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► To cite this version:

Stéphane Avril, Pierre Badel, Ambroise Duprey. Anisotropic and hyperelastic identification of in vitro human arteries from full-field optical measurements. *Journal of Biomechanics*, Elsevier, 2010, 43 (15), pp.2978-2985. <10.1016/j.jbiomech.2010.07.004>. <hal-00543316>

HAL Id: hal-00543316

<https://hal.archives-ouvertes.fr/hal-00543316>

Submitted on 6 Dec 2010

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Anisotropic and hyperelastic identification of *in vitro* human arteries from full-field optical measurements

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Abstract

In this paper, we present a new approach for the bi-axial characterization of *in vitro* human arteries and we prove its feasibility on an example. The specificity of the approach is that it can handle heterogeneous strain and stress distributions in arterial segments. From the full-field experimental data obtained in inflation/extension tests, an inverse approach, called the virtual fields method (VFM), is used for deriving the material parameters of the tested arterial segment. The obtained results are promising and the approach can effectively provide relevant values for the anisotropic hyperelastic properties of the tested sample.

1 Introduction

It is well assessed that, despite biochemical and hemodynamical factors play a primary role in the development of most vascular disorders, solid mechanics models may contribute to understand their genesis and progression. The realism of models in solid mechanics depends significantly on the mechanical properties used as input parameters. Therefore, characterizing the biomechanical properties of arteries remains an essential issue.

In vivo measurements with ultrasounds or magnetic resonance imaging (MRI) techniques provide relevant information on the vascular behavior [Slager et al., 2000, Masson et al., 2008, Avril et al., 2009] although they are not sufficient for a rigorous determination of arterial wall constitutive equations. To investigate the passive structural response of the arterial tissue, a large variety of *in vitro* experimental protocols are available [Humphrey, 2002].

Among them, inflation/extension of arterial segments are physiologically meaningful tests since the *in vivo* loading conditions may be reproduced and the native geometry is preserved [Hayashi, 1993]. Classically, for the analysis, it is assumed that the artery is a perfect cylinder and that the loading induces a homogeneous stress and strain distribution [Humphrey, 2002, Fung, 1993]. These assumptions provide a framework for deriving stress-strain curves and to fit them by appropriate constitutive models.

33 However, inverse approaches should generally be employed since stresses and strains are always het-
34 erogeneous, because of noncylindrical shape [Holzapfel, 2004, Humphrey, 1999], locally varying material
35 properties or experimental artifacts like edge effects [Arimitsu et al., 1995].

36 The combination of 3-D deformation measurements [Rastogi, 1999, Foster, 1978, Viotti et al., 2008,
37 Matthys et al., 1991, Genovese, 2007, Genovese, 2009, Sutton et al., 2007] and inverse approaches [Avril
38 et al., 2008] is now very common in solid mechanics but it is still under-employed for identifying the
39 anisotropic hyperelastic properties of the arterial tissues [Seshaiyer and Humphrey, 2003, Einstein et al.,
40 2005]. Moreover, the Virtual Fields Method [Grédiac et al., 2006], which is an inverse method specifically
41 dedicated to full-field data, has never been used for the mechanical identification of arterial tissues
42 although it has very relevant assets: insensitivity to the uncertainty of boundary conditions [Grédiac
43 et al., 2006], robustness [Avril et al., 2004], fast convergence [Avril and Pierron, 2007].

44 This paper attempts to give a new contribution for addressing the mechanical identification of arterial
45 tissues by presenting an implementation of the Virtual Fields Method to full-field experimental data
46 measured on the whole surface of an arterial segment during inflation/extension tests. The objective of
47 the paper is to present the principle of the approach and to prove its feasibility on an example.

48 **2 Materials and Methods**

49 **2.1 Materials**

50 Results reported in this paper are obtained on a fairly straight segment of a human ascending aorta (initial
51 length: $L_0 \simeq 35$ mm, initial average radius: $R_0 \simeq 10$ mm, initial average thickness: $e_0 \simeq 1.3$ mm). This
52 vascular segment has been obtained from a cadaveric 65 years old female donor. All procedures were
53 carried out in accordance with the guidelines of the Institutional Review Board of the University Hospital
54 Centre of Saint-Etienne, France. After gently cleaning the artery with physiological solution, two side
55 branches were clamped with surgical suture threads (Fig. 1).

56 **2.2 Experiments**

57 This study results from a collaboration between the University of Basilicata (Italy) and Ecole des Mines
58 de Saint-Etienne (France). The experimental setup was developed at the University of Basilicata and
59 presented in a companion paper [Genovese, 2009]. The current paper focuses on the inverse approach
60 and all the details about the experimental techniques may be found in [Genovese, 2009].

61 The sample was mounted on the *in vitro* rig and preconditioned *via* 8 pressurization cycles from $p =$

62 0 to $p = 80$ mmHg (10.5 kPa) and then 8 cycles of axial stretching from $L/L_0 = 1$ to $L/L_0 = 1.4$. Then,
 63 different sets of inflation/extension tests were performed.

64 Let Ψ_t denote the geometrical deformation of the artery induced by the loading at time t , such as
 65 $\mathbf{x}(t) = \Psi_t(\mathbf{x}_0)$, where $\mathbf{x}(t)$ is the position vector of a material point at t and \mathbf{x}_0 is the initial position
 66 vector of the same material point.

67 The vascular segment is covered with N ($N \simeq 300$) black spherical markers (Fig. 1). The markers
 68 were fixed with cyanoacrylate adhesive which was shown not to diffuse in the tissues [Holzapfel et al.,
 69 2007]. The optical measurement presented in [Genovese, 2009] provides a point-wise evaluation of Ψ_t at
 70 all the points where a marker has been bonded, with an accuracy of 0.17 mm.

71 During all the inflation/extension tests, the artery remains in a bath of physiological solution. The
 72 optical technique was calibrated accounting for the bath of physiological solution around the artery
 73 [Genovese, 2009]. However the artery was not hydrated while we bonded the markers. This lasted less
 74 than one hour. This could be reduced in the future by using glue that activates with water.

75 **2.3 Derivation of the field of Green-Lagrange strain tensors**

76 The surface of the artery is meshed for interpolating Ψ_t across the whole artery from the data measured
 77 at the marker position. Triangular elements with linear interpolation functions are used. The mesh is
 78 defined using a Delaunay triangularization algorithm applied onto the marker initial positions (actually
 79 the marker position means the contact position of the marker onto the surface of the artery). Each
 80 triangular element is labeled by an index denoted q . Let $\mathbf{x}^q(t)$ denote the position vector of the center of
 81 gravity of each element at time t

82 Eventually, Ψ_t is known everywhere across the meshed area of the artery. Let Ω be this domain across
 83 which the deformation is measured. Ω is defined in the initial configuration because the deformation is
 84 Lagrangian. Let $\omega(t)$ be the counterpart of Ω in the deformed configuration at time t .

85 All the derivations presented in this paper are based on the theory of finite deformations [Ogden,
 86 1997]. Let $\mathbf{F}(\mathbf{x}_0, t)$ denote the deformation gradient:

$$87 \quad \mathbf{F}(\mathbf{x}_0, t) \equiv \nabla \Psi_t(\mathbf{x}_0)$$

88 where ∇ is the gradient operator.

89 Then we define the right Cauchy-Green tensor:

$$90 \quad \mathbf{C}(\mathbf{x}_0, t) \equiv \mathbf{F}^T \mathbf{F}$$

91 where \mathbf{F}^T is the transpose of \mathbf{F} .

92 Eventually, we define the Green-Lagrange strain tensor:

$$93 \quad \mathbf{E}(\mathbf{x}_0, t) \equiv [\mathbf{C} - \mathbf{I}]/2$$

94 where \mathbf{I} is the identity tensor.

95 2.4 Constitutive equations

96 Arteries *in vitro* are usually considered as anisotropic visco-hyperelastic materials. Here, the viscoelas-
97 tic properties are not considered and only the instantaneous and monotonic response is modeled, after
98 preconditioning. Pseudo-elasticity can therefore be applied [Fung, 1993]. Hyperelastic models are con-
99 sidered.

100 In these models, the Cauchy stress tensor, denoted \mathbf{S} , is deduced from the Green-Lagrange strain
101 tensor such as:

$$\mathbf{S} = \mathbf{F} : d\Phi/d\mathbf{E} : \mathbf{F}^T + \lambda \mathbf{I} \quad (1)$$

102 where Φ is the strain energy function depending only on \mathbf{E} and on material parameters. The term $\lambda \mathbf{I}$
103 comes from the incompressibility and λ is assessed by writing the plane stress condition ($S_{33} = 0$).

104 Different strain energy functions for the anisotropic hyperelastic behavior of arteries may be found
105 in the literature. Some of them are phenomenological [Fung, 1993, Sun and Sacks, 2005] and others are
106 based on the microstructure [Holzapfel et al., 2000]. In this study, we consider 3 different models:

1. the Delfino model [Holzapfel et al., 2000]:

$$\Phi' = \frac{1}{2}\beta' [\exp(\alpha'(I_1 - 3)) - 1] \quad (2)$$

107 where β' and α' are material parameters and $I_1 = \text{tr}(\mathbf{C})$.

2. the 2-D Fung model [Fung, 1993], which may be written as:

$$\Phi = \frac{1}{2}\beta [\exp(\alpha_{11}(E_{11})^2 + \alpha_{22}(E_{22})^2 + \alpha_{12}E_{11}E_{22}) - 1] = \frac{1}{2}\beta [\exp(Q) - 1] \quad (3)$$

109 where β , α_{11} , α_{22} and α_{12} are material parameters and E_{ij} are the components of the \mathbf{E} matrix in
110 the material coordinate system (indice 1 refers to the circumferential direction and indice 2 refers
111 to the axial direction in the local coordinate system).

3. the Holzapfel model [Holzapfel et al., 2000]. This model assigns separate strain energy functions to the media layer and the adventitia layer. The simplest form of the strain energy function may be written:

$$\Phi'' = \frac{c''}{2}(I_1 - 1) + \frac{k_1''}{2k_2''} [\exp(k_2''(I_4 - 1)^2) - 1] + \frac{k_1''}{2k_2''} [\exp(k_2''(I_6 - 1)^2) - 1] \quad (4)$$

112 where c'' and k_1'' are stress-like material parameters, k_2'' is a dimensionless material parameter; I_4
 113 and I_6 are the squares of the stretch component in the two families of symmetric tissue fibers.
 114 Hence, this model prescribes a fiber orientation angle ϕ for each layer, based usually on analysis of
 115 microscopy data. The fiber angles for the specimen used in this work were not known, and so the
 116 fiber angles are another material parameter to be determined from the experimental data.

117 2.5 Inverse approach

118 The Virtual Fields Method (VFM) is an approach dedicated to the identification of material parameters
 119 from full-field measurements [Grédiac et al., 2006, Avril et al., 2004, Avril and Pierron, 2007, Promma
 120 et al., 2009]. It is based on the weak form of equilibrium equations.

121 Assuming quasi-static conditions and no body forces, the weak form of equilibrium in the deformed
 122 configuration may be written such as:

$$-\int_{\omega(t)} \mathbf{S} : \mathbf{E}^* dv + \int_{\partial\omega(t)} \mathbf{t} \cdot \mathbf{u}^* ds = 0 \quad (5)$$

123 where $\partial\omega(t)$ is the boundary of the volume in the deformed configuration, vector \mathbf{t} denotes the tractions
 124 applied onto this boundary (pressure forces and axial load), \mathbf{u}^* is a displacement virtual field which acts
 125 as a test function in Eq. 5). \mathbf{E}^* is the strain virtual field deduced from the gradient of \mathbf{u}^* :

$$\mathbf{E}^* = \frac{1}{2} \left(\nabla \mathbf{u}^* + \nabla^T \mathbf{u}^* \right) \quad (6)$$

126 At position \mathbf{x} where a marker is located, \mathbf{u}^{*1} and \mathbf{u}^{*2} take the following values:

$$\begin{cases} \mathbf{u}^{*1} = \sin\left(\pi \frac{z - z_b}{z_t - z_b}\right) \mathbf{i} \\ \mathbf{u}^{*2} = \frac{z - z_t}{z_b - z_t} \mathbf{k} \end{cases} \quad (7)$$

127 where:

128 • vectors \mathbf{i} and \mathbf{k} are respectively the radial and longitudinal axes in the global cylindrical coordinate
 129 system defined in Fig. 2;

130 • z is the component of \mathbf{x} along the longitudinal axis: $z = \mathbf{x} \cdot \mathbf{k}$;

131 • z_b and z_t are defined in Fig. 2.

132 The rationale behind this choice of virtual displacements is:

133 • one involves mainly the radial properties (\mathbf{u}^{*1})

134 • one involves mainly the axial properties (\mathbf{u}^{*2})

135 • both zero the action of unknown reaction forces or involve their resultant if the resultant is measured
 136 [Grédiac et al., 2006].

137 Given the previous points, infinity of virtual deformation may be used. The rationale behind the
 138 current choice is that the best results are always obtained with the simplest functions. This was proved
 139 in different papers [Grédiac et al., 2006, Avril et al., 2004, Avril and Pierron, 2007, Promma et al., 2009].
 140 This can be interpreted like this: a virtual deformation is a test function. If the test function is very
 141 smooth, it will average out the randomly distributed errors contained in the experimental data.

142 Eq. 7 only gives the values of the virtual displacement at marker positions. Between the marker
 143 positions, the values of \mathbf{u}^{*1} and \mathbf{u}^{*2} are interpolated linearly.

144 Moreover, regarding the external virtual work, it can be written:

$$\begin{cases} \int_{\partial\omega(t)} \mathbf{t} \cdot \mathbf{u}^{*1} ds = \int_{\partial\omega(t)} \mathbf{k}_t \cdot \mathbf{u}^{*1} p(t) ds \\ \int_{\partial\omega(t)} \mathbf{t} \cdot \mathbf{u}^{*2} ds = \int_{\partial\omega(t)} \mathbf{k}_t \cdot \mathbf{u}^{*2} p(t) ds + \mathcal{F}(t) \end{cases} \quad (8)$$

145 where \mathbf{k}_t is the unit outer normal vector at t , $\mathcal{F}(t)$ is the axial resultant load measured by a load cell.

146 As \mathbf{S} is deduced from experimental data, Eq. 5 cannot be satisfied exactly and the principle of the
 147 inverse method [Grédiac et al., 2006] is to minimize the squares of residuals, as defined in the following
 148 cost function:

$$\zeta(\beta, \alpha_{11}, \alpha_{22}, \alpha_{12}) = \sum_k \sum_m \left[- \int_{\omega(t_m)} \mathbf{S} : \mathbf{E}^{*k} dv + p(t_m) \int_{\partial\omega(t_m)} \mathbf{k}_{t_m} \cdot \mathbf{u}^{*k} ds + \delta_{k2} \mathcal{F}(t) \right]^2 \quad (9)$$

149 where m labels the time when a measurement is achieved and k labels the virtual field ($k = 1, 2$); $\delta_{k2}=0$
 150 if $k = 1$ and $\delta_{k2}=1$ if $k = 2$.

151 Considering the interpolation of all quantities with the triangular mesh, the previous integrals in the
 152 principle of virtual work may be changed into discrete sums.

$$\zeta(\beta, \alpha_{11}, \alpha_{22}, \alpha_{12}) = \sum_k \sum_m \left[- \sum_q \mathbf{S}(\mathbf{x}^{q,m}) : \mathbf{E}^{*k}(\mathbf{x}^{q,m}) A(\mathbf{x}^{q,m}) e(\mathbf{x}^{q,m}) + p(t_m) \sum_q \mathbf{k}_t(\mathbf{x}^{q,m}) \cdot \mathbf{u}^{*k}(\mathbf{x}^{q,m}) A(\mathbf{x}^{q,m}) + \delta_{k2} \mathcal{F}(t) \right]^2 \quad (10)$$

153 where $A(\mathbf{x}^{q,m})$ denotes the area of triangular element q at time t_m and $e(\mathbf{x}^{q,m})$ denotes the thickness of
 154 the artery¹ at \mathbf{x}^q at time t_m . \mathbf{E}^{*k} is derived from the virtual displacements \mathbf{u}^{*k} using Eq. 6. The virtual
 155 displacements \mathbf{u}^{*k} are defined in Eq. 7. The value of p are experimental data and \mathbf{S} is derived using
 156 Eq. 1. It must be noted that the model parameters are involved in \mathbf{S} . All the incremental steps are taken
 157 into account as the squares of residuals of each incremental step, denoted m , are summed up.

Cost function ζ figures the quadratic gap between the internal virtual work (IVW) and the external
 virtual work (EVW) where:

$$\begin{aligned} \text{IVW}^{k,m} &= e_0 \sum_q \mathbf{S}(\mathbf{x}^{q,m}) : \mathbf{E}^{*k}(\mathbf{x}^{q,m}) A(\mathbf{x}^{q,0}) \\ \text{EVW}^{k,m} &= p(t_m) \sum_q \mathbf{k}_t(\mathbf{x}^{q,m}) \cdot \mathbf{u}^{*k}(\mathbf{x}^{q,m}) A(\mathbf{x}^{q,m}) + \delta_{k2} \mathcal{F}(t) \end{aligned} \quad (11)$$

158 Cost function ζ and IVW are driven by the choice of the unknown material parameters. Eventually,
 159 the cost function can be minimized through an iterative scheme using the Nelder-Mead algorithm [Nelder
 160 and Mead, 1965]. This yields the unknown material parameters.

161 3 Results

162 A sample was tested for proving the feasibility of the approach (Fig. 1 and Fig. 2). We report data
 163 corresponding to 8 pressure levels distributed between 0 and 150 mmHg at $L/L_0 = 1.1$ (Fig. 3 and
 164 Fig. 4).

165 The results obtained with the VFM are reported in Tab. 1 for the Fung model. Convergence of the
 166 Nelder Mead optimization routine was reached in nearly 10 minutes. The identified values for the Fung
 167 material parameters of the artery are consistent with the range orders reported in the literature [Holzapfel

¹Due to the incompressibility assumption, $A(\mathbf{x}^{q,m})e(\mathbf{x}^{q,m}) = A(\mathbf{x}^{q,0})e_0$, with $\mathbf{x}^{q,m} = \mathbf{x}^q(t_m)$.

parameter	β	α_{11}	α_{22}	α_{12}
identified value	5 kPa	14.5	7	0.1

Table 1: Results obtained with experimental data for the Fung model.

et al., 2000, Fung, 1993]. Parameter β is usually around 10 kPa. It is interesting to notice the large difference between the value of α_{11} and α_{22} . It means that there is a large anisotropy in this specimen, which has already been reported in the literature [Holzapfel et al., 2000].

It can be observed in Fig. 5 that the gap between the external virtual work and the internal virtual work is very low for the obtained material parameters.

4 Discussion

4.1 Application to the other constitutive models

4.1.1 The Delfino model

The material parameters of Eq. 2, namely α' and β' can be identified using only \mathbf{u}^{*1} . This means that only the circumferential response is used for the identification. The results obtained with the VFM for these two material parameters are: $\beta' = 1.5$ kPa and $\alpha' = 4.6$.

However, it is interesting to notice in Fig. 6 that the gap between IVW and EVW remains very large for \mathbf{u}^{*2} when computed with the constitutive equations of Eq. 2 and the material parameters reported above. This indicates irrelevancy of the isotropic assumption. This justifies the choice of anisotropic laws.

4.1.2 The Holzapfel model

Here, having access only to measurements on the surface of the artery, it is only possible to identify the average strain energy function across the whole thickness, but not separately strain energy functions of the media layer and of the adventitia layer. If, as in [Marra et al., 2006], the strain energy function is assumed homogeneous across the thickness, then a mean fiber angle and mean values of the other parameters of the Holzapfel model can be identified by the VFM. The obtained results are: $c'' = 0.4$ kPa, $k_1'' = 8.7$ kPa, $k_2'' = 5.4$ and $\phi = 27^\circ$. These results show that parameters of a Holzapfel model may be retrieved by the VFM.

Fiber angles and material parameters in human arteries are usually different for the media and the adventitia. Only average values are reported here. A separate identification of media and adventitia constitutive equations would be possible by mechanical separation of the layers of the human artery, as

194 shown in [Holzapfel et al., 2000].

195 **4.1.3 Material heterogeneities**

196 Arteries may also have properties varying along the axis. The VFM was already applied successfully to the
197 identification of nonlinear heterogeneous behavior for metals [Sutton et al., 2008]. The implementation
198 of a similar approach for arteries is possible but it will require that the spatial resolution is adapted to
199 the length scale of local variations of the material properties. This may be achieved by employing 3-D
200 digital image correlation, as shown in [Sutton et al., 2007].

201 **4.2 Discussions for α_{12} in the Fung model**

202 Parameter α_{12} is the coupling factor between E_{11} and E_{22} in the exponential of the strain energy function.
203 It means that α_{12} characterizes the effect of the circumferential strain onto the axial response, or vice-
204 versa. A value of 0.1 was found for parameter α_{12} (Tab. 1). For obtaining this result, the algorithm was
205 initialized according to values² reported in [Fung, 1993].

206 However, it must be noted that the results regarding α_{12} are significantly affected by the value input
207 for initializing the optimization algorithm. This may indicate a lack of sensitivity of the cost function to
208 α_{12} , inducing the existence of a valley in the cost function.

209 The dependence of the VFM to the initializing values was not observed for the Holzapfel model.

210 Nevertheless, in the Holzapfel model, the exponential part is driven by only 3 material parameters,
211 whereas the exponential part of the Fung model is driven by 4 material parameters. This supplementary
212 material parameter in the Fung model does not affect sufficiently the value of cost function ζ in Eq. 10,
213 inducing the lack of sensitivity to α_{12} . If one would like to increase the sensitivity to α_{12} , one would
214 have to consider the response of the artery not only to incrementally varying pressures, but also to
215 incrementally varying stretches in the axial direction, and to include these responses in the definition of
216 cost function ζ . This will be considered in future studies.

217 **4.3 Implementation of the identified model in a FE code**

218 In [Sun and Sacks, 2005], the implementation of the Fung model in a FE code was discussed. In order
219 to check the feasibility of utilizing this model with the parameters that we identified here (Tab. 1),
220 computations were achieved on the geometry of the artery with the Abaqus[®] software. The geometry of
221 the model is a 33.8 mm long cylinder with an initial diameter of 21.5 mm, and initial thickness of 1.3 mm.

²Initial values: $\beta=29\text{kPa}$, $\alpha_{11}=2.5$, $\alpha_{22}=0.5$ and $\alpha_{12}=0.17$

222 It was meshed with 1276 membrane elements (M3D4R type in Abaqus[®]). The Fung model is a built-in
223 feature of Abaqus[®], which requires the definition of a local coordinate system related to the anisotropy
224 directions and the constitutive parameters reported in Tab. 1. No residual stress was incorporated in this
225 model. One end of the cylinder was clamped. Regarding the other end, radial displacements were fixed
226 and a longitudinal displacement was prescribed, corresponding to $L/L_0 = 1.1$. The different experimental
227 pressure steps were applied successively. The resolution of the problem was performed using an implicit
228 scheme accounting for large strains.

229 A comparison of the average radii computed by Abaqus[®] to the measured average radii is shown in
230 Fig. 7. The comparison is made for the cross section located at $z = (z_b + z_t)/2$, *i.e.* at midspan. The
231 results show a good agreement.

232 However, the radii computed by FE model are slightly larger than the experimental ones. It may be
233 induced by the fact that, in the model, we neglected the pressure applied by the physiological bath on
234 the external surface of the artery (about 5 mmHg \simeq 0.7 kPa at the midspan of the arterial segment).
235 Discrepancies may also be induced by a difference between the experimental boundary conditions and
236 the actual ones at the ends. For the FE model, the boundary conditions were of clamping type at both
237 ends, whereas the actual ones are less rigid.

238 It is worth noting here that this discussion about the boundary conditions at both ends does not affect
239 the results obtained by the VFM. Indeed, the effect of unknown reaction forces and parasitic motions at
240 both ends is filtered out by the VFM. This is an essential asset of the VFM, as shown in [Grédiac et al.,
241 2006].

242 4.4 Derivation of average stress/strain curves

243 It was shown that the identified Fung model can be used to derive stress/strain curves of the artery, both
244 in the circumferential direction and in the axial direction. Results are presented in the appendices.

245 5 Conclusion

246 In conclusion, results presented in this work are promising regarding the application of the virtual fields
247 method (VFM) for identifying the anisotropic hyperelastic properties of arteries. An innovative experi-
248 mental device has been set up, calibrated and validated for characterizing arterial mechanical properties.

249 It is now important to carry out a large number of experiments for validating the approach with
250 different sets of data and different tests. Other perspectives concern more complex loading conditions

251 and more sophisticated models. More especially, next steps will consist in (i) considering the possible
 252 heterogeneity of the properties and geometry of the artery wall, (ii) processing data from different loading
 253 cases (including torsion), (iii) and considering independently the different layers of arteries and the
 254 residual stresses. Achievement of these steps will require optimal spatial resolution and accuracy of the
 255 measurement technique, employing for instance 3-D digital image correlation [Sutton et al., 2007].

256 6 Acknowledgements

257 The authors would like to thank Professeur Jean-Pierre Favre and to his staff in the department of
 258 cardiovascular surgery at the University Hospital of Saint-Etienne (France), for their help in preparing
 259 the specimen tested in this study. The authors are also very grateful to Dr. Katia Genovese for the
 260 experimental work. This study is part of the Imandef project (Grant ANR-08-JCJC-0071) funded by the
 261 ANR (French National Research Agency).

262 7 Appendices

263 7.1 Appendix A

264 In this section, we show that the identified Fung model can be used to derive a stress/strain curve of the
 265 artery in the circumferential direction and that this curve is in agreement with the average stress/strain
 266 curve deduced from experimental data.

267 The average circumferential Green-Lagrange strain is defined by:

$$\tilde{E}_{11}(t) = \frac{1}{2} \left(\left(\frac{\bar{R}}{R_0} \right)^2 - 1 \right) \quad (12)$$

268 where $\bar{R}(t)$ is the average radius of the best fitting circle at time t at $x_3 = (x_3^b + x_3^t)/2$ and R_0 is $\bar{R}(t)$ at
 269 $t = 0$.

270 The circumferential stress component, denoted \tilde{S}_{11} , should satisfy the Laplace law everywhere:

$$\tilde{S}_{11}(t) = \frac{p(t)\bar{R}(t)}{\bar{e}(t)} \quad (13)$$

271 where $\bar{e}(t)$ is the average thickness at the midspan of the arterial segment.

272 We compare the value of $\tilde{S}_{11}(t)$ and $\bar{S}_{11}(t)$, where $\bar{S}_{11}(t)$ is the value deduced from \tilde{E}_{11} by applying
 273 directly the Fung model of Eq. 3 to \tilde{E}_{11} with the assumption $\tilde{E}_{22} = 0$. The parameters for the Fung's
 274 model are the ones that are reported above in Tab. 1.

275 Results shown in Fig. 8a are in agreement. This means that the material parameters identified in
 276 the circumferential direction are consistent with a standard procedure (plotting stress/strain curves from
 277 pressure/diameter measurements). The advantage of the VFM is that the properties in the axial direction
 278 were also identified simultaneously.

279 7.2 Appendix B

280 In this appendix, we show that the identified Fung model can be used to derive a stress/strain curve of
 281 the artery subjected to a tensile test in the axial direction and that this curve is in agreement with the
 282 average stress/strain curve deduced from experimental data.

283 The axial Green-Lagrange strain is defined by:

$$\tilde{E}_{22}(t) = \frac{1}{2} \left(\left(\frac{L}{L_0} \right)^2 - 1 \right) \quad (14)$$

284 where $L(t)$ is the length of the arterial segment at time t .

285 The axial stress component, denoted \tilde{S}_{22} should satisfy:

$$\tilde{S}_{22}(t) = \frac{\mathcal{F}(t)}{2\pi\bar{R}(t)\bar{e}(t)} \quad (15)$$

286 We compare the value of $\tilde{S}_{22}(t)$ and $\bar{S}_{22}(t)$, where $\bar{S}_{22}(t)$ is the value deduced from \tilde{E}_{22} by applying
 287 directly the Fung model of Eq. 3 to \tilde{E}_{22} with the assumption $\tilde{E}_{11} = 0$. The parameters for the Fung's
 288 model are reported in Tab. 1.

289 Results shown in Fig. 8b are in agreement. This means that the material parameters identified in the
 290 axial direction are in agreement with a classical analysis consisting in plotting stress/strain curves. The
 291 advantage of the VFM is that the properties in the axial direction were identified using only one stretch
 292 value, whereas a full tensile test was required in the axial direction for plotting the stress/strain curve
 293 with the standard analysis.

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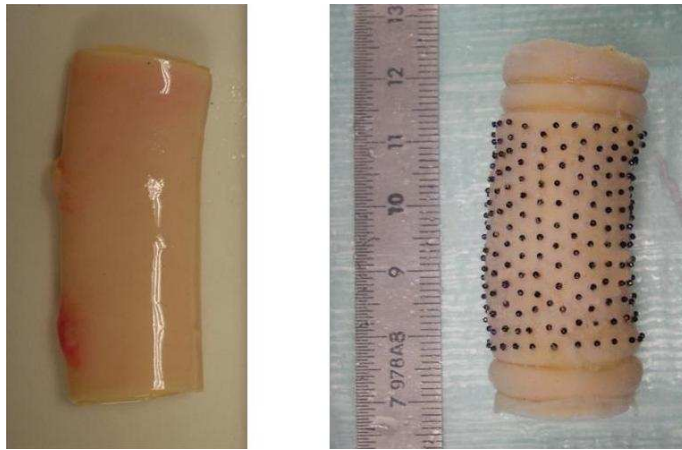


Figure 1: Picture of the arterial segment used in the tests.

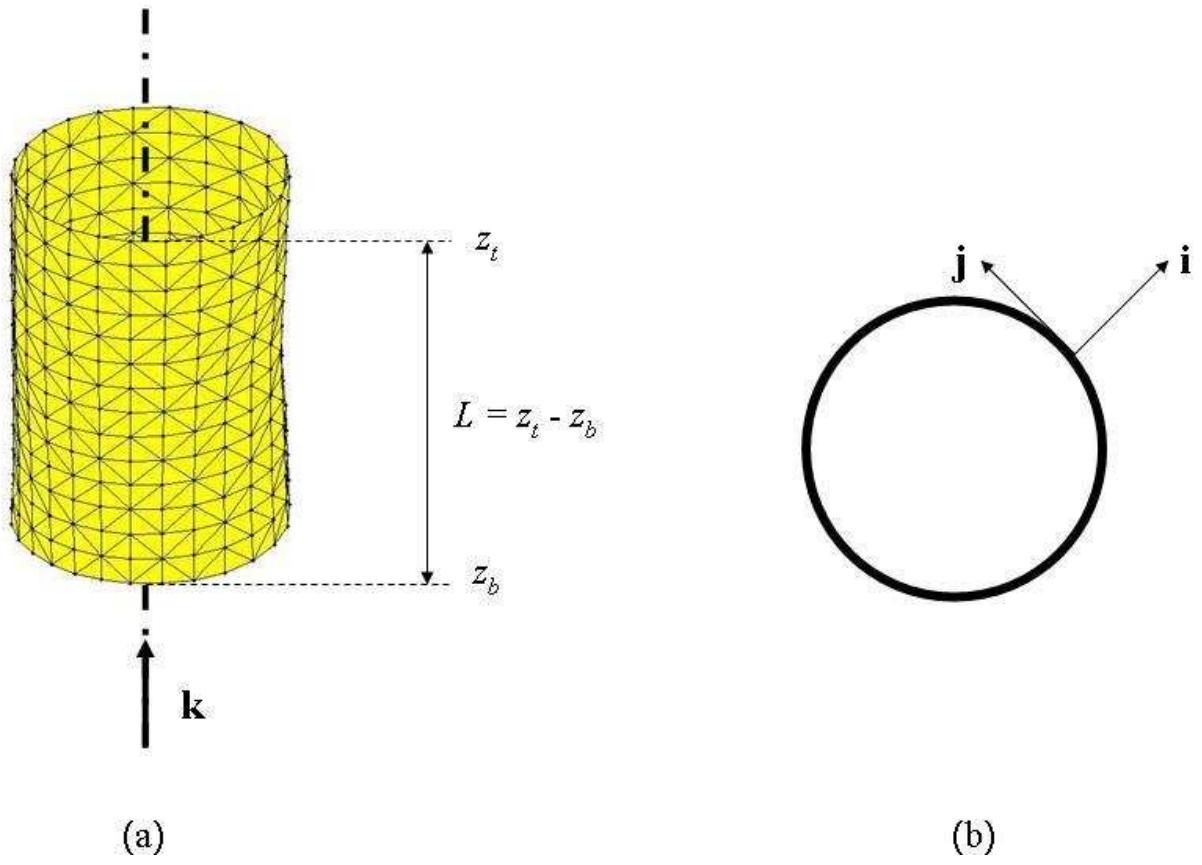


Figure 2: Meshing and schematic of the initial geometry: (a) 3D view (b) cross sectional view.

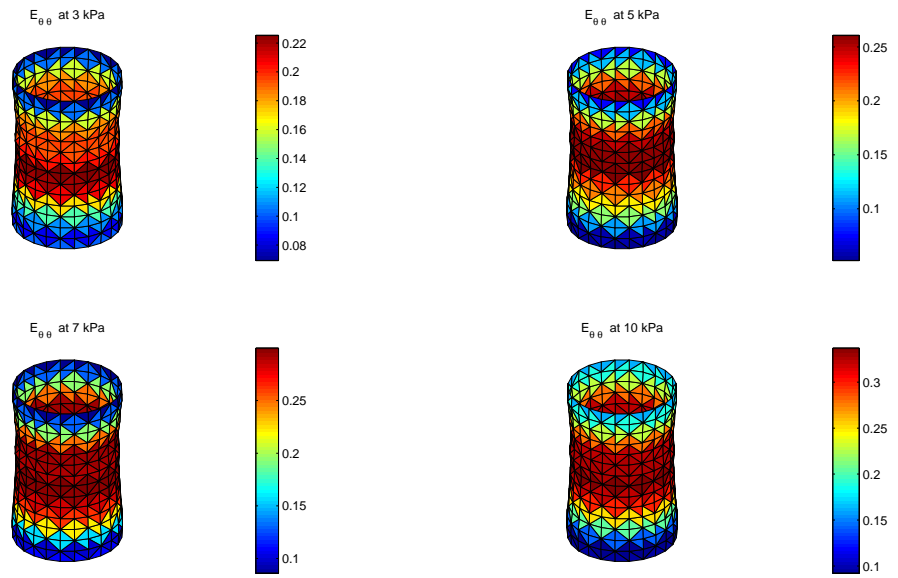


Figure 3: Plot of the circumferential components of the Green-Lagrange strain tensor for different values of p . The plot is displayed in the undeformed configuration.

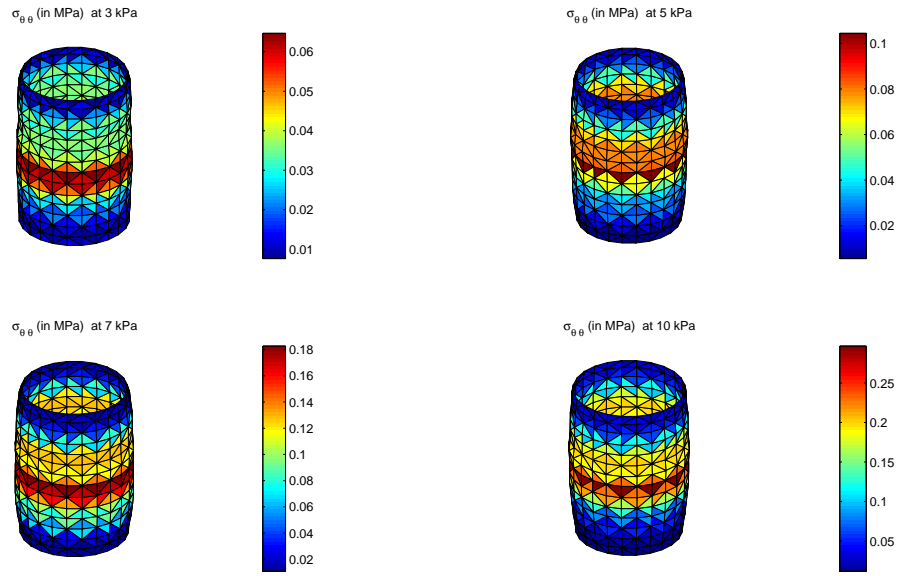


Figure 4: Plot of the circumferential components of the Cauchy stress tensor for different values of p . The plot is displayed in the deformed configuration. The Cauchy stress are calculated with the Fung model using the identified values reported in Tab.1

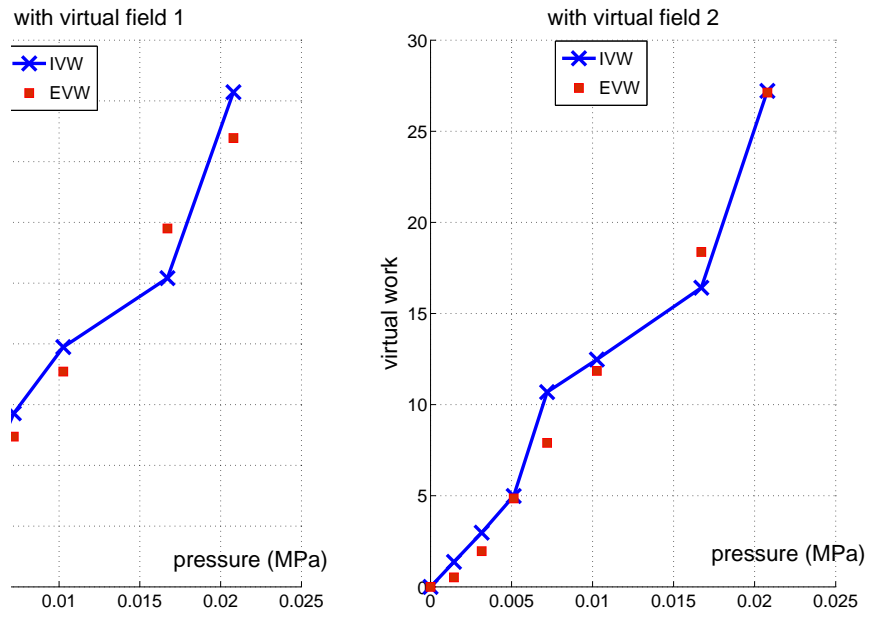


Figure 5: Comparison of the internal and external virtual work for virtual field 1 (a) and virtual field 2 (b) when an anisotropic model is identified

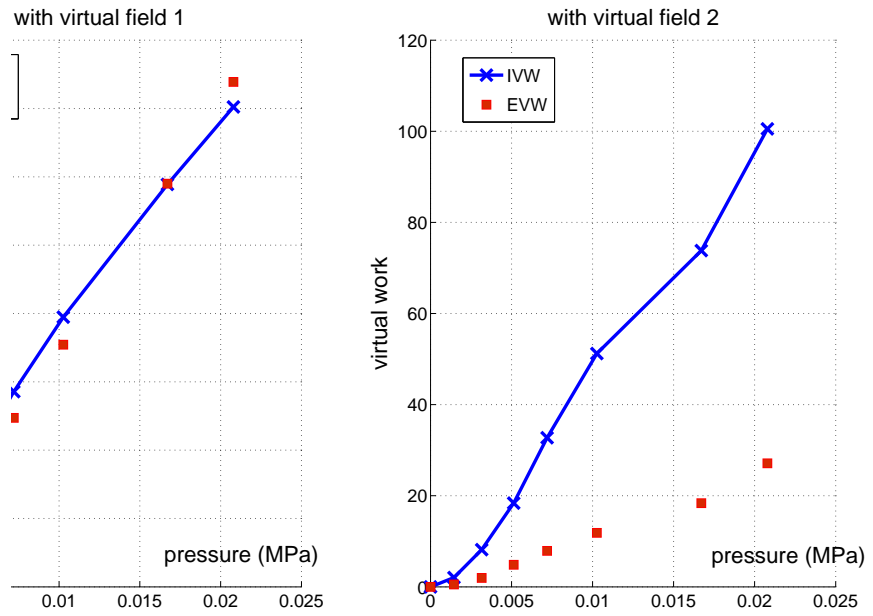


Figure 6: Comparison of the internal and external virtual work for virtual field 1 (a) and virtual field 2 (b) when an isotropic model is identified

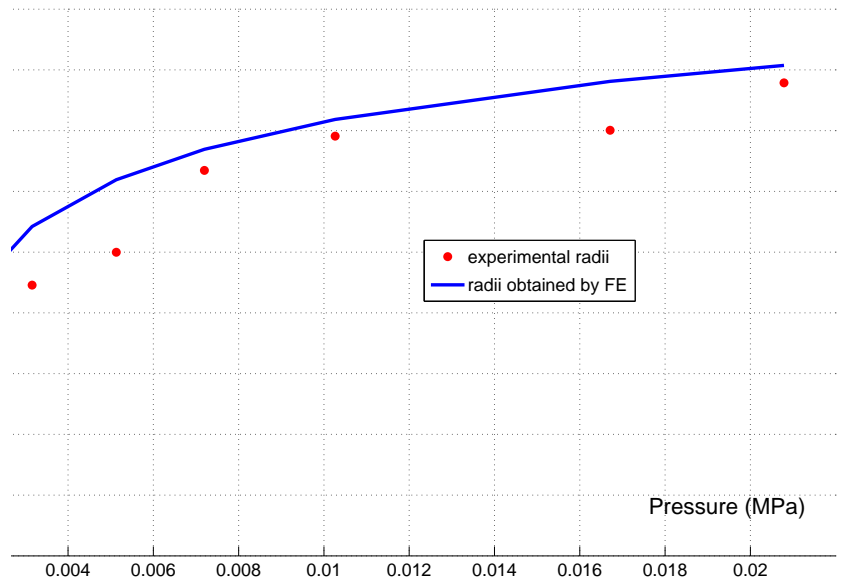


Figure 7: Comparison of the deformed geometry between a FE model and experimental data

frag replacements

S_{11}
 E_{11}
 S_{22}
 E_{22}

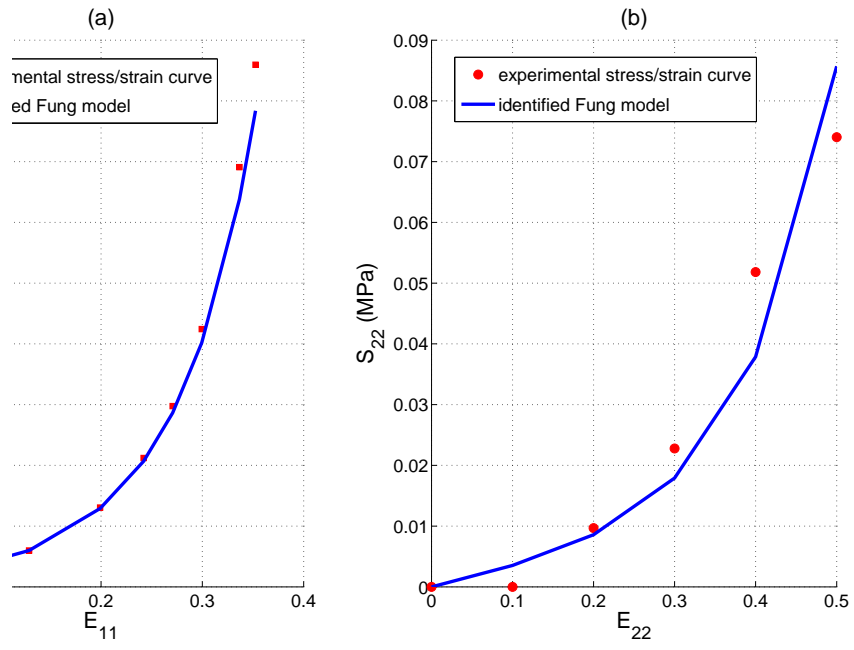


Figure 8: Obtained stress/strain curves: (a) in the circumferential direction and (b) in the axial direction