Discussion of "Investigation of Flow Upstream of Orifices" by D. B. Bryant, A. A. Khan and N. M. Aziz, Journal of Hydraulic Engineering, January 1, 2008, Vol. 134, No. 1, pp. 98-104

G. Belaud, X. Litrico

Discussion of "Investigation of Flow Upstream of Orifices" by D. B. Bryant, A. A. Khan and N. M. Aziz

Journal of Hydraulic Engineering, January 1, 2008, Vol. 134, No. 1, pp. 98-104.

DOI: 10.1061/(ASCE)0733-9429(2008)134:1(98)

Gilles Belaud[1] and Xavier Litrico[2]

In their paper (Bryant et al., 2008) the authors use potential flow theory to study flow upstream of orifices, and compare theoretical results to a large set of experimental data. As pointed out previously by different authors among which Shammaa et al. (2005) and Belaud and Litrico (2007,2008), potential flow assumptions give a quick and efficient method to estimate the velocity field generated by an orifice and may be sufficient for many engineering applications that do not require to take all the real fluid effects into account, such as vortices. This paper therefore provides another interesting application of the potential flow theory to real cases.

However, there seems to be some confusion about the notions used in the paper, especially the radial velocity and the velocity magnitude, and this discussion aims at bringing some complements to clarify these points. We also clarify the way one should apply the principle of superposition, and finally provide some theoretical background for the study of the velocity distribution.

[1]Researcher, UMR Gestion de l'Eau, Acteurs, Usages, IRD, Maison des Sciences de l'Eau, 300 av. Emile Jeanbrau 34095 Montpellier Cedex 5, France, belaud@msem.univ-montp2.fr

[2]Researcher, UMR Gestion de l'Eau, Acteurs, Usages, Cemagref, 361, rue JF Breton, B.P. 5095, 34196 Montpellier Cedex 5, France. E-mail: xavier.litrico@cemagref.fr

Potential Flow in Polar Coordinates

The paper uses the potential method described in Shammaa et al. (2005). According to Bryant et al. (2008), the application of this method gives a good description of the flow far from the orifice but a rather bad description of the flow pattern near the orifice. The authors therefore propose a so-called "new model" that better reproduces the flow pattern. This discrepancy between the two models is surprising, since the method used by Bryant et al. (2008) to derive the potential function is exactly the same as the one originally proposed by Shammaa et al. (2005). In fact, the model proposed by the authors is obtained from the original model of Shammaa et al. (2005) via a simple change of coordinates, which does not justify in our view the denomination of "new model".

Moreover, the discrepancy appears to be due to a misunderstanding of the original model of Shammaa et al. (2005) which correctly represents the velocity pattern (see Fig. 3 in Shammaa et al. (2005)).

First of all, let us recall that the flow potential Φ, in the plane $x - z$ (the same notation as Bryant et al. is used), is a function of r and θ, contrarily to the notation used by the authors in Eq. (1). The radial and transverse velocities are then obtained as follows (see e.g., Batchelor, 1967, pp.100, 600):

$$\begin{aligned} V_r &= \frac{\partial \Phi}{\partial r} \\ V_\theta &= \frac{\partial \Phi}{r \partial \theta} \end{aligned}$$

and $\vec{V} = V_r \vec{e_r} + V_\theta \vec{e_\theta}$. We recall in Figure 1 the definition of the radial and transverse velocities V_r and V_θ and the unit vectors $\vec{e_r}$ and $\vec{e_\theta}$.

In the vertical plane $x - z$, the velocity magnitude V can be computed as follows:

$$V = \sqrt{V_r^2 + V_\theta^2} \tag{1}$$

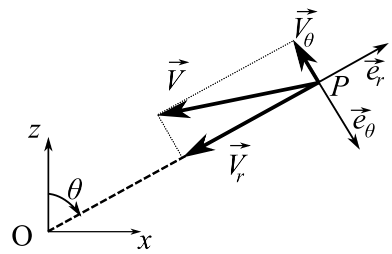

Figure 1: Definition of velocity components in a vertical plane $(x - z)$

Assuming that the potential depends only on r leads to assume that V_θ is zero, which in turn gives $V = |V_r|$. Note that V_r should be negative in the case of a sink.

In fact, it is not clear which quantity is displayed in the figures of the paper. The legend states V_r, i.e. the radial velocity, but the discussers think that the plot corresponding to the "new solution" and the experimental results in fact represent the velocity magnitude, while the "original solution" corresponds to the magnitude of the radial velocity.

Therefore, we think that the apparent discrepancy between the original and new solutions is due to a confusion between radial velocity and velocity magnitudes. Provided the origin of the polar coordinates is in the orifice, these two quantities are close to each other far from the orifice as pointed out by the authors, but largely deviate as we approach the orifice.

We now use an analytical model derived by Belaud and Litrico (2007) to illustrate these concepts.

Radial Velocity and Velocity Magnitude

Let us consider a square orifice of side $2c$ centered in 0, and use potential flow theory to express the velocity components in Cartesian coordinates. As shown by Belaud and Litrico (2007) for any rectangular orifice, the potential flow solution can be expressed in closed-form for the velocity components V_x, V_y and V_z. To simplify the writing, we calculate the velocity components in $x-z$ plane. Due to symmetry, the component V_y is null. We introduce

$$
\begin{aligned}
M &= -\frac{Q}{8\pi c}, && (2)\\
r_1 &= \sqrt{c^2+x^2+(c-z)^2}, && (3)\\
r_2 &= \sqrt{c^2+x^2+(c+z)^2}, && (4)\\
\lambda_1 &= \frac{\sqrt{x^2+(c-z)^2}+(c-z)}{x}, && (5)\\
\lambda_2 &= \frac{\sqrt{x^2+(c+z)^2}-(c+z)}{x}, && (6)\\
X_1 &= \frac{c}{r_1+\sqrt{x^2+(c-z)^2}}, && (7)\\
X_2 &= \frac{c}{r_2+\sqrt{x^2+(c+z)^2}}, && (8)
\end{aligned}
$$

in which Q is the discharge through the orifice.

Using the results developed in Belaud and Litrico (2007), V_x and V_z can be computed using the following analytical expressions:

$$
\begin{aligned}
V_x &= 2M\left[\arctan(\lambda_1 X_1)-\arctan\left(\frac{X_1}{\lambda_1}\right)-\arctan(\lambda_2 X_2)+\arctan\left(\frac{X_2}{\lambda_2}\right)\right] && (9)\\
V_z &= M\log\left[\frac{(r_1+c)(r_2-c)}{(r_1-c)(r_2+c)}\right]. && (10)
\end{aligned}
$$

From this set of equations, we can compute the radial velocity V_r and the velocity magnitude V as follows:

$$
\begin{aligned}
V_r &= \frac{\vec{V}.\overrightarrow{\mathrm{OP}}}{|\overrightarrow{\mathrm{OP}}|}=\frac{xV_x+zV_z}{\sqrt{x^2+z^2}} && (11)\\
V &= \sqrt{V_x^2+V_z^2}. && (12)
\end{aligned}
$$

These velocities are plotted at different distances from the orifice in Figure 2. The distances are normalized by the equivalent diameter $d=2\sqrt{4/\pi}c$ which gives the same area for the square orifice as for a circular orifice of diameter d. The plots are very similar to those of Bryant et al. (2008) (Fig. 4). In the paper, the radial velocity is calculated using the method of Shammaa

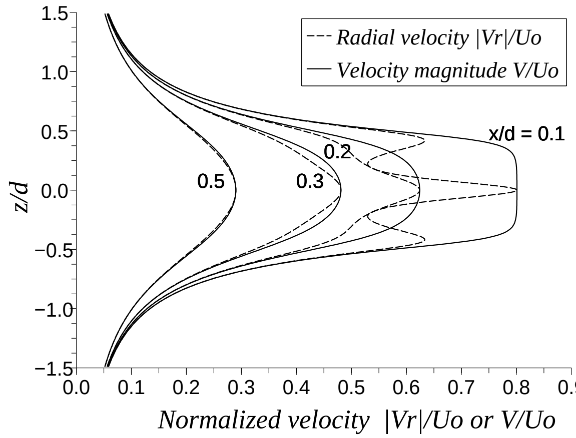

Figure 2: Velocity values along z coordinate at different distances x from the orifice plane.

et al. (2005), while the solution presented as the "new solution" appears to be the velocity magnitude. Far from the orifice, the transverse component of the velocity becomes small and both quantities become very close. Figure 3 shows the iso-velocity lines in the plane $x - z$ and can be compared to Figure 3 of Bryant et al. (2008): the dotted lines depict the radial velocity V_r, while the plain lines depict the velocity magnitude. We can also point out that the orifice shape has little influence in the domain of study.

As a side remark, we note that using Eq. (1), we have:

$$V_r^2 \leq V^2. \tag{13}$$

This inequality explains why, in Figures 3 and 4 of Bryant et al. (2008), the original solution is always lower than the so-called new solution, in which the velocity magnitude V is calculated from Eq. (12) of this discussion and Eqs. (5) to (7) of Bryant et al. (2008).

Principle of Superposition

According to the principle of superposition, the potential function, the stream function and the velocity vectors can be added when superposing different sinks, but the velocity magnitudes

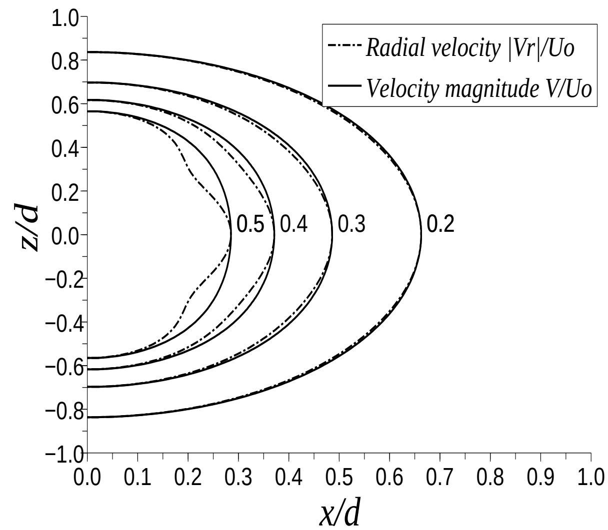

Figure 3: Iso-velocity lines in $x - z$ plane

can certainly not be added.

Let us illustrate this point in the case of the superposition of two velocity fields. We want to find the resulting velocity fields by superposition of the fields $\vec{V}_1$ and $\vec{V}_2$. The resulting field is given by:

$$\vec{V} = \vec{V}_1 + \vec{V}_2.$$

This can be expressed in any coordinate system by adding the components of the velocity vector along the coordinate basis. In Cartesian coordinates, we get:

$$V_x = V_{1x} + V_{2x}$$

$$V_z = V_{1z} + V_{2z}.$$

In polar coordinates, we get:

$$V_r = V_{1r} + V_{2r}$$

$$V_\theta = V_{1\theta} + V_{2\theta}.$$

Therefore, the radial velocity components can be added while superposing different sinks, but the transverse component V_θ is needed in order to compute the velocity vector. This is valid only if the same coordinate system is used for all the superposed sinks. In the polar coordinates case, it is important to check that the two systems are described using the same origin.

The expressions provided by Shammaa et al. (2005) in Eqs. (10–11) are correct and lead to the same results as in Cartesian coordinates. The discrepancy between the original solution and the new solution in Figures 10, 12, 13 and 14 of Bryant et al. (2008) is again probably due to a confusion between radial velocities and velocity magnitudes. According to the discussers' calculations, these plots ("original solution") may result from the addition of the velocity magnitudes or from addition of radial velocities calculated with a different origin for both sources.

Effect of the Velocity Distribution

The authors used an empirical method to determine the limit when the velocity distribution should be considered. The present discussion brings some theoretical elements consistent with the experimental results. To simplify, we consider a square orifice of height $2c$. The effect of the orifice size can be analyzed by considering that an orifice is composed of two parts, an upper part of height c, centered in $+c/2$, and a lower part of height c, centered in $-c/2$.

The mean head above the upper orifice is $h_0 - c/2$, while the mean head above the lower orifice is $h_0 + c/2$. Since the orifice strength is proportional to its discharge and therefore to the square root of the head, the mean error ϵ on the strength between the lower and the upper orifices can be estimated by :

$$\epsilon = \frac{\sqrt{h_0 + c/2} - \sqrt{h_0 - c/2}}{\sqrt{h_0}} \tag{14}$$

which gives

$$\epsilon \simeq \frac{c}{2h_0} \tag{15}$$

for small values of c/h_0.

If $d = 2c$ is the height of the orifice and if we take $\epsilon = 0.02$ as the authors do in their paper, corresponding to an error of 2%, we find a limit of $h_0/d = 12.5$, which is exactly the result obtained experimentally by the authors.

The same analysis conducted with a circular orifice would lead to a slightly different result, but the analysis gives a rough and rapid estimation of the effect of the velocity distribution within the orifice. This may be helpful to decide whether, or not, the velocity distribution should be used in the potential flow solution. If not, we may use closed-form expressions for the velocity field (see equations (9) and (10) of this discussion for the square orifice).

References

Batchelor, G.K. (1967). *An introduction to fluid mechanics.* Cambridge University Press.

Belaud G. and Litrico X. (2007). 3D velocity field generated by a side orifice in an open-channel. *32th Congress IAHR, Venice, 1-4 July 2007.*

Belaud G. and Litrico X. (2008). Closed-form solution of the potential flow in a contracted flume *J. of Fluids Mechanics*, 599:299–307.

Bryant D. B., Khan, A. A. and Aziz, N. M. (2008). Investigation of Flow Upstream of Orifices. *J. of Hydraulic Engineering*, 134(1):98–104.

Shammaa, Y., Zhu, D. Z. and Rajaratnam, N. (2005). Flow upstream of orifices and sluice gates. *J. of Hydraulic Engineering*, 131(2):127–133.