Spectral functions from the Functional Renormalization Group*

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We present a method to obtain spectral functions at finite temperature and chemical potential from the Functional Renormalization Group (FRG) approach. Our non-perturbative method is thermodynamically consistent, symmetry preserving and based on an analytic continuation from imaginary to real time on the level of the flow equations for two-point functions. In order to demonstrate the feasibility of our method, we apply it to the quark-meson model and present results for the pion and sigma spectral function at finite temperature and density.

The calculation of real-time quantities like spectral functions or transport coefficients for strongly interacting matter is difficult and often hampered by the analytic continuation problem. This problem arises for example in lattice QCD, where discrete numerical data has to be used to reconstruct real-time correlation functions. Within our new approach for the Functional Renormalization Group (FRG), the analytic continuation can be performed on the level of the flow equations, without the need for any numerical reconstruction technique.

In particular, the flow equations for the retarded 2-point functions are obtained from their Euclidean counterparts via the following two-step procedure. First, the periodicity of the bosonic and fermionic occupation numbers with respect to the discrete Euclidean energy $p_0 = 2n\pi T$ is used, i.e.

$$n_{B,F}(E + ip_0) \to n_{B,F}(E).$$

As a second step, the Euclidean energy p_0 is replaced by a continuous real frequency ω as follows,

$$\Gamma^{(2),R}(\omega, \vec{p}) = -\lim_{\epsilon\to 0}\Gamma^{(2),E}(p_0 = -i(\omega + i\epsilon), \vec{p}).$$

The spectral functions are then given by the imaginary part of the retarded propagator, which is given by the inverse of the retarded two-point function $\Gamma^{(2),R}(\omega, \vec{p})$.

In Fig. 1 we show the pion spectral function at $T = 100$ MeV and vanishing chemical potential. Due to thermal processes, the pion is unstable at this temperature, as represented by a peak with finite width, that is Lorentz-boosted to higher energies as the spatial momentum increases. Decay channels at higher energies and additional processes at space-like 4-momenta give rise to non-zero values of the spectral function away from the pion peak.

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In Fig. 2 we show the sigma spectral function for different spatial momenta near the critical endpoint. We observe that the sigma meson is stable and exhibits an almost vanishing mass near this second order phase transition, as expected.With increasing spatial momenta, the sigma peak is Lorentz-boosted to higher energies and space-like processes modify the spectral function at $\omega < |\vec{p}|$.

As an outlook, we note that our approach also allows to calculate transport coefficients like the shear viscosity via Green-Kubo formulas. Moreover, the calculation of quark spectral functions and the inclusion of vector and axial-vector mesons represent interesting extensions.

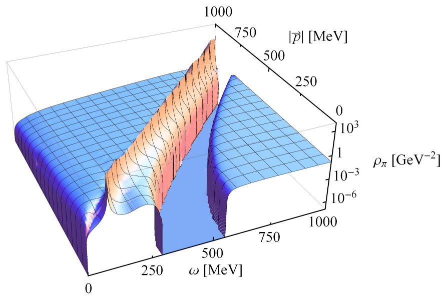

Figure 1: The pion spectral function ρ_π is shown vs. energy ω and spatial momentum $\vec{p}$ at $T = 100$ MeV and $\mu = 0$.

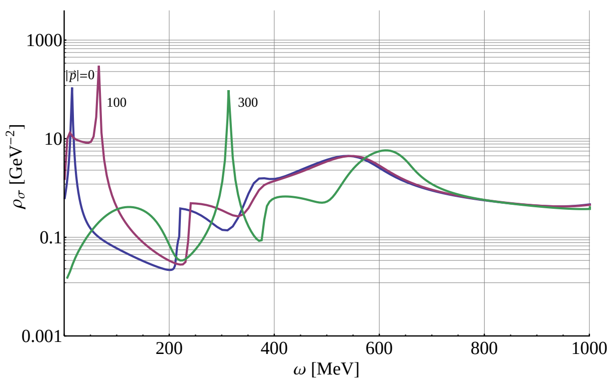

Figure 2: The sigma spectral function is shown vs. energy ω at $T = 10$ MeV and $\mu = 292.97$ MeV for different spatial momenta $\vec{p}$, as indicated by inset labels.

References

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