A BOOTSTRAP APPROACH TO THE PRICE UNCERTAINTY OF WEATHER DERIVATIVES

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ABSTRACT

This paper investigates price uncertainties in weather derivatives contracts through a bootstrap approach. Futures prices are computed under a periodic ARMA model in an actuarial framework for two different locations, Paris and Chicago. We show that statistical errors may lead to substantial uncertainties on futures prices with confidence intervals up to 10% of the assessed prices.

KEYWORDS.

Weather derivatives, temperature, ARMA model, parameter uncertainty, model risk, bootstrap.

1. Introduction

The market for weather derivatives was launched by investment banks, insurance companies and utilities in the late 90's. Most of the contracts are OTC though some can be traded on future exchanges (CME, LIFFE-Euronext). These products provide protection against losses due to non-catastrophic climatic events. End-users are, mostly, energy companies but also theme parks, breweries, winter shipment manufacturers, leisure resorts, fertiliser manufacturers… The underlying climatic risks are measured by means of indexes built from available meteorological data.

Since these markets are currently quite illiquid and not very transparent, it is difficult to mark to market the products and calibrate some parameters from market prices. Thus, market participants rather use econometric models plus a pricing rule and then mark to model. The outcome of the paper is to assess the impact of estimation error and some model error on the prices of weather futures. We provide some confidence intervals for prices; this can be used for determining a reserve policy and better cope with model risk.

The article is organised as follows. In section 2, we introduce the data, the temperature model, the specifications and the valuation of temperature future contracts. In section 3, we present the bootstrap methodology for the assessment of price uncertainty. Results are presented in sections 4 (LIFFE case) and 5 (CME). Finally in section 6, the price uncertainties are compared with those obtained with an asymptotic delta-method methodology.

2. Presentation of the data, the temperature model, the specifications and the valuation of temperature future contracts

2.1. The data

We use daily average temperatures from the 1st of January 1979 to the 31st of December 1999. The average temperature is the common "underlying" for weather derivative contracts as proposed by the Chicago Mercantile Exchange for instance. We consider two meteorological stations: Paris-Montsouris and O'Hare Airport, near Chicago. The data come from Météo France for Paris and the website of the Chicago Mercantile Exchange for Chicago. In order to facilitate the treatment of data, we removed the 29th of February, which corresponds to remove 5 values per station for a total of 7665 temperatures.

2.2. The temperature model

The model takes into account the major characteristics of temperature: seasonality of the values and of the dispersion, quick reversion to the mean, correlations from the days before and today... It has been presented by Cao and Wei (2000) or Roustant (2002). We refer to Dischel (1998), Dornier and Quéruel (2000), Moréno (2000), Brody, Syroka, Zervos (2001), Davis (2001) or Campbell, Diebold (2001) for related specifications. It is a linear model with a periodic variance:

$$X_t = m_t + s_t + \sigma_t Z_t$$

with:

- m_t represents the trend;
- s_t the seasonal component;
- σ_t a deterministic and periodic process with an annual periodicity representing the standard deviation of X_t
- Z_t an ARMA process:

$$Z_t = \phi_1 Z_{t-1} + ... + \phi_p Z_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + ... + \theta_q \varepsilon_{t-q}$$

where (ε_t) is a Gaussian white noise. The variance of Z_t is set to 1 (the variance of ε_t is then a function of $\phi_1,...,\phi_p,\theta_1,...,\theta_q$, see Brockwell, Davis, 1991, §3.3.).

Moreover, we assume the following parametric forms:

- $m_t = dt + e$
- $s_t = \sum_{i=1}^{N_f} (a_i \cos(i\omega t) + b_i \sin(i\omega t))$
- $\sigma_t = a + b\cos(\omega t) + c\sin(\omega t)$

with $\omega = 2\pi/365$.

Such expressions are justified by basic fitting based on data, and allow easy computation of maximum likelihood estimator. For the seasonal component, the choice of frequencies is achieved by means of a preliminary spectral analysis of the normal temperature of each series. In the case of Chicago, we only kept the fundamental (annual) frequency and s_t is simply parameterised by $s_t = a_1.\cos(\omega.t) + b_1.\sin(\omega.t)$. In addition, the discrete curve of the normal temperature of Paris is asymmetric which forces the use of at least two frequencies. Finally, we retained the form

$$s_t = \sum_{i=1}^{2} a_i \cos(i\omega t) + b_i \sin(i\omega t).$$

The selection of p and q, the orders of the ARMA model is accomplished by standard procedures (see Brockwell, Davis, 1991), after a preliminary estimation of m_t, s_t and σ_t. It leads to the choice $p = 3$, $q = 0$ for the two stations.

Despite its relative simplicity, this linear model for temperature is not far from being correct as one can see on Figure 1. In the case of Paris, correlograms of residuals and squared residuals show that the dependence between residuals can be considered as independent. In the case of Chicago, the first squared residuals autocorrelations are significant, which is consistent with the conditional heteroskedasticity model of Campbell and Diebold (2001). But, even in that case, the dependence is small. Some departures from normality are observed in the Paris case, for which the distribution of residuals exhibits fat tails for low temperatures.

2.3. Specifications of temperature indexes

In this paper, we will focus on the weather futures contracts offered by the CME and LIFFE exchanges. They are built on temperature indexes that are also widely used on the over-the-counter (OTC) market. On the LIFFE, *modulo* a constant, the temperature index is simply the average temperature (expressed in degree Celsius):

$$AVE = 100 + \frac{1}{L}\sum_{t=t_1}^{t_2} X_t$$

where L is the length of the risk exposure period $[t_1;t_2]$. On the CME, two indexes are used, namely the Heating Degree-Day (HDD) index and the Cooling Degree-Day (CDD) index (expressed in degree Fahrenheit):

$$HDD = \sum_{t=t_1}^{t_2} (65 - X_t)^+ \qquad CDD = \sum_{t=t_1}^{t_2} (X_t - 65)^+$$

The value of 65°F (18°C) is a benchmark in the energy industry, since it is usually considered that heating starts when temperatures go below 18°C. Since it takes into account all temperatures below 18°C, the HDD index is suitable for cool months (when heating is "necessary"); it is used from October to April. On the other hand, the CDD index is used from May to September.

2.4. Specifications of futures contracts; futures prices

A weather future is a financial product that provides or demands reimbursement according to the level of a weather index. The risk exposure period is generally a month, or, more rarely, a season. Precise specifications are given in Table 1. The size of this payment is calculated as such no initial premium is required. For instance, the payment of a HDD-future contract is

$$F - N \times HDD$$

where F is the future price, and N is the contract size (on the CME for instance, N equals 100$ per degree). Thus for a specified month, the payment is all the more important than the HDD index is low or, equivalently, than the temperatures are mild. On the other hand, the "payment" can be negative, and therefore required, for colder temperatures than usual. Typically, such a contract may be entered by an energy company to protect against mild winters; however, it will have to pay for cold winters…

There are currently a number of pricing methodologies for such contracts, see Carr, Geman and Madan (1999), Davis (2001), Musiela and Zariphopoulou (2001), Schweizer (2001) or Barrieu and El Karoui (2002). Roughly speaking, the different valuation approaches depend on whether or not the underlying weather variables can be "linked" to the financial market: for instance, energy commodities may be correlated with temperature, and might be used for the valuation of weather derivatives. Most practitioners use an actuarial framework and the "standard deviation principle" (see Goovaerts, DeVylder, Haezendonck, 1984, or Bühlmann, 1996). They calculate the net premium as

$$P = E[\text{discounted payment}] + \lambda \times \sigma[\text{discounted payment}] \qquad (*)$$

In the following, we will restrict to pure premiums, i.e. $\lambda = 0$ in (*) ; we refer to Denneberg (1990), Schweizer (2001), Hürlimann (2001) or Moller (2001, 2003a and 2003b) for more discussions about the valuation rule.

For a weather future, assuming deterministic interest rates for simplicity, we have (see e.g. Duffie, 2001):

$$0 = E[F - N \times TI]$$

where TI denotes a temperature index and F the futures price. Therefore,

$$F / N = E[TI]$$

When temperature is modelled by the ARMA model presented above, we will denote by $P(\Theta)$ the futures prices corresponding to the temperature parameters Θ:

$$P(\Theta) = E[TI(\Theta)]$$

Note that expectations should be computed conditionally to available information. That includes the set of past temperatures. Actually, these have very little influence when the risk exposure period begins more than 20 days after the present date (Roustant, 2002), and will be further neglected.

When the assumptions of the model are verified, futures prices are obtained in closed-form. In the LIFFE case, we have $P(\Theta) = 100 + \frac{1}{L}\sum_{t=t_1}^{t_2}(m_t + s_t)$, while analytical expressions in the CME case can be found in (Cao, Wei, 2000).

3. ASSESSMENT OF PRICE UNCERTAINTY BY A BOOTSTRAP METHODOLOGY

In practise, futures prices are only estimated, and the estimated price is a function of the estimate of the temperature parameters. Thus, the estimation errors of the temperature model result in estimation errors on the corresponding prices. This is that kind of uncertainty that we want to assess now. Examples and further discussions about this price uncertainty approach can be found in (Campbell, Lo, MacKinlay, 1997, §9.3.3.) or (Cairns, 2000). To do this, we want, in addition, to account for the model misspecification relative to the normality assumption in the residuals ε_t. Therefore, we will not assume that the temperature residuals are Gaussian; either, we will work with the empirical distribution of centred residuals:

$$\hat{F}_T(x) = \frac{1}{T}\sum_{t=1}^{T} 1_{(-\infty,x]}(\hat{\varepsilon}_t - \overline{\varepsilon})$$

where T is the data size, $\hat{\varepsilon}_t$ are the "initial" estimated residuals - that is the residuals corresponding to the initial parameters Θ_0 obtained by maximum likelihood estimation (MLE) based on the data, and $\overline{\varepsilon}$ their mean. The residuals are centred to have the same mean as the innovations of the model. This has a very minor effect since the mean of $\hat{\varepsilon}_t$ is nearly 0, and results in closed form expressions for futures prices (see section 4); it is also a current practise when bootstrapping with time-series, see for instance (Davinson, Hinkley, 1997, chapter 8). On a statistical point of view, our objective is equivalent to assess the precision of the estimator:

$$\hat{P}(\hat{\Theta})$$

where $\hat{\Theta}$ is the maximum likelihood estimator, and $\hat{P}$ is the future price under the empirical distribution of the (initial) residuals:

$$\hat{P} = E_{\hat{F}_T, \hat{\Theta}}\left[\text{Temperature index}\right]$$

This objective can be achieved by a bootstrap technique. The idea of bootstrap is to generate from the available data, solely, a sample of independent realisations of some statistic of interest. In our case, we would like to have a sample of price values $\hat{P}\left(\Theta^{*1}\right),...,\hat{P}\left(\Theta^{*R}\right)$ corresponding to a huge sample of temperature parameters values $\Theta^{*1},...,\Theta^{*R}$; price uncertainty could therefore be assessed by the 2.5% and 97.5% quantiles of $\hat{P}\left(\Theta^{*1}\right),...,\hat{P}\left(\Theta^{*R}\right)$. When bootstrapping with time-dependent data, a three-stage procedure is currently done (see Efron, Tibshirani, 1986, §6 or Davinson, Hinkley, 1997). Firtsly, the structure of the model is used (in the "reverse" sense) to extract a white noise from the data: this is *pre-whitening*; then, *bootstrap* is made on this noise and, finally, new paths are generated by reconstitution, or *post-blackening*, using again the structure of the model (in the "natural" sense). The bootstrap itself can be done in several ways, whether one assumes independence of the data - then a simple resampling can be used - or suspects some residual dependency: in that case, the *block bootstrap* for instance may be preferred (see Davinson, Hinkley, 1997 or Bühlmann, 2000 for a more extensive presentation of bootstrap techniques). In our case, it seems that there is no real need to use such techniques (see again Figure 1) and we will restrict to simple resampling. Eventually, we do the following operations:

for $r = 1,...,R$ *do* :

1. Simulate independently $\varepsilon_1^*,...,\varepsilon_n^*$ from the centred empirical distribution of residuals
2. Reconstitute the corresponding temperature path $x_1^*,...,x_n^*$ using the temperature model with the initial parameters (MLE based on the data)
3. Calculate the MLE Θ^{*r} from the new data $x_1^*,...,x_n^*$
4. Calculate the corresponding future price $\hat{P}\left(\Theta^{*r}\right)$

In practise, to insure stationarity of $x_1^*,...,x_n^*$, a longer sample is generated and first values are discarded (see e.g. Davinson, Hinkley, 1997, §8.2.2.). Of course Θ^{*r} is not issued exactly from the distribution of the temperature parameter MLE estimator $\hat{\Theta}$, since resampling is made in the empirical distribution of a particular sample issued from it. However, the statistical bias vanishes as the sample size tends to infinity (see e.g. Davinson, Hinkley, 1997). As in our case the data size $T = 7.665$ is rather large, we can reasonably think that this bias is small and, consequently, that the empirical properties of $\hat{P}\left(\Theta^{*1}\right),...,\hat{P}\left(\Theta^{*R}\right)$ will give accurate estimates of the statistical properties of $\hat{P}(\hat{\Theta})$. We will assess the future price uncertainty by the 95% confidence interval of $\hat{P}\left(\Theta\right)$ given by the 2.5% and 97.5% quantiles of $\hat{P}\left(\Theta^{*1}\right),...,\hat{P}\left(\Theta^{*R}\right)$.

4. Price uncertainty of the LIFFE weather futures.

For the LIFFE weather futures, the underlying index is a linear function of temperature. Thus, futures prices can be obtained in closed-form under the empirical distribution of (centred) residuals:

$$\hat{P}(\Theta) = 100 + \frac{1}{L}\sum_{t=t_1}^{t_2}(m_t + s_t)$$

where L is the length of the risk exposure period $[t_1;t_2]$. Hence, the methodology described in the previous section can be applied directly. We set $R = 10.000$, and with the prices sample obtained by bootstrap $\hat{P}\left(\Theta^{*1}\right),...,\hat{P}\left(\Theta^{*R}\right)$, we calculated the median, the standard deviation and a 95% confidence interval obtained with the 2.5% and 97.5%-quantiles. Results are shown in Table 2. We also indicate the price uncertainty, expressed in percentage (last column) and the theoretical price, computed under the normality assumption and based on the initial parameters value Θ_0 (first column).

Note that the “bootstrap” future price estimated by bootstrap is not different from the expected value under the normality assumption for the temperature process. It shows that non-normalities have no impact on futures prices. The form of the index may help to understand this: the average over a long period of time may result in smoothing the daily differences that could exist. A more statistical explanation is that the linearity of the index implies the linearity of the future price as a function of the temperature parameters estimator. Now, anticipating the asymptotic results of section 6, this one is approximately normal, which explains that the futures prices sample is nearly equal to its mean, and may be unbiased, which explains that this mean is equal to the theoretical value. This asymptotic property may also explain the symmetry of confidence intervals observed in column 4 or 5.

In absolute value, price uncertainty is constant over months (column 3). Its severity then depends on the value of the future price. We see that price uncertainty does not represent more than 5% from April to October, but is larger in winter with more than 10% for winter months, and 8% for the winter whole season. Estimation error may be taken into account for these months.

5. PRICE UNCERTAINTY OF THE CME WEATHER FUTURES.

At the CME exchange, weather products are based on HDD or CDD indexes, which are non-linear functions of temperature. In this situation, futures prices have no analytical expressions under the empirical distribution and must be estimated by Monte Carlo simulations. To reduce time computation, we used the control variates technique. Then, only 1.000 simulations are required to give correct estimates for $\hat{P}\left(\Theta^{*r}\right)$, and the methodology of section 2 can be achieved.

The results are shown on Table 3. Firstly, we observe a seasonal pattern in the price uncertainty (column 3). This departs from the LIFFE case where all price uncertainties have the same order of magnitude. It may be a consequence of the non-linearity of the HDD and CDD indexes. Indeed the temperature threshold of 65°F does not “cut” temperatures in the same way in winter or in summer or in the “shoulder months” of May and September. When these indexes can be approximated by a linear one, as in winter (where temperature rarely goes above 65°F), the estimation error is nearly constant. Finally, when thinking of price uncertainty in terms of percentage, worst results are associated with lower prices, corresponding to May and September. For these months, price uncertainty goes beyond 15%. Excepting these particular cases, price uncertainty is about 10% in summer and 5% in winter. Let us remark that this “5%” is associated with a high future price and therefore gives a large confidence interval.

6. ASSESSMENT OF PRICE UNCERTAINTY BY THE DELTA-METHOD

One drawback with the bootstrap methodology presented in section 3 is that it requires the estimation of extreme quantiles. This estimation may be inaccurate and, as an alternative, we propose to estimate the asymptotic distribution of futures prices. We show that this distribution is approximately normal, so that quantiles can be related to the standard deviation, which is estimated more accurately. In addition, let us remark that asymptotic results may give good approximations here since the number of available data is large. To use this method, we need to assume that the ARMA model is well specified, and thus that the residuals are normally distributed. From now on, we will make this assumption and will denote $P(\Theta)$ the corresponding futures prices.

We then argue that the asymptotic distribution of the MLE temperature parameters estimator $\hat{\Theta}$ is approximately normal. The idea is that the model used for temperature departs from an ARMA only by deterministic terms (trend, seasonality, seasonal volatility). Therefore the asymptotic normality satisfied by the maximum likelihood estimator of an ARMA process (see for example Gourieroux, Montfort, 1997, §9.2.F.) may be verified as well. To check this assumption, we applied statistical tests of normality to a sample of size $R = 10.000$ of MLE estimations obtained by resampling in the Gaussian distribution of residuals (of the temperature model). We used the Kolmogorov test to check the normality of the marginal distributions, and skewness and kurtosis tests for the normality of the multidimensional distribution (see Lütkepohl, 1993, § 4.5., formula 4.5.4., 4.5.5. and 4.5.8.). The results shown in Table 5 prove that the distributional assumption for $\hat{\Theta}$ is satisfactory (nearly all marginal distributions pass the normality test, p-values are correct for the kurtosis test; the little weak p-value of the skewness test may then rather reveal the slow rate of convergence to the asymptotic distribution than a non-normality). More precisely, we will assume that (as for a pure ARMA process),

$$\sqrt{n}\left(\hat{\Theta} - \Theta\right) \xrightarrow[n \to +\infty]{} N(0; \Gamma)$$

where Θ is the “true” vector of parameters, and Γ is the asymptotic covariance matrix of $\hat{\Theta}$. Then, the *delta-method* consists of using a first-order Taylor expansion (Campbell, Lo, and MacKinlay, 1997, section A.4. of the appendix) to derive an asymptotic pivotal distribution of $P(\Theta)$:

$$\sqrt{n}\left(P(\hat{\Theta}) - P(\Theta)\right) \xrightarrow[n \to +\infty]{} N\left(0; \frac{\partial P}{\partial \Theta'} \Gamma \frac{\partial P}{\partial \Theta}\right)$$

If derivatives $\frac{\partial P}{\partial \Theta}$ can be evaluated, then price uncertainty can be assessed as a function of the standard deviation of the asymptotic normal distribution of $P(\hat{\Theta})$.

The delta-method methodology can be summarized as follows. For $r = 1,...,R$ do :

1'. Simulate independently $\varepsilon_1^*,...,\varepsilon_n^*$ from the estimated Gaussian distribution of residuals

2. Reconstitute the corresponding temperature path $x_1^*,...,x_n^*$ using the temperature model with the initial parameters (maximum likelihood estimation based on the data)

3. Calculate the maximum likelihood estimate Θ^{*r} from the new data $x_1^*,...,x_n^*$

4'. Calculate the empirical covariance matrix $\Gamma(\Theta)$ to estimate the asymptotic normal distribution of Θ^*

5'. [Delta-method] deduce the asymptotic normal distribution of $P(\Theta^*)$, and calculate the 95% confidence interval of $P(\Theta)$ based on the standard deviation of the distribution of $P(\Theta^*)$

Note that there is no bootstrap here since resampling is made in the estimated Gaussian distribution of residuals. Indeed, the property of asymptotic normality may not be shared for any distribution assumption. In addition, the derivatives $\frac{\partial P}{\partial \Theta}$ can then be obtained in closed-form (this is obvious for LIFFE futures; we report to appendix for the CME case).

In the LIFFE case, P is a linear function of Θ. Therefore, the delta-method will give identical results as if one had assumed that $\hat{\Theta}$ were exactly normally distributed, the first-order expansion being equality for linear functions. Thus, as departures from normality of the temperature residuals are not too important, we expect to obtain similar results as with the bootstrap methodology. In the CME case, P is a non-linear function of Θ, and the differences between the two methods may appear in a more striking manner. The results obtained with the delta-method confirm these insights (Table 4), but also show that there are very few differences between the results of the two methods.

7. Conclusion

We studied the impact of temperature estimation error on prices of some standard futures contracts of the LIFFE and CME exchanges. By considering the price as a random variable function of the temperature parameters, price uncertainty was assessed in two manners. By extreme quantiles, first, using a bootstrap technique; and by the standard deviation of a normal asymptotic distribution, using the delta-method. Both methodologies lead to the conclusions that price uncertainty is not negligible in general, and may be important for some months. For LIFFE contracts, and, at least, for CME winter contracts (where the HDD index is approximately linear), errors are coming exclusively from the estimation errors made on the mean of the temperature process. This suggests that the modelling of trend and seasonality in temperature is a key feature of the valuation of weather derivatives.

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REFERENCES

BARRIEU P., EL KAROUI N. (2002). Optimal design of derivatives in illiquid markets. *Quantitative Finance*, **2**, 181-188.

BROCKWELL, P.J., DAVIS, R.A.(1991). *Time Series: Theory and Methods.* Springer Verlag.

BRODY D.C., SYROKA, J., ZERVOS, M. (2001). Dynamical Pricing of Weather Derivatives. Working Paper, Imperial College, London. Submitted to: Quantitative Finance.

BÜHLMANN H. (1996). *Mathematical methods in risk theory*. Springer-Verlag.

BÜHLMANN P. (2000). Bootstrap for Time Series. Seminar für Statistik, ETH Zürich. Revised Version.

CAIRNS A.J.G. (2000). A discussion of parameter and model uncertainty in insurance. *Insurance: Mathematics and Economics* **27**, 313-330.

CAMPBELL J.Y., LO A.W., MACKINLAY A.C. (1997). *The Econometrics of Financial Markets*. Princeton University Press.

CAMPBELL S., DIEBOLD F.X. (2001). Weather Forecasting for Weather Derivatives. Working Paper, University of Pennsylvania, Philadelphia.

CAO M., WEI J. (2000). Pricing Weather Derivatives: an Equilibrium Approach. Working Paper, Queen's University Kingston and University of Toronto, Ontario, Canada.

CARR P., GEMAN H., MADAN D. (1999). Pricing and Hedging in Incomplete Markets. Working paper.

DAVIS M. (2001). Pricing weather derivatives by marginal value. *Quantitative Finance*, **1**,1-4.

DAVINSON A.C., HINKLEY D.V. (1997). *Bootstrap Methods and their Applications*. Cambridge University Press.

DENNEBERG D. (1990). Premium calculation: why standard deviation should be replaced by absolute deviation. *Astin Bulletin*, **20**, 181-190.

DISCHEL R. (1998). The fledgling weather market takes off, Part. 2. *Applied Derivatives Trading*.

DORNIER F., QUERUEL M. (2000). Caution to the Wind. *Risk, Energy and Power Risk Management*, August.

DUFFIE D. (2001). *Dynamic asset pricing theory*. Princeton University Press.

EFRON B., TIBSHIRANI R. (1986). Bootstrap Methods for Standard Errors, Confidence Intervals, and Other Measures of Accuracy. *Journal of Statistical Science* **1**(1), 54-77.

GOOVAERTS M.J., DEVYLDER F., HAEZENDONCK J. (1984). *Insurance Premiums*. Elsevier.

GOURIEROUX C., MONTFORT A. (1997). *Time Series and Dynamic Models*. Cambridge University Press.

HÜRLIMANN W. (2001). Distribution free comparison of pricing principles. *Insurance: Mathematics and Economics*, **28**, 351-360.

LÜTKEPOHL H. (1993). *Introduction to Multiple Time Series Analysis.* Springer Verlag.

MOLLER T. (2001). On transformations of actuarial valuation principles. *Insurance: Mathematics and Economics*, **28**, 281-303.

MOLLER T. (2003). Indifference pricing of insurance contracts in a product space model: applications. *Insurance: Mathematics and Economics*, **32**, 295-315.

MOLLER T. (2003). Indifference pricing of insurance contracts in a product space model. *Finance and Stochastics*, **7**, 197-217.

MORENO M. (2000). Riding the Temp. *Weather Derivatives, FOW Special Supplement*, December.

MORENO M., ROUSTANT O., (2002). *Modélisation des températures,* in Blondeau J., Partrat C., *La réassurance : approche technique*, Economica, 2003.

MUSIELA M., ZARIPHOPOULOU T. (2001). Pricing and Risk Management of Derivatives Written on Non-Traded Assets. Working paper.

ROUSTANT O. (2002). Une application de deux modèles économétriques de température à la gestion des risques climatiques, 1ère partie. *Banque & Marchés*, **58**, 22-29.

ROUSTANT O. (2002). Une application de deux modèles économétriques de température à la gestion des risques climatiques, 2ème partie. *Banque & Marchés*, **59**, 36-44.

SCHWEIZER M. (2001). From actuarial to financial valuation principles, *Insurance : Mathematics and Economics*, **28**, 31-47.

APPENDIX

Analytical expressions of the derivatives of HDD and CDD futures prices

Let us consider, for example, the case of future contracts based on HDD. With the notation $L_t = \dfrac{K-(m_t+s_t)}{\sigma_t}$, the corresponding prices given by (Cao et Wei, 2000) can be expressed as:

$$P(\Theta)=\sum_{t=t_1}^{t_2} f_t = \sum_{t=t_1}^{t_2} \sigma_t \left(L_t N(L_t) + \frac{1}{\sqrt{2\pi}} \exp\left(-L_t^{\,2}/2\right)\right)$$

where $N(.)$ is the cumulative density function of the $N(0;1)$ distribution.

Denote $\Theta=(\theta_1,...,\theta_K)$, and $\dfrac{\partial P}{\partial \Theta}=\left(\dfrac{\partial P}{\partial \theta_1},...,\dfrac{\partial P}{\partial \theta_K}\right)'$ the derivatives on the vectorial form.

An immediate calculation then gives $\dfrac{\partial f_t}{\partial \theta_k} = -N(L_t)\dfrac{\partial (m_t+s_t)}{\partial \theta_k}$ if θ_k is relative to the process mean m_t+s_t, $\dfrac{\partial f_t}{\partial \theta_k}=\dfrac{1}{\sqrt{2\pi}}\exp\left(-L_t^{\,2}/2\right)\dfrac{\partial \sigma_t}{\partial \theta_k}$ if θ_k is relative to the process volatility σ_t and $\dfrac{\partial f_t}{\partial \theta_k}=0$ otherwise. Therefore, we obtain:

$$\frac{\partial P}{\partial \Theta}=\left(-\left[N\left(L_{t_1}\right),...,N\left(L_{t_2}\right)\right]\times M_\mu \quad \frac{1}{\sqrt{2\pi}}\left[\exp\left(-L_{t_1}^{\,2}/2\right),...,\exp\left(-L_{t_2}^{\,2}/2\right)\right]\times M_\sigma \quad 0_p\right)'$$

where M_μ and M_σ are matrices relative to the (linear) process mean and process volatility:

$$M_\mu = \begin{pmatrix} t_1 & 1 & \cos(\omega t_1) & \sin(\omega t_1) & ... & \cos(N_f \omega t_1) & \sin(N_f \omega t_1) \\ | & | & | & | & ... & | & | \\ t_2 & 1 & \cos(\omega t_2) & \sin(\omega t_2) & ... & \cos(N_f \omega t_2) & \sin(N_f \omega t_2) \end{pmatrix}, \quad M_\sigma = \begin{pmatrix} 1 & \cos(\omega t_1) & \sin(\omega t_1) \\ | & | & | \\ 1 & \cos(\omega t_1) & \sin(\omega t_2) \end{pmatrix}$$

Similarly, the derivatives of futures prices on CDD are:

$$\frac{\partial P}{\partial \Theta}=\left(\left[N\left(-L_{t_1}\right),...,N\left(-L_{t_2}\right)\right]\times M_\mu \quad -\frac{1}{\sqrt{2\pi}}\left[\exp\left(-L_{t_1}^{\,2}/2\right),...,\exp\left(-L_{t_2}^{\,2}/2\right)\right]\times M_\sigma \quad 0_p\right)'$$

Figure 1 – Diagnostic checking for the temperature model.

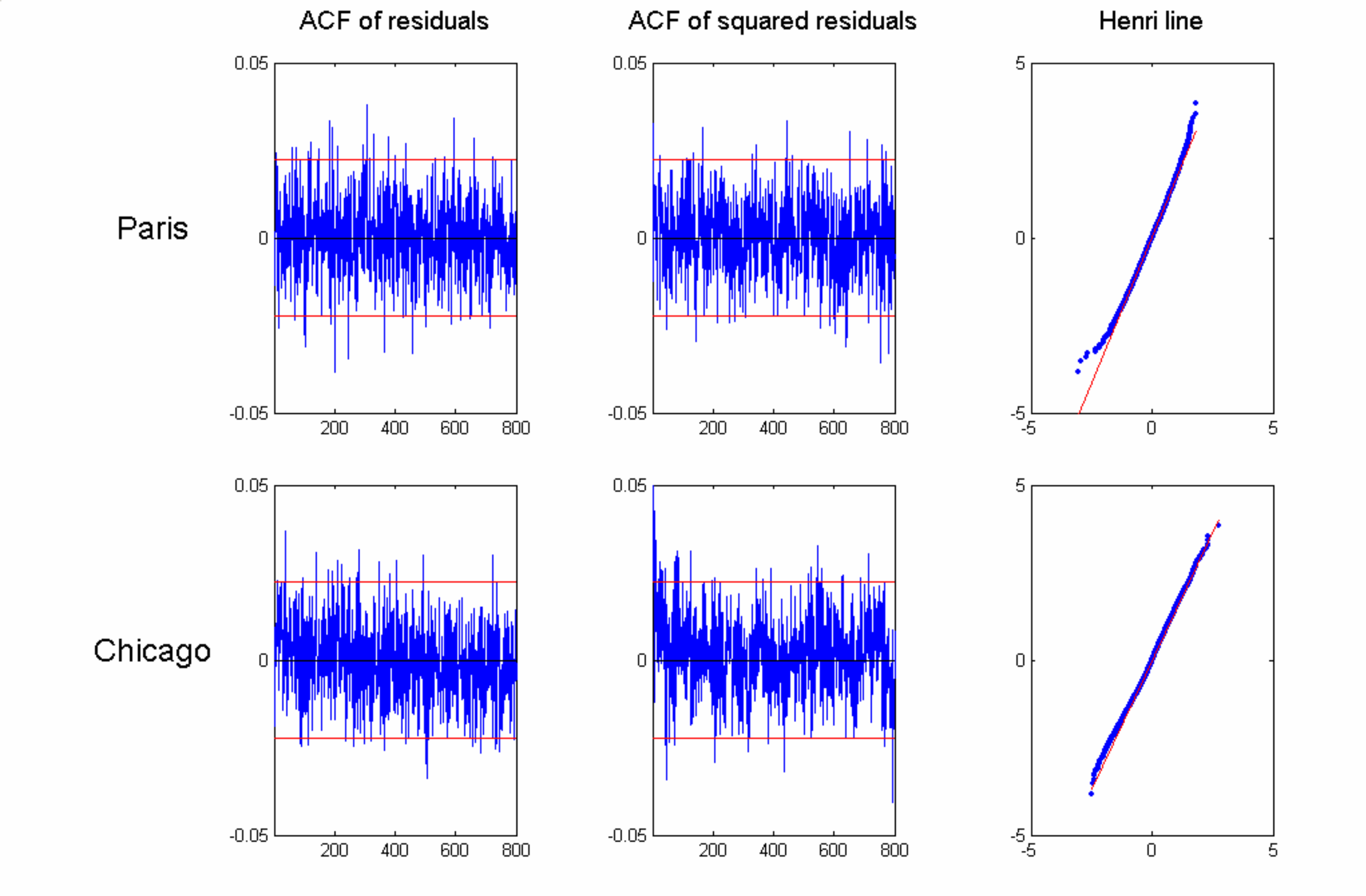

Table 1 – Contract specifications on the CME and LIFFE Exchanges.

	CME	**LIFFE-Euronext**
Temperature index	HDD, CDD	100 + Average temperature
Length of the risk exposure period	1 month	a) 1 month b) Winter season (01/11-31/03)
Location	Atlanta, Chicago, Cincinnati, Dallas, Des Moines, Las Vegas, New York, Philadelphia, Portland, Tucson.	London, Paris, Berlin
Contract type	Futures and options	Futures
Maturity	1 to 12 months	a) 1 to12 months b) 1 to 2 consecutive seasons
Contract value	1°F = 100$	1°C = 3.000€ (£ for London)

Table 2 – Assessment of future price and future price uncertainty by bootstrap. Station = Paris-Orly.

	Theoretical future price	Future price median	Future price std. dev.	Future price 95% confidence interval	Relative 95% confidence interval (%)
	$P(\Theta_0)$	$F_{\text{bootstrap}}$		$I_{\text{bootstrap}}$	$(I_{\text{bootstrap}} / F_{\text{bootstrap}} - 1) \times 100$
			Weather futures – Monthly indices		
January	104.55	104.55	0.30	[103.96 ; 105.14]	[-12.8 ; 13.0]
February	105.67	105.67	0.30	[105.08 ; 106.26]	[-10.4 ; 10.4]
Mars	107.96	107.96	0.30	[107.37 ; 108.54]	[-7.4 ; 7.3]
April	111.09	111.09	0.29	[110.51 ; 111.66]	[-5.2 ; 5.1]
May	114.70	114.70	0.29	[114.13 ; 115.26]	[-3.9 ; 3.8]
June	118.15	118.16	0.28	[117.59 ; 118.72]	[-3.1 ; 3.1]
July	120.34	120.34	0.29	[119.78 ; 120.91]	[-2.8 ; 2.8]
August	120.14	120.14	0.29	[119.58 ; 120.72]	[-2.8 ; 2.9]
September	117.26	117.26	0.29	[116.70 ; 117.84]	[-3.3 ; 3.4]
October	112.68	112.68	0.30	[112.10 ; 113.27]	[-4.6 ; 4.7]
November	108.17	108.18	0.31	[107.58 ; 108.77]	[-7.3 ; 7.3]
December	105.30	105.30	0.30	[104.70 ; 105.93]	[-11.3 ; 11.8]
			Weather futures – Winter season index		
1^{st} Nov – 31^{st} March	106.37	106.37	0.26	[105.87 ; 106.89]	[-7.9 ; 8.1]

Table 3 – Assessment of future price and future price uncertainty by bootstrap. Station = O'Hare Airport.

	Theoretical future price $P(\Theta_0)$	Future price median $F_{\text{bootstrap}}$	Future price std. dev.	Future price 95% confidence interval $I_{\text{bootstrap}}$	Relative 95% confidence interval (%) $(I_{\text{bootstrap}} / F_{\text{bootstrap}} - 1) \times 100$
			CDD season		
May	53.56	52.77	5.37	[42.7 ; 63.8]	[-19.1 ; 20.9]
June	195.22	196.06	11.95	[172.4 ; 219.5]	[-12.1 ; 12.0]
July	314.90	316.04	14.50	[286.9 ; 344.1]	[-9.2 ; 8.9]
August	256.14	257.09	13.54	[230.8 ; 283.8]	[-10.3 ; 10.4]
September	90.77	90.32	7.68	[75.9 ; 106.2]	[-16.0 ; 17.6]
			HDD season		
October	397.87	396.90	15.36	[367.1 ; 427.5]	[-7.5 ; 7.7]
November	752.31	752.03	16.87	[719.4 ; 785.5]	[-4.4 ; 4.5]
December	1079.00	1079.05	18.27	[1043.2 ; 1114.8]	[-3.4 ; 3.4]
January	1215.41	1215.35	18.12	[1179.4 ; 1250.5]	[-3.0 ; 2.9]
February	1036.39	1036.22	16.58	[1003.5 ; 1068.3]	[-3.2 ; 3.1]
Mars	901.86	901.42	18.12	[865.7 ; 936.9]	[-4.0 ; 4.0]
April	518.69	517.91	16.40	[486.2 ; 550.9]	[-6.2 ; 6.4]

Table 4 – Assessment of the future price uncertainty: comparison of the bootstrap and the delta-method methodologies.

	Price uncertainty (%) (bootstrap)	Price uncertainty (%) (delta-method)	Price uncertainty (%) (bootstrap)	Price uncertainty (%) (delta-method)
	LIFFE Weather futures *station: Paris-Orly*		CME Weather futures *station: O'Hare Airport*	
May	[-3.9 ; 3.8]	± 3.9	[-19.1 ; 20.9]	± 19.7
June	[-3.1 ; 3.1]	± 3.1	[-12.1 ; 12.0]	± 12.0
July	[-2.8 ; 2.8]	± 2.7	[-9.2 ; 8.9]	± 9.1
August	[-2.8 ; 2.9]	± 2.8	[-10.3 ; 10.4]	± 10.4
September	[-3.3 ; 3.4]	± 3.3	[-16.0 ; 17.6]	± 16.6
October	[-4.6 ; 4.7]	± 4.6	[-7.5 ; 7.7]	± 7.6
November	[-7.3 ; 7.3]	± 7.3	[-4.4 ; 4.5]	± 4.5
December	[-11.3 ; 11.8]	± 11.4	[-3.4 ; 3.4]	± 3.4
January	[-12.8 ; 13.0]	± 13.1	[-3.0 ; 2.9]	± 2.9
February	[-10.4 ; 10.4]	± 10.5	[-3.2 ; 3.1]	± 3.1
Mars	[-7.4 ; 7.3]	± 7.3	[-4.0 ; 4.0]	± 3.9
April	[-5.2 ; 5.1]	± 5.2	[-6.2 ; 6.4]	± 6.1
1st Nov – 31st March	[-7.9 ; 8.1]	± 7.9		

Table 5 – Normality testing of the MLE estimator distribution

	Paris	Chicago
Marginal distributions	Kolmogorov statistic	
$\hat{d}$	.0060	.0068
$\hat{e}$	.0059	.0066
$\hat{a}_1$	.0046	.0054
$\hat{b}_1$	.0042	.0049
$\hat{a}_2$	.0047	-
$\hat{b}_2$	.0060	-
$\hat{a}$	.0081	.0045
$\hat{b}$	.0040	.0094*
$\hat{c}$	.0043	.0066
$\hat{\phi}_1$	.0059	.0080
$\hat{\phi}_2$	.0084	.0089
$\hat{\phi}_3$	.0035	.0055
Multidimensional distribution	p-value	
Skewness test	.0251*	.0538
Kurtosis test	.4684	.4192
Joint test	.0678	.1022

For kolmogorov test, at 5% confidence level, normality is rejected here when the statistic value is superior to 0.00895. Stars indicate the cases of rejection at level 5%.