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## Abstract

The grand piano action has been developped empirically and provides a remarkably accurate control of the hammer velocity and the impact time [1]. The numerical simulation of the action is necessary to improve numerical keyboards and to better understand haptic controllability. Following previous efforts [2, 3], we propose a simulation based on an explicit model and a solver of non-smooth dynamical problems. We also discuss the physical quantity – force or displacement – with which the model is to be controlled.

A scheme of the grand piano key is presented in Figure 1, with the proposed corresponding kinematical diagram in Figure 2. The 2D model [3] includes six rigid bodies in rotation with dry and viscous articular friction, thirteen contact zones with nonlinear springs (Equation 1), three of them (hammer-jack, jack-escapement button, hammer-check) being also subject to Coulomb friction. The reaction force $F_{\text{felts}}$ of a felt to a displacement $\delta$ is given by:

$$F_{\text{felts}}(\delta) = k\,\delta^r + p\,\dot{\delta}\,\delta^2 \tag{1}$$

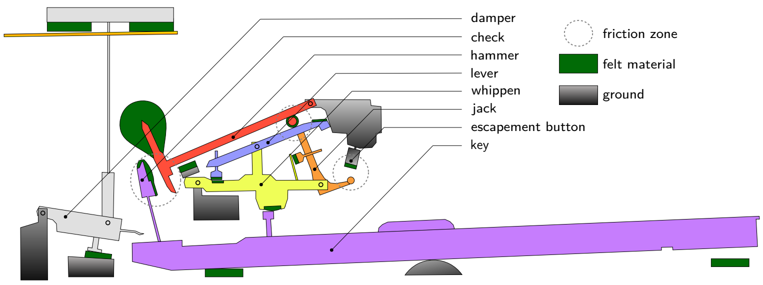


**Figure 1.** Grand piano action.

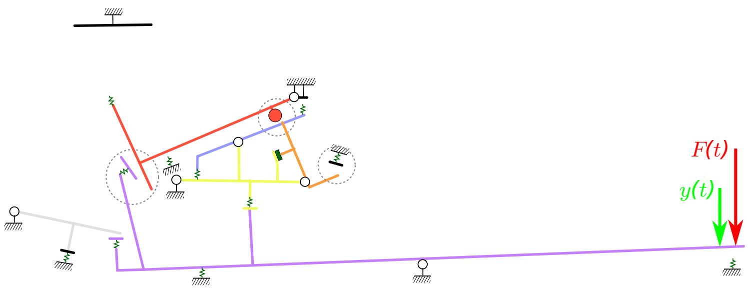


**Figure 2.** Kinematic diagram of the grand piano action. $F(t)$ is the force exerted/felt by the player, $y(t)$ is the displacement of the key.

Dry articular friction and Coulomb friction induce stick-slip transitions and therefore velocity discontinuities. Other potential discontinuities also arise when contacts occur. Therefore, the behaviour of the rigid bodies can only be described with piecewise regular functions. A decomposition in regular phases would be extremely tedious because of the numerous contacts and because dry friction cannot be written as a function of velocities. We overcome this difficulty by using a non-smooth approach [4]. The dynamics are written as a measure differential inclusion of the form:

$$M(q)\,\mathrm{d}\dot{q} + F(t, q, \dot{q})\,\mathrm{d}t = \mathrm{d}i \tag{2}$$

where $q$ is the vector of generalised coordinates made of the angles of each rigid body, $M(q)$ is the mass matrix, $F(t, q, \dot{q})$ are the internal forces, d$t$ is the Lebesgue measure and d$i$ is the reaction measure. The Lebesgue's decomposition theorem yields:

$$\mathrm{d}\dot{q} = \gamma\,\mathrm{d}t + (\dot{q}^{+} - \dot{q}^{-})\,\mathrm{d}\nu + \mathrm{d}v_s \tag{3}$$

$$\mathrm{d}i = f\,\mathrm{d}t + p\,\mathrm{d}\nu + \mathrm{d}i_s \tag{4}$$

where d$\nu$ is a countable sum of weighted Dirac distributions and d$v_s$ and d$i_s$ are neglected singular measures. Equation 3 is discretized in time using a theta-method [5]. Typical results of the simulation are presented in Figure 3, either for a force-driven simulation (top frames) or for a position-driven simulation (bottom frames). We discuss the fact that, for the same model, the qualitative differences between the simulation and the measurements are greater for position-driven simulations.

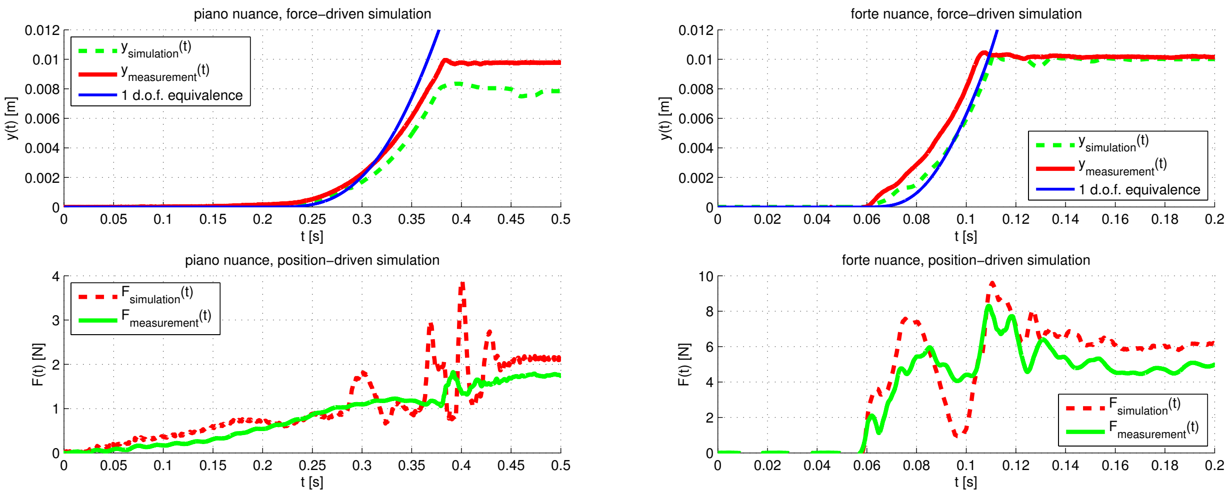


**Figure 3.** Experimental and simulation results $F(t)$ and $y(t)$ (see Figure 2). Left: $p$ nuance. Right: $f$ nuance. Top: control by the measured $F(t)$. Bottom: control by the measured $y(t)$. In blue: the displacement calculated from the measured $F(t)$ on a simple model of the key with only one degree of freedom.

The time series $F(t)$ or $y(t)$ which has not been used for driving the simulation is considered as the output of the system. The dynamics of the whole mechanism is dominated by inertia and gravity (see the equivalent system of 1 d.o.f. subject only to inertia and gravity, blue curve in figure 3). Therefore, it mainly acts as a low-pass second order filter of the exerted force $F(t)$ or as a second-order high-pass filter of the displacement $y(t)$. Playing on recent numerical keyboards with "heavy touch" demonstrates that inertia and gravity alone are not enough to provide a realistic haptic feedback. Therefore, we hypothesize that the realistic quality of a key relies on small variations, of high-frequency content, in the time-series $\{F(t), y(t)\}$. Since the force control is mostly insensitive to such details (by virtue of the low-pass filtering), it follows that the model must be position-driven in order to account for significant details.

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