General adaptive neighborhood-based Minkowski maps for gray-tone image analysis.

Séverine Rivollier, Johan Debayle, Jean-Charles Pinoli

GENERAL ADAPTIVE NEIGHBORHOOD-BASED MINKOWSKI MAPS FOR GRAY-TONE IMAGE ANALYSIS

SEVERINE RIVOLLIER, JOHAN DEBAYLE AND JEAN-CHARLES PINOLI

Ecole Nationale Supérieure des Mines de Saint-Etienne, Centre Ingénierie et Santé - LPMG, UMR CNRS 5148, 158 cours Fauriel, 42023 Saint-Etienne cedex 2, France
e-mail: rivollier@emse.fr, debayle@emse.fr, pinoli@emse.fr

ABSTRACT

In quantitative image analysis, Minkowski functionals are standard parameters for topological and geometrical measurements. Nevertheless, they are often limited to binary images and achieved in a global and monoscale way. The use of General Adaptive Neighborhoods (GANs) enables to overcome these limitations. The GANs are spatial neighborhoods defined around each point of the spatial support of a gray-tone image, according to three (GAN) axiomatic criteria: a criterion function (luminance, contrast, . . .), an homogeneity tolerance with respect to this criterion, and an algebraic model for the image space. Thus, the GANs are simultaneously adaptive with the analyzing scales, the spatial structures and the image intensities.
The aim of this paper is to introduce the GAN-based Minkowski functionals, which allow a gray-tone image analysis to be realized in a local, adaptive and multiscale way. The Minkowski functionals are computed on the GAN of each point of the image, enabling to define the so-called Minkowski maps which assign the geometrical or the topological functional to each point. The impact of the GAN characteristics, as well as the impact of multiscale morphological transformations, is analyzed in a qualitative way through these maps. The GAN-based Minkowski maps are illustrated on the test image ’Lena’ and also applied in the biomedical and materials areas.

Keywords: General adaptive neighborhood, GLIP Mathematical morphology, Minkowski functionals, Minkowski maps, Multiscale image representation, Pattern analysis.

INTRODUCTION

This paper aims to introduce a novel approach for analyzing a gray-tone image in a local, adaptive and multiscale way. A segmentation process, generally used before quantitative image analysis, is not here required. The quantitative description is directly applied on the raw gray-tone images. Geometrical and topological measurements, through Minkowski functionals, are performed on spatial neighborhoods associated to each point of the image. These specific neighborhoods, named General Adaptive Neighborhoods (GANs) (Debayle and Pinoli, 2006), are simultaneously adaptive with the analyzing scales, the spatial structures and the image intensities. It enables to define the so-called GAN-based Minkowski maps which assign a measurement (based on the local Minkowski functionals) to each point of the image to be studied.

First, this paper recalls the notions of general adaptive neighborhood and of Minkowski functionals. Then, the next section introduces the GAN-based Minkowski maps. Thereafter, the impact of the GAN axiomatic criteria (analyzing criterion, homogeneity tolerance, algebraic model), as well as the impact of a multiscale morphological transformation is analyzed in a qualitative way through these maps. The GAN-based Minkowski maps are illustrated on the test image ’Lena’ and also in both the biomedical and materials areas.

GENERAL ADAPTIVE NEIGHBORHOODS

The GANIP (General Adaptive Neighborhood Image Processing) approach (Debayle and Pinoli, 2006) provides a general and operational framework for adaptive processing and analysis of gray-tone images. It is based on an image representation by means of spatial neighborhoods, named General Adaptive Neighborhoods (GANs). Indeed, GANs are simultaneously adaptive with:

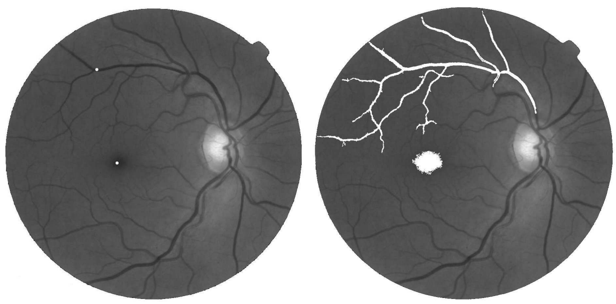
- the spatial structures: the size and the shape of the neighborhoods are adapted to the local context of the image,
- the analyzing scales: the scales are given by the image itself, and not *a priori* fixed,
- the intensity values: the neighborhoods are defined according to the GLIP (Generalized Linear Image Processing) mathematical framework, enabling to consider the physical and/or psychophysical settings of the image class.

For each point x of the image f, a GAN belongs to the spatial support $D \subseteq \mathbb{R}^2$ of f. The neighborhood of x, denoted $V_m^h(x)$, is a connected set and homogeneous with respect to an analyzing criterion h (such as luminance, contrast, thickness, ...) using a tolerance $m_{\bigcirc}$ within a GLIP (Generalized Linear Image Processing) framework (Oppenheim, 1967; Pinoli, 1997), i.e. in a vector space with its vector addition $\oplus$ and its scalar multiplication $\otimes$. The GAN of a point x is mathematically defined as follows:

$$V_{m_{\bigcirc}}^h(x) := C_{h^{-1}([h(x) \ominus m_{\bigcirc}; h(x) \oplus m_{\bigcirc}])}(x) \qquad (1)$$

where $C_X(x)$ denotes the path-connected component (with the usual Euclidean topology on D) of $X \subseteq D$ holding x.

Figure 1 illustrates the GANs of two points computed with the luminance criterion in the CLIP (Classical Linear Image Processing) framework (the operations $\oplus$ and $\otimes$ correspond to the usual operations between images, $+$ and $\times$ respectively) on a human retina image.



(a) original retina image with two (white) points x and y (b) GANs $V_{20}^h(x)$ and $V_{20}^h(y)$ respectively

Fig. 1. *The GANs of the two points x and y of the original image (a) are connected and homogeneous (b) with respect to the luminance criterion using the tolerance value $m = 20$ within the CLIP framework.*

The General adaptive neighborhoods are intrinsically defined by the local structures of the image. Thus, the GANs $\{V_{m_{\bigcirc}}^h(.)\}_{m_{\bigcirc}}$ allow a new multi-scale representation of gray-tone images to be defined. On the contrary, the shape and size of the classical neighborhoods $\{B_r(.)\}_r$ (centered homothetic isotropic discs, of radius r), generally used as analyzing windows for image transforms, are *a priori* fixed (Fig. 2).

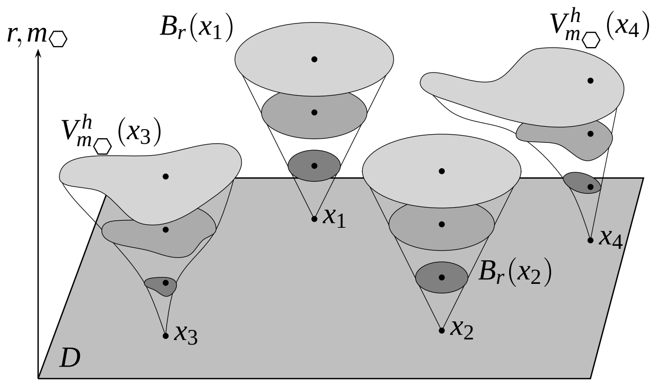


Fig. 2. *Classical neighborhoods $B_r(x)$ vs. general adaptive neighborhoods $V_{m_{\bigcirc}}^h(x)$.*

MINKOWSKI FUNCTIONALS AND DENSITIES

In quantitative image analysis, Minkowski functionals are standard parameters for topological and geometrical measurements (Minkowski, 1903). A description of the shape of a two-dimensional pattern is provided, using the three 2D Minkowski functionals (up to a constant): area, perimeter, and Euler number, denoted A, P and χ, respectively.

These functionals are defined on the class of the nonempty compact convex sets (convex bodies) of $\mathbb{R}^2$, and satisfy five properties (increasing, invariance under rigid motions, homogeneity, C-additivity, continuity vs. the Hausdorff metric). They have been extended (excluding the properties of increasing and continuity) to the convex ring (Mecke and Stoyan, 2000), i.e. the set of all finite unions of convex bodies of $\mathbb{R}^2$, which may be considered as a realistic Euclidean model for digital planar images.

In this paper, the densities of these functionals, i.e. the ratio of the Minkowski functionals by the area of the spatial support of the image, will be used. These densities are called the specific area, the specific perimeter, and the specific Euler number, and are denoted, A_A, P_A and χ_A, respectively.

GAN-BASED MINKOWSKI MAPS

GANs measurements enable a gray-tone image analysis, in a local, adaptive and multiscale way to be defined. For each point x of the image f, various measurements, such as area, orientation, concavities number, ...(Coster and Chermant, 1985; Rivollier, 2006), of the GAN $V_{m_{\bigcirc}}^h(x)$, can be computed. In this paper, only the (densities of) Minkowski functionals are considered.

DEFINITION

A GAN-based Minkowski map is defined in assigning a value for each point x which represents the GAN Minkowski density of $V^h_{m_\bigcirc}(x)$. In a more explicit way, the GAN-based Minkowski map, denoted $\mu^h_{m_\bigcirc}(f)$, of an image f, with respect to the Minkowski density μ (area: $\mu \equiv A_A$, perimeter: $\mu \equiv P_A$, Euler number: $\mu \equiv \chi_A$), is defined by:

$$\mu^h_{m_\bigcirc}(f) : \begin{cases} D & \rightarrow & \mathbb{R} \\ x & \mapsto & \mu\left(V^h_{m_\bigcirc}(x)\right) \end{cases} \tag{2}$$

where $V^h_{m_\bigcirc}(x)$ is the GAN of the point x with respect to the analyzing criterion h using the homogeneity tolerance $m_\bigcirc$ in a GLIP framework.

The densities of the Minkowski functionals of the GAN $V^h_{m_\bigcirc}(x)$ can be estimated in an efficient way (Osher and Mücklich, 2001).

Figure 3 illustrates some GAN-based Minkowski maps of the image 'Lena' f. The GANs are homogeneous with respect to the luminance criterion ($h \equiv f$) using the tolerance value $m = 40$ in the CLIP framework. Therefore, the value $\mu^h_{m_\bigcirc}(x)$ of each point x of the Minkowski map corresponds to the density μ of the GAN $V^f_{40}(x)$.

(a) original image f

(b) $(\chi_A)^f_{40}$ (c) $(P_A)^f_{40}$ (d) $(A_A)^f_{40}$

Gray-scale lower and upper bound values

(a)	0	255
(b)	-812.10^{-5}	1.10^{-5}
(c)	4.10^{-5}	9686.10^{-5}
(d)	1.10^{-5}	42926.10^{-5}

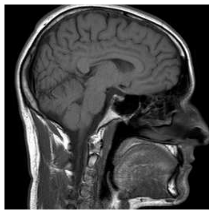
Fig. 3. *GAN-based Minkowski maps (b-d) of the image 'Lena' (a) with respect to the luminance criterion $h \equiv f$ using the homogeneity tolerance $m = 40$ in the CLIP mathematical framework. Gray-scale bound values gives the extrema of the Minkowski densities.*

The GANs depend on three axiomatic criteria (eq. 1): an analyzing criterion (luminance, contrast, ...), an homogeneity tolerance with respect to this criterion, and an algebraic model for the criterion mapping space. The following three sections of this paper will show, from a visual point of view, the impact of these characteristics on the Minkowski maps (eq. 2).

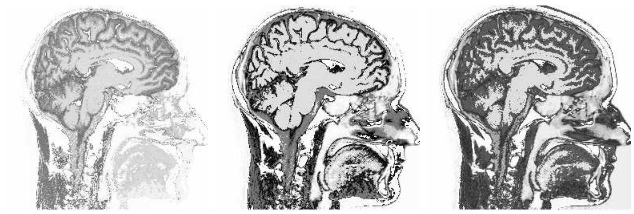
IMPACT OF THE HOMOGENEITY TOLERANCE

The GANs are homogeneous regions with respect to an analyzing criterion using a tolerance $m_\bigcirc$ within a GLIP framework (Fig. 2).

Figure 4 illustrates the GAN-based Minkowski maps of a brain MR image f with respect to the luminance criterion f using various homogeneity tolerance values: $m = 20$, $m = 30$ and $m = 40$ in the CLIP framework. This figure illustrates the fact that the application $m \mapsto (A_A)^f_m(.)$ increases, contrary to $m \mapsto (P_A)^f_m(.)$ and $m \mapsto (\chi_A)^f_m(.)$. This property can be generalized to the others GLIP frameworks and criterions. This is due to the tortuosity of the adaptive neighborhood's boundary and the number of holes of this one.



(a) original image f



(b) $(\chi_A)^f_{20}$ (c) $(\chi_A)^f_{30}$ (d) $(\chi_A)^f_{40}$

(e) $(P_A)^f_{20}$ (f) $(P_A)^f_{30}$ (g) $(P_A)^f_{40}$

(h) $(A_A)^f_{20}$ (i) $(A_A)^f_{30}$ (j) $(A_A)^f_{40}$

Gray-scale lower and upper bound values		
(a)	0	255
(b-d)	-955.10^{-5}	1.10^{-5}
(e-g)	4.10^{-5}	14765.10^{-5}
(h-j)	1.10^{-5}	57701.10^{-5}

Fig. 4. *GAN-based Minkowski maps (b-j) of a brain image (a) with respect to the luminance criterion f using the homogeneity tolerance values $m = 20$ (b,e,h), $m = 30$ (c,f,i) and $m = 40$ (d,g,j) in the CLIP framework.*

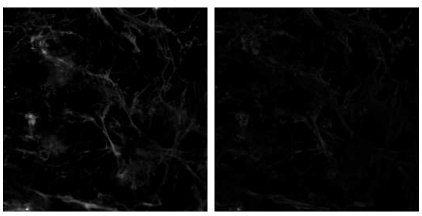
IMPACT OF THE ANALYZING CRITERION

The GANs are homogeneous with respect to a criterion function h (such as luminance, contrast, thickness, ...). For instance, the luminance criterion is defined by $h \equiv f$ where f is the original image. A contrast criterion can also be used. For instance, the image contrast, denoted c, can be defined as:

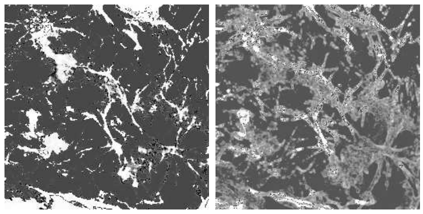
$$c : \begin{cases} D & \rightarrow & \mathbb{R} \\ x & \mapsto & \dfrac{1}{\#N(x)} \displaystyle\sum_{y \in N(x)} |f(x) - f(y)| \end{cases} \tag{3}$$

where $N(x)$ is a neighborhood of the point x (for instance, points in the window 3×3 centered on x).

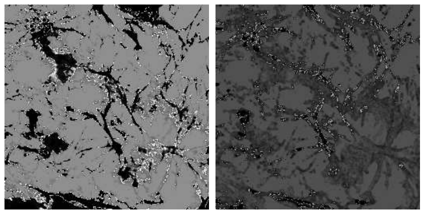
Figure 5 illustrates the GAN-based Minkowski maps of a fibronectin image f with respect to the luminance criterion f and the contrast criterion c using the homogeneity tolerance $m = 10$ in the CLIP framework.



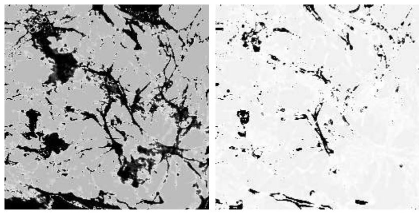
(a) original image f (b) criterion mapping c



(c) $(\chi_A)^f_{10}$ (d) $(\chi_A)^c_{10}$



(e) $(P_A)^f_{10}$ (f) $(P_A)^c_{10}$



(g) $(A_A)^f_{10}$ (h) $(A_A)^c_{10}$

Gray-scale lower and upper bound values		
(a-b)	0	255
(c-d)	-970.10^{-5}	1.10^{-5}
(e-f)	4.10^{-5}	23247.10^{-5}
(g-h)	1.10^{-5}	99213.10^{-5}

Fig. 5. *GAN-based Minkowski maps (c-h) of a fibronectin image (a) with respect to the luminance criterion f (c,e,g) and the contrast criterion c (d,f,h) using the homogeneity tolerance $m = 10$ in the CLIP framework.*

IMPACT OF THE GLIP MATHEMATICAL FRAMEWORK

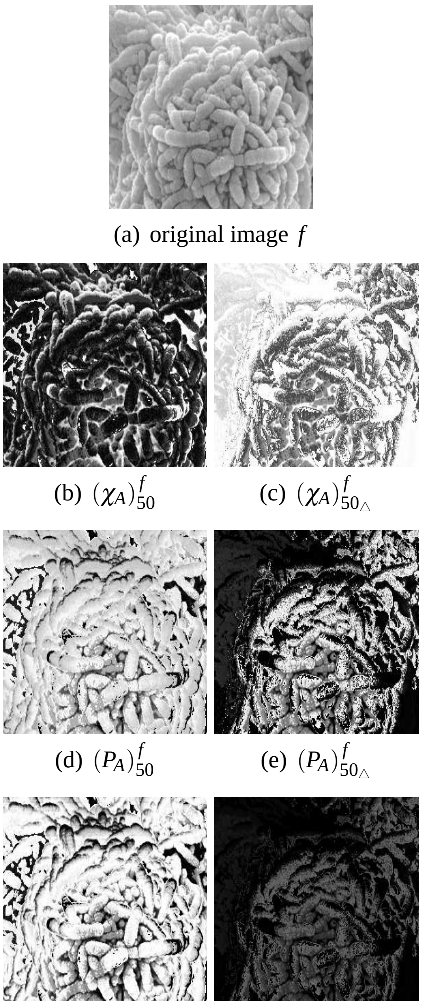
The criterion function h is represented in a GLIP model, i.e. a vector space with its vector addition $\oplus$ and its scalar multiplication $\otimes$. For instance, the operations $\oplus$ and $\otimes$ of the usual CLIP (Classical Linear Image Processing) framework correspond to the usual operations between images, $+$ and $\times$ respectively. The vector addition and the scalar multiplication of the LIP (Logarithmic Image Processing) framework (Pinoli, 1997), denoted $\triangle\!\!\!+$ and $\triangle\!\!\!\times$ respectively, are defined as following:

$$f \,\triangle\!\!\!+\, g = f + g - \frac{fg}{M} \tag{4}$$

$$\alpha \,\triangle\!\!\!\times\, f = M - M\left(\frac{M - f}{M}\right)^{\alpha} \tag{5}$$

where f and g are intensity gray-tone images, $\alpha \in \mathbb{R}$ is a scalar, and $M \in \mathbb{R}$ denotes the upper bound of the range where intensity images are digitized and valued. The LIP framework has been proved to be consistent with the transmittance and the multiplicative reflectance/transmittance image formation model, and with several laws and characteristics of human brightness perception (Pinoli, 1997).

Figure 6 illustrates the GAN-based Minkowski maps of a zinc sulfide image f acquired by scanning electron microscopy imaging with respect to the luminance criterion f using the homogeneity tolerance value 50 in both the CLIP and LIP mathematical frameworks, respectively.



(a) original image f

(b) $(\chi_A)_{50}^{f}$ (c) $(\chi_A)_{50_\triangle}^{f}$

(d) $(P_A)_{50}^{f}$ (e) $(P_A)_{50_\triangle}^{f}$

(f) $(A_A)_{50}^{f}$ (g) $(A_A)_{50_\triangle}^{f}$

Gray-scale lower and upper bound values

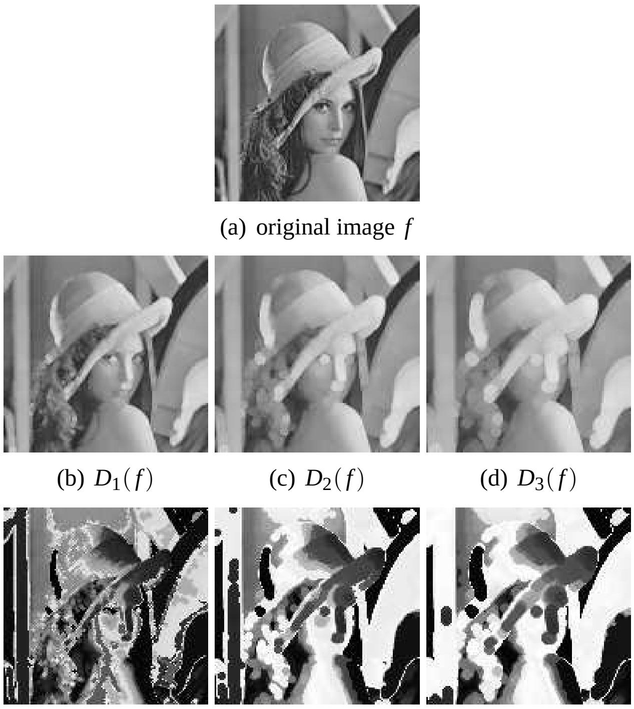
(a)	0	255
(b-c)	-656.10^{-5}	1.10^{-5}
(d-e)	4.10^{-5}	13867.10^{-5}
(f-g)	1.10^{-5}	85569.10^{-5}

Fig. 6. *GAN-based Minkowski maps (b-g) of a zinc sulfide image (a) with respect to the luminance criterion f using the homogeneity tolerance $m = 50$ in the CLIP (b,d,f) and LIP (c,e,g) framework.*

IMPACT OF A MULTISCALE MORPHOLOGICAL TRANSFORMATION

Mathematical morphology (Serra, 1982) is an important and nowadays a traditional theory in image processing, particularly used for geometrical image analysis. The elementary morphological operators of dilation and erosion (and thus the combined operators of closing and opening) act on image intensities within the use of an operational window named structuring element.

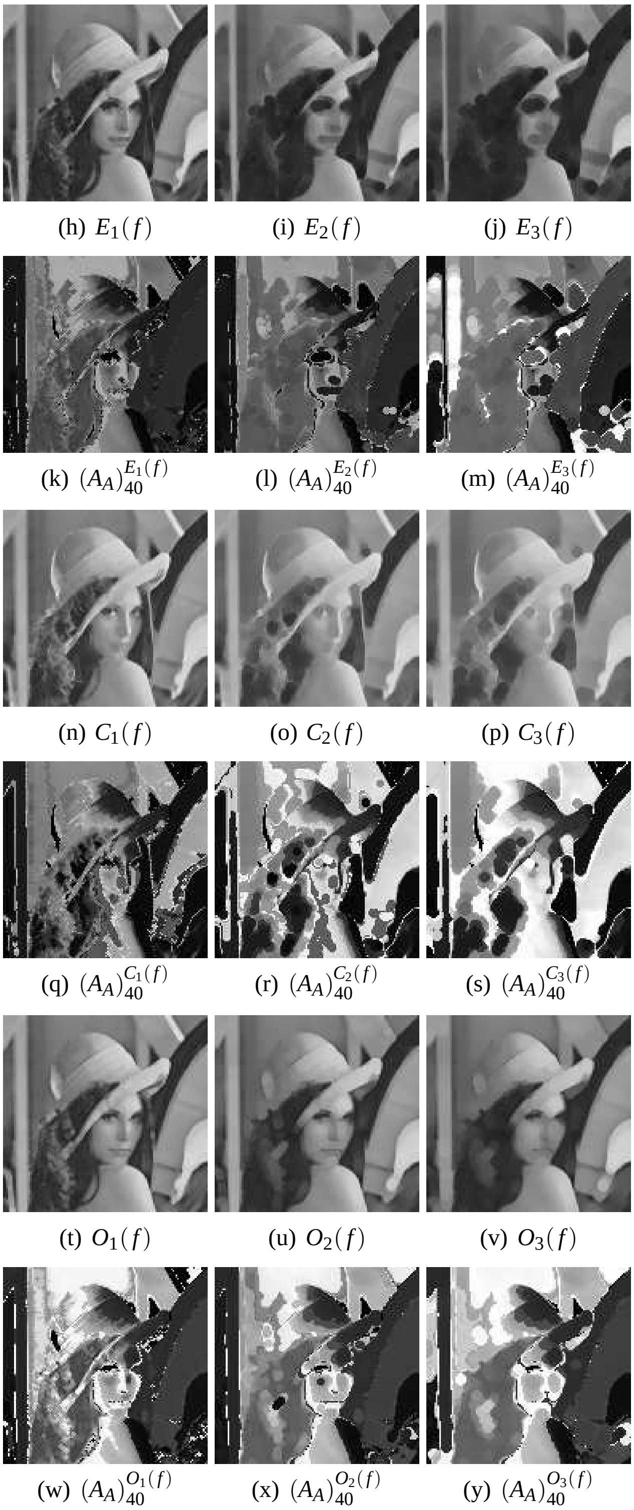
The GAN-based Minkowski maps can be computed on transformed images by morphological operators. For instance, figure 7 first illustrates the morphological tranforms using a disk of radius r as structuring element (dilation D_r, erosion E_r, closing C_r, opening O_r) of the image 'Lena' f. Thereafter, the GAN-based Minkowski maps (using the area functional) are computed with respect to the luminance criterion f using the homogeneity tolerance $m = 40$ in the CLIP framework. This figure illustrates the fact that the application $r \mapsto (A_A)_{m_\bigcirc}^{T_r(f)}(.)$ is non-monotonous, contrary to $r \mapsto T_r(f)(.)$.



(a) original image f

(b) $D_1(f)$ (c) $D_2(f)$ (d) $D_3(f)$

(e) $(A_A)_{40}^{D_1(f)}$ (f) $(A_A)_{40}^{D_2(f)}$ (g) $(A_A)_{40}^{D_3(f)}$



(h) $E_1(f)$ (i) $E_2(f)$ (j) $E_3(f)$

(k) $(A_A)_{40}^{E_1(f)}$ (l) $(A_A)_{40}^{E_2(f)}$ (m) $(A_A)_{40}^{E_3(f)}$

(n) $C_1(f)$ (o) $C_2(f)$ (p) $C_3(f)$

(q) $(A_A)_{40}^{C_1(f)}$ (r) $(A_A)_{40}^{C_2(f)}$ (s) $(A_A)_{40}^{C_3(f)}$

(t) $O_1(f)$ (u) $O_2(f)$ (v) $O_3(f)$

(w) $(A_A)_{40}^{O_1(f)}$ (x) $(A_A)_{40}^{O_2(f)}$ (y) $(A_A)_{40}^{O_3(f)}$

Gray-scale lower and upper bound values

(a,b-d,h-j,n-p,t-v)	0	255
(e-g)	6.10^{-5}	67316.10^{-5}
(k-m)	6.10^{-5}	63232.10^{-5}
(q-s)	6.10^{-5}	68439.10^{-5}
(w-y)	6.10^{-5}	45911.10^{-5}

Fig. 7. *GAN-based Minkowski maps (e-g,k-m,q-s,w-y) of the image 'Lena' (a) dilated (b-d), eroded (h-j), closed (n-p), opened (t-v) with a disk of radius r, with respect to the luminance criterion f using the homogeneity tolerance $m = 40$ in the CLIP framework.*

CONCLUSION

In this paper, a novel approach for analyzing a gray-tone image in a local, adaptive and multiscale way is proposed. The so-called GAN-based Minkowski maps assign a geometrical or topological measurement of a spatial neighborhood (GAN) associated to each point of the image to be studied.

The influence of the GAN axiomatic criteria (analyzing criterion, homogeneity tolerance, algebraic model), and the impact of a multiscale morphological transformation on the image to be studied, is analyzed in a qualitative way through these maps. Currently, the authors investigate a quantitative analysis through the histogram of these maps.

ACKNOWLEDGMENTS

The authors wish to thank the University Hospital Center (ophtalmology and radiology) of Saint-Etienne in France, the UMR CNRS 5148, and the INSERM U890, who have kindly provided the original images.

REFERENCES

Coster M, Chermant JL (1985). Précis d'analyse d'image. Hermès.

Debayle J, Pinoli JC (2006). General Adaptive Neighborhood Image Processing. Journal of Mathematical Imaging and Vision 25:245–86.

Mecke KR, Stoyan D (2000). Statistical physics and spatial statistics. Tech. rep., Berlin, Heidelberg, New York.

Minkowski H (1903). Volumen und Oberfläche. Mathematische Annalen 57:447–95.

Oppenheim A (1967). Generalized Superposition. Information and Control 11:528–36.

Osher J, Mücklich F (2001). Statistical Analysis of Microstructures in Materials Science. John Wiley and Sons.

Pinoli JC (1997). A General Comparative Study of the Multiplicative Homomorphic, Log-Ratio and Logarithmic Image Processing Approaches. Signal Processing 58:11–45.

Rivollier S (2006). Analyse spectrale morphologique adaptative d'image. Master's thesis, Ecole Nationale Supérieure des Mines, Saint-Etienne, France.

Serra J (1982). Image Analysis and Mathematical Morphology, vol. 1. Academic Press.