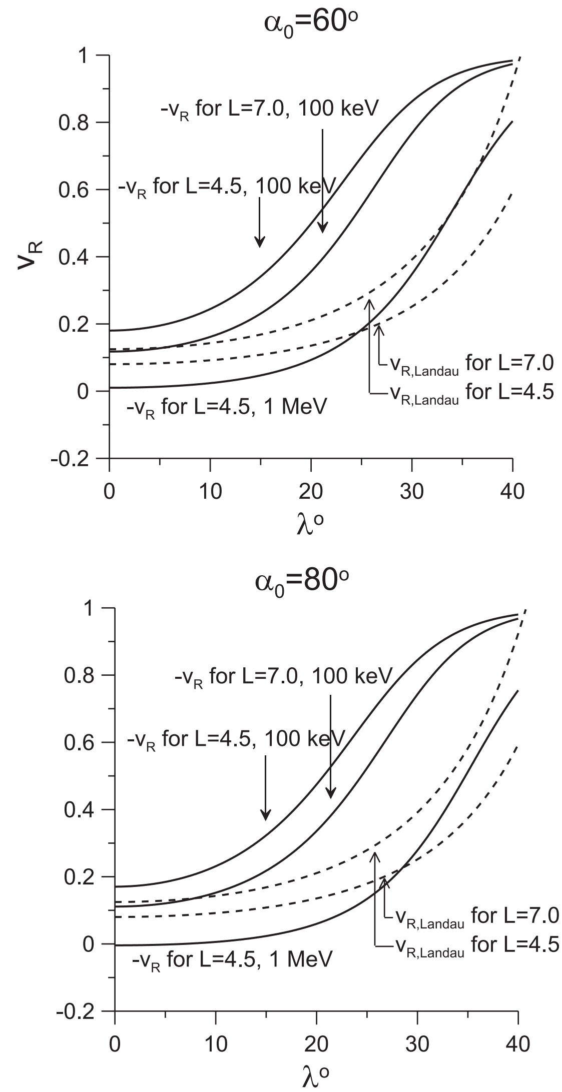

FIG. 4. Profiles of v_R defined by Eq. (6) for various system parameters and profiles of resonant velocity for the Landau resonance $v_R = b/(2\omega_{pe})$. Wave frequency is $\omega_m = 0.35$.

$$
\begin{aligned}
(\gamma/k_\|)\ddot{\phi} &= -\mathrm{A} + \mathrm{B}\cos\phi,\\
\mathrm{A} &= \gamma_R^2 \frac{I_x \chi b'}{\gamma} + \gamma_R^2 \gamma v_R v_R',\\
\mathrm{B} &= \gamma_R^2\left(1 - v_R^2 \frac{\omega\gamma}{\chi b - \omega\gamma}\right)\varepsilon k_\| u_1,\\
\gamma &= \gamma_R\sqrt{1 + 2I_x \chi b(z)}.
\end{aligned}
$$

One can see that the general form of the equation for ϕ coincides with the one derived for Landau resonance (see Sec. III). We rewrite coefficients A and B for a wave propagating at the Gendrin angle such that $k_\| = k_0/b(z) = 2\omega_{pe}\omega_m\chi/b(z)$. To this aim, we first derive an expression for v_R from the equation $1 - 2\omega_{pe}v_R = b^2/(\omega_m\gamma)$

$$
\begin{aligned}
v_R &= \frac{v_{R0} - \sqrt{v_{R0}^2 + (a - v_{R0}^2)(1+a)}}{1+a},\\
a &= \left(\frac{b^2}{2\omega_{pe}\omega_m}\right)^2 \frac{1}{1 + 2\chi I_x b}, \quad v_{R0} = \frac{b}{2\omega_{pe}}.
\end{aligned}
\tag{6}
$$

We plot profiles of v_R in Fig. 4 and compare them with profiles of the resonant velocity for the Landau resonance $v_R = b/(2\omega_{pe})$. One can see that for 100 keV particles, the cyclotron resonant velocity v_R is always negative. For large energy ($\sim$1 MeV) factor, $\gamma\omega_m$ can be larger than b and, as a result, this resonant velocity becomes positive.[38] The cyclotron resonant velocity $|v_R|$ is larger in general than the corresponding resonant velocity for the Landau resonance.

For Gendrin wave propagation, we can now rewrite the expressions for A and B as

$$
\begin{aligned}
\mathrm{A} &= \frac{\gamma_R^2}{\gamma} b'\left(I_x\chi + \frac{1}{2}\gamma^2 \frac{\partial v_R^2}{\partial b}\right),\\
\mathrm{B} &= \frac{\gamma_R^2}{b}\left(1 - v_R^2 \frac{\omega_m\gamma}{b - \omega_m\gamma}\right)\varepsilon k_0 u_1.
\end{aligned}
$$

Corresponding profiles of A and B are shown in Fig. 5 for various equatorial pitch-angles. Ranges of latitudes where particles can become trapped (B > A) are similar to the corresponding latitude ranges for the Landau resonance (compare with Fig. 1).

However, in contrast with the Landau resonance, now Fig. 5 provides only the positions of trapping, but cannot

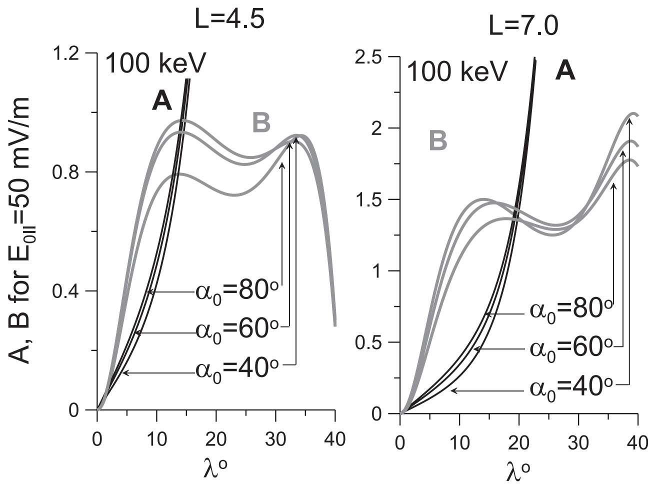

FIG. 5. Profiles of A and B for various system parameters (cyclotron resonance case). The factor $\sim J_1$ is taken into account ($\omega_m = 0.35$).

give the position of particle escape from the resonance, because profiles of A, B are shown for constant χI_x. In the course of trapped particle motion, χI_x changes (see system (5)) and, as a result, we should take into account the evolution of A, B with χI_x (see Appendix B).

To determine the gain of energy in the fundamental cyclotron resonance and corresponding probability of capture, we apply a similar approach as the one used for the Landau resonance in Sec. III. We plot 2D maps of gain of energy $\Delta\gamma$ for various system parameters (see Fig. 6). The analytical expressions for the probability Π of capture are derived in Appendix B and presented as 2D maps in Fig. 6.

Now, we solve the system (4) numerically and plot the particle trajectory in Fig. 7 to show the main features of

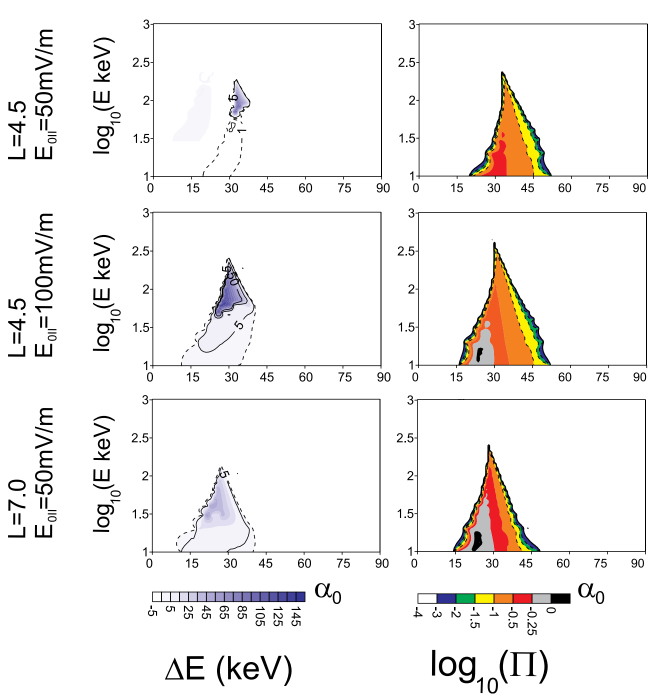

FIG. 6. 2D maps of energy gain and probability of trapping for various system parameters (cyclotron resonance case). The factor $\sim J_1$ is taken into account ($\omega_m = 0.35$).

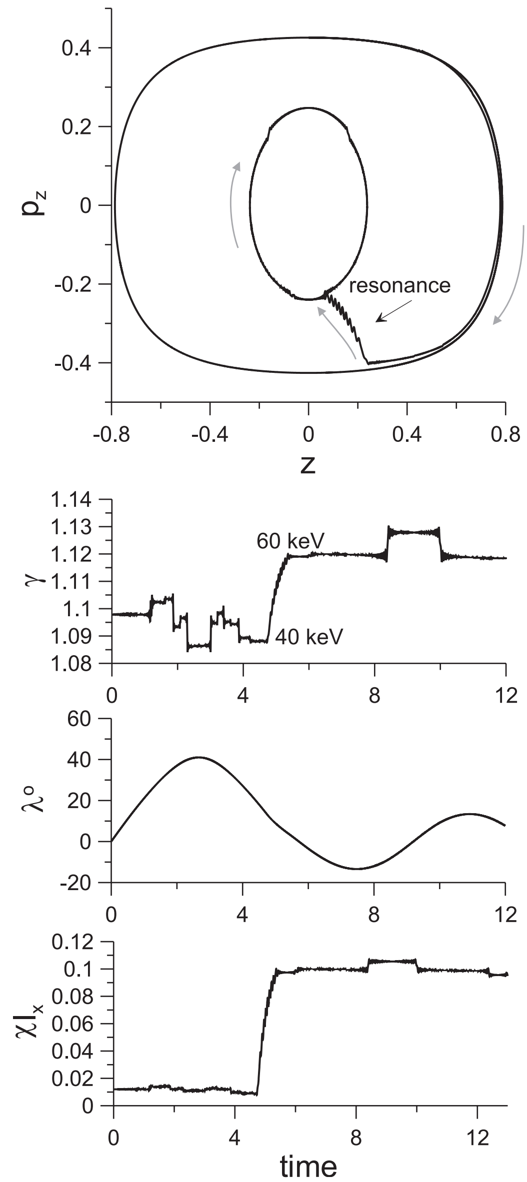

FIG. 7. The particle trajectory, energy and latitude λ as functions of time (cyclotron resonance trapping). Initial energy of the particle is 50 keV, initial pitch-angle is 20°, system parameters are: $E_{0\|} = 100\,\mathrm{mV/m}$, $L = 4.5$, $\omega_m = 0.35$. The factor $\sim J_1(\eta)$ is taken into account.

acceleration. Initially, the particle oscillates along the bounce trajectory (closed trajectory in the plane (z, p_z)). Then, after a certain time the particle becomes trapped by the wave and transported to lower latitudes with an increase of its energy. A comparison with Fig. 3 shows that electron acceleration in the fundamental cyclotron resonance corresponds to motion in the direction opposite to the wave.

V. PITCH-ANGLE JUMPS

Besides the energy changes, particles can also change their pitch-angles during their resonant interaction with the waves, which corresponds to a modification of the energy distribution between parallel and perpendicular motions. To estimate these jumps of pitch-angles $\Delta\alpha_0$, we use the definition of the I_x invariant: $2\chi I_x = (\gamma^2 - 1)\sin^2\alpha_0$. Thus, for the Landau resonance (when $I_x = \text{const}$) we have

$$\Delta\alpha_0 = \arcsin\left(\sin\alpha_{0,init}\sqrt{\frac{\gamma_{init}^2 - 1}{\gamma^2 - 1}}\right) - \alpha_{0,init},$$

where $\alpha_{0,init}$ and γ_{init} are initial values of equatorial pitch-angle and relativistic gamma factor.

For the fundamental resonance, conversely, I_x changes and we can write

$$\Delta\alpha_0 = \arcsin\sqrt{\frac{2\chi I_x}{\gamma^2 - 1}} - \alpha_{0,init}.$$

Both these jumps of the equatorial pitch-angles are shown in Fig. 8. One can see that a typical value of $\Delta\alpha_0$ is about 30°. Pitch-angles are seen to decrease for the Landau resonance while they increase for the fundamental cyclotron resonance.

VI. COMPARISON OF LANDAU AND CYCLOTRON RESONANCES

In this section, we compare the effectiveness of Landau and cyclotron resonances in particle acceleration and pitch-angle change. To this aim, we make use of the probabilities of trapping Π, gains of energy ΔE, and jumps of pitch-angle $\Delta\alpha_0$ shown in Figs. 2, 6, and 8. For each value of the initial electron energy E, we calculate the maximum gain of energy $\max_{\alpha_0}(\Delta E)$ and the maximum value of pitch-angle jump $\max_{\alpha_0}(\Delta\alpha_0)$ (we find the maximum values over the whole pitch-angle range $\alpha_0 \in [0^\circ, 90^\circ]$). We also calculate the corresponding probabilities of trapping $\Pi_{\max\Delta E}$ and $\Pi_{\max\Delta\alpha_0}$. Then, we compare dependencies of $\max_{\alpha_0}(\Delta E)$, $\max_{\alpha_0}(\Delta\alpha_0)$ on E for both resonances. In addition, we plot the ratio of probabilities $\Pi_{\max\Delta E}$, $\Pi_{\max\Delta\alpha_0}$ for cyclotron and Landau resonances.

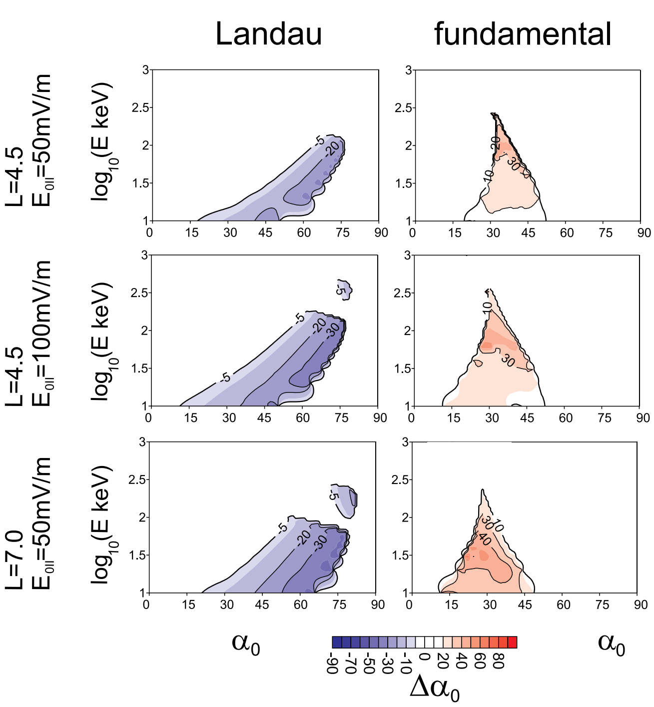

FIG. 8. 2D maps of jumps of the equatorial pitch-angles for Landau and fundamental cyclotron resonances. Terms $\sim J_0(\eta)$ and $\sim J_1(\eta)$ are taken into account.

Fig. 9 displays the maximum values of electron energy gain for Landau and fundamental cyclotron resonances (top panels). For initially ∼100 keV electrons, the fundamental cyclotron resonance can provide a similar acceleration as the Landau resonance. However, the probability of trapping is substantially (∼ ten times) larger for the Landau resonance (see bottom panels). The exception is the case at $L = 7$ where $\Pi_{\rm Lan} < \Pi_{\rm fun}$ for $E \sim 100$ keV, but in this case the potential gain of energy in the Landau resonance is larger than for the fundamental cyclotron resonance. For electrons with initial energy less than 100 keV, acceleration in the Landau resonance is potentially much more effective: the maximum gain of energy is 10–100 times larger for the Landau resonance than for the cyclotron one. Moreover, in this energy range, the probability of such trapping-acceleration via Landau resonance is ∼100 times higher than the same probability for cyclotron resonance. For $E \sim 100$ keV electrons, pitch-angle jumps are larger for the fundamental cyclotron resonance, while for $E < 100$ keV, pitch-angle jumps are again larger for the Landau resonance.

VII. DISCUSSION

A comparison of Figs. 2 and 6 shows the main difference between Landau and fundamental cyclotron resonances for electron trapping by intense, strongly oblique quasi-electrostatic waves: the ranges of initial energies and pitch-angles with a positive probability of capture are wider for the Landau resonance (see also Fig. 9). Moreover, <100 keV electrons can gain more energy through Landau resonance acceleration, while gain of energy for ∼100 keV electrons is similar for both resonances. The fundamental cyclotron resonance is likely more important for changing particle pitch-angles than for their acceleration. Indeed, trapping into the fundamental resonance results in an increase of pitch-angles (see Fig. 8). Thus, this mechanism can help to transport some accelerated electrons away from the loss-cone, thereby contributing essentially to their sustained acceleration via successive trapping events in the Landau resonance.

The simple approximation of a dipolar geomagnetic field has been used here. However, for large L-shells (∼7) the deviation of the magnetic field configuration from the dipole model can be substantial. Such a magnetic field deformation is especially significant on the night-side where local currents of hot ions often deform the dipolar magnetic field.[63,64] This deformation influences particle quasi-linear scattering[65,66] and, as a result, can be important for nonlinear wave-particle interaction as well. In this paper, general expressions for probabilities of particle trapping have been obtained for a simplified model of magnetic field which magnitude is a function of magnetic latitude (see Appendices A and B). Thus, our expressions for probabilities can be used for any distorted magnetic field which can be roughly described by such a simple model. For example, we plan to apply our model of nonlinear wave-particle interactions to the problem of electron acceleration by whistler waves in the Earth's magnetotail. In this region, strong emissions of

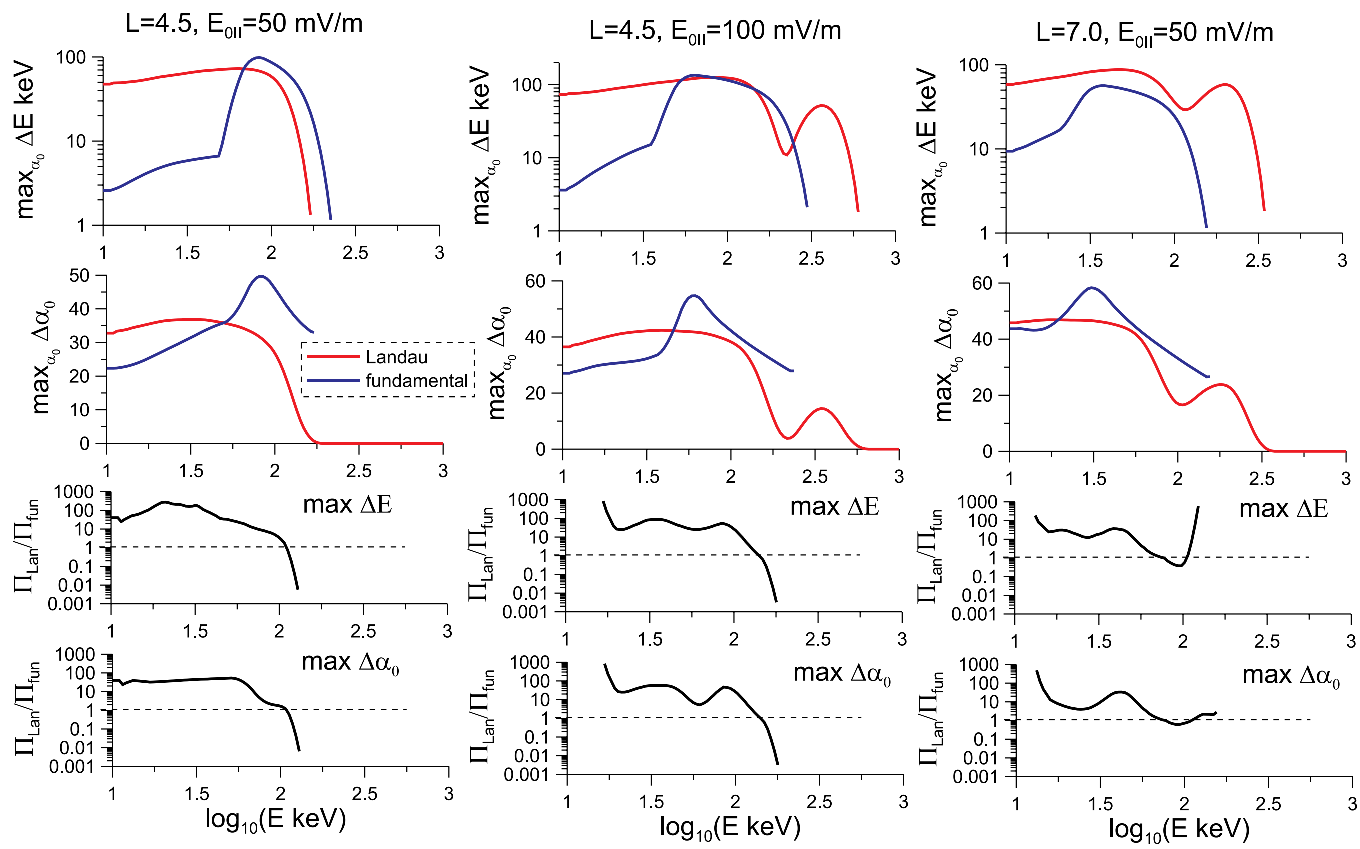

FIG. 9. The top panels show the maximum value of the gain of energy of trapped electrons as a function of their initial energy. The middle panels show maximum values of pitch-angle jumps as a function of the initial electron energy (for Landau resonance $|\Delta\alpha_0|$ is presented). The bottom panels show ratios of probabilities of trapping for Landau and fundamental cyclotron resonances.

whistler waves are associated with fast plasma flows (e.g., Ref. 67)propagating from the deep tail towards the Earth. Electrons can be effectively accelerated by these waves in the magnetic field configuration with stretched field lines[68] or with depolarized current sheet.[69]

In this paper, the Landau and fundamental cyclotron resonances have been considered separately. This is possible because the distance between these resonances in phase space is usually much larger than their widths (see Ref. 51). However, an important deformation of the magnetic field configuration and an increase of plasma density due to particle injections into the inner magnetosphere can substantially increase resonance width. This is especially true for waves propagating with normal-angles close to the resonance cone at large L-shells (where such waves are often observed, see Ref. 47). In this case, resonance merging may result in the appearance of an "integrated" Cherenkov resonance $\mathbf{vk} = \omega$, which cannot be expanded over cyclotron resonances. This is a classical situation for wave-particle interaction in a weak magnetic field[70,71] when the so-called surfatron acceleration is realized (e.g., Refs. 72–74). Thus, further consideration of nonlinear resonance acceleration in a nondipole magnetic field configuration seems to be an important task for the future.

For very oblique whistler waves, higher-order cyclotron resonances may also become important. The situation of overlapping cyclotron resonances would require that the width of resonances $\sim\sqrt{\Phi_0}$ be larger than the distance between resonances $\sim\Omega_c/k_\parallel$. For Gendrin angle propagation, this condition corresponds to very high wave amplitudes, larger than 1 V/m for $L \leq 7$. In the latter case, quasi-linear (stochastic) diffusion should again prevail. However, no overlap with the Landau resonance should generally occur for $\omega \leq \Omega_c/3$. A full consideration of higher-order cyclotron harmonics is beyond the scope of the present paper and it will be addressed elsewhere.

The limit of high wave-amplitudes has been considered here to study nonlinear trapping of electrons into the Landau and fundamental resonances. However, we should mention that even waves with smaller amplitudes can interact with particles in the nonlinear regime. Trapped particles oscillate in the effective potential well created by the wave electric field. If the duration of particle trapping is much longer than the period of such oscillations, one can consider nonlinear acceleration as described in this paper. On the other hand, if the duration of particle trapping is comparable with (or shorter than) the period of oscillations, then we deal rather with nonlinear scattering.[6,75,76] Moreover, particle trapping is a probabilistic process and particles cannot be trapped at each time of passage through the resonance, even for high wave-amplitudes. If particles are not trapped, they are scattered at resonance.[77,78] The timescale of such a scattering depends on the wave amplitude and, as a result, this scattering cannot be described in the frame of quasi-linear theory.[79] To complete the picture of nonlinear wave-particle interaction, we should include effects of such a scattering into diffusion equations. Fortunately, the corresponding diffusion coefficients are already derived (see Ref. 6 and references therein) and one only needs to apply the general expressions describing these coefficients to a particular system with oblique electrostatic waves. We leave this piece of work for further publications.

The acceleration of electrons should correspond to some wave damping due to the conservation of energy inside the system. This is especially true for very oblique waves propagating at the Gendrin angle (see Ref. 80). However, there exists also a population of transient particles which are responsible for wave amplification[81] In reality, the considered waves play the role of intermediates between a population of transient electrons losing their energy and a trapped population of electrons gaining energy.[82] The description of this complicated self-consistent system can be based on a model of resonant currents of trapped and transient populations (see Ref. 83). The analytical expressions for the probability of trapping obtained here can already be considered as an important step on this way.

VIII. CONCLUSIONS

In this paper, we have considered nonlinear particle interactions with high-amplitude strongly oblique whistler waves. Using analytical estimates, the corresponding particle energy gains ΔE and jumps of pitch-angle $\Delta\alpha_0$ have been obtained as functions of initial electron energy and pitch-angle in a very wide parameter range. Analytical formulas for the probability of relativistic electron trapping into the Landau and fundamental cyclotron resonances have also been derived. To obtain analogous results by means of numerical simulations, one would have to run millions of particle trajectories. Thus, the analytical expressions derived here for the variation of ΔE and $\Delta\alpha_0$ as functions of L-shell, wave amplitude $E_{0\|}$, and wave-frequency ω_m, are both useful and insightful. In particular, they can be applied to estimate the role of nonlinear trapping in the dynamics of large electron populations. For $E < 100$ keV electrons, Landau resonance looks more effective than the fundamental cyclotron resonance, i.e.,

1. The range of initial pitch-angles with a positive probability of capture is wider for the Landau resonance.
2. The energy gained by trapped electrons is larger for the Landau resonance for $E < 100$ keV and it is comparable for both resonances for $E \sim 100$ keV.
3. The amplitude of pitch-angle jumps due to trapping is comparable for both resonances. These jumps result in pitch-angle increase for the fundamental cyclotron resonance and lead to pitch-angle decrease for the Landau resonance.

ACKNOWLEDGMENTS

The work of A.A.V., A.O.V., K.V. was supported by CNES through the grant Modele dondes. Work of V.A.A. and A.A.V. was also supported by Russian Foundation for Basic Research (Projects 13-01-00251) and by Russian Academy of Science (OFN-15).

APPENDIX A: PROBABILITIES OF TRAPPING FOR THE LANDAU RESONANCE

In this Appendix, we derive an expression for the probability of particle trapping by a wave in the case of the Landau resonance. To this aim, we apply the general approach developed for resonant wave-particle interaction (e.g., Refs. 68 and 84). We renormalize the dimensionless time $t \to t/\varepsilon$, coordinate $z \to z/\varepsilon$, frequency $\omega \to \varepsilon\omega$ and wavenumber $k_\| \to \varepsilon k_\|$ are rewrite the Hamiltonian (3) as

$$\begin{aligned} H &= \gamma - \varepsilon u_0(\varepsilon z)\sin\phi, \\ \gamma &= \sqrt{1 + p_z^2 + \xi b(\varepsilon z)}, \\ \phi &= \phi_0 + \int^{\varepsilon z} k_\|(\varepsilon z')dz' - \omega t, \end{aligned} \tag{A1}$$

where we have introduced the parameter $\xi = 2I_x\chi$. Now, we introduce the new variable ϕ with the help of the canonical transform with the generating function $G = pz + I\phi$, where I is a new variable canonically conjugate to ϕ. For new variables p, I, we have $p = p_z - k_\| I$ and the Hamiltonian (A1) takes a form

$$\begin{aligned} H &= -\omega I + \gamma - \varepsilon u_0(\varepsilon z)\sin\phi, \\ \gamma &= \sqrt{1 + (p + k_\|(\varepsilon z)I)^2 + \xi b(\varepsilon z)}. \end{aligned} \tag{A2}$$

We use the renormalization $q = \varepsilon z$ and rewrite the Hamiltonian (A2),

$$\begin{aligned} H &= -\omega I + \gamma - \varepsilon u_0(q)\sin\phi = H_0 + \varepsilon H_1, \\ \gamma &= \sqrt{1 + (p + k_\|(q)I)^2 + \xi b(q)}. \end{aligned} \tag{A3}$$

Here, the conjugate variables are (ϕ, I) and $(q/\varepsilon, p)$. The corresponding equations of motion are

$$\begin{aligned} \dot{I} &= -\frac{\partial H}{\partial \phi} = \varepsilon u_0(q)\cos\phi, \\ \dot{\phi} &= \frac{\partial H}{\partial I} = -\omega + \frac{pk_\| + k_\|^2 I}{\gamma}, \\ \dot{p} &= -\varepsilon\frac{\partial H}{\partial q} = -\varepsilon\frac{\xi b' + 2pIk_\|' + 2k_\|k_\|' I}{2\gamma}, \\ \dot{q} &= \varepsilon\frac{\partial H}{\partial p} = \varepsilon\frac{p + k_\| I}{\gamma}, \end{aligned} \tag{A4}$$

where $' = d/dq$. The resonance $\dot{\phi} = 0$ corresponds to the condition

$$p + k_\|(q)I_R = \frac{\omega}{k_\|(q)}\sqrt{1 + \xi b(q) + (p + k_\|(q)I_R)^2}. \tag{A5}$$

Equation (A5) defines a certain surface in the 3D space (q, p, I). This surface intersects with the surface of constant energy $H_0 = h$ along the so-called resonant curve,

$$p + k_\|(q)I_R = \frac{\omega}{k_\|(q)}(h + \omega I_R). \tag{A6}$$

The resonant condition (A5) gives the expression for I_R

$$I_R = \frac{\omega/k_\parallel}{\sqrt{k_\parallel^2-\omega^2}}\sqrt{1+\xi b} - \frac{p}{k_\parallel}. \tag{A7}$$

We expand the Hamiltonian (A3) around $I = I_R$,

$$\begin{aligned} H &= \Lambda(p,q) + \frac{1}{2}g(p,q)(I-I_R)^2 - \varepsilon u_0(q)\sin\phi, \\ \Lambda &= H_0\Big|_{I=I_R} = \frac{1}{\gamma_R}\sqrt{1+\xi b} + v_R p, \\ g &= \frac{\partial^2 H_0}{\partial I^2}\Big|_{I=I_R} = \frac{k_\parallel^2}{\gamma_R^3\sqrt{1+\xi b}}, \end{aligned} \tag{A8}$$

where $v_R = \omega/k_\parallel(q)$, $\gamma_R = 1/\sqrt{1-v_R^2}$. We introduce the new canonical variable $K = I - I_R$ with the help of generating function $G_{Landau} = pq\varepsilon^{-1} + (K + I_R)\phi$,

$$H = \Lambda(p,q) + \frac{1}{2}g(p,q)K^2 - \varepsilon u_0(q)\sin\phi + \varepsilon d\phi, \tag{A9}$$

where

$$d(p,q) = \{I_R, \Lambda\} = \frac{\partial I_R}{\partial q}\frac{\partial \Lambda}{\partial p} - \frac{\partial I_R}{\partial p}\frac{\partial \Lambda}{\partial q} \tag{A10}$$

with $\{,\}$ the Poisson bracket

$$\{I_R, \Lambda\} = \frac{\gamma_R}{k_\parallel}\frac{\xi b' + 2v_R v_R' \gamma_R^2(1+\xi b)}{2\sqrt{1+\xi b}} \tag{A11}$$

and $v_R' = -\omega k_\parallel'/k_\parallel^2 = -v_R(k_\parallel'/k_\parallel)$. We use variables $P = K/\sqrt{\varepsilon}$, $t \to t\sqrt{\varepsilon}$ and $F = H/\varepsilon$. In this case, the conjugate pairs are (P,ϕ) and $(p, \varepsilon^{-3/2}q)$. Thus, we can rewrite the Hamiltonian (A9) in the form

$$\begin{aligned} F &= \varepsilon^{-1}\Lambda(p,q) + F_0(P,\phi,p,q), \\ F_0 &= \frac{1}{2}g(q)P^2 - u_0(q)\sin\phi + d(q)\phi. \end{aligned} \tag{A12}$$

The Hamiltonian F_0 is similar to the classical Hamiltonian of a nonlinear pendulum. If $u_0 > d$, then there is a region of oscillations in the phase plane (P,ϕ). The area of this region is

$$S = \frac{2^{3/2}}{\sqrt{g}}\int_{\phi_s}^{\phi_m}\sqrt{d(\phi_s-\phi) + u_0\sin\phi - u_0\sin\phi_s}\,d\phi, \tag{A13}$$

where $\phi_s = -\arccos(d/u_0)$ and ϕ_m is a root of the equation $d(\phi_s - \phi_m) + u_0\sin\phi_m - u_0\sin\phi_s = 0$ different from ϕ_s. To determine the probability of trapping Π, one needs to compare the velocity of evolution of S with the total phase flux Υ. For systems with $d > u_0/\sqrt{\varepsilon}$, we have

$$\Upsilon = \int_0^{2\pi}\dot{p}_\phi d\phi = 2\pi|d| \tag{A14}$$

and the corresponding probability Π is (for $\{S,\Lambda\} > 0$),

$$\Pi = \sqrt{\varepsilon}\frac{\{S,\Lambda\}}{2\pi|d|} = \frac{\sqrt{\varepsilon}v_R}{2\pi|d|}\frac{\partial S}{\partial q}. \tag{A15}$$

For systems with $d < u_0/\sqrt{\varepsilon}$, the equation for Π has the form[85]

$$\Pi = \begin{cases} 2W/(W+4\pi|d|) & \text{if} \quad W < 4\pi|d| \\ 1 & \text{if} \quad W > 4\pi|d|, \end{cases} \tag{A16}$$

where $W = \sqrt{\varepsilon}(\partial S/\partial q)v_R$. Equation (A16) transforms to Eq. (A15) for $4\pi d/\sqrt{\varepsilon} \gg (\partial S/\partial q)v_R$. In both Eqs. (A15) and (A16), all the variables are calculated at the resonance.

Now, we need to derive expressions for d, g, S in the case of waves propagating at the Gendrin angle. For such waves, we have $k_\parallel = \varepsilon k_0/b$ and $v_R = b/(2\omega_{pe})$, leading to

$$\begin{aligned} d &= \frac{b}{k_0\varepsilon}\frac{\gamma_R^2 b'}{\gamma}\left(\frac{1}{2}\xi + \frac{b}{(2\omega_{pe})^2}\gamma^2\right), \\ \frac{1}{\sqrt{g}} &= \frac{\gamma_R\gamma^{1/2}b}{k_0\varepsilon}, \end{aligned} \tag{A17}$$

where $\gamma = \gamma_R\sqrt{1+\xi b}$. In the vicinity of the equator (where $b' = 0$), the probability is defined by Eq. (A16) and it can reach a value of 1, i.e., all the resonant particles can be captured in this region.

Equation (A13) can be rewritten with coefficients A and B from Sec. III,

$$\tilde{S} = \tilde{g}\int_{\phi_s}^{\phi_m}\sqrt{\frac{\mathrm{A}}{\mathrm{B}}(\phi_s-\phi) + \sin\phi - \sin\phi_0}\,d\phi, \tag{A18}$$

where $\tilde{S} = S\varepsilon k_0$, $k_\parallel d = \mathrm{A}$ and $k_\parallel u_0 = \mathrm{B}$ and

$$\tilde{g} = \frac{2^{2/3}\varepsilon k_0}{\sqrt{g/u_0}} = 2^{2/3}\gamma_R b\sqrt{u_0\gamma}. \tag{A19}$$

Therefore, the final expression for the probability Π is

$$\Pi = \begin{cases} 2\tilde{W}/\sqrt{k_0} & \text{if} \quad \sqrt{\varepsilon}\mathrm{A} < \mathrm{B} \\ 2\tilde{W}/(\tilde{W}+\sqrt{k_0}) & \text{if} \quad W < \sqrt{k_0} \\ 1 & \text{if} \quad W > \sqrt{k_0}, \end{cases} \tag{A20}$$

where

$$\tilde{W} = \frac{\sqrt{k_0\varepsilon}v_R(\partial S/\partial q)}{4\pi|d|} = \frac{k_\parallel v_R(\partial\tilde{S}/\partial q)}{4\pi\sqrt{k_0\varepsilon}\mathrm{A}} = \frac{\sqrt{k_0\varepsilon}\tilde{S}'}{8\pi\omega_{pe}\mathrm{A}}. \tag{A21}$$

The first regime in Eq. (A20) corresponds to $\sqrt{k_0\varepsilon}\mathrm{A}/\mathrm{B} < 1/\sqrt{k_0}$ (this condition is satisfied almost everywhere; only for particles trapped in the close vicinity of the equator do we get $\mathrm{A} \sim b' \sim 0$). In this regime, it is convenient to use $\tilde{\Pi} = \Pi\sqrt{\omega_m}$ (with ω the wave frequency normalized on Ω_{c0}), because for the Gendrin angle of propagation, $\tilde{\Pi}$ does not depend on wave frequency.

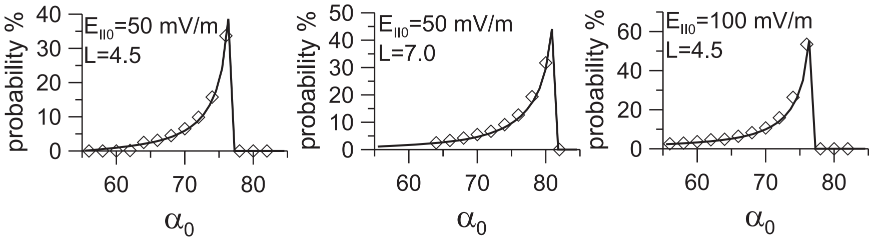

FIG. 10. Comparison of numerical probabilities of trapping in Landau resonance (diamonds) with the corresponding analytical expressions (A20) for 100 keV particles in three different systems.

The obtained expressions (A21) for the probabilities of trapping can be checked by means of test-particle simulations. For each value of the electron energy and equatorial pitch-angle, we run 10^4 particles from the equator in the system, with various initial wave phases. Then, we integrate each trajectory during 1/4 of the bounce period and determine the number of particles which become trapped. The percentage of particles getting trapped gives the numerical probability. A comparison between the numerical results and the analytical expression (A21) shows that the latter is very accurate (see Fig. 10).

In the Earth's radiation belts, the wave amplitude $u(z)$ is often observed to grow as the wave propagates from the equator in the latitude range $\lambda < 20^\circ$ (e.g., Ref. 23). For reasonable values of the plasma density,[86] we have then $\omega_{pe} > 2$ and $v_R^2 = (b/2\omega_{pe})^2 < 0.12$. Therefore, we can neglect the term $\sim\gamma_R$ in Eq. (A17) (at least for particles with energies >100 keV, see Ref. 28).

APPENDIX B: PROBABILITIES OF TRAPPING FOR THE FUNDAMENTAL CYCLOTRON RESONANCE

In this Appendix, we derive an expression for the probability of electron trapping by a wave in the case of the fundamental cyclotron resonance. We use the same dimensionless variables and parameters as in A and rewrite the Hamiltonian (4) as

$$\begin{aligned} H &= \gamma - \varepsilon u(\varepsilon z) J_1(\eta) \sin\phi(\varepsilon z, t, \theta), \\ \gamma &= \sqrt{1 + p_z^2 + I b(\varepsilon z)}, \end{aligned} \tag{B1}$$

where $I_x \to 2\chi I_x$ and $\theta \to \theta/(2\varepsilon\chi)$. Then we use the generating function R, which defines the transformation to new variables $I = I_x$, $p = p_z - k_\| I$,

$$R = \left(\int^{\varepsilon z} k_\|(\varepsilon z') d\varepsilon z' + \theta - \omega t \right) I + pz. \tag{B2}$$

We introduce the $q = \varepsilon z$ and write the Hamiltonian in the new variables

$$\begin{aligned} H &= -\omega I + \gamma - \varepsilon u(q) J_1(\eta) \sin\phi, \\ \gamma &= \sqrt{1 + (p + k_\| I)^2 + Ib}. \end{aligned} \tag{B3}$$

Here, the pairs of conjugate variables are $(p, \varepsilon^{-1} q)$ and (I, ϕ). The Hamiltonian (B3) can be written as $H = H_0 + \varepsilon H_1$. The corresponding equations of motion are

$$\begin{aligned} \dot I &= -\frac{\partial H}{\partial \phi} = \varepsilon u(q) J_1(\eta) \cos\phi, \\ \dot\phi &= \frac{\partial H}{\partial I} = -\omega + \frac{b + 2pk_\| + 2k_\|^2 I}{2\gamma} + O(\varepsilon), \\ \dot p &= -\varepsilon \frac{\partial H}{\partial q} = -\varepsilon \frac{Ib' + 2pIk_\|' + 2k_\| k_\|' I}{2\gamma} + O(\varepsilon^2), \\ \dot q &= \varepsilon \frac{\partial H}{\partial p} = \varepsilon \frac{p + k_\| I}{\gamma}, \end{aligned} \tag{B4}$$

where $' = d/dq$. The resonance $\dot\phi = 0$ corresponds to the condition

$$b + 2k_\| (p + k_\| I_R) = 2\omega \sqrt{1 + I_R b(q) + (p + k_\| I_R)^2}. \tag{B5}$$

The solution of Eq. (B5) is

$$I_R = -\frac{b}{2k_\|^2} - \frac{p}{k_\|} \pm \frac{\omega\sqrt{4k_\|^2 - 4bpk_\| - b^2}}{2k_\|^2 \sqrt{k_\|^2 - \omega^2}}. \tag{B6}$$

Only the solution with "+" can be used. Moreover, for the fundamental resonance, we obtain the additional condition

$$4k_\|^2 - 4bpk_\| - b^2 > 0. \tag{B7}$$

The Hamiltonian (B3) can be expanded around the resonance as

$$H = \Lambda(p, q) + \frac{1}{2} g(p, q)(I - I_R)^2 - \varepsilon u(q) J_1(\eta_R) \sin\phi, \tag{B8}$$

where $\eta_R = \eta_R(p, q)$ is evaluated at $I = I_R$. Functions $\Lambda(p, q)$ and $g(p, q)$ are defined as

$$\begin{aligned} \Lambda &= H_0\Big|_{I=I_R} = \sqrt{1 - v_{R0}^2}\sqrt{1 - \frac{bp}{k_\|} - \frac{b^2}{4k_\|^2}} + \left(\frac{2b}{k_\|} + p\right) v_{R0}, \\ g &= \frac{\partial^2 H_0}{\partial I^2}\Big|_{I=I_R} = \frac{2\left(k_\|^2 - \omega^2\right)^{3/2}}{\sqrt{4k_\|^2 - 4bpk_\| - b^2}}, \end{aligned} \tag{B9}$$

where $v_{R0} = \omega/k_\|$. We use the generating function $G_{fundamental}$ to introduce the new variable $K = I - I_R$,

$$G_{fundamental} = p\varepsilon^{-1} q + (K + I_R)\phi. \tag{B10}$$

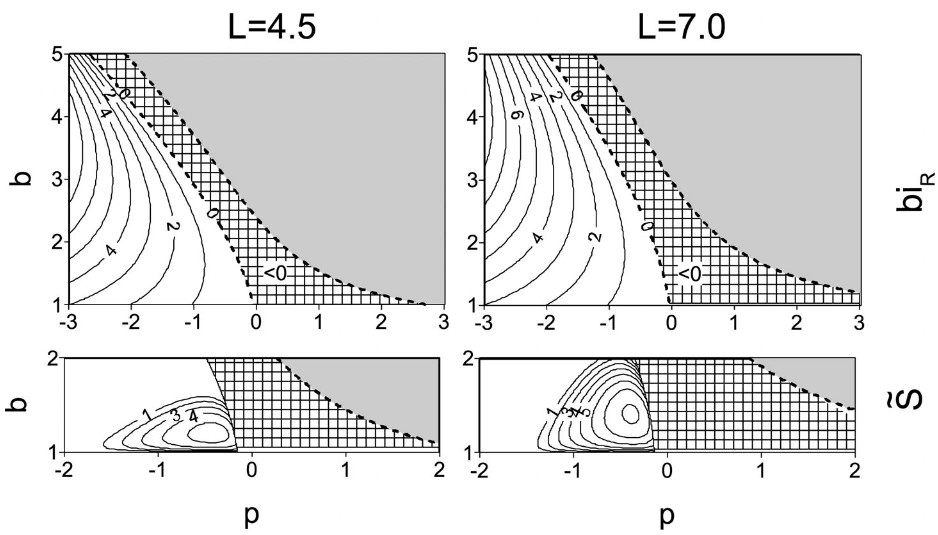

FIG. 11. 2D maps of coefficients I_R, $\tilde{S}$ for two values of L and $E_{\|0} = 50\,\mathrm{mV/m}$ (cyclotron resonance case). Grey color shows region where condition (B7) is not satisfied.

In the variables, the Hamiltonian is

$$H = \Lambda + \frac{1}{2} g K^2 - \varepsilon u(q) J_1(\eta_R) \sin\phi + \varepsilon d\phi, \tag{B11}$$

where $d(p, q) = \{I_R, \Lambda\}$.

We introduce new variables $P = K/\sqrt{\varepsilon}$, $t \to t\sqrt{\varepsilon}$, $F = H/\varepsilon$. Corresponding conjugate variables are (P, ϕ), $(p, \varepsilon^{-3/2} q)$. New Hamiltonian takes the form

$$\begin{aligned} F &= \varepsilon^{-1}\Lambda(p,q) + F_0(P, \phi, p, q), \\ F_0 &= \frac{1}{2} g(p,q) P^2 - u(q) J_1(\eta_R) \sin\phi + d\phi. \end{aligned} \tag{B12}$$

This is the classical Hamiltonian system for the mathematical pendulum. However, in contrast to A, parameters g and d now depend on both coordinates (p,q). Thus, we get a two-dimensional graph for the area S surrounded by the separatrix

$$S = 2^{3/2} \sqrt{\frac{|u(q) J_1(\eta_R)|}{g(p,q)}} \times \int_{\phi_s}^{\phi_m} \sqrt{\sin\phi - \sin\phi_s - \frac{d(p,q)}{u(q) J_1(\eta_R)}(\phi - \phi_s)}\, d\phi. \tag{B13}$$

To plot $S(p,q)$, we should first calculate all the coefficients. We start with $I_R(p,q)$, Λ, and $g(p,q)$ described by Eqs. (B6) and (B9). These equations can be rewritten as

$$\begin{aligned} k_\| I_R &= -(v_0 + p) + \frac{v_{R0}}{\sqrt{1 - v_{R0}^2}} \sqrt{1 - 2 v_0 p - v_0^2}, \\ \Lambda &= \sqrt{1 - v_{R0}^2} \sqrt{1 - 2 v_0 p - v_0^2} + (v_0 + p) v_{R0}, \\ g &= \frac{2 k_\|^2 \left(1 - v_{R0}^2\right)^{3/2}}{\sqrt{1 - 2 v_0 p - v_0^2}}, \end{aligned} \tag{B14}$$

where $v_{R0} = \omega b / k_0$ and $v_0 = \bar{b}/2k_\| = b^2/(2\omega_{pe}\omega_m)$. In dimensional variables, we can write

$$\begin{aligned} \tilde{S} &= \tilde{g} \int_{\phi_s}^{\phi_m} \sqrt{\sin\phi - \sin\phi_s - a(p,q)(\phi - \phi_s)}\, d\phi, \\ a(p,q) &= \frac{d}{u(q) J_1(\eta_R)} = \frac{\{b i_R, \Lambda\}}{k_0 \varepsilon u(q) J_1(\eta_R)}, \\ \tilde{g} &= 2\sqrt{|u(q) J_1(\eta_R)|}\, \frac{b\left(1 - 2 v_0 p - v_0^2\right)^{1/4}}{\left(1 - v_{R0}^2\right)^{3/4}}, \\ i_R &= -(v_0 + p) + v_{R0} \frac{\sqrt{1 - 2 v_0 p - v_0^2}}{\sqrt{1 - v_{R0}^2}}, \end{aligned} \tag{B15}$$

where $\tilde{S} = k_0 \varepsilon S$. We plot the corresponding surfaces in Fig. 11.

We rewrite Eq. (A20) for the probability of capture Π as

$$\Pi = \begin{cases} 2\tilde{W}/\sqrt{k_0} & \sqrt{\varepsilon} a < 1 \\ 2\tilde{W}/(\tilde{W} + \sqrt{k_0}) & \tilde{W} < \sqrt{k_0} \\ 1 & \tilde{W} > \sqrt{k_0}, \end{cases} \tag{B16}$$

with

$$\tilde{W} = \frac{\sqrt{k_0 \varepsilon}\{S, \Lambda\}}{4\pi |d|} = \frac{\sqrt{k_0 \varepsilon}}{4\pi} \frac{\{\tilde{S}, \Lambda\}_{b,p}}{\{b i_R, \Lambda\}_{b,p}}, \tag{B17}$$

where $\{,\}_{b,p} = \{,\}\partial b/\partial q$ is the Poisson bracket with derivative $\partial/\partial q$ replaced by $\partial/\partial b$. Here, $\tilde{W}$ should be evaluated at the resonance, i.e., for $I = I_R$.

To estimate the energy gain due to resonant interaction with the wave, one needs to determine the coordinates (q, p) where particles actually escape from the resonance. In this point, the area S should have the same value as in the trapping point. To determine the evolution of the momentum p in the resonance, we substitute the expression for I_R into Eq. (B3), where we can omit the term $\sim\varepsilon$ and assume $H = \mathrm{const}$,

$$H = v_{R0}(v_0 + p) + \sqrt{1 - v_{R0}^2} \sqrt{1 - 2 v_0 p - v_0^2}. \tag{B18}$$

This equation can be solved with respect to p

$$p = \frac{Hv_{R0} - v_0 + \sqrt{1 - v_{R0}^2}\sqrt{v_0^2 + v_{R0}^2 - 2v_0 v_{R0} H}}{v_{R0}^2}. \quad \text{(B19)}$$

Calculating S along the resonant curve determined by Eq. (B19), we get points of trapping and escape.

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