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HAL Id: hal-01162465

https://hal.archives-ouvertes.fr/hal-01162465

Submitted on 10 Jun 2015

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Alexandre Bayen and Xavier Litrico — March 10, 2009

With a population of more than six billion people, food production from agriculture has to be raised to meet the increasing demand of food supply. While irrigated agriculture provides 40% of the total food production, it represents 80% of the freshwater consumption worldwide. In summer and drought conditions, efficient management of the scarce water resources becomes crucial. The majority of open-channel irrigation canals are managed manually, however, with large water losses leading to low water efficiency. The present article focuses on the development of algorithms which could contribute to more efficient management of irrigation canals, used to convey water from a source (generally a dam or reservoir located upstream) to water users.

Figure 1 illustrates the type of setting considered for this study. The intake structure, figure 1a, withdraws water from the source of supply (river, lake, etc.) and directs it to the irrigation system. The withdrawn water is conveyed to a distribution system by a conveyance canal, as shown in figure 1b. The distribution system consists of canals with lateral offtakes used to irrigate the fields. Gate structures, figure 1c, are used to control the water distribution.

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These cross-structures are operated in order to ensure an accurate water delivery for different users and to guarantee the infrastructure safety. In particular, they are used to maintain minimum water levels inside the reaches above the off-takes supply depths, and to prevent the canal from overtopping. In this article, we present results from the implementation of an algorithm, derived to perform real-time operations using a *Supervision, Control And Data Acquisition* (SCADA) system with automatic centralized controller.

Irrigation canals can be viewed (and modeled) as delay systems since it takes time for the water released at the upstream end to reach the user located downstream. In particular, open-loop control can greatly improve real-time operations. We present an open-loop controller which can deliver water at a given location in a given time. The design of such an open-loop controller requires a method to invert the system of equations that describe the dynamics of the open channel in order to parametrize the controlled input as a function of the desired output. The Saint-Venant equations are widely used to describe water flow dynamics in a canal, yet these equations are not easy to invert. We use a simplified model, the Hayami model, and differential flatness, a method which can be used to design an open-loop controller and invert the dynamics of the system.

Modeling of Open Channel Flow

Saint-Venant Equations

The Saint-Venant equations are named after Adhémar Jean-Claude Barré de Saint-Venant, a French engineer who first derived these equations in 1871 in a note to the *Comptes-Rendus de*

l'Académie des Sciences de Paris [1]. This model assumes one-dimensional flow, with uniform velocity over the cross-section. The effect of boundary friction is accounted for through an empirical law such as the Manning-Strickler friction law [2]. The average channel bed slope is assumed to be small, and the pressure to be hydrostatic. Under these assumptions, the Saint-Venant equations are written as follows:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{1}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial (Q^2/A)}{\partial x} + gA\frac{\partial H}{\partial x} = gA(S_b - S_f) \tag{2}$$

with $A(x,t)$ the wetted cross-sectional area (m^2), $Q(x,t)$ the discharge (m^3/s) across section $A(x,t)$, $H(x,t)$ the water depth (m), $S_f(x,t)$ the friction slope, S_b the bed slope and g the gravitational acceleration (m^2/s). Equation (1) expresses conservation of mass and equation (2) expresses conservation of momentum.

These equations are completed by boundary conditions at cross structures, where the Saint-Venant equations are not valid. The cross-structure at the downstream end of the canal can be modeled by a static relation between the discharge, $Q(L,t)$, and the water depth, $H(L,t)$, at $x = L$, i.e:

$$Q(L,t) = W(H(L,t)) \tag{3}$$

where $W(\cdot)$ is an empirical function or a law derived from hydrostatics, usually written analytically. For a weir overflow structure, this relation is given by $Q(L,t) = C_w\sqrt{2g}L_w\left(H(L,t) - H_w\right)^{3/2}$ where g is the gravitational acceleration, L_w is the weir length, H_w is the weir elevation, and C_w is the weir discharge coefficient.

A Simplified Linear Model

A simplified version of the Saint-Venant equations is obtained by neglecting inertia terms in the momentum equation (2), which leads to the diffusive wave equation (see [3] for more details). Linearizing the Saint-Venant equations around a reference uniform flow Q^0, and water depth H^0 leads to the Hayami equations:

$$D_0 \frac{\partial^2 q}{\partial x^2} - C_0 \frac{\partial q}{\partial x} = \frac{\partial q}{\partial t} \tag{4}$$

$$B_0 \frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{5}$$

where $C_0 = C_0(Q^0)$ and $D_0 = D_0(Q^0)$ are the nominal celerity and diffusivity which depend on Q^0, and B_0 is the average bed width. The quantities $q(x,t)$, $h(x,t)$ are the deviations from the nominal uniform flow and water depth, respectively. Figure 2 illustrates the notations used in this work.

The linearized boundary condition at the downstream end $x = L$ is:

$$q(L,t) = bh(L,t) \tag{6}$$

where b is the linearization constant equal to $\frac{\partial W}{\partial H}(H^0)$ in our case. The value of this constant depends on the hydraulic structure geometry: length, height, and discharge coefficient.

The initial conditions are defined by the deviations from their nominal values, which are assumed to be zero initially.

$$q(x,0) = 0 \tag{7}$$

$$h(x,0) = 0 \tag{8}$$

Flatness-based Open-loop Control

Open-loop Control of a Canal Pool

We consider the design of a feed-forward controller of water discharge of an open-channel hydraulic system. The system of interest is a one-dimensional hydraulic canal with a cross-structure at the downstream end as shown in Figure 2. We assume that the desired downstream discharge $q_{\mathrm{d}}(t)$ is known, based on scheduled user demands. The control problem consists in determining the upstream discharge $q(0, t)$ that has to be delivered in order to meet this downstream discharge $q_{\mathrm{d}}(t)$. This is an inverse problem, or an open-loop control problem. Note that by linearization, this is equivalent to determining $Q(0, t)$ as a function of $Q_{\mathrm{d}}(t) = Q^0 + q_{\mathrm{d}}(t)$ in the flow regime considered here.

The appropriate upstream discharge control $q(0, t)$ is the solution of the open-loop control problem defined by the Hayami model equations (4)-(5), initial conditions (7)-(8), boundary condition (6). Differential flatness, for details see "What is Differential Flatness?", provides a way to solve this open loop control problem [4], [3] in the form of a parametrization of the input $u(t) = q(0, t)$ as a function of the desired output $y(t) = q_{\mathrm{d}}(t)$:

$$u(t) = e^{\left(-\frac{\alpha^2}{\beta^2}t - \alpha L\right)} \left(T_1(t) - \kappa T_2(t) + \frac{B_0}{b}T_3(t) \right). \tag{9}$$

where

$$T_1(t) = \sum_{i=0}^{+\infty} \frac{d^i(e^{\frac{\alpha^2}{\beta^2}t}y(t))}{dt^i} \frac{\beta^{2i}L^{2i}}{(2i)!}, \tag{10}$$

$$T_2(t) = \sum_{i=0}^{+\infty} \frac{d^i(e^{\frac{\alpha^2}{\beta^2}t}y(t))}{dt^i} \frac{\beta^{2i}L^{2i+1}}{(2i+1)!}, \tag{11}$$

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$$T_3(t) = \sum_{i=0}^{+\infty} \frac{d^{i+1}(e^{\frac{\alpha^2}{\beta^2}t}y(t))}{dt^{i+1}} \frac{\beta^{2i}L^{2i+1}}{(2i+1)!},$$ (12)

$\alpha = \frac{C_0}{2D_0}$, $\beta = \frac{1}{\sqrt{D_0}}$, and $\kappa = \frac{B_0}{b}\frac{\alpha^2}{\beta^2} - \alpha$.

The convergence of the infinite series is guaranteed for output Gevrey functions $y(t)$ of order lower than 2 [4], [3]. The choice of the output, $y(t)$, is defined by the demand, imposed on the system and is discussed below.

Assessment of the Performance of the Method in Simulation

Before field implementation, it is needed to test the method in simulation, which we now do. For this, we perform a simulation in which we apply the controller defined by equation (9), called hereafter the Hayami controller, to the full nonlinear Saint-Venant model.

Simulation of Irrigation Canals (SIC)

The simulations are carried out using the software *Simulation of Irrigation Canals* (SIC). SIC is developed by Cemagref and implements a semi-implicit Preissmann scheme to solve the nonlinear Saint-Venant equations for open-channel one-dimensional flow [5], [6]. The test canal has a length $L = 4940\ m$, an average bottom slope $3.8 \times 10^{-4}\ m/m$, an average bed width $B_0 = 2\ m$ and Manning coefficient $0.024\ m^{-1/3}s$.

Parameter Identification

The purpose of model identification is to identify the parameters, C_0, D_0 and b corresponding to the Hayami model, that would best approximate the flow governed by the Saint-

Venant equations. This is done with an upstream discharge in the form of a step input. The flow discharges are monitored at the upstream and downstream positions. The hydraulic identification is performed classically by finding the values of model parameters that minimize the least-square error of the downstream discharge computed by the Hayami model and simulated by SIC. The identification is performed using Saint-Venant equations generated data by SIC and around a steady flow regime $Q^0 = 0.400 \ m^3/s$. This leads to the following parameters $C_0 = 0.84 \ m/s$, $D_0 = 634 \ m^2/s$, $b = 0.61 \ m^2/s$.

Desired Water Demand

The water demand curve is approximated from predicted consumption or by acknowledgment of farmers of their consumption intentions. The requirements of consumption at offtakes are usually modeled by a demand curve in the form of step function . However, depending on the canal model used, such demand may require high values of upstream discharge. We define our demand curve to be a linear transformation of a Gevrey function of the form $y(t) = q_1 \phi(t/T)$ where q_1, T are constants and $\phi(t)$ is a dimensionless bump function. The chosen Gevrey function allows a transition from zero discharge flow for $t \leq 0$ to a discharge flow equal to q_1 for $t \geq T$. The function $q_1 \phi(t)$ is depicted in figure 3, (desired output).

Simulation Results

The Hayami control (equation (9)) is computed using the corresponding identified parameters. The downstream discharge is defined by $y(t) = q_1 \phi(t)$, where $q_1 = 0.1 \ m^3/s$ and $T = 3 \times 3600 \ s$. Figure 3 shows the control $u(t)$ and the desired output $y(t)$.

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The control input $u(t)$ is simulated with SIC to compute the corresponding downstream discharge. Figure 4 shows the actual downstream discharge and the desired one.

The open-loop control based on the Hayami model is able to achieve an accurate motion planning. The output discharge follows closely the desired one, even if the model is here different from the Hayami model. We implemented this open loop controller on a real canal.

Implementation on Gignac Canal in Southern France

The experiments are performed on the Gignac Canal, located northwest of Montpellier, south of France. The main canal is 50 km long, with a common feeder (8 km long) and two branches on the left and the right bank of Hérault river (respectively 27 km and 15 km long). Figure 5 shows a map of the feeder canal with its left and right branches.

The canal separates into two branches (figure 6a), the right bank and the left bank, at Partiteur station. The canal is equipped at each branch with:

- Automatic regulation gate with position sensors (figure 6b).
- Water level sensors: The water level sensors are piezo-resistive sensors, whose resistance varies with the water level.
- Ultrasonic velocity sensor (figure 6c): The ultrasonic velocity sensor measures the average velocity of the water flow. The velocity measurement, water level measurement and the geometric properties of the canal at the gate determine the flow discharge.

We are interested in controlling the water discharge of the right bank of the canal. The cross section of the right bank is trapezoidal with average slope of $0.00035 \ m/m$. The canal is managed

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by a SCADA system, which enables the monitoring and control of water discharge. Data from sensors and actuators of the four gates at Partiteur is collected by a control station at the left bank as shown in figure 7. The information is communicated by radio frequency signals every few minutes (five minutes in our case) to the main center which is two kilometers away. The data is displayed, saved in a database, and commands to the actuators are sent back to the local controllers at the gates. In our experiment, we are interested in controlling the gate at the right bank of Partiteur station to achieve a desired downstream discharge five kilometers downstream at Avencq station. The gate opening at Partiteur is computed to deliver the control discharge, for details see "How to Impose a Discharge at a Gate?".

Results Obtained Assuming Constant Lateral Discharge Withdrawals

The identification of the parameters of the canal between Partiteur and Avencq is performed. The steady flow regime is $Q^0 = 0.640 \ m^3/s$. This lead to the following parameters $C_0 = 1.35 \ m/s$, $D_0 = 893 \ m^2/s$, $b = 0.17 \ m^2/s$. The downstream discharge is defined by $y(t) = q_1 \phi_\sigma(t)$, where $q_1 = -0.1 \ m^3/s$, and $T = 3.2 \times 3600 \ s$. The control input $u(t)$ is computed using equations (9) for the Hayami model. Figure 8 shows the desired downstream discharge and the desired control (upstream discharge) to be applied at the upstream with the measured discharges at each location, respectively.

Given the actuator limitations, a dead-band in the gate opening of $2.5 \ cm$ and unpredicted disturbances like friction in the gate opening mechanism, the results are very satisfactory. The downstream discharge is tracked well until $t \sim 3.4 \ h$, yet there is a steady state error. This error does not seem to be due to the actuator limitations, but to simplifications in the model

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assumptions, not necessarily satisfied in practice. Indeed, this steady state error may be due to the fact that the lateral discharge was assumed to be constant, whereas it in fact depends on the water level in the canal pool.

Modeling the Effects of Lateral Discharge Withdrawals

The discharge in a lateral offtake is a function of the water level in the canal just upstream of the offtake. Typically, the flow through a gravitational underflow offtake is proportional to the square root of the upstream water level. In a first assumption, we may linearize this relation, and assume that the offtakes are located at the downstream end of the canal. Then, instead of being constant, the lateral flow is proportional to the downstream water level. This can be seen as a local feedback between the level and the outflow. This modifies the dynamical model of the canal as follows:

$$q_l(t) = b_1 h(L, t) \tag{13}$$

We combine the output equation $y(t) = q_d(t) = bh(L, t)$ with the conservation of flow at $x = L$, $q(L, T) = q_l(t) + q_d(t) = (b + b_1)h(L, t)$, to obtain:

$$y(t) = Gq(L, t)$$

where $G = \frac{b}{b+b_1}$. Therefore, the effect of gravitational lateral discharge is expressed by a gain factor G which is lower than 1. This means that, in order to deliver a given discharge at the downstream end, one should release a larger discharge, to account for the lateral losses.

The open-loop control is readily deduced from equation (9) by replacing b with $b + b_1$,

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and $y(t)$ by $q(L,t) = G^{-1}y(t)$:

$$u(t) = \frac{1}{G}e^{\left(-\frac{\alpha^2}{\beta^2}t - \alpha L\right)}\left(T_1(t) - \kappa T_2(t) + \frac{B_0}{b + b_1}T_3(t)\right). \tag{14}$$

In the case of gravitational lateral discharge, the open loop control depends on four parameters G, C_0, D_0 and $b + b_1$ and these parameters need to be identified using the same method outlined for the constant lateral discharge withdrawals.

Results Obtained Accounting for Lateral Discharge Withdrawals

The open-loop control input is simulated on the Saint-Venant equations using SIC software, in order to evaluate the impact of lateral discharge withdrawals on the output.

Simulation Results

The simulations are carried out on a test canal of length $L = 4940\ m$, average bottom slope 3.8×10^{-4}, average bed width $B_0 = 2\ m$, Manning coefficient $0.024\ m^{-1/3}s$, and gravitational lateral off-takes distributed along its length. Identification is performed around a steady flow regime $Q^0 = 0.400\ m^3/s$. This leads to the following parameters: $G = 0.90$, $C_0 = 0.87\ m/s$, $D_0 = 692.34\ m^2/s$, and $b = 0.62\ m^2/s$ for the gravitational lateral discharge model, and to $C_0 = 0.84\ m/s$, $D_0 = 1100.72\ m^2/s$, and $b = 0.75\ m^2/s$ for the constant lateral discharge model. The downstream discharge is defined by $y(t) = q_1\phi(t)$, where $q_1 = 0.1\ m^3/s$, and $T = 3 \times 3600\ s$. Figure 9 shows the control $u(t)$ and the desired output $y(t)$ for each model (Hayami with and without gravitational lateral discharges).

We notice that the open-loop control with gravitational lateral discharges has a steady-

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state above the desired output to account for the variable withdrawal of water. The control input, $u(t)$, is simulated with SIC to compute the corresponding downstream discharge. Figure 10 shows the SIC simulation results.

Experimental Results

The identification of the parameters of the canal between Partiteur and Avencq is performed as described above for the models with gravitational lateral discharge. The steady flow regime is $Q^0 = 0.480 m^3/s$. This lead to the following parameters of the Hayami model $G = 0.70$, $C_0 = 1.08\ m/s$, $D_0 = 444\ m^2/s$, $b = 0.27\ m^2/s$. The downstream discharge is defined by $y(t) = q_1\phi(t)$, where $q_1 = 0.1\ m^3/s$, and $T = 5 \times 3600\ s$. The control input $u(t)$ is computed using equations (14) for the Hayami model. Figure 11 shows the desired downstream discharge and the desired control (upstream discharge) to be applied at the upstream with the actual discharges at each location.

Conclusion

This article introduced a flatness based controller for an open channel hydraulic canal. The controller was tested by computer simulation (SIC) and real experimentation (Gignac canal, France) using a SCADA system for the implementation. The initial model which assumed constant lateral outflows has been improved to take into account the variation of lateral outflow with the water level. This new model has shown to accurately represent the system dynamics. This paper has described a realistic application of a flatness based open-loop controller that is able to compute the upstream flow corresponding to a desired outflow, taking into account the

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gravitational offtakes along the canal reach.

Acknowledgments

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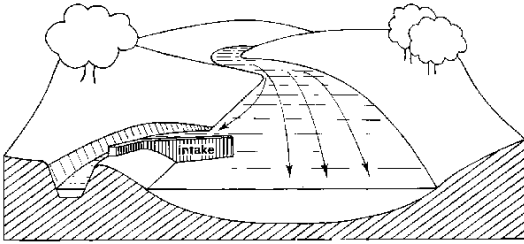
(a) Intake structure.
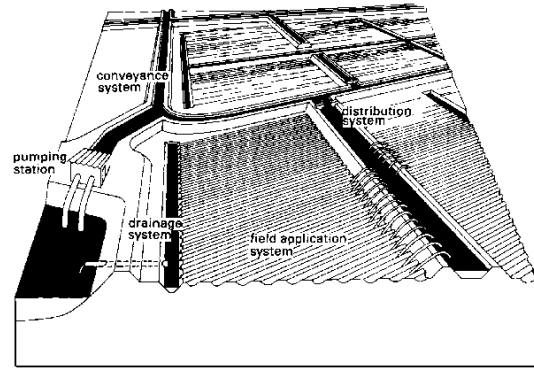
(b) An irrigation system.
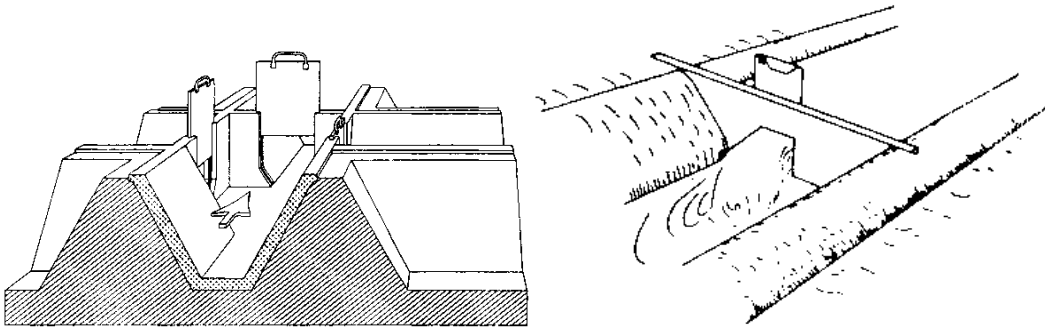
(c) Gate structures.

Figure 1: An irrigation canal system with intake and gate structures [7].
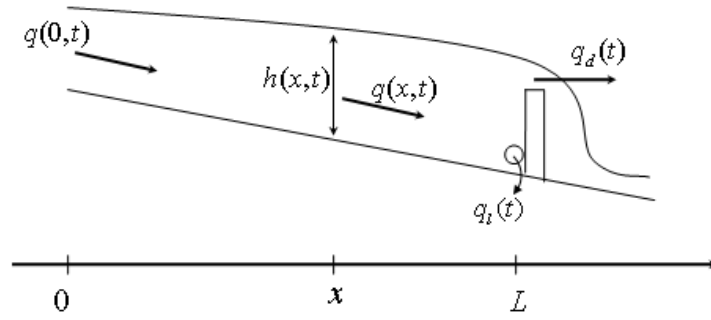
Figure 2: One-dimensional hydraulic canal with an off take at the downstream end of the canal. $q(x,t)$, $h(x,t)$, $q_\text{d}(t)$ and $q_\text{l}(t)$ are the deviations from the nominal values of water discharge, water depth, desired downstream discharge and lateral discharge, respectively.
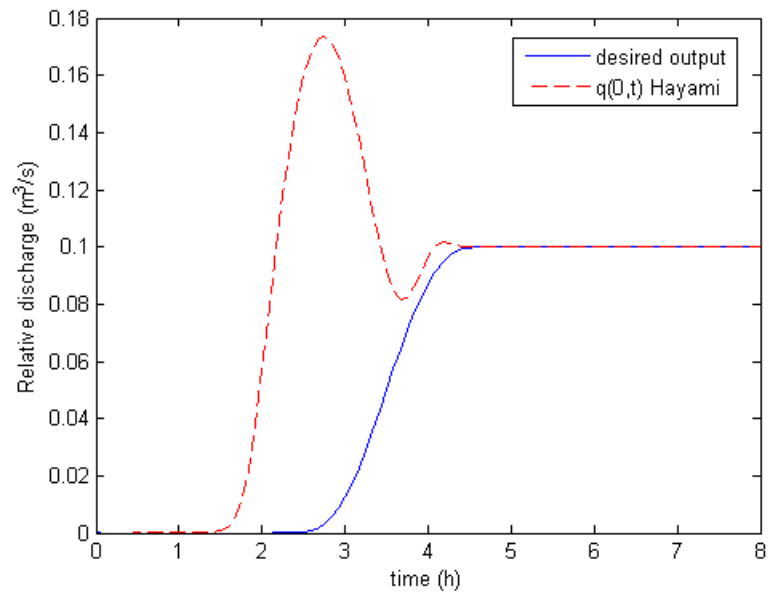
Figure 3: Control input, $u(t) = q(0, t)$ computed using differential flatness on Hayami model for a desired output discharge $y(t)$.
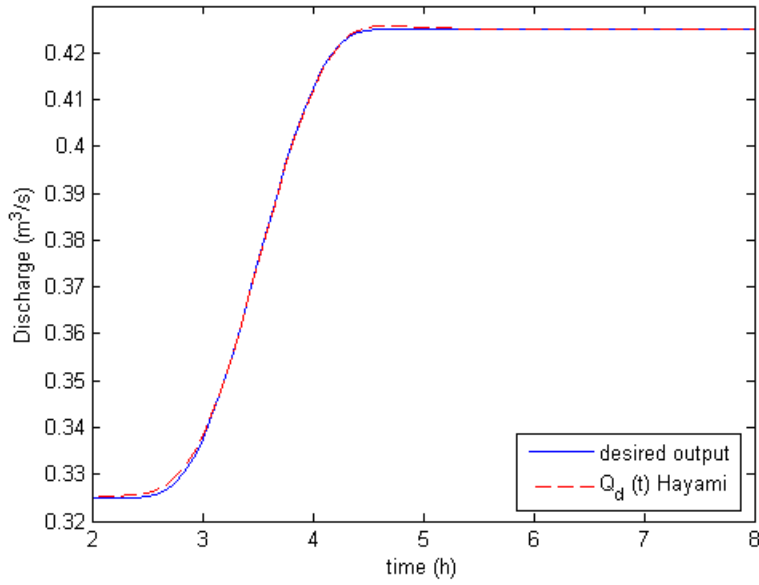
Figure 4: Desired downstream discharge and actual downstream discharge computed by SIC. Note that $Q_d(t)$ is the output obtained by applying the Hayami control on the full nonlinear model (Saint-Venant model).
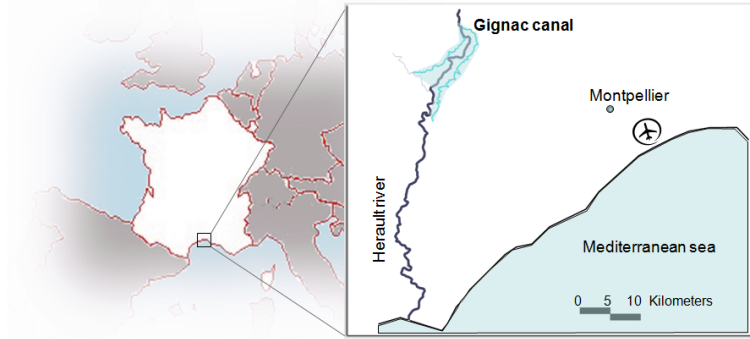
Figure 5: Location of Gignac canal in southern France. The canal takes water from the Hérault river, with two branches irrigating a total area of 3000 ha, where mainly vineyard is grown.

(a) The right bank (RB) and the left bank (LB) of the canal at Partiteur station.

(b) Partiteur right bank regulation

gate.

(c) Ultrasonic velocity sensor.

Figure 6: The left and right bank of Partiteur canal with actuators (submerged gates) and sensors (velocity sensors).
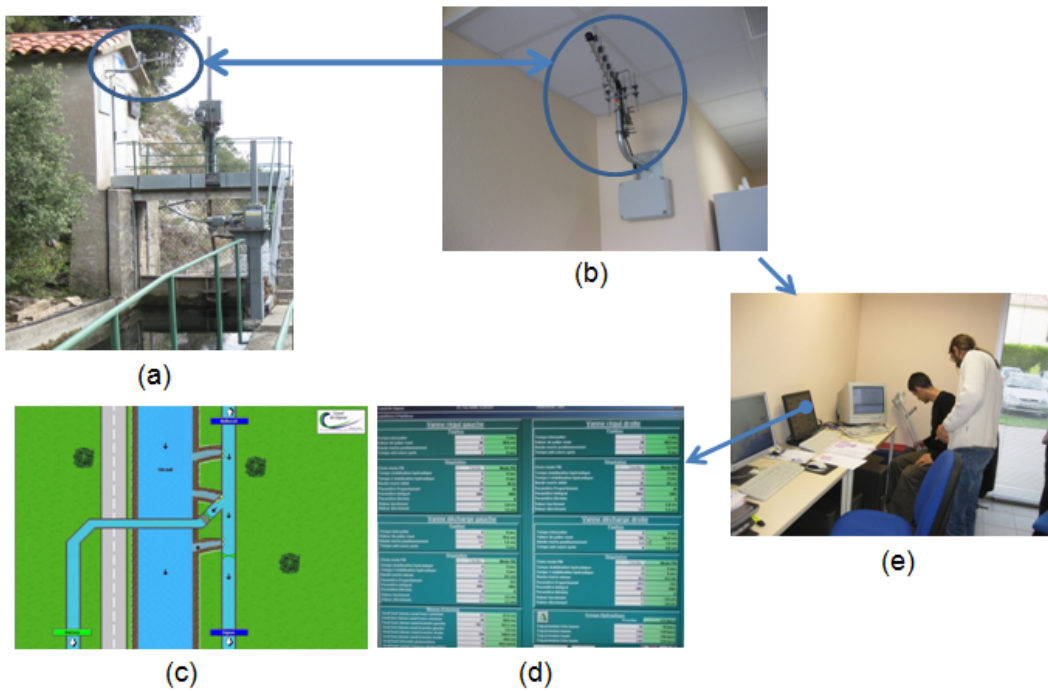
Figure 7: SCADA (Supervision, Control And Data Acquisition) system. (a) Wireless radio transmitter. (b) Wireless radio receiver. (c)-(d) SCADA computer interface. (e) Control center.
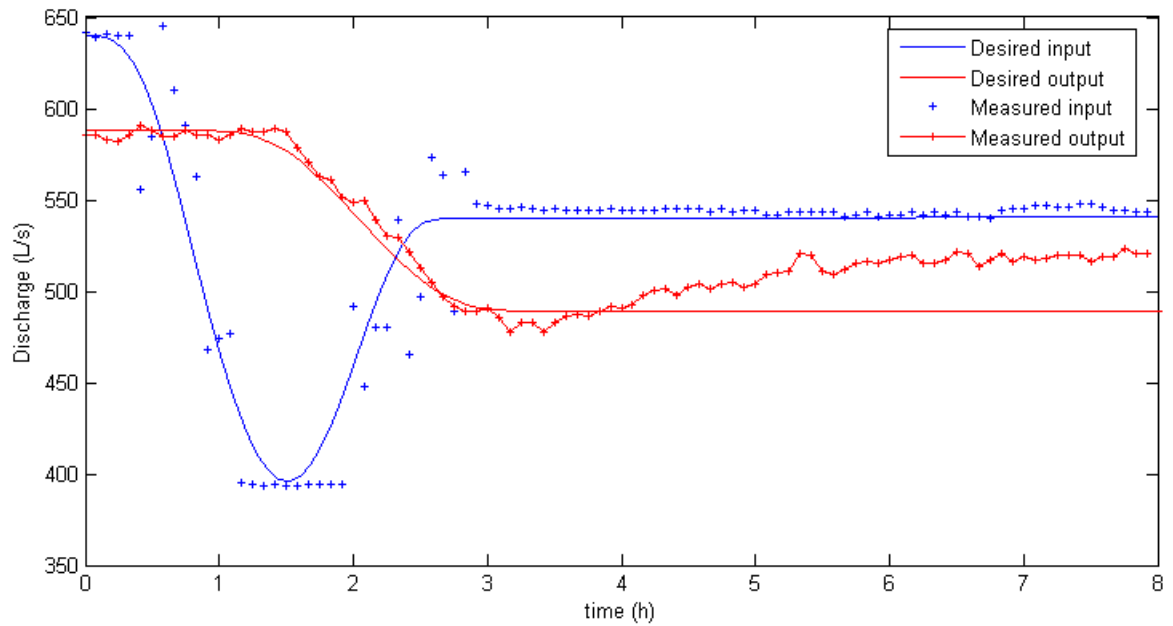
Figure 8: Open-loop control of the Gignac canal using Hayami model flatness based controller. The measured output flow is rather close to the desired one, except at the end of the simulation. This discrepancy cannot be explained solely by the actuator limitations, but to simplifications in the model assumptions.
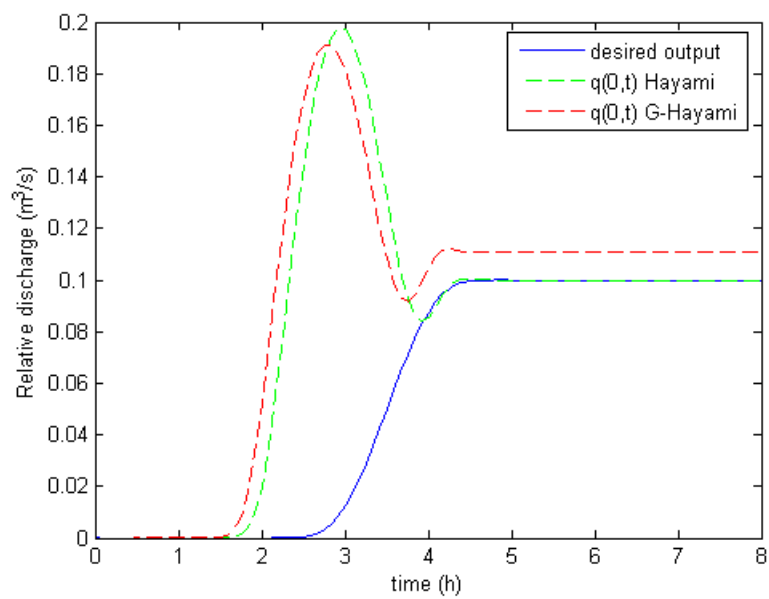
Figure 9: Open-loop control input $u(t)$ computed by Hayami model (with and without gravitational lateral discharges) for a desired output discharge, $y(t)$.
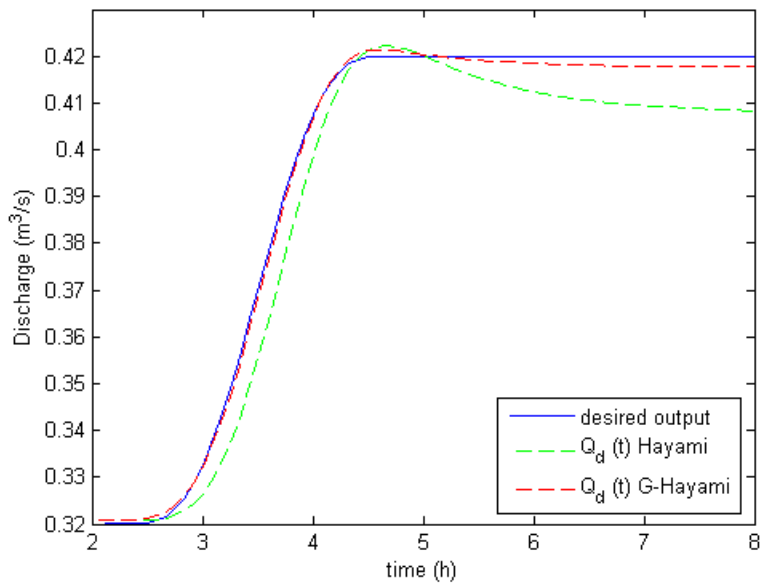
Figure 10: Desired downstream discharge and actual downstream discharge computed by SIC. Taking into account the effect of gravitational lateral discharge withdrawals enables to accurately follow the desired outflow. Note that this result is obtained on a realistic model of open channel flow, which is different from the simplified model used for control design.
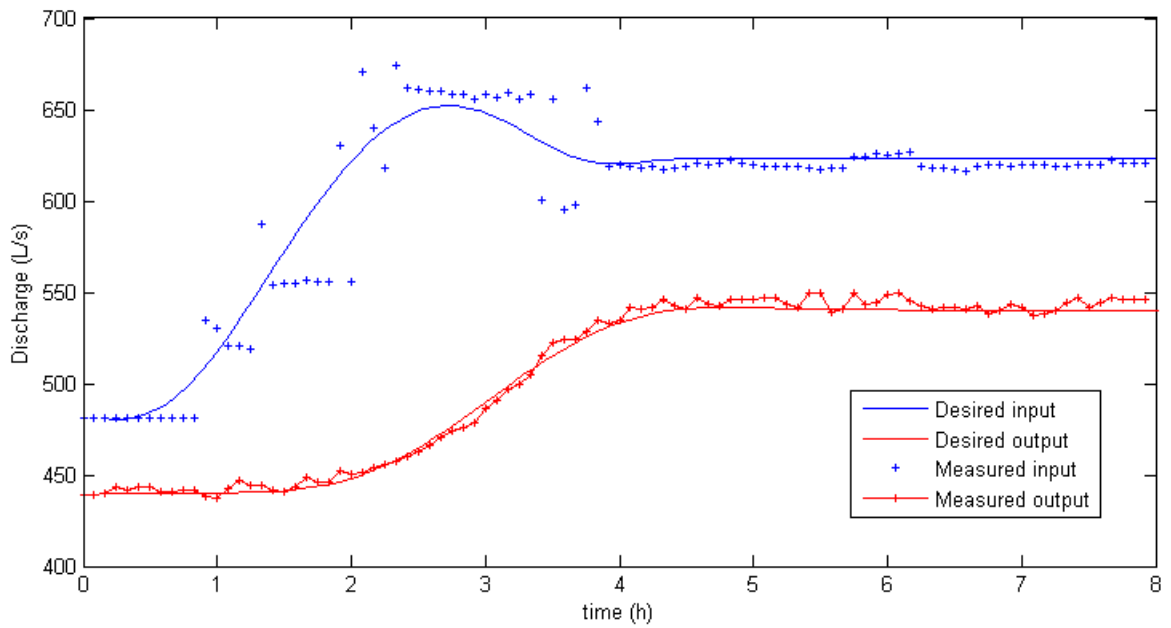
Figure 11: Implementation of open-loop control based on Hayami model with gravitational lateral discharge withdrawals. The measured output flow is very close to the desired one, even if the delivered upstream inflow is perturbed due to the actuator limitations.

Sidebar 1: What is Differential Flatness?

The theory of differential flatness, which was first developed in [8], consists in a parametrization of the trajectories of a system by one of its outputs, called the "flat output" and its derivatives. Let us consider a system $\dot{x} = f(x, u)$, where the state x is in $\mathbb{R}^n$, and the control input u ranges in $\mathbb{R}^m$. The system is said to be *flat*, and admits z (where $\dim(z) = \dim(u)$) for *flat output* if all the quantities can be parametrized by z and its derivatives. More specifically:

- The state x can be written as $x = h(z, \dot{z}, \cdots, z^{(n)})$.
- The equivalent dynamics can be written as $u = g(z, \dot{z}, \cdots, z^{(n+1)})$.

In the context of partial differential equations, the vector x can be thought of as "infinite dimensional". The notion of differential flatness extends to this case, and for a differentially flat system of this type, the evolution of x can be parametrized using an input u which often is the value of x at a given point. A system with a flat output can then be parametrized as a function of this output. This enables the solution of open loop control problems, if this flat output is the one that needs to be controlled. The open loop control input can then directly be expressed as a function of the flat output. This also enables the solution of motion planning problems, where one wants to steer the system from one state to another. Differential flatness was used to investigate the related problem of heavy chains motion planning [9], as well as the Burgers equation [10], the telegraph equation [11], the Stefan equation [12] and the heat equation [13].

Methods for parametrization can be achieved in different ways depending on the type of the problem. Laplace transform is widely used [9], [10], [11] to invert the system. The

equations can be transformed back from the Laplace domain to the time domain, thus resulting in the flatness parametrization. Other methods can be used to compute the parametrization in the time domain directly. For example, the Cauchy-Kovalevskaya form [13], [14] can be used to parametrize the trajectories of a distributed parameter system $X(\zeta, t)$ in $\zeta \in [0, 1]$ as a power series in space multiplied by a time function, i.e. $X(\zeta, t) = \sum_{i=0}^{+\infty} a_i(t) \frac{\zeta^i}{i!}$ where $X(\zeta, t)$ is the state of the system, and $a_i(t)$ is a time function. The usual approach consists in substituting the Cauchy-Kovalevskaya form in the governing partial differential equation and boundary conditions; a relation between $a_i(t)$ and the flat output $y(t)$ or its derivatives can then be found (for example, $a_i(t) = y^{(i)}(t)$ where $y^{(i)}(t)$ is the ith derivative of $y(t)$) which leads to the final parametrization.

Sidebar 2: How to Impose a Discharge at a Gate?

Once a desired open-loop discharge is computed, it needs to be imposed at the upstream end of the canal. In open channel flow, it is not easy to impose a discharge at a gate. Indeed, once the gate is opened or closed, the upstream and downstream gate water levels react instantly, and modify the discharge: this discharge is among other a function of the water levels on both sides of the gate. One possibility would be to use a local slave controller that operates the gate in order to deliver a given discharge. But due to operational constraints, it is usually not possible to operate the gate at a very high sampling period. As an example, some large gates may not be operated more than a few times an hour, which directly limits the operation of the local controller.

Several methods have been developed by hydraulic engineers to perform this control input based on the gate equation, which provides a good model for the flow through the gate [15]. The problem can be described as depicted in figure S1. Two pools are interconnected with a hydraulic structure, in our case a submerged orifice (also applicable for more complex structures). The gate opening is to be controlled to deliver a required flow from pool 1 to pool 2.

The hydraulic cross-structure is assumed to be modeled by a static relation between the discharge flowing through the gate Q, the water levels upstream and downstream of the gate Y_1, Y_2, respectively, and the gate opening W:

$$Q = f(Y_1, Y_2, W)$$

In our case, $f(Y_1, Y_2, W) = C_d\sqrt{2g}L_g W \sqrt{Y_1 - Y_2}$ where C_d is a discharge coefficient, L_g

is the gate width, and g is the gravitational acceleration. This nonlinear model can be linearized for small deviations q, y_1, y_2, w from the reference discharge value Q, water levels Y_1, Y_2, and gate opening W, respectively.

$$q = k_u \left(y_1 - y_2 \right) + k_w w$$

where the coefficients k_u, and k_w are obtained by differentiating $f(Y_1, Y_2, W)$ with respect to its first, second and third arguments, respectively.

Different inversion methods can be applied either to the nonlinear or to the linear model to obtain a gate opening W necessary to deliver a desired discharge through the gate, usually during some sampling period T_s. The static approximation method, assumes constant water levels Y_1, and Y_2 during the gate operation period T_s. This leads an explicit solution of the gate opening W in the linear model assumption. The characteristic approximation method uses the characteristics for zero slope rectangular frictionless channel to approximate the water levels. The linear version of the model also leads to an explicit expression of the gate opening. The dynamic approximation method uses the linearized Saint-Venant equations to predict the water levels. This method can be thought of a global method, because it considers the global dynamics of the canal to predict the gate opening necessary to deliver the desired flow. In [15], the three methods are compared and tested by simulation and on the Gignac canal. The dynamic approximation methods has been shown to better predict the gate opening necessary to obtain a desired average discharge.
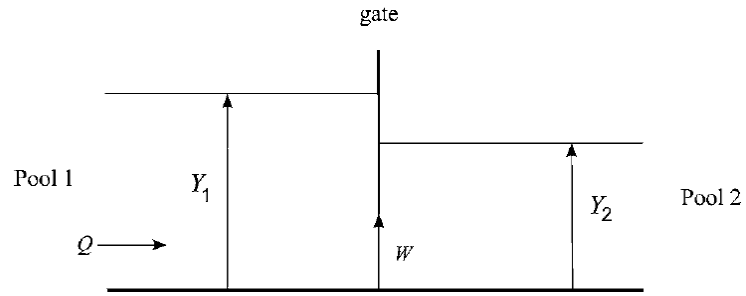
Figure S1: Schematic representation of a gate between two canal pools [15].

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