

A novel Monte-Carlo approach to particle-field dynamics

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Motivated by the problem to identify possible observables suitable for the exploration of the phase diagram of strongly interaction matter in heavy-ion collisions, we have developed a novel algorithm to address the non-equilibrium kinetics of phase transitions of the bulk medium created in such reactions. As known from lattice-QCD calculations at low net-baryon densities and high temperatures, the deconfinement-confinement as well as the chiral phase transition is a smooth cross-over transition. At higher net-baryon numbers (finite baryon chemical potential) one expects a 1st-order transition line with a tri-critical 2nd-order endpoint. In heavy-ion collisions, however the produced medium is a rapidly expanding fireball consisting of strongly interacting matter. Thus the question arises, whether possible signatures of equilibrium phase transitions, including critical fluctuations of conserved charge-like quantum numbers and critical slowing-down close to a 2nd-order endpoint can be expected and, which are the pertinent signatures for such structures in the phase diagram.

The key idea of our novel Monte-Carlo simulation method [1] is to use transition rates of such creation and annihilation processes, $q\bar{q} \leftrightarrow \sigma$ given by the Feynman rules of a quark-meson linear- σ model. The quarks and antiquarks are described as test particles of a classical molecular-dynamics simulation, while the σ field is treated on the classical mean-field level. Monte-Carlo techniques are used to simulate the pair creation and annihilation processes according to these transition-probability rates on a space-time grid.

For a particle annihilation, the corresponding energy and momentum are then transferred to the corresponding mean σ -field as local disturbances in terms of Gaussian wave “packlets”. To take into account the inverse process of pair creation as demanded by the principle of detailed balance, the mean field is mapped locally to a momentum distribution with a local temperature, allowing for the “particlelization” of the field. In this way in another Monte-Carlo step a σ -particle with some definite energy and momentum can be chosen according to this local equilibrium distribution, and the decay-probability rate to a $q\bar{q}$ pair can be used to add the corresponding quark and anti-quark to the test-particle ensemble. Note that in addition the relevant net-quark number conservation is fulfilled in the sense of a microcanonical description.

Besides other simple test cases like a simple harmonic oscillator or a 1+1-dimensional Klein-Gordon field coupled to a fluctuating heat bath, we have demonstrated that with such a description the particles reach thermal equilibrium, and the mean field develops the corresponding

equilibrium spectrum, preventing the ultraviolet catastrophe due to an effective cutoff due to the finite spatial extent of the interactions with the particles (cf. Fig. 1).

Applications to descriptions of the chiral phase transition in heavy ion collisions are current work of progress.

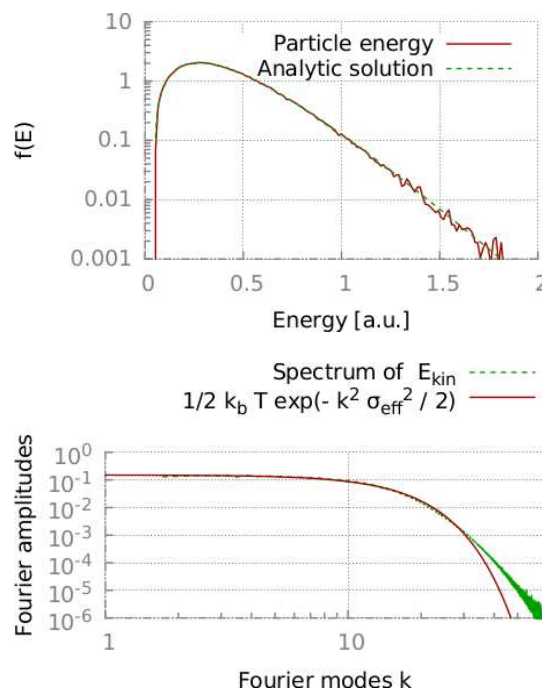


Figure 1: Upper panel: The quark-energy distribution from the kinetic Monte-Carlo simulation after a stationary state has been reached compared to the expected equilibrium-Boltzmann distribution $\propto \exp(-E/T)$. Lower panel: The Fourier spectrum of the mean field in comparison to the classical equipartition theorem, modified by an effective ultra-violet cutoff due to the finite extent of the interaction volume between particle creation and annihilation processes.

References

- [1] C. Wesp, H. van Hees, A. Meistrenko, and C. Greiner (2014), [arXiv: 1411.7979](https://arxiv.org/abs/1411.7979) [hep-ph].