PARAMETER OPTIMISATION IN GROUNDWATER USING PROPER ORTHOGONAL DECOMPOSITION AS A REDUCED MODELLING TECHNIQUE.

John N. Wise*¹, G. Venter¹, Mireille Batton-Hubert² and Eric Touboul²

¹ Mechanic and Mechatronic engineering University of Stellenbosch P.O.BOX 1400, 26500 Stellenbosch, South Africa *e-mail: jnwise@sun.ac.za

²Ecole des Mines de Saint Etienne, Institut Fayol, France

Keywords: POD, Richards Equation, Darcy flow, Parameter optimization, Finite Element Method.

Abstract. This paper deals with different approaches of applying Proper Orthogonal Decomposition in the field of groundwater flow, specifically the Richards equation, which is a convection-diffusion partial differential equation governing the behaviour of unsaturated fluid flow through a porous medium. The motivation for this research is the need to reduce computational complexity in inverse modelling studies, where a significant number of simulations are needed to determine suitable model parameters. Three different methods of implementing Proper Orthogonal Decomposition are explored. The first method is the Petrov-Galerkin method, a method well suited to speeding up linear problems. The second method is a "Hybrid" method, and proposes a linearization of all non-linear functions, building upon the Petrov-Galerkin approach. As such, it is suitable for use in the non-saturated groundwater zone. The third method combines the use of kriging and Proper Orthogonal to create a non-intrusive model for comparison purposes. With these three methods, the suitability of Proper Orthogonal as a reduced modelling method for unsaturated groundwater flow is shown.

REFERENCES.

[1] Segal, I. A., *Finite element methods for the incompressible Navier-Stokes equations*. Delft University of Technology, Faculty of Electrical Engineering, Mathematics and Computer Science, 2011

[2] de Vuyst, F., *PDE Metamodeling using Principal Component Analysis*, Multidisciplinary optimization, 2009 (Unpublished)

[3] Vrugt, J.A. et al., *Modeling of Subsurface Flow and Transport Properties: A review with new developments*, Vadose zone journal, 2008

[4] Mcphee, J. and Yeh, W.W., *Groundwater Management Using Model Reduction via Empirical Orthogonal Functions*. Journal of Water Resources Planning and Management, 2008

[5] Siade, A. J., Putti, M. and Yeh W. W. G., Snapshot selection for groundwater model reduction using proper orthogonal decomposition WATER RESOURCES RESEARCH, VOL. 46, W08539, 2010

[6] Astrid, P., *Reduction of Process Simulation Models: a proper orthogonal decomposition approach*, Technische Universiteit Eindhoven, 2004.

[7] Todd, D. K. and Mays, L.W.: Groundwater hydrology. John Wiley and Sons, Inc, 3rd Edition, 2005.

[8] Lophaven, N. S., Nielson, H. B., Sondergaard, J., *DACE : A MATLAB Kriging Toolbox, Version 2.,* August 2002, Technical University of Denmark

[9] Hecht, F., *FreeFEM++ Third Edition Version 3.11-1*, Laboratoire Jacques Louis Lions, Universite Pierre et Marie Curie, Paris, 2011