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# Symmetric cumulants and event-plane correlations

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The ALICE Collaboration has recently measured the correlations between amplitudes of anisotropic flow in different Fourier harmonics, referred to as symmetric cumulants. We derive approximate relations between symmetric cumulants involving  $v_4$  and  $v_5$  and the event-plane correlations measured by ATLAS. The validity of these relations is tested using event-by-event hydrodynamic calculations. The corresponding results are in better agreement with ALICE data than existing hydrodynamic predictions. We make quantitative predictions for three symmetric cumulants which are not yet measured.

Anisotropic flow is the key observable showing that the matter produced in an ultrarelativistic nucleus-nucleus collision behaves collectively as a fluid [1]. Following the discovery of flow fluctuations [2] and triangular flow [3], a “flow paradigm” has emerged, which states that particles are emitted independently (up to short-range correlations) but with a momentum distribution that fluctuates event to event [4]. The azimuthal ( $\varphi$ ) distribution in a given event is written as a Fourier series:

$$P(\varphi) = \frac{1}{2\pi} \sum_{n=-\infty}^{+\infty} V_n e^{-in\varphi}, \quad (1)$$

where  $V_n = v_n \exp(in\Psi_n)$  is the (complex) anisotropic flow coefficient in the  $n$ th harmonic, and  $V_{-n} = V_n^*$ . Both the magnitude [5] and phase [2, 6] of  $V_n$  fluctuate event to event. In the last five years or so, an extremely rich phenomenology has emerged from this simple paradigm. RMS values of  $v_n$  have been measured up to  $n = 6$  [7–10], and more recently, the full probability distribution of  $v_n$  [11]. An even wider variety of new observables can be constructed by combining different Fourier harmonics [12–14]. This new direction was pioneered by the ATLAS collaboration which has measured fourteen mixed correlations involving relative phases between Fourier harmonics, dubbed event-plane correlations [15].

Recently, the ALICE collaboration has taken a new step in this direction [16] by measuring the correlation between the magnitudes of different Fourier harmonics using a cumulant analysis [17]. We define the symmetric cumulant  $SC(n, m)$ <sup>1</sup> with  $n \neq m$  by

$$SC(n, m) \equiv \frac{\langle v_n^2 v_m^2 \rangle - \langle v_n^2 \rangle \langle v_m^2 \rangle}{\langle v_n^2 \rangle \langle v_m^2 \rangle}. \quad (2)$$

ALICE has measured  $SC(3, 2)$  and  $SC(4, 2)$  as a function of centrality. While these two quantities are formally similar, the hydrodynamic mechanisms giving rise to these

correlations differ. Elliptic flow,  $v_2$ , and triangular flow,  $v_3$ , are both determined to a good approximation by linear response to the anisotropies of the initial density profile in the corresponding harmonics [18, 19]. Therefore,  $SC(3, 2)$  directly reflects correlations present in the initial spatial density profile, which are preserved by the hydrodynamic evolution as the spatial anisotropy is converted into a momentum anisotropy. Standard models for the initial density indeed reproduce the negative sign and overall (small) magnitude of the measured  $SC(3, 2)$  for all centralities [16]. By contrast,  $V_4$  gets a significant non-linear contribution proportional to  $V_2^2$  generated by the hydrodynamic evolution [20–22] in addition to the linear contribution from the initial anisotropy in the fourth harmonic [23, 24]. The nonlinear response explains [25] the large event-plane correlation between  $V_2$  and  $V_4$ . It also explains qualitatively why  $SC(4, 2)$  is positive.

In this paper, we derive a proportionality relation between  $SC(4, 2)$  and the corresponding event-plane correlation, where the proportionality constant involves the fluctuations of  $v_2$ . Using this, we are able to relate recent ALICE measurements with previously measured quantities, which circumvents the most typical limitation of hydrodynamic predictions that depend on initial conditions or medium properties [26–32]. The sole assumption underlying our derivation is that the linear and nonlinear contributions to  $V_4$  are independent. The validity of this assumption is tested using hydrodynamic calculations. The value of  $SC(4, 2)$  derived using our relation and previous ATLAS measurements is compared with the recent direct measurement by ALICE. We make predictions along the same lines for  $SC(5, 2)$ ,  $SC(5, 3)$  and  $SC(4, 3)$ , which are not yet measured.

We decompose  $V_4$  and  $V_5$  into linear and non-linear parts [21]

$$\begin{aligned} V_4 &= V_{4L} + \chi_4 (V_2)^2 \\ V_5 &= V_{5L} + \chi_5 V_2 V_3. \end{aligned} \quad (3)$$

We define  $\chi_4$  and  $\chi_5$  in such a way that the linear correlations between linear and nonlinear parts vanish, that is,  $\langle V_{4L} (V_2)^{*2} \rangle = \langle V_{5L} V_2^* V_3^* \rangle = 0$ . We now introduce a measure of the relative magnitude of the linear and nonlinear parts via the Pearson correlation coefficients

<sup>1</sup> Note the ALICE collaboration uses the same notation for the numerator only.

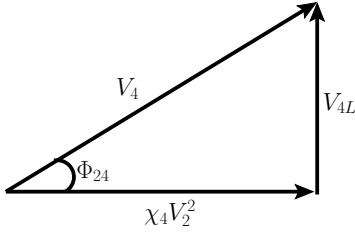


FIG. 1. (Color online) Schematic picture of the relation between the event-plane angle  $\Phi_{24}$  in Eq. (4) and the decomposition Eq. (3). The legs of the triangle correspond to the rms values of the linear and nonlinear parts, and the hypotenuse is the rms  $v_4$ . A similar figure can be drawn for  $V_5$ .

between  $V_4$ , or  $V_5$ , and their nonlinear parts:

$$\begin{aligned} \cos \Phi_{24} &\equiv \frac{\text{Re}\langle V_4(V_2^*)^2 \rangle}{\sqrt{\langle v_4^2 \rangle \langle v_2^4 \rangle}} \\ \cos \Phi_{235} &\equiv \frac{\text{Re}\langle V_5 V_2^* V_3^* \rangle}{\sqrt{\langle v_5^2 \rangle \langle v_2^2 v_3^2 \rangle}}, \end{aligned} \quad (4)$$

where  $\Phi_{24}$  and  $\Phi_{235}$  lie between 0 and  $\pi$ . The first angle  $\Phi_{24}$  corresponds precisely to the event-plane correlation measured by ATLAS [15] and denoted by  $\langle \cos(4(\Phi_2 - \Phi_4)) \rangle_w$ .<sup>2</sup> The second angle  $\Phi_{235}$  almost corresponds to the quantity denoted by  $\langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle_w$ . The only difference is that the latter has  $\langle v_2^2 \rangle \langle v_3^2 \rangle$  in the denominator, instead of  $\langle v_2^2 v_3^2 \rangle$  [21]. Therefore the precise relation is

$$\cos \Phi_{235} = \frac{\langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle_w}{\sqrt{1 + SC(3,2)}}, \quad (5)$$

where  $SC(3,2)$  is defined in Eq. (2).

Inserting Eq. (3) into Eq. (4), one obtains

$$\begin{aligned} \chi_4^2 \langle v_2^4 \rangle &= \langle v_4^2 \rangle \cos^2 \Phi_{24} \\ \chi_5^2 \langle v_2^2 v_3^2 \rangle &= \langle v_5^2 \rangle \cos^2 \Phi_{235}. \end{aligned} \quad (6)$$

These equations are exact and simply follow from the definition of  $\chi_4$  and  $\chi_5$ . They are depicted in Fig. 1.

We now assume that the linear parts  $V_{4L}$  and  $V_{5L}$  are statistically independent of  $V_2$  and  $V_3$ . This is a stronger statement than just assuming that the linear correlation vanishes. As will be shown below, it is a reasonable approximation in hydrodynamics. Then, only the nonlinear response contributes to the correlation between  $v_4$  and  $v_2$ , and Eq. (3) gives:

$$\langle v_4^2 v_2^2 \rangle - \langle v_4^2 \rangle \langle v_2^2 \rangle = \chi_4^2 (\langle v_2^6 \rangle - \langle v_2^4 \rangle \langle v_2^2 \rangle). \quad (7)$$

<sup>2</sup> We only consider the event-plane correlations measured using the scalar-product method, which are denoted by the subscript “w” in the ATLAS paper and have a clear interpretation in terms of  $V_n$ , in contrast to the results obtained using the event-plane method [33].

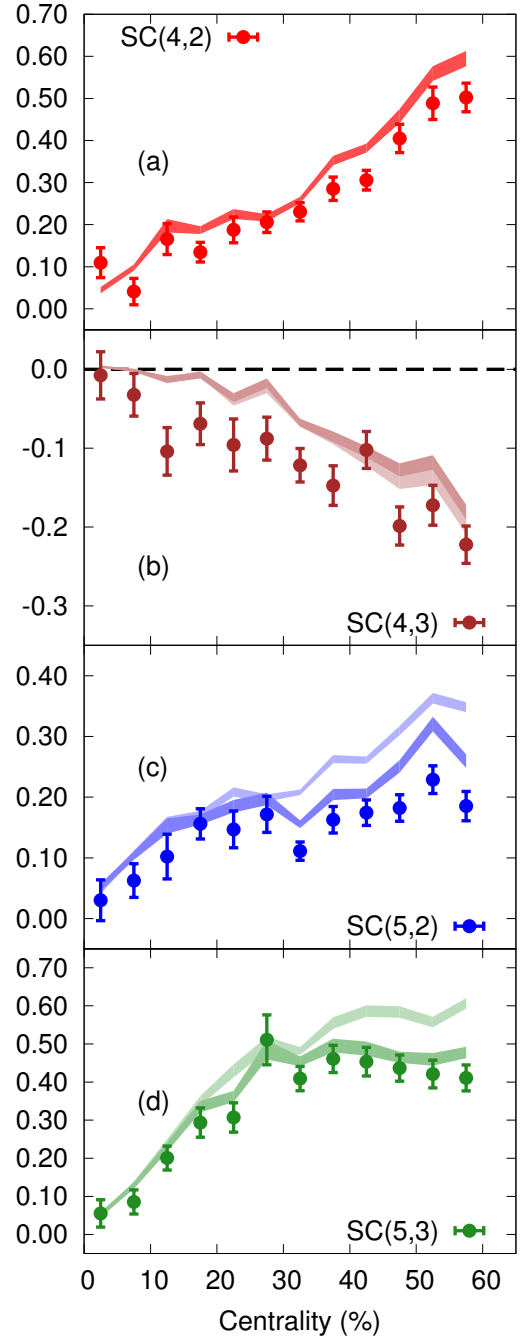


FIG. 2. (Color online) Test of Eqs.(8) using hydro calculations. Symbols correspond to the left-hand sides of Eqs. (8), dark shaded bands to the right-hand sides. Light-shaded bands correspond to Eqs. (9) and (12). Errors are statistical and estimated via jackknife resampling.

Similar relations can be written for the correlations between  $v_4^2$  and  $v_3^2$ ,  $v_5^2$  and  $v_2^2$  or  $v_3^2$ . Substituting in  $\chi_4$  and  $\chi_5$  extracted from Eqs. (6), one obtains

$$SC(4,2) = \left( \frac{\langle v_2^6 \rangle}{\langle v_2^4 \rangle \langle v_2^2 \rangle} - 1 \right) \cos^2 \Phi_{24}$$

$$\begin{aligned}
SC(4, 3) &= \left( \frac{\langle v_2^4 v_3^2 \rangle}{\langle v_2^4 \rangle \langle v_3^2 \rangle} - 1 \right) \cos^2 \Phi_{24} \\
SC(5, 2) &= \left( \frac{\langle v_2^4 v_3^2 \rangle}{\langle v_2^2 v_3^2 \rangle \langle v_2^2 \rangle} - 1 \right) \cos^2 \Phi_{235} \\
SC(5, 3) &= \left( \frac{\langle v_2^2 v_3^4 \rangle}{\langle v_2^2 v_3^2 \rangle \langle v_3^2 \rangle} - 1 \right) \cos^2 \Phi_{235} \quad (8)
\end{aligned}$$

These equations express symmetric cumulants in terms of event-plane correlations and moments of  $v_2$  and  $v_3$ . Based on these equations, one expects symmetric cumulants involving  $v_4$  or  $v_5$  to increase with viscosity, in the same way as event-plane correlations [34, 35].

In order to test Eqs. (8), we carry out event-by-event hydrodynamic calculations using the same setup as in Ref. [36]: initial conditions are given by the Monte-Carlo Glauber model [37], the shear viscosity over entropy ratio is  $\eta/s = 0.08$  [38] within the viscous relativistic hydrodynamical model v-USPhydro [39, 40], and  $V_n$  is calculated at freeze-out [41] for pions. Note, however, that the particular setup used, and whether or not it quantitatively reproduces experimental data, is irrelevant in this context, since the statement is that Eqs. (8) should hold to a good approximation for *any* hydrodynamic calculation. In hydrodynamics,  $V_n$  can be computed exactly from the one-particle momentum distribution for each event [42–44]. Therefore, reasonable accuracy is obtained with fewer events than in an actual experiment. We generate 1000 events for each 5% centrality bin. Figure 2 displays the comparison between the left-hand side (symbols) and the right-hand side (dark shaded bands) of Eqs. (8). Agreement is good for all four quantities and all centralities, in the sense that the absolute difference is typically a few  $10^{-2}$ . The values of  $SC(n, m)$  derived using Eqs. (8) tend to be above the actual values. This shows that the magnitude of  $V_{4L}$  (or  $V_{5L}$ ) and that of  $v_2$  (or  $v_3$ ) are not quite independent in hydrodynamics, but have a slight negative correlation. However, Eqs. (8) correctly capture the sign, magnitude and centrality dependence of symmetric cumulants.

The equation for  $SC(4, 2)$  can also be tested against existing data. The moments of  $v_2$  are not directly measured but they can be expressed [21] as a function of cumulants, which have also been measured by ATLAS [45]. Figure 3 displays the comparison between the left-hand side of Eq. (8) measured by ALICE [16] and the right-hand side using ATLAS data. Agreement is reasonable for all centralities. In particular, our data-driven approach gives a better result for  $SC(4, 2)$  than existing hydrodynamic predictions [16, 35]. Based on the hydrodynamic calculation of Fig. 2, one would expect that the right-hand side of Eq. (8) is larger than the left-hand side. However, it is the other way around above 30% centrality. One reason may be that the event-plane correlation for ATLAS uses a much larger pseudorapidity window ( $|\eta| < 4.8$ ) than ALICE ( $|\eta| < 0.8$ ). Now, the phase of  $V_n$  depends slightly on rapidity [46–48], which induces a decoherence of azimuthal correlations for larger  $\Delta\eta$  [49, 50]. Due to these longitudinal flow fluctuations, the event-plane correlation

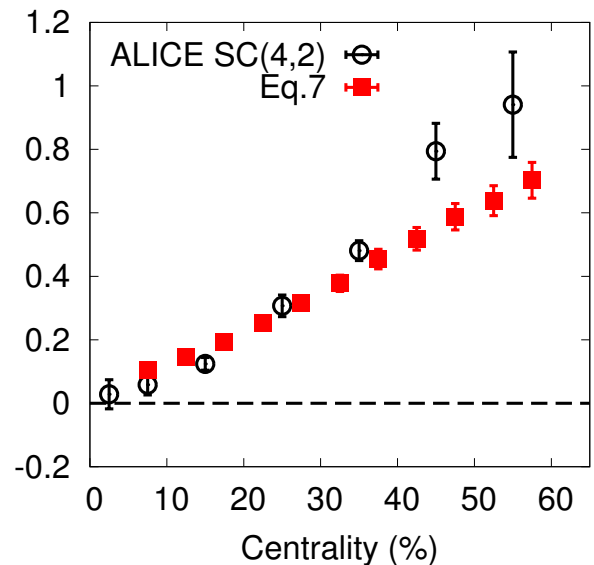


FIG. 3. (Color online) Open symbols: ALICE data for  $SC(4, 2)$  [16]. Closed symbols: value obtained using the right-hand side of Eq. (8) using ATLAS data for the moments of  $v_2$  [45] and the event-plane correlation [15].

measured by ATLAS is smaller than what ALICE would measure in a more central rapidity window. Ideally, the comparison between the two sides of Eq. (8) should be done in the exact same rapidity window.

We now make predictions for  $SC(4, 3)$ ,  $SC(5, 2)$  and  $SC(5, 3)$  using Eqs. (8). The right-hand sides involve the mixed moments  $\langle v_2^4 v_3^2 \rangle$  and  $\langle v_2^2 v_3^4 \rangle$  which could be measured directly [14] but are not yet measured. However, the ALICE collaboration measures  $|SC(3, 2)| \ll 1$  for all centralities [16], which implies  $\langle v_2^2 v_3^2 \rangle \approx \langle v_2^2 \rangle \langle v_3^2 \rangle$ . Therefore, one can assume, as a first approximation, that  $v_2^2$  and  $v_3^2$  are independent. Out of curiosity's sake, we also neglect the correlation in evaluating  $\Phi_{235}$ , i.e., we make the approximation  $\cos \Phi_{235} \approx \langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle_w$  (see Eq. (5)). Eqs. (8) then give

$$\begin{aligned}
SC(5, 2) &\approx \left( \frac{\langle v_2^4 \rangle}{\langle v_2^2 \rangle^2} - 1 \right) \langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle_w^2 \\
SC(5, 3) &\approx \left( \frac{\langle v_3^4 \rangle}{\langle v_3^2 \rangle^2} - 1 \right) \langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle_w^2. \quad (9)
\end{aligned}$$

The validity of Eqs. (9) can again be tested using event-by-event hydrodynamics. The right-hand sides are shown as light-shaded bands in Figs. 2 (c) and (d). Agreement is excellent for central collisions but becomes worse as the centrality percentile increases, as expected since we have neglected  $SC(3, 2)$  which becomes sizable for peripheral collisions.

If one assumes that  $v_2^2$  and  $v_3^2$  are independent, the second line of Eqs. (8) gives  $SC(4, 3) = 0$ . In order to obtain a non-trivial prediction for  $SC(4, 3)$ , we need to take into account the small correlation between  $v_2^2$  and  $v_3^2$ . We do this by assuming that  $v_3^2$  can be decomposed

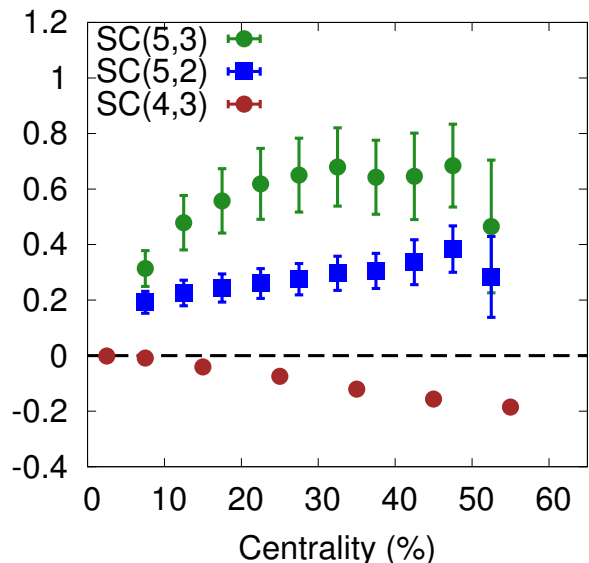


FIG. 4. (Color online) Predictions using the right-hand sides of Eqs. (9) and (12), using ATLAS data for the moments of  $v_2$  and  $v_3$  [45] and the event-plane correlations [15], and ALICE data for  $SC(3,2)$  [16].

as

$$v_3^2 = cv_2^2 + \beta, \quad (10)$$

where  $c$  is the same for all events in a centrality class, and  $\beta$  is independent of  $v_2^2$ . Using Eq. (10), the correlation between an arbitrary moment of  $v_2$  and  $v_3^2$  is given in terms of moments of  $v_2$ :

$$\begin{aligned} \langle v_2^2 v_3^2 \rangle - \langle v_2^2 \rangle \langle v_3^2 \rangle &= c (\langle v_2^4 \rangle - \langle v_2^2 \rangle^2) \\ \langle v_2^4 v_3^2 \rangle - \langle v_2^4 \rangle \langle v_3^2 \rangle &= c (\langle v_2^6 \rangle - \langle v_2^4 \rangle \langle v_2^2 \rangle). \end{aligned} \quad (11)$$

The first equation relates  $c$  with  $SC(3,2)$  through Eq. (2). Taking the ratio of Eqs. (11) and inserting into Eq. (8), one obtains

$$SC(4,3) \approx \frac{\langle v_2^2 \rangle (\langle v_2^6 \rangle - \langle v_2^4 \rangle \langle v_2^2 \rangle)}{\langle v_2^4 \rangle (\langle v_2^4 \rangle - \langle v_2^2 \rangle^2)} SC(3,2) \cos^2 \Phi_{24}. \quad (12)$$

The right-hand side of this equation is shown as a light-shaded band in Fig. 2 (b). It is very close to the dark-shaded banded for all centralities, thus showing that the decomposition in Eq. (10) appropriately takes into account the correlation between  $v_2$  and  $v_3$ .

Figure 4 displays our predictions for  $SC(5,3)$ ,  $SC(5,2)$  and  $SC(4,3)$  using Eqs. (9) and (12), where we use ATLAS data for the quantities in the right-hand side. Since  $\langle v_3^4 \rangle$  is not measured below 15% centrality, we assume  $\langle v_3^4 \rangle \approx 2\langle v_3^2 \rangle^2$ , i.e., Gaussian fluctuations [51] for  $SC(5,3)$  in the most central bins.<sup>3</sup> For  $SC(4,3)$ , we use ALICE data for  $SC(3,2)$ , and the other quantities in the right-hand side of Eq. (12) (moments of  $v_2$  and  $\cos \Phi_{24}$ ) are interpolated from ATLAS data, since ALICE and ATLAS use different centrality bins.

We have derived proportionality relations between symmetric cumulants involving  $v_4$  or  $v_5$  and event-plane correlations. These relations link correlations of different orders (symmetric cumulants are 4-particle correlations, while event-plane correlations are 3-particle correlations) and are fully non trivial. They are satisfied to a good approximation in event-by-event hydrodynamics, and thus offer a direct test of hydrodynamic behavior, which does not rely on a specific model of initial conditions and medium properties. The recent measurement of  $SC(4,2)$  by ALICE passes the test. We have made predictions for  $SC(5,2)$ ,  $SC(5,3)$  and  $SC(4,3)$  which can be measured in the near future. These new observables will allow to test hydrodynamic behavior directly, provided that one also measures higher-order correlations between  $v_2$  and  $v_3$  such as  $\langle v_2^4 v_3^2 \rangle$ .

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