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Ghost Dark Energy with Variable Gravitational Constant

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ABSTRACT

In a bid to resolve lingering problems in modern Cosmology, Cosmologist are turning to models in which Physical constants of nature are not real constants but vary as function of the scale factor of the universe. In this work, a cosmological model is developed by incorporating Ghost Dark Energy model with Barrow's ansatz for varying Gravitational Constant. A model obtained is free from initial Big Bang Singularity and Inflation Problem. A test of Causality showed that the model depends of the constant m and is classically stable at $-6 \geq m > -3$

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1. INTRODUCTION

In physical cosmology and astronomy, dark energy is an unknown form of energy that affects the universe on the largest scales. The first observational evidence for its existence came from measurements of supernovae, which showed that the universe does not expand at a constant rate; rather, the expansion of the universe is accelerating [1]. Understanding the evolution of the universe requires knowledge of its starting conditions and its composition. Before these observations, it was thought that all forms of matter and energy in the universe would only cause the expansion to slow down over time. Measurements of the cosmic microwave background suggest the universe began in a hot Big Bang, from which general relativity explains its evolution and the subsequent large-scale motion. Since the 1990s, dark energy has been the most accepted premise to account for the accelerated expansion. Assuming that the lambda cold dark matter (Λ-CDM) model of cosmology is correct, the best current measurements indicate that dark energy contributes 68% of the total energy in the present-day observable universe [2]. The ΛCDM model is a parameterization of the Big Bang cosmological model in which the universe contains three major components: first, a cosmological constant denoted by Lambda (Greek Λ) associated with dark energy; second, the postulated cold dark matter (CDM); and third, ordinary matter. It is frequently referred to as the standard model of Big Bang cosmology because it is the simplest model that provides a reasonably good account of the following properties of the cosmos: the existence and structure of the cosmic microwave background, the large-scale structure in the distribution of Galaxies, the observed abundances of hydrogen (including deuterium), helium, and lithium, the accelerating expansion of the universe observed in the light from distant galaxies and supernovae.

The simplest model of DE is the cosmological constant, which is a key ingredient in the ΛCDM model. Although the ΛCDM model is consistent very well with all observational data, it faces the fine-tuning problem

[3]. Plenty of other DE models have also been proposed [4 – 7], but almost all of them explain the acceleration expansion either by introducing new degree(s) of freedom or by modifying gravity. Recently a new DE model, the so-called Veneziano ghost DE, has been proposed [8]. The key ingredient of this new model is that the Veneziano ghost, which is unphysical in the usual Minkowski space-time, exhibits important physical effects in dynamical space-time or space-time with non-trivial topology. Veneziano ghost is supposed to exist for solving the problem in low-energy effective theory of Quantum Chromodynamics [9 – 11]. The ghost has no contribution to the vacuum energy density in Minkowski space-time, but incurved space-time it gives rise to a small vacuum energy density.

A dynamical dark energy model, known as Veneziano ghost dark energy (GDE) has been introduced in the late 70s by Veneziano [10]. This has significant non-trivial physical properties for the expanding universe or in space-time having nontrivial topological formation. The existence of Veneziano GDE is supposed to resolve the coincidence problem [11]. The GDE has little contribution to the vacuum energy density in curved space-time.

According to [12], observational data relating the luminosity-redshift from type Ia supernovae is now at range $z\sim1$. The varying speed of light theory is geared towards explaining: hard breaking the Lorentz invariance, bimetric theories (with postulates that the speeds of gravity and light are not the same), locally Lorentz invariance. By varying the speed of light, the principle of general covariance is violated and the laws of Physics are now valid only in some special frames. Varying Speed of Light (VSL) theory has successfully handled some fundamental contention in cosmology such as flatness, horizon, and Lambda problem.

The Newtonian gravitational constant G occurs in the source term of Einstein's field equation of the general theory of relativity, which is a fundamental equation for developing every model of cosmology. In Einstein's field equation, G acts as a coupling constant between the geometry of spacetime and matter. In quantum mechanics, G is essential in the definition of the Planck constant [13]. While in SCM, G is an invariant quantity. It has been noted that there is significant evidence that the gravitational constant G can vary in time [14]. When either G or the cosmological constant Λ is varied with time, the Einstein field equation is still preserved. It has been shown that the variable Newtonian gravitation constant can account for dark energy and most of its effects, and current dynamical dark energy models using time-dependent cosmological constant terms are being considered [15].

The structure of this paper is as follows: In section I, we gave a brief introduction, section II contents the ghost dark energy model. The evolution of the Universe studied in section III. We tested the classical causality in section IV and conclude in section V.

2. Ghost Dark Energy Model

In this work, we shall use the action of variable speed of light given by:

$$s = \int dx^4 \sqrt{-g}\,(\frac{R+\lambda}{16\pi G} + L_m) \tag{1}$$

Where λ= cosmological constant which serves as vacuum energy, L_m= matter Lagrange density, R = Ricci scalar, G = Newton's gravitational constants, and g = The determinant of the metric tensor. Varying the action for the metric and ignoring surface terms leads to

$$G_{\mu v} - g_{\mu v} = \frac{8\pi G T_{\mu v}}{C^4} \tag{2}$$

Where $G_{\mu v=}$ = Einstein tensor $T_{\mu v}$= Energy momentum tensor. The Greek indices μ *and* v run from 0 – 3. In the cosmological context, the Friedmann Robertson Walker metric is:

$$dS^2 = -c^2 dt^2 + a(t)^2 \left[\frac{dr^2}{1+Kr^2} + r^2 dn\right] \tag{3}$$

Where $dn = d\theta^2 + sin^2\theta d\emptyset^2$

$a(t) = scalefactor\ of theuniverse$t is the comoving time and K= 0, 1, -1 representing flat, closed and open universe respectively. The Friedmann equations are;

$$H^2 = \left(\dot{a}/a\right)^2 = \frac{8\pi G(t)}{3}\rho_{GDE} \tag{4}$$

$$\ddot{a}/a = \frac{4\pi G(t)}{3}\,(\rho_{GDE} + 3P_\lambda) \tag{5}$$

Here we will consider the energy density of ghost dark energy given by:

$$\rho_{GDE=\alpha H} \tag{6}$$

And Chaplygin gas equation of state (EoS)

$$P_{\lambda=\frac{-B}{\rho_{GDE}}} \tag{7}$$

We shall use the conservation equation given by

$$\dot{\rho}_{GDE=}\,{}^{3\dot{a}}/_{a}(\rho_{GDE}+P_{\lambda})-\rho_{GDE}{}^{\dot{G}}/_{G}+\frac{3K}{4\pi c^2a^2} \tag{8}$$

For flat FRW universe ($k = 0$). The conservation equation can be rewritten as

$$\dot{\rho}_{GDE} + {}^{3\dot{a}}/_{a}\,(\rho_{GDE} + p_{\lambda}) + \rho_{GDE}\frac{\dot{G}}{G} = 0 \tag{9}$$

For varying$G$, we shall assume the Barrow ansatz [16] given by

$$G = G_0 a^m \tag{10}$$

We shall use this to solve for the scale factor and use the scale factor to determine physical properties of cosmic importance such as deceleration parameter equation of state parameter e.t.c

I. Evolution of the Scale Factor

For this model, the scale factor of the universe for a flat space-time is computed using the conservation given by

$$\dot{\rho}_{GDE} + {}^{3\dot{a}}/_{a}\,(\rho_{GDE} + p_{\lambda}) + \rho_{GDE}\frac{\dot{G}}{G} = 0 \tag{11}$$

Taking the time derivative of equation (10) and substituting it into Equation (11) we obtain:

$$\frac{\rho_0 m\left(\frac{d}{dt}a(t)\right)}{a(t)^4} = 0 \tag{12}$$

Where $\rho_{GDE} = \frac{\rho_0}{a(t)^3}$ and $\rho_{GDE} = 0$ for dust particles. The solution to (12) is

$$a(t) = C_1 \tag{13}$$

C_1 is a constant, indicating an empty universe.
Now applying the energy density for the Ghost Dark Energy model (6) and the Chaplygin gas E0S (7) into (11) we obtained a differential equation of the form:

$$\frac{\alpha\left(\frac{d^2}{dt^2}a(t)\right)}{a(t)} - \frac{\alpha\left(\frac{d}{dt}a(t)\right)^2}{a(t)^2} + \frac{3\left(\frac{d}{dt}a(t)\right)\left(\frac{\alpha\left(\frac{d}{dt}a(t)\right)}{a(t)} - \frac{B\,a(t)}{\alpha\left(\frac{d}{dt}a(t)\right)}\right)}{a(t)} + \frac{\alpha\left(\frac{d}{dt}a(t)\right)^2 m}{a(t)^2} = 0 \tag{14}$$

The solution of the above equation is :

$$a(t) = \frac{12^{-\frac{1}{2(m+3)}} \left(\frac{\alpha^2 \left(e^{\frac{2\sqrt{3}\sqrt{B}\sqrt{m+3}\,t}{\alpha}} C \right)^2 (m+3)}{B} \right)^{\frac{1}{2(m+3)}}}{e^{\frac{\sqrt{3}\sqrt{B}\,t}{\alpha\sqrt{m+3}}}} \quad (15)$$

Where C is a constant. A simple case is when $B = 1, \alpha = 1, m = 1$ and $C = 1$, the scale factor becomes:

$$a(t) = \frac{1}{3} 3^{7/8} e^{\frac{1}{2}\sqrt{3}\,t} \quad (16)$$

The evolution of the scale factor is shown in figure 1 below. It is observed that the model is free from the initial Big Bang singularity and the universe ends in a big rip. The figure also shows that there was no inflation as alleged by the standard model

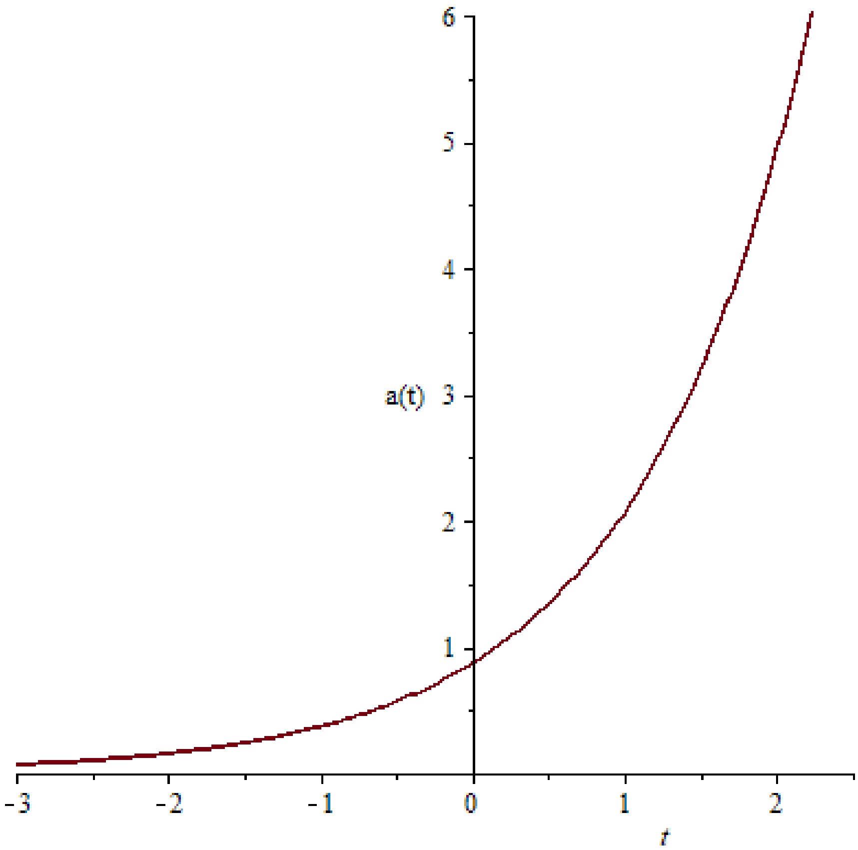

Figure 1. Evolution of the Scale factor of the Universe

The Hubble parameter is :

$$H = \frac{\dot{a}}{a} = \frac{\sqrt{3}\sqrt{B}}{\alpha\sqrt{m+3}} \quad (17)$$

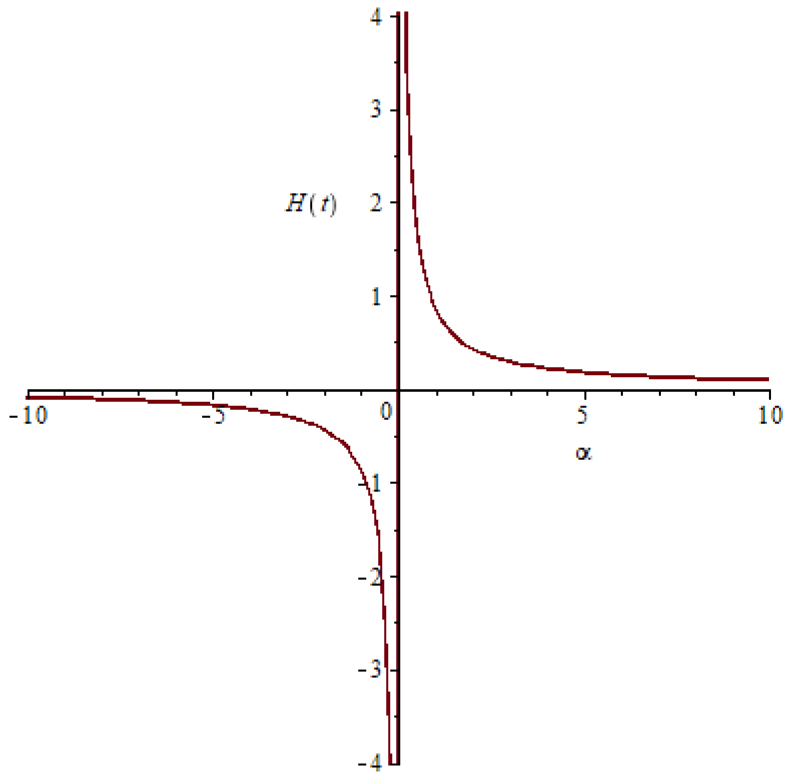

Figure 2. Variation of the Hubble Parameter

From figure 2, it is observed that the Hubble parameter diverges at the beginning and end of the universe. Such a universe starts with a positive deceleration parameter indicating deceleration expansion and transits to a negative deceleration parameter indicating accelerating expansion phase

$$\rho_{GDE} = \sqrt{3}\frac{\sqrt{B}}{\sqrt{m+3}} \tag{18}$$

The deceleration parameter is

$$q = -\frac{\ddot{a}}{a^2}a = -1 \tag{19}$$

The universe is decelerating when q >0 and accelerating when q <0. The model agrees with the recent accelerating expansion of the universe since q < 0.

3. Test of Causality

For any cosmological model to survive, it is established that the speed of sound cannot exceed the local speed of light, cs≤ 1. The second requirement for stability is that the square of the speed of sound must be positive, i.e., c2s>0. In case the model is classically we obtain:

$$c_2^2 = \frac{dP}{d\rho} = -\frac{1}{3}m - 1 \tag{20}$$

A requirement of the classical stability$c_s^2 > 0$ is $m > -3$ and the causality $c_s \leq 1$ is $m \leq -6$. Hence, the best value of m, for both stability conditions are $-6 \geq m > -3$

4. CONCLUSION

Cosmic models in which physical constants of nature are not constant but vary with time have been of interest in recent years. Indeed, the speed of light in our universe which has gone through various phase transition due to various content might not have been a constant, especially at the early stage. The Newtonian gravitational constant which acts as a coupling term in the Einstein Field Equation used here varies as a function of the scale factor of the universe to obtain a solution to the Friedmann equation that governs the dynamics of the universe from the very beginning, through various phase transitions to the present epoch.

The value of the scale factor obtained indicates that the size of the universe was never zero. By incorporating Ghost dark energy and variable gravitational constant, a model is presented free from the initial big bang singularity and inflation. The model further revealed that the Hubble parameter diverges at the beginning and end of the universe. Such a universe starts with a positive deceleration parameter indicating deceleration expansion and transits to a negative deceleration parameter indicating accelerating expansion phase. Furthermore, studying the stability of this model it is observed that it depends on the constant m and is classically stable at $-6 \geq m > -3$.

REFERENCES

[1] D. Overbye, "One cosmic question, too many answers," New York Times, pp. 2–9, 2003.
[2] P. J. E. Peebles and B. Ratra, "The cosmological constant and dark energy," Rev. Mod. Phys., vol. 75, no. 2, p. 559, 2003.
[3] D. H. Weinberg, R. Dave, N. Katz, and J. A. Kollmeier, "The Lyman-α Forest as a Cosmological Tool," in AIP Conference Proceedings, 2003, vol. 666, no. 1, pp. 157–169.
[4] R. A. Copeland, D. L. Pompliano, and T. D. Meek, "Drug–target residence time and its implications for lead optimization," Nat. Rev. Drug Discov., vol. 5, no. 9, pp. 730–739, 2006.
[5] E. Curiel, "Singularities and black holes," 2009.
[6] P. J. Steinhardt, L. Wang, and I. Zlatev, "Cosmological tracking solutions," Phys. Rev. D, vol. 59, no. 12, p. 123504, 1999.
[7] S. Capozziello, V. F. Cardone, S. Carloni, and A. Troisi, "Curvature quintessence matched with observational data," Int. J. Mod. Phys. D, vol. 12, no. 10, pp. 1969–1982, 2003.
[8] F. R. Urban and A. R. Zhitnitsky, "Cosmological constant from the ghost: a toy model," Phys. Rev. D, vol. 80, no. 6, p. 63001, 2009.
[9] E. Witten, "Cosmic separation of phases," Phys. Rev. D, vol. 30, no. 2, p. 272, 1984.
[10] C. Beisbart, "Can we justifiably assume the cosmological principle in order to break model underdetermination in cosmology?," J. Gen. Philos. Sci., vol. 40, no. 2, pp. 175–205, 2009.
[11] K. Kawarabayashi and N. Ohta, "On the partical conservation of the U (1) current," Prog. Theor. Phys., vol. 66, no. 5, pp. 1789–1802, 1981.
[12] A. K. Singha and U. Debnath, "Varying speed of light, modified Chaplygin gas and accelerating universe," Int. J. Mod. Phys. D, vol. 16, no. 01, pp. 117–122, 2007.
[13] L. Kantha, "A time-dependent and cosmological model consistent with cosmological constraints," Adv. Astron., vol. 2016, 2016.
[14] S. Nesseris and L. Perivolaropoulos, "Evolving Newton's constant, extended gravity theories, and SnIa data analysis," Phys. Rev. D, vol. 73, no. 10, p. 103511, 2006.
[15] R. Dungan and H. B. Prosper, "Varying-G cosmology with type Ia supernovae," Am. J. Phys., vol. 79, no. 1, pp. 57–62, 2011.
[16] B. Borah and M. Ansari, "Holographic Dark Energy Model with Time Varying G, as well as c 2 Parameter," Int. J. Theor. Phys., vol. 53, no. 4, pp. 1217–1224, 2014.