MINIMIZING SETUP COST FOR MULTI-PART PRODUCTION LINES

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Abstract:
An industrial engineering problem is under study. It consists in optimal configuration of a multi-part production line. A line is a sequence of workstations, at which the manufacturing operations at each workstation are executed in parallel. Parallel execution of operations is inherent for multi-spindle heads in mechanical industry, for example. Each machine part of a specific type f requires a specific set of operations. Different sets can contain common operations. The total number of operations per workstation cannot exceed a given upper bound. Parts move along the stations in the same direction. If there is at least one operation attributed to the part type arriving at a station, this station is set up. Setup costs are part type dependent and they appear due to additional resources required for processing of a part at stations. There are two criteria: minimization of the number of stations, and minimization of the total setup cost. A solution of the problem implies the number of stations and the assignment of operations to these stations. Properties of an optimal solution were established. Basing on these properties, optimal algorithms were developed for the cases $f = 2$, $f = 3$, and an arbitrary f. The algorithms employ combinatorial optimization and linear algebra techniques. They run in a constant time if f is a constant.

Keywords:
line balancing, multi-part production line, setups, combinatorial optimization, linear algebra.
Proposed topics: Design for Manufacturing, Production/Manufacturing Systems and Processes

1 Introduction

We study an industrial engineering problem in which a production line consisting of several (work)stations has to be configured for a manufacturing of f types of machine parts, $f \geq 2$. Let $F = \{1, \ldots, f\}$. A given set of operations N_t is attributed to each part of type $t \in F$. Different sets N_t can contain common operations. Let $\mathcal{N} := \cup_{t=1}^{f} N_t = \{1, \ldots, n\}$ denote the superset of all the required operations. The number of operations at each station is limited by an upper bound r. Each operation must be executed exactly once. Machine parts move along the stations in the same direction one after another, and a station is set up if at least one operation must be executed at this station. A setup cost a_t is associated with part type t, $t = 1, \ldots, f$. Setup costs appear due to the additional resources required for starting and finishing part processing at a station. No station can process more than one part at the same time. Let x_t denote the number of setups for any single part of type t, $t = 1, \ldots, f$. A solution of the problem under study implies the number of stations and an assignment of operations to the stations. The primary criterion is the minimization of the number of stations and the secondary criterion is the minimization of the total setup cost, $a_1 x_1 + \cdots + a_f x_f$. We denote this problem as P.

Problem P belongs to the class of assembly line balancing problems, in which minimizing the production cycle time and minimizing the number of stations are commonly studied criteria. Reviews of assembly line balancing research are provided by Erel and Sarin [3], Rekiek et al. [6], and Boysen et al. [1]. In problem P, it is assumed that any operation can be assigned to any station, all operations assigned to the same station are executed simultaneously and, if there is an upper bound on the production cycle time, then the maximum operation execution time plus the maximum setup time does not exceed this bound. Therefore, operation execution times and setup times are omitted from the problem formulation. Parallel execution of operations is typical for multi-spindle heads in mechanical industry, see, for example, Dolgui et al. [2], and Guschinskaya and Dolgui [4].

Note that the minimum number of stations, m^*, can be determined as $m^* = \lceil \frac{n}{r} \rceil$, where $\lceil \cdot \rceil$ is the rounding up operator. Thus, problem P reduces to finding an assignment of operations of the set $\mathcal{N}$ to m^* stations such that $a_1 x_1 + \cdots + a_f x_f$ is minimized.

In the next section, properties of an optimal solution of problem P are given. Section 3 describes solution algorithms for the cases $f = 2$, $f = 3$ and arbitrary f. Section 4 contains conclusions and suggestions for future research.

2 Properties of an optimal solution

Consider a subset of types $S \subseteq F$. We call operation i an *S-operation* if $i \in \cup_{t \in S} N_t$ and $i \notin \mathcal{N} \backslash (\cup_{t \in S} N_t)$. Verbally, i is S-operation if it is required for parts of types $t \in S$ and it is not required for parts of other types. In the case of two part types, there can be {1}-operations, {2}-operations and {1,2}-operations. Introduce set of type subsets $Z = \{S \in F \mid \text{there exists at least one } S - \text{operation}\}$. We have $|Z| \leq 2^f - 1$ where $|Z|$ is the cardinality of Z.

We call a station a *full station* if it is assigned r operations. We call a station an *S-station* if it is assigned S-operations. By this definition, the same station can be S-station and S'-station, $S \neq S'$, at the same time. We call a station a *pure S-station* if it is assigned only S-operations.

Given an assignment of operations to the stations, denote by $T(u)$ the set of part types associated with operations assigned to station u, and by $k_S(u)$ the number of S-operations assigned to station u. For example, if station u is assigned two $\{1,2\}$-operations and three $\{1,3\}$-operations, then $T(u) = \{1,2,3\}$, $k_{\{1,2\}} = 2$ and $k_{\{1,3\}} = 3$.

Let $U_f := 2^f - 2^{f/2+1} + 2$ and n_S be the number of S-operations in problem P.

The following properties of an optimal solution have been established:

Property 1 *For $f = 2$ and $f = 3$, there exists an optimal solution, which contains at least $\lfloor \frac{n_S}{r} \rfloor$ full pure S-stations for each $S \in Z$.*

Property 2 *For $f \geq 4$, there exists an optimal solution, which contains at most U_f non-full or non-pure S-stations and at least $m_S := \max\left\{0, \left\lceil \frac{n_S - U_f(r-1)}{r} \right\rceil\right\}$ full pure S-stations for each $S \in Z$.*

3 Algorithms

This section contains descriptions of algorithms for the cases $f = 2$, $f = 3$ and $f \geq 4$ of problem P. Assume without loss of generality that $a_1 \geq \cdots \geq a_f$.

3.1 Two parts

Property 1 justifies the following solution algorithm for the case $f = 2$.

Algorithm RE-2 (Round and Enumerate for the case $f = 2$)

Step 1. Assign S-operations to $k_S := \lfloor \frac{n_S}{r} \rfloor$ full pure S-stations for $S \in Z \subseteq \Big\{\{1\},\{2\},\{1,2\}\Big\}$. Note that the assigned operations use the minimum number of stations.

Calculate numbers of unassigned S-operations: $u_S = n_S - k_S r$ for $S \in Z$. Observe that $u_S < r$ for $S \in Z$. Hence, there are at most three stations to be filled by the unassigned operations.

Step 2. If $\sum_{S \in Z} u_S \le r$, then assign remaining operations to the same single station.

If $r < \sum_{S \in Z} u_S \le 2r$, then assign remaining operations to two stations. There are the following three cases to consider:

(i) $u_{\{1,2\}} + u_{\{1\}} \le r$. In this case, assign all {1,2}-operations and {1}-operations to one station, and assign all {2}-operations to another station.

(ii) $u_{\{1,2\}} + u_{\{1\}} > r$ and $u_{\{1,2\}} + u_{\{2\}} \le r$. In this case, assign all {1,2}-operations and {2}-operations to one station, and assign all {1}-operations to another station.

(iii) $u_{\{1,2\}} + u_{\{1\}} > r$ and $u_{\{1,2\}} + u_{\{2\}} > r$. In this case, assign operations arbitrarily to two stations.

If $u_{\{1\}} + u_{\{2\}} + u_{\{1,2\}} > 2r$, then assign all remaining {1,2}-operations to one station, all remaining {1}-operations to a second station and all remaining {2}-operations to a third station.

The assignment in this step minimizes the number of stations as the primary criterion and it minimizes $a_1 x_1 + a_2 x_2$ as the secondary criterion if $a_1 \ge a_2$.

Observe that the run time of algorithm RE-2 and the length of the description of the corresponding optimal solution is a constant number.

3.2 Three parts

Property 1 can also be used to solve problem P in the case $f = 3$. However, we verified that in this case calculations similar to Step 2 of algorithm RE-2 should consider too many cases, which include various assignments of S-operations, $S \in Z \subseteq \Big\{\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\}\Big\}$ to at most seven non-full or non-pure stations. Therefore, we decided to go for an integer programming formulation of problem P and to show that its solution can be obtained by an enumerative algorithm in combination with linear algebra tools.

Algorithm RE-3 (Round and Enumerate for the case $f = 3$)

Step 1. Assign S-operations to $k_S := \lfloor \frac{n_S}{r} \rfloor$ full pure S-stations for $S \in Z \subseteq \Big\{\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\}\Big\}$.

Step 2. The number of stations to assign remaining operations is equal to $m := m^* - \sum_{S \in Z} k_S \le 7$. Let these stations be numbered $1,\ldots,m$. It is easy to show that there exists an optimal solution, in which $u_{\{1,2,3\}} := n_{\{1,2,3\}} - k_{\{1,2,3\}} r$ remaining $\{1,2,3\}$-operations are assigned to the same station. Assume without loss of generality that they are assigned to station 1. Re-set $Z := Z \backslash \{1,2,3\}$ and re-number subsets of this set. If the original set Z contained seven subsets, then re-set $Z := \{1,\ldots,6\}$, see Table 1. If it contained $q < 7$ subsets, then re-set $Z := \{1,\ldots,q-1\}$. In the sequel, assume that the original set Z contained seven subsets.

Step 3. Calculate numbers of unassigned i-operations: $u_i = n_i - k_i r$, $i = 1,\ldots,6$.

Table 1: Subsets $S \in Z$ and their new notation.

{1}	{2}	{3}	{1,2}	{1,3}	{2,3}
1	2	3	4	5	6

Let variable y_{ij} denote the number of i-operations assigned to station j, $i = 1, \ldots, 6$, $j = 1, \ldots, m$. For $y \geq 0$ define function $\mathrm{sgn}(y) = 0$ if $y = 0$ and $\mathrm{sgn}(y) = 1$ if $y > 0$. Given y_{ij}, the numbers of setups associated with the remaining operations, x'_t, $t = 1, 2, 3$, can be calculated as follows:

$$x'_1 = \sum_{j=2}^{m} \mathrm{sgn}(y_{1j} + y_{4j} + y_{5j}),\ x'_2 = \sum_{j=2}^{m} \mathrm{sgn}(y_{2j} + y_{4j} + y_{6j}),\ x'_3 = \sum_{j=2}^{m} \mathrm{sgn}(y_{3j} + y_{5j} + y_{6j}).$$

The problem of optimal (with respect to problem P) assignment of i-operations, $i = 1, \ldots, 6$, to m stations can be formulated as the following integer problem IP-3.

Problem IP-3:

$$\min\left\{a_1 \sum_{j=2}^{m} \mathrm{sgn}(y_{1j}+y_{4j}+y_{5j}) + a_2 \sum_{j=2}^{m} \mathrm{sgn}(y_{2j}+y_{4j}+y_{6j}) + a_3 \sum_{j=2}^{m} \mathrm{sgn}(y_{3j}+y_{5j}+y_{6j})\right\}, \quad (1)$$

subject to

$$\sum_{i=1}^{6} y_{i1} \leq r - u_{\{1,2,3\}},\ \sum_{i=1}^{6} y_{ij} \leq r,\ j = 2, 3, \ldots, m, \quad (2)$$

$$\sum_{j=1}^{m} y_{ij} = u_i,\ i = 1, \ldots, 6, \quad (3)$$

$$y_{ij} \geq 0 \text{ and integer},\ i = 1, \ldots, 6,\ j = 1, \ldots, m. \quad (4)$$

Given feasible with respect to (2)-(4) values y_{ij}, consider a collection $c(y)$ of $3(m-1)$ triples

$$c(y) = \{(y_{1j}, y_{4j}, y_{5j}), (y_{2j}, y_{4j}, y_{6j}), (y_{3j}, y_{5j}, y_{6j}) \mid j = 2, 3, \ldots, m\}.$$

Note that objective function (1) is not linear. The important information for its minimization is which triples of the collection $c(y)$ are equal to (0,0,0). If a triple from $c(y)$ is not equal to (0,0,0), then its specific content does not affect the objective function value. Let C denote the set of collections $c(y)$ with various specified triples (0,0,0). We have $|C| \leq 2^{3(m-1)} \leq 2^{18} = 262144$.

The problem reduces to finding an arbitrary solution of the system of integer linear inequalities (2)-(4) for each $c(y) \in C$ and selecting $c(y)$ and corresponding values y_{ij} which minimize objective function (1).

Observe that if we use single-index variables ($y_{6(i-1)+j} := y_{ij}$), then the matrix of coefficients in the left-hand side of (2)-(4) is *totally unimodular*, because every entry is 0 or 1, every column contains at most two non-zero entries, and rows can be partitioned into two disjoint sets such that any two non-zero entries in a column belong to different sets of rows, see Heller and Tompkins [5]. Therefore, a *basic solution* of the system (2)-(4) relaxed to fractional variables is actually integral. It can be obtained by any method of solving a system of linear inequalities in time depending solely on the numbers of variables ($6m \leq 42$) and basic inequalities ($m + 6 \leq 13$).

The run time of algorithm RE-3 is determined by the time of solving problem IP-3, which is $O(2^{3(m-1)}T)$ where T is the time to find a basic solution of (2)-(4). Since $m \leq 7$, algorithm RE-3 runs in a constant time.

3.3 f parts

An algorithm similar to algorithm RE-3 can be developed for the case of arbitrary f. It employs Property 2.

Recall that $|Z| \leq 2^f - 1$. Let Z_t, $Z_t \subseteq Z$, be the set of subsets in Z each of which includes part type t. For example, set $\{1, 2, 3\}$ belongs to Z_1, Z_2 and Z_3.

Algorithm RE (Round and Enumerate for the case of arbitrary f)

Step 1. Assign S-operations to m_S full pure S-stations for $S \in Z$, see Property 2.

Step 2. The number of stations to assign remaining operations is equal to

$$m := m^* - \sum_{S \in Z} m_S \leq \left\lceil \frac{\sum_{S \in Z} n_S}{r} \right\rceil - \sum_{S \in Z} \left\lceil \frac{n_S - U_f(r-1)}{r} \right\rceil \leq$$

$$\frac{\sum_{S \in Z} n_S}{r} + 1 - \sum_{S \in Z} \frac{n_S - U_f(r-1)}{r} = 1 + \frac{|Z| U_f(r-1)}{r}.$$

Note that m is bounded by a function solely depending of f.

Let these stations be numbered $1, \ldots, m$. Re-number subsets of the set Z such that $Z := \{1, \ldots, |Z|\}$.

Step 3. Calculate numbers of unassigned i-operations: $u_i = n_i - m_i r$, $i = 1, \ldots, |Z|$.

Let y_{ij} denote the number of i-operations assigned to station j, $i = 1, \ldots, |Z|$, $j = 1, \ldots, m$. Problem P can be formulated as the following integer problem IP.

Problem IP:

$$\min \Big\{ \sum_{t=1}^{f} a_t \sum_{j=1}^{m} \text{sgn}\Big(\sum_{i \in Z_t} y_{ij} \Big) \Big\}, \tag{5}$$

subject to

$$\sum_{i \in Z} y_{ij} \leq r, \ j = 1, \ldots, m, \tag{6}$$

$$\sum_{j=1}^{m} y_{ij} = u_i, \ i = 1, \ldots, |Z|, \tag{7}$$

$$y_{ij} \geq 0 \text{ and integer}, \ i = 1, \ldots, |Z|, \ j = 1, \ldots, m. \tag{8}$$

Given feasible with respect to (6)-(8) values y_{ij}, consider a collection $d(y)$ of mf number of $|Z_t|$-tuples

$$d(y) = \{(y_{ij} \mid i \in Z_t) \mid t = 1, \ldots, f, \ j = 1, \ldots, m\}.$$

Let D denote the set of collections $d(y)$ with various specified $|Z_t|$-tuples $(0, \ldots, 0)$. We have $|D| \leq 2^{mf}$.

The problem reduces to finding an arbitrary solution of the system (6)-(8) for each $d(y) \in D$ and selecting $d(y)$ and corresponding values y_{ij} which minimize objective function (5). This can be done in $O(2^{mf} E)$ time where E is the time to find a basic solution of (6)-(8).

The run time of algorithm RE is a constant if f is a constant because m and E are bounded by functions solely depending of f.

4 Conclusions and suggestions for future research

The industrial engineering problem consisting in optimal configuration of a multi-part production line has been studied. It has been modeled as problem P, in which the minimization of the number of stations is the primary criterion and the minimization of the total setup cost is the secondary criterion. Properties of an optimal solution have been established, and constant time solution algorithms have been developed for the cases $f = 2$, $f = 3$ and arbitrary given f.

Problem P can be generalized to include precedence, inclusion and exclusion relations on the set of operations. However, at present, we do not see an efficient solution algorithm for this generalization.

We believe that the considered criterion of minimizing a total setup cost is important and it deserves to be studied in other settings of the line balancing problems.

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