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Research Paper

The problem of production-distribution under uncertainty based on Vendor Managed Inventory

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ABSTRACT

In this paper, a problem of managed inventory by the vendor in the production-distribution supply chain is presented based on the scenario. The main purpose of presenting the model of maximizing producer profit in a three-level supply chain network consisting of various strategic and tactical decisions under uncertainty. Due to the nonlinearity and NP-Hardness of the problem, meta-heuristic genetic algorithms, Whale optimization algorithm and league champions algorithm have been used. The results of problem solving show the high efficiency of meta-heuristic algorithms compared to accurate methods in solving the above model. So that the maximum percentage of relative differences between the methods mentioned with GAMS is less than 1%.Also, by solving the sample problems in larger sizes, it was observed that the league champions algorithm has the highest efficiency in terms of achieving the optimal value of the target function in a shorter time than the other algorithms used, with a useful weight of 0.998.

1. Introduction

In any industry, the relationship between the manufacturer and the customers is most important. In the traditional inventory control system, the customer manages his inventory and the place of reprocessing his inventory by ordering it from the manufacturer. More recently, supply chain initiatives such as VMI have brought about changes in traditional customer-producer relationships (Pol & Inamdar, 2012). Salesman inventory management is one of the most popular partnership methods to increase supply chain efficiency, the principles of which were established in the late 1980s. Nowadays, VMI plays a very key and critical role in small and large supply chains that have the feature of "fast customer response".In this partnership, the supplier, who is sometimes the producer, vendor, or distributor, makes decisions about inventory control for the consumer (Chakraborty et al., 2015). Sales inventory management is a logistical strategy. The traditional supply chain is a system consisting of suppliers and customers that these members are connected to each

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other through the forward flow of materials and the reverse flow of information (Nozari et al., 2022). In the traditional supply chain, each actor is solely responsible for inventory control. The main challenge facing all members - retailers, distributors, manufacturers and suppliers - is how much the manufacturing system has to order to meet customer demand, which is the classic problem of controlling production and inventory. (Oláh et al., 2017- Nozari et al., 2022). Nowadays, VMI has become a tool that is widely used to improve supply chain performance. However, not all VMIs work successfully (Lee et al., 2015). Inventory is one of the most valuable assets for any company, but the results show that most companies fail to manage it effectively. The majority of manufacturers and distributors rely on outdated and very simple policies or inventory maintenance.

Vendor Managed Inventory (VMI) is a logistics strategy in which the supplier can control and manage its customers' inventory with the information provided by them. In today's competitive world, companies are increasingly looking to reduce costs and inventory levels to maximize profits, and VMI plays a key role in achieving these goals (Ghahremani et al., 2022). With the implementation of the vendor's VMI by the buyer organization, the decision is made to replenish the inventory. The difference between VMI and other models is that in the VMI system, the retailer, in addition to sharing accurate information about its inventory level and customer demand from the supplier, leaves all inventory management decisions to the supplier. The benefits of using VMIs for retailers include reducing overhead costs and transferring inventory costs to the supplier. The benefits of using a VMI for the supplier are not explicitly and directly generated, and by long-term use of a Vendor Managed Inventory system and linking shipments between the supplier and a number of retailers, inventory costs can be reduced. The stages of completing an order in a supply chain without / with VMI have some differences, which are discussed below.

1.1. Steps to complete ordering in a small supply chain without VMI

In the process of completing the order, the amount of sales is predicted using old data. The manufacturer then tracks inventory sales information (usually the amount of inventory in hand) and forecasts orders. Purchasing officials also look at the order information and place orders accordingly. The orders are then notified to the manufacturer, and the manufacturer looks at its current inventory and determines whether it can cover the order. If the vendor's inventory is sufficient, the product is transferred to the warehouse or retail store. The vendor then sends an invoice to the manufacturer. Upon receipt of the product, the manufacturer matches the invoice with the received items and pays the amount (Darvish and Oda, 2010).

1.2. Steps to complete ordering in a small supply chain despite VMI

In the process of meeting demand using VMI, forecasting activities and the creation of purchase orders are usually performed by the vendor or supplier and not by the retailer. Electronic data interchange (EDI) is an essential component of the VMI process and plays an essential role in the data communication process. The retailer sends inventory and sales information to the vendor via EDI or other B2B collaboration capabilities, and the supplier executes purchase orders based on inventory levels. In the VMI process, the retailer is free to anticipate and generate orders (quantities); Because the vendor determines the orders (quantities). The vendor is responsible for creating and maintaining an inventory program for the retailer. The vendor sends the invoices to the retailer or warehouse before sending the product. Soon after, the vendor sends the invoice to the retailer. By receiving the product, the retailer checks the validity of the invoice and controls the payments across their billing payment systems (Marquès et al., 2010). In a supply chain without VMI, the supplier sees only the goods requested by the customer and has no information about the customer's sales or inventory level, while in a supply chain with VMI, the supplier observes the customer's actual inventory and can supply at any time. Commodities put the customer's inventory at a minimum and maximum inventory. The supplier sends the goods to the customer at any time when inventory is needed, taking into account the supply time, and thus keeps the inventory level lower.

Considering the above-mentioned issues regarding VMI, it can be said that the costs in such a system are much lower than traditional systems and have been considered by managers and companies. Therefore, in this paper, an inventory management model is designed by the vendor in a three-level supply-chain supply-production network based on a scenario. Due to the importance of this issue, the producer's profit has been maximized. Due to the NP-Hard nature of the inventory management problem model by the vendor, this article uses meta-innovative algorithms such as (genetic algorithm, whale optimization algorithm and league champions algorithm) with a very efficient initial answer to obtain near-optimal results. The structure of the article is as follows, in the second part, the research literature is reviewed and the research gap is identified. The third section defines the problem and mathematical modeling of the problem. In the fourth section, the initial solution used to solve the problem is shown and the parameterization of the meta-heuristic algorithms is performed. In the fifth section, the experiments are analyzed. Finally, the sixth section concludes and presents future research proposals.

2. Literature Review

VMI is a term in the supply chain, when the supplier takes full responsibility for controlling the vendor's inventory level (Yao et al., 2007). VMI is a term for the inventory management system where the vendor controls the customer's daily activities and inventory. In the VMI, the manufacturer or vendor assumes responsibility for managing his customers' inventory, and most of the common inventory maintenance costs are transferred from the customer to the vendor. However, by taking over inventory management, the producer will be able to coordinate inventory control and production decisions (Archetti et al., 2007). VMI is one of the most widely discussed initiatives to improve the supply chain efficiency of several companies. This initiative, known as continuous refill or Vendor Managed Inventory, was popularized in the late 1980s by WalMart and Procter & Gamble. VMI has become one of the most important programs in the food industry. As Tyan & Wee (2003) showed in their research on the food industry, VMIs can not only reduce costs, but also help improve service levels and create job opportunities for both sides of the supply chain. Many leading companies in various industries, such as Campbell Soup, Johnson & Johnson, and Barilla Waller et al., Have implemented Electrolux Italy, Nestlé and Texo, Boeing, and Aqua VMI. With VMI implemented, the vendor makes the decision to replenish inventory on behalf of the buyer organization. This means that the vendor monitors the buyer's inventory (either in person or by e-mail) and makes periodic resale decisions based on order quantity, shipping, and time. Pasandideh et al. (2011) in an article presented a two-level supply chain network model with one manufacturer and several retailers. In this system, the vendor operates using retail information about the supply of orders. Given the nonlinearity of the proposed supply chain network model, they used a genetic algorithm to solve the problem. In a paper modeling a supply chain network based on Vendor Managed Inventory, Yu et al. (2012) presented how the vendor decides how to manage the inventory system to convert raw materials into products.

Sadeghi et al. (2013) modeled a customer-centered inventory management problem in a two-level supply chain network with multiple vendors and multiple retailers. Their main purpose was to determine the amount of orders along with the number of shipments received by retailers and vendors, taking into account the reduction in the cost of the entire inventory system. They used genetic algorithms and particle swarm optimization to solve the problem. Nia et al. (2014) considered a inventory management model by the vendor with one buyer and one vendor with several products in fuzzy conditions. There was a shortage in their model and the storage capacity, delivery, orders and number of pallets were limited. Also in this model, demand and storage capacity and volume of orders were considered fuzzy. They used the ant algorithm to solve the problem and compared the model with a detailed genetic and evolutionary algorithm for validation. Taleizadeh et al. (2015) presented a two-level inventory model with one vendor and several retailers in a non-competitive environment with degradability of raw materials and final product. Market demand for the final product was considered definitive. The decisions of the retailer's final pricing model were the raw material reprocessing sequence, the product reprocessing cycle, and the production rate, so as to maximize the profits of the entire supply chain network. In their modeling, they used game theory with a Stockelberg approach, and the vendor was considered the leader and the retailer the follower. Khan et al. (2016) proposed a vendor-based supply chain model of inventory management to reduce system costs. In this paper, the supply chain considered included one vendor and several buyers. Finally, the impact of different parts of defective materials, storage costs as well as disposal plans were investigated. The results showed that the salience of the proposed storage plan compared to a conventional contract is proportional to the size and number of shipments per cycle. Kaasgari et al. (2017) modeled a two-level supply chain network with one vendor and several retailers in which product life was limited and the distribution function was followed. In this article, due to the corruption of products, the use of discounts for earlier supply of products was considered.

Babazadeh & Torabi (2018) in a paper presented a complex integer linear programming model for integrated production and distribution planning in a closed loop supply chain network. The model they envisioned included several products, several time periods, and several direct and indirect vehicles. In an article on a supply chain issue with one supplier and several retailers, Pasandideh et al. (2018) modeled several products based on the vendor inventory policy. The main purpose of this article was to find the desired number of products in order to consider two policies. To solve this problem, they used the teacher-student algorithm and implemented their designed model on a real case study. Soni et al. (2018) examined a vendor inventory system issue in which vendor decisions were considered instead of buyer decisions. The main purpose of this article was to compare the traditional system and VMI with the lowest system cost. After problem solving and sensitivity analysis, they showed that VMI incurs lower costs compared to the traditional system. Sainathan et al. (2019) in an article examined the supply chain management based on VMI. They provided a mathematical model for analyzing retail and customer inventory contracts. De Giovanni et al. (2019) presented an integrated model of production and inventory management with pricing and advertising considerations to evaluate the effects of advertising programs. They examined a supply chain based on VMI along with a transportation contract for supply chain coordination. Mishra & Talati (2019) proposed an evolutionary algorithm to minimize the costs of the production-inventory problem. To this end, they considered time-dependent failure as a Weibull distribution in their integrated model, including manufacturer and retailer, and used a genetic algorithm to minimize total costs. Pramudyo & Luong (2019) presented a vendor-based supply chain model of inventory management for a system with one vendor and several retailers. The main purpose of this model was to determine the optimal amount of production and distribution to reduce system costs. To solve the problem, they used genetic algorithms to obtain near-optimal results. Dai et al. (2020) proposed a cyclical inventory routing-inventory model under the VMI policy for perishable products with supply-side demand and price in the supply chain. They used a hybrid algorithm to solve the problem. Çömez-Dolgan et al. (2021) examined the impact of two-level supply chain coordination, including one vendor and one buyer in the presence of late delivery costs. Fang & Chen (2021) implemented an integrated VMI Hub to solve the inventory problem. They introduced two different models including the stock transfer order structure (STO) and the purchase order structure (SO). Lotfi et al. (2022) proposed three fuzzy and data-driven combined optimization models for the Resilience Supply and Sustainable Health Care Supply Chain (RSHCSC) with the VMI approach. The first model is the mean absolute function, the second model is the conditional risk value (CVaR) and the third model is the traditional inventory model. Modares et al. (2022) used redundancy allocation (RAP) as an effective technique to increase vendor reliability and also considered retailer reliability as well as the relationship between retailers and vendors.

After reviewing the literature and examining the research gap, in this paper, an inventory management problem by the vendor in a supply chain network in the scenario-driven mode is reviewed and modeled. Genetic algorithms, whale optimization algorithm and the league champions algorithm have also been used to solve the problem.

3. Problem Definition

This paper considers a three-level supply chain issue involving several external suppliers, several distributors, and one manufacturer. The manufacturer buys the raw materials needed to produce different products from foreign suppliers and produces various products according to the consumption coefficient of raw materials and production limitations. The products are sent to the distributors from the production center. Distributors can sell each product at a different retail price. Therefore, the agreement between the manufacturer and the distributors will be based on the VMI. Based on an agreement reached between the manufacturer and each distributor. Each distributor must pay a cost to the manufacturer for managing the inventory of each product.

According to the following assumptions, the production-distribution problem can be modeled in scenario-based mode based on VMI:

- Discounts are not allowed and the level of manpower is assumed to be constant.
- Shortages are allowed for distributors.
- Demand is calculated as a nonlinear function of the retail price.
- The cost of maintaining the final product is the same for the distributor and manufacturer in each time period.
- The ordering period of each product is the same for all distributors.
- Therefore, the main purpose of this article is to determine the wholesale and retail prices of each distributor as well as the amount of product produced, the amount of shortage of each product and the amount of inventory in such a way as to maximize profits from manufacturers selling products to distributors and general level wholesale and retail prices. To perform modeling, the set, parameters and decision variables are defined as follows.

3.1. Sets

S	Set of foreign suppliers $s = \{1,2,\dots,S\}$
C	Distributor set $c = \{1,2,\dots,C\}$
M	Raw material set $m = \{1,2,\dots,M\}$
T	Time period set $t = \{1,2,\dots,T\}$
I	Product set $i = \{1,2,\dots,I\}$
Se	Set of scenarios $se = \{1,2,\dots,Se\}$

3.2. Parameters

Pr^{se}	Probability of occurrence of scenario se
K_c^{se}	Market Index for Distributor c in Scenario se
E_c^{se}	Price elasticity in relation to the distributor market c in scenario se
ζ_{ict}	The cost of inventory management of each product i relates to distributor c in period t
Se_{it}	Fixed cost of product preparation i for the manufacturer in period t
Cr_{it}^{se}	Cost of production of each unit of product i at normal time in time period t in scenario se
Co_{it}^{se}	Cost of production of each unit of product i in overtime in period t in scenario se
Cc_{it}^{se}	The cost of producing each unit of product i in the ancillary contract over time t in scenario se
a_i^{se}	The time required to produce each unit of product i in scenario se
Hm_{mt}^{se}	The cost of maintaining each unit of raw materials m produced in period t in scenario se

Hp_{it}^{se}	The cost of maintaining each unit of product i the manufacturer in period t in scenario se
Sr_{ct}	Fixed cost of ordering distributor c in time period t
H_{ict}^{se}	The cost of maintaining each unit of product i distributor c in period t in scenario se
π_{ict}^{se}	Cost of shortage of each unit of product i distributor c in time period t in scenario se
ε_{mi}	Number of raw materials required m to produce each unit of product i
Vo_i	Volume of each unit of final product i
Tcs_{smt}^{se}	The cost of distributing each unit of raw material m from supplier s to producer in time period t in scenario se
Tcc_{ict}^{se}	The cost of distributing each unit of product i from producer to distributor c over time t in scenario se
Cm_{smt}^{se}	The cost of raw material m from supplier s in time period t in scenario se
$TCAR_t$	Maximum time available for production at normal time in period t
$TCAO_t$	Maximum time available for production in overtime during period t
$TCAC_t$	Maximum production capacity of ancillary contract in time period t
$CAPM_m$	Maximum storage capacity of raw materials for the producer in the period t
$CAPS_{smt}$	Maximum raw material delivery capacity m supplier s in time period t
τ_s	Duration of transportation of each raw material from supplier s to manufacturer
υ_c	Shipping time of each unit of final product from manufacturer to distributor c
A_1	Maximum budget available for distributor costs
A_2	Maximum time available for distribution of raw materials and final products
BT_t	The total volumetric space of production of products in the period t

3.3. Decision Variables

P_{ict}^{se}	Retail price of each unit of product i related to distributor c in period t in scenario se
W_{ict}^{se}	The wholesale price of each unit of product i relates to distributor c in period t in scenario se
B_{ict}^{se}	Deficit rate of each product unit i related to distributor c in time period t in scenario se
CY_{it}^{se}	General ordering period of each product unit i in time period t in scenario se
XR_{it}^{se}	The amount of product i produced in normal time in time period t in scenario se
XO_{it}^{se}	The amount of product i produced during overtime in time period t in scenario se
XC_{it}^{se}	The amount of product i produced in the ancillary contract in time period t in scenario se
SUP_{smt}^{se}	The amount of raw material m distributed from supplier s to producer in time period t in scenario se
IM_{mt}^{se}	The amount of raw material inventory stored m for the producer at the end of time period t in scenario se
D_{ict}^{se}	Demand for each unit of final product i Distributor c in time period t in scenario se

p_{it}^{se}	Final product production rate i in time period t in scenario se
IP_{it}^{se}	Average inventory level during the general order period of the final product i for the producer in period t in scenario se
${TC_{VMI}}^{se}$	Costs resulting from inventory management by the vendor in scenario se

In the general order period policy, the order period of each product in each time period is the same for all distributors and the orders are sent to the manufacturer in an integrated manner. This policy provides the ability for the manufacturer to respond to all orders at the same time and have more effective control over orders. In this case, the cost of ordering will be reduced compared to the case of multiple order periods. In this paper, the general ordering period of the decision variable is assumed and must be selected in such a way that the goals of the model are achieved; Therefore, for each product in each time period, general ordering periods are considered. The order quantity of each retailer in each public order period is equal to the amount of demand sent to that distributor in that time period during the general order period. For each product in each time period, the manufacturer sends the product in its entirety to the distributors for the total amount of the distributors' orders during a public order period, and each distributor receives orders depending on the amount of demand in that public order period. Figure (1) shows the average inventory and shortage of product for distributors over the time period of that product in the public order period policy. Figure (2) also shows the level of inventory of the product during the planning period of that product for the manufacturer in this policy.

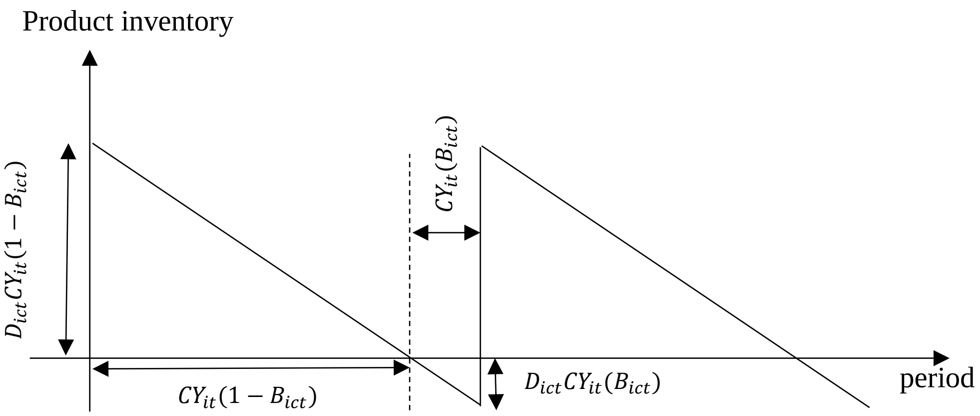

Fig. 1. Product inventory chart for the distributor in the general order period policy

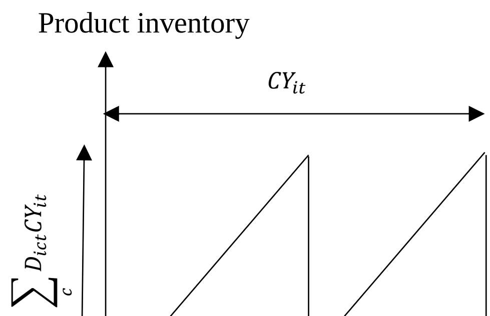

Fig. 2. Inventory chart for the manufacturer in the general order period policy

3.4. Modeling

$$MaxZ = \sum_{se} Pr^{se} \Bigg(\sum_{i,c,t} W_{ict}^{se} D_{ict}^{se} - \sum_{i,t} Cr_{it}^{se} XR_{it}^{se} - \sum_{i,t} Co_{it}^{se} XO_{it}^{se} - \sum_{i,t} Cc_{it}^{se} XC_{it}^{se} - \sum_{s,m,t} Tcs_{smt}^{se} SUP_{smt}^{se} - \sum_{s,m,t} Tcc_{ict}^{se} D_{ict}^{se} - \sum_{i,t} \frac{Se_{it}}{CY_{it}^{se}} - \sum_{s,m,t} Cm_{smt}^{se} SUP_{smt}^{se} - \sum_{i,t} Hp_{it}^{se} IP_{it}^{se} - \sum_{m,t} Hm_{mt}^{se} IM_{mt}^{se} - {TC_{VMI}}^{se} \Bigg) \quad (1)$$

$s.t.$:

$$D_{ict}^{se} = K_c^{se} . {P_{ict}^{se}}^{-E_c^{se}}, \quad \forall i, c, t, se \quad (2)$$

$$p_{it}^{se} = XR_{it}^{se} + XC_{it}^{se} + XO_{it}^{se}, \quad \forall i, t, se \quad (3)$$

$$IP_{it}^{se} = \left(\frac{\sum_c {D_{ict}^{se}}^2 \, {CY_{it}^{se}}^2}{2 p_{it}^{se} CY_{it}^{se}} \right), \quad \forall i, t, se \quad (4)$$

$$P_{ict}^{se} > W_{ict}^{se} - \zeta_{ict}, \quad \forall i, c, t, se \quad (5)$$

$$IP_{it}^{se} = IP_{it-1}^{se} + p_{it}^{se} - \sum_{c} D_{ict}^{se}, \quad \forall i, t, se \quad (6)$$

$$IM_{mt}^{se} = IM_{mt-1}^{se} + \sum_{s} Sup_{smt}^{se} - \sum_{i} \varepsilon_{mi} \left(XR_{it}^{se} + XC_{it}^{se} + XO_{it}^{se} \right), \quad \forall m, t, se \quad (7)$$

$$\sum_{i} a_i^{se} XR_{it}^{se} \leq TCAR_t, \quad \forall i, t, se \quad (8)$$

$$\sum_{i} a_i^{se} XC_{it}^{se} \leq TCAC_t, \quad \forall i, t, se \quad (9)$$

$$\sum_{i} a_i^{se} XO_{it}^{se} \leq TCAO_t, \quad \forall i, t, se \quad (10)$$

$$\sum_{m} IM_{mt}^{se} \leq CAPM_t, \quad \forall t, se \quad (11)$$

$$Sup_{smt}^{se} \leq CAPS_{smt}, \quad \forall s, m, t, se \quad (12)$$

$$\sum_{c} D^{se}_{ict} \le p^{se}_{it}, \quad \forall i, t, se \tag{13}$$

$$\sum_{i,c,t} D^{se}_{ict}\left(W^{se}_{ict} - P^{se}_{ict} + \zeta_{ict}\right) \le A_1, \quad \forall se \tag{14}$$

$$\sum_{s,m,t} \tau_s \, Sup^{se}_{smt} + \sum_{i,c,t} \upsilon_c \, D^{se}_{ict} \le A_2, \quad \forall se \tag{15}$$

$$\sum_{i} Vo_i\left(XR^{se}_{it} + XC^{se}_{it} + XO^{se}_{it}\right) \le BT_t, \quad \forall t, se \tag{16}$$

$$\begin{aligned} TC_{VMI}{}^{se} = \sum_{i,c,t} \frac{Sr_{ct}}{CY^{se}_{it}} + \sum_{i,c,t} H^{se}_{ict}\left(\frac{D^{se}_{ict}\left(1 - B^{se}_{ict}\right)^2 CY^{se\,2}_{it}}{2CY^{se}_{it}}\right) + \sum_{i,c,t} \pi^{se}_{ict}\left(\frac{D^{se}_{ict} B^{se\,2}_{ict} CY^{se\,2}_{it}}{2CY^{se}_{it}}\right) \\ - \sum_{i,c,t} \zeta_{ict} D^{se}_{ict}, \quad \forall se \end{aligned} \tag{17}$$

$$0 \le B^{se}_{ict} \le 1, \quad \forall i, c, t, se \tag{18}$$

$$P^{se}_{ict}, W^{se}_{ict}, B^{se}_{ict}, CY^{se}_{it}, XR^{se}_{it}, XO^{se}_{it}, XC^{se}_{it}, SUP^{se}_{smt}, IM^{se}_{mt}, D^{se}_{ict}, p^{se}_{it}, IP^{se}_{it} \ge 0 \quad \forall i, s, t, c, m, se \tag{19}$$

Equation (1) shows the objective function of the problem and its purpose is to maximize the profit of the producer. This profit is the difference between the revenue from the sale of products to distributors and the costs associated with it. Equation (2) calculates the amount of problem demand from the Cobb-Douglas relation. Equation (3) shows the production rate of the final product in each time period. Equation (4) shows the average inventory level during the general ordering period in each time period. Equation (5) calculates the retail and wholesale prices of each distribution center. Equation (6) shows the equilibrium relationship of inventory per final product. Equation (7) also shows the equilibrium relationship of inventory per raw material. Equations (8) to (10) show the maximum time available for production in normal time, overtime and ancillary contract. Equation (11) limits the producer's capacity to store raw materials and Equation (12) limits the distribution capacity of raw materials from each supplier. Equation (13) shows the producer rate of production. Equation (14) limits the total costs of distributors from purchasing products. Equation (15) limits the maximum distribution time of raw materials and final products. Equation (16) shows the production volume limit. Equation (17) calculates the costs of inventory management by the vendor. Relationships (18) and (19) show the type and gender of decision variables.

After modeling the production-distribution problem based on inventory management by the vendor in scenario-based mode, genetic algorithms, league champions algorithm and whale optimization algorithm have been used to solve the problem in larger sizes. Therefore, in this part of the article, the design of the primary chromosome or the initial solution used in solving the problem with meta-heuristic algorithms is discussed and by providing a numerical example, the designed chromosome size and how to decode it is described. For example, a production-distribution problem based on inventory management by the vendor with 3 different products ($i = 1,2,3$), 2 retailers ($c = 1,2$), 3 different time periods ($t = 1,2,3$) and consider in 2 different scenarios ($se = 1,2$). Therefore, the initial answer to the problem can be provided in Figure (3).

Average inventory level				Retail price			
$t = 3$	$t = 2$	$t = 1$	Period / product	$c = 2$	$c = 1$	retail vendor / product	$t = 1,2,3$
			$i = 1$			$i = 1$	

			$i = 2$			$i = 2$	
			$i = 3$			$i = 3$	

$se = 1,2$

Fig. 3. The initial solution (chromosome) was designed to solve the production-distribution problem

According to Figure (3), it can be seen that the initial answer is to generate real random numbers for the two variables of retail price and average inventory level in different scenarios. Other decision variables can be calculated based on the random data generated in Figure (3). However, in the production of initial answers or the production of new answers, the solution space may be unjustified, and a penalty function has been used to resolve this issue. The following steps show how to decode the initial answer and solve the proposed model.

Step 1: Calculate the decision variable D_{ict}^{se} from the following equation due to the availability of the variable P_{ict}^{se}.

$$D_{ict}^{se} = K_c^{se} . P_{ict}^{se^{-E_c^{se}}}, \quad \forall i, c, t, se \tag{20}$$

Step 2: Calculate the W_{ict}^{se} decision variable from Equation (21) due to the availability of the variable P_{ict}^{se}.

$$P_{ict}^{se} > W_{ict}^{se} - \zeta_{ict}, \quad \forall i, c, t, se \tag{21}$$

Step 3: Calculate the value of the p_{it}^{se} decision variable from Equations (22) and (23).

$$IP_{it}^{se} = IP_{it-1}^{se} + p_{it}^{se} - \sum_c D_{ict}^{se}, \quad \forall i, t, se \tag{22}$$

$$\sum_c D_{ict}^{se} \leq p_{it}^{se}, \quad \forall i, t, se \tag{23}$$

Step 4: Calculate the value of the decision variable CY_{it}^{se} from the relation (24).

$$IP_{it}^{se} = \left(\frac{\sum_c D_{ict}^{se\,2} \, CY_{it}^{se\,2}}{2 p_{it}^{se} CY_{it}^{se}} \right), \quad \forall i, t, se \tag{24}$$

Step 5: Calculate the value of the decision variables XR_{it}^{se} ،XC_{it}^{se} and XO_{it}^{se} according to the following equations:

$$p_{it}^{se} = XR_{it}^{se} + XC_{it}^{se} + XO_{it}^{se}, \quad \forall i, t, se \tag{25}$$

First, it should be noted that the sum of the decision variables presented in step 5 should be equal to the decision variable obtained in step 3. Therefore, first the decision variable XR_{it}^{se} is set in such a way that relation (26) is not violated. If the value of XR_{it}^{se} is not equal to the value of p_{it}^{se}; The decision variable XO_{it}^{se} will not be set as long as the relation is not violated (27) and finally the decision variable XC_{it}^{se} will not be set as long as the relation is not violated (28).

$$\sum_i a_i^{se} XR_{it}^{se} \leq TCAR_t, \quad \forall i, t, se \tag{26}$$

$$\sum_i a_i^{se} XO_{it}^{se} \leq TCAO_t, \quad \forall i, t, se \tag{27}$$

$$\sum_i a_i^{se} XC_{it}^{se} \leq TCAC_t, \quad \forall i, t, se \tag{28}$$

Step 6: Calculate the value of the decision variable Sup_{smt}^{se} with respect to non-violation of relations (29) and (30).

$$\sum_{s,m,t} \tau_s \, Sup_{smt}^{se} + \sum_{i,c,t} \upsilon_c \, D_{ict}^{se} \leq A_2, \quad \forall se \tag{29}$$

$$Sup_{smt}^{se} \leq CAPS_{smt}, \quad \forall s, m, t, se \tag{30}$$

Step 7: Calculate the decision variable IM_{mt}^{se} from relation (31) with the condition of non-violation of relation (32).

$$IM_{mt}^{se} = IM_{mt-1}^{se} + \sum_{s} Sup_{smt}^{se} - \sum_{i} \varepsilon_{mi}(XR_{it}^{se} + XC_{it}^{se} + XO_{it}^{se}), \quad \forall m, t, se \tag{31}$$

$$\sum_{m} IM_{mt}^{se} \leq CAPM_t, \quad \forall t, se \tag{32}$$

Step 8: Calculate the decision variable B_{ict}^{se} from the following equation:

$$0 \leq B_{ict}^{se} \leq 1, \quad \forall i, c, t, se \tag{33}$$

Step 9: Consider the penalty function for not establishing the following relationships:

$$\sum_{i,c,t} D_{ict}^{se} \, (W_{ict}^{se} - P_{ict}^{se} + \zeta_{ict}) \leq A_1, \quad \forall se \tag{34}$$

$$\sum_{i} Vo_i(XR_{it}^{se} + XC_{it}^{se} + XO_{it}^{se}) \leq BT_t, \quad \forall t, se \tag{35}$$

Step 10: Calculate the value of the objective function taking into account the penalty.

Due to the NP-Hard nature of the problem, genetic algorithms, whale optimization algorithm and the league champions algorithm have been used to solve the sample problems in larger sizes. Before solving the problem, first the initial parameters of the mentioned algorithms should be set in the most optimal way, which is done by Taguchi method. The parameter values set for genetic algorithms, whale optimization algorithm, and the league champions algorithm using the Taguchi method are presented in Table (1).

Table 1. Optimal operating levels for meta-heuristic algorithms

Algorithm	parameters	Operating levels 1	2	3	Optimal parameter
GA	p_c	0.2	**0.5**	0.7	0.5
	p_M	**0.2**	0.5	0.7	0.2
	$Npop$	100	200	**300**	300
	$Max\ it$	100	200	**300**	300
WOA	A	0.2	**0.5**	0.7	0.5
	C	**0.2**	0.5	0.7	0.2
	$Nwhale$	100	200	**300**	300
	$Max\ it$	100	200	**300**	300
LCA	$LeagueSize$	100	200	**300**	300
	$Max\ it$	100	200	**300**	300
	ψ_1	2	**4**	6	4
	ψ_2	**0.9**	1	1.1	0.9
	p_c	-6	-4	**-2**	-2

4. Analysis of results

4.1. Solve the sample problem based on the scenario in small size

Due to the uncertainty of considering some of the model parameters, first a sample problem is shown in a small design size and the output of the problem is shown. Table (2) shows the sample size of the sample designed in small size and Table (3) shows the limits of the problem parameter ranges based on uniform distribution.

Table 2. Sample problem in small size

Set	Description	Size
S	Set of foreign suppliers	2
C	Set of distributors	2
M	Set of raw materials	2
T	Period set	2
I	Product Set	3
Se	Set of scenarios	3

Table 3. Limits of problem parameter intervals based on uniform distribution

Parameter	Approximate range		
	Scenario 1	Scenario 2	Scenario 3
K_c^{se}	~U(20000,30000)	~U(30000,50000)	~U(50000,70000)
E_c^{se}	~U(12,1.5)	~U(1.5,1.8)	~U(۱/۸ ،۲/۱)
Cr_{it}^{se}	~U(۰/۵ ،۰/۷۵)	~U(۰/۷۵ ،۱)	~U(۱ ،۱/۲۵)
Co_{it}^{se}	~U(۱ ،۱/۵)	~U(۱/۵ ،۲)	~U(۲ ،۲/۵)
Cc_{it}^{se}	~U(۲ ،۲/۵)	~U(۲/۵ ،۳)	~U(۳ ،۳/۵)
a_i^{se}	~U(۱ ،۲)	~U(۲ ،۳)	~U(۳ ،۴)
Hm_{mt}^{se}	~U(۱/۵ ،۲)	~U(۲ ،۳)	~U(۳ ،۳/۵)
Hp_{it}^{se}	~U(۲ ،۳)	~U(۳ ،۴)	~U(۴ ،۵)
H_{ict}^{se}	~U(۲ ،۳)	~U(۳ ،۴)	~U(۴ ،۵)
π_{ict}^{se}	~U(۲۰ ،۳۰)	~U(۳۰ ،۴۰)	~U(۴۰ ،۵۰)
TCS_{smt}^{se}	~U(۰/۵ ،۱)	~U(۱ ،۱/۵)	~U(۱/۵ ،۲)
Tcc_{ict}^{se}	~U(۰/۵ ،۱)	~U(۱ ،۱/۵)	~U(۱/۵ ،۲)
Cm_{smt}^{se}	~U(۲ ،۴)	~U(۴ ،۶)	~U(۶ ،۸)
Pr^{se}	0.2	0.3	0.5

Parameter	Approximate range	Parameter	Approximate range
$\zeta_{ict}, \tilde{\alpha}_i$	~U(۳ ،۷)	ε_{mi}	~U(۱ ،۳)
Se_{it}	~U(۱۰ ،۱۵)	Vo_i	~U(۱ ،۳)
υ_c, τ_s	~U(۰/۵ ،۲)	$TCAR_t$	~U(۱۰ ،۲۰)
Sr_{ct}	~U(۱۰ ،۲۰)	$TCAO_t$	~U(۶۰ ،۸۰)
A_1	۳۴۰	$TCAC_t$	~U(۴۰ ،۵۰)
A_2	۱۲۰	$CAPM_m$	~U(۱۴۰ ،۱۸۰)
BT_t	~U(۵۰ ،۱۰۰)	$CAPS_{smt}$	~U(۱۵ ،۲۰)

The optimal value of the objective function obtained from solving the problem of the above sample is 13983.152. For further analysis of the problem, the probability of occurrence of each modified scenario and the value of the objective function obtained from the model are shown in Table (3).

Table 3. The values of the objective functions obtained in different probabilities of the scenario

problem	Probability of different scenarios			The value of the objective function
	Probability 1	Probability 2	Probability 3	
۱	۰/۲	۰/۳	۰/۵	۱۳۹۸۳/۱۵۲
۲	۰/۲	۰/۲	۰/۶	۱۲۴۳۸/۸۵۸
۳	۰/۳	۰/۳	۰/۴	۱۶۸۸۱/۱۴۴
۴	۰/۳	۰/۴	۰/۳	۱۸۴۲۵/۷۳۸
۵	۰/۴	۰/۳	۰/۳	۱۹۷۷۹/۱۳۶
۶	۰/۴	۰/۲	۰/۴	۱۸۲۳۴/۸۴۲
۷	۰/۵	۰/۴	۰/۱	۲۴۲۲۱/۴۲۳
۸	۰/۵	.۰/۳	۰/۲	۲۲۶۷۷/۱۲۹
۹	۰/۶	۰/۲	۰/۲	۲۴۰۳۰/۸۲۷
۱۰	۰/۶	۰/۱	۰/۳	۲۲۴۸۶/۵۳۳

4.2. Solving sample problems with meta-heuristic algorithms

After setting the parameter of the meta-heuristic algorithms, first the problem of the designed sample in small size is solved with the mentioned meta-heuristic algorithms to investigate the difference between the values of the objective functions and also the computational time between them. Therefore, Table (4) shows the values of the objective functions and the average computational time for the designed sample problems with the probability of different events.

Table 4 - Values of objective functions obtained by solving a small sample size problem

problem	GA	WOA	LCA	GAMS
۱	۱۳۸۷۷/۷۲	۱۳۸۶۹/۷۶	۱۳۸۸۴/۱۴	۱۳۹۸۳/۱۵۲
۲	۱۲۳۳۶/۶۳	۱۲۳۱۸/۹۵	۱۲۴۳۲/۲۷	۱۲۴۳۸/۸۵۸
۳	۱۶۷۲۲/۱۶	۱۶۷۹۹/۵۲	۱۶۸۵۰/۸۳	۱۶۸۸۱/۱۴۴
۴	۱۸۴۰۰/۷۱	۱۸۳۸۲/۶۵	۱۸۴۰۹/۹۸	۱۸۴۲۵/۷۳۸
۵	۱۹۶۲۲/۵۸	۱۹۷۲۶/۷۲	۱۹۶۲۲/۸	۱۹۷۷۹/۱۳۶
۶	۱۸۱۸۸/۱۸	۱۸۱۰۱/۶۶	۱۸۱۳۷/۸۵	۱۸۲۳۴/۸۴۲
۷	۲۴۱۳۰/۷۹	۲۴۱۸۸/۳۲	۲۴۱۱۹/۸۲	۲۴۲۲۱/۴۲۳
۸	۲۲۵۲۱/۶۹	۲۲۴۷۳/۶۴	۲۲۵۲۵/۱۵	۲۲۶۷۷/۱۲۹
۹	۲۳۸۹۷/۶۷	۲۳۸۸۹/۰۸	۲۳۸۱۶/۴۷	۲۴۰۳۰/۸۲۷
۱۰	۲۲۳۰۴/۷۱	۲۲۲۷۲/۳۲	۲۲۳۷۹/۳۸	۲۲۴۸۶/۵۳۳
Computational time	۳۷/۷۱	۲۷/۰۶	۱۷/۸۶	۱۵۷/۶۱

According to the results of Table (4), it can be seen that the meta-heuristic algorithms have achieved very acceptable results in obtaining near-optimal solutions. According to the calculations, the maximum relative percentage difference of the optimal answer obtained from GAMS and meta-heuristic algorithms is less than 1%. Also, all three algorithms were able to achieve the desired answers in much less time than GAMS software. The trend of changes in genetic meta-heuristic algorithms, whale optimization algorithm and league champions algorithm in different iterations for the first problem is shown in Figure (3).

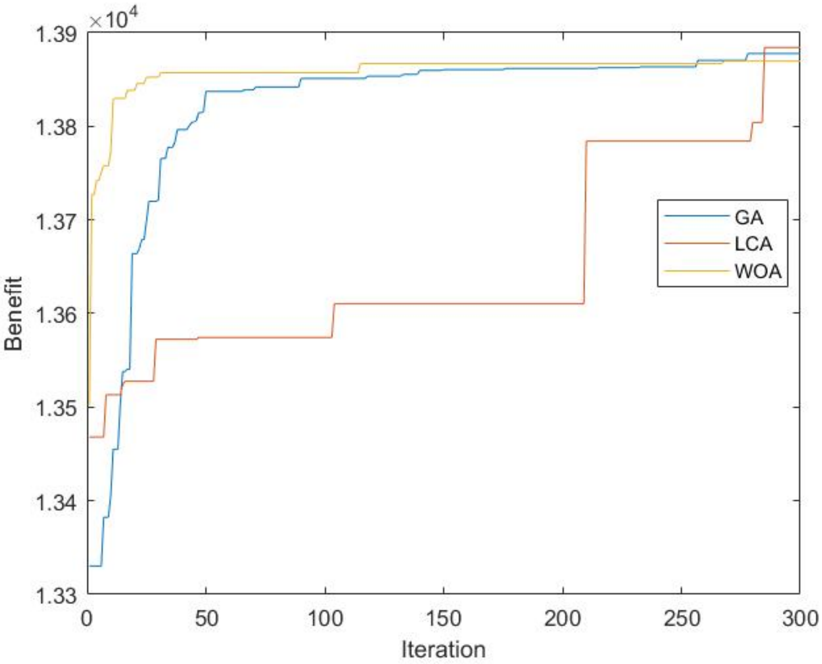

Fig. 3. The process of changing the results in different iterations with meta-heuristic algorithms

In order to solve the sample problems in larger sizes, only meta-innovative algorithms of genetic algorithm, whale optimization algorithm and league champions algorithm have been used. The size of different sample problems designed in larger size with 3 scenarios is in accordance with Table (5) and the range of parameters of the generated parameters is based on the uniform distribution of Table (2).

Table 5. Size of sample problems designed in larger size

Sample problem	*S*	*C*	*M*	*T*	*I*	Sample problem	*S*	*C*	*M*	*T*	*I*
۱	۴	۴	۳	۵	۴	۹	۱۲	۱۲	۸	۸	۶
۲	۶	۶	۴	۵	۴	۱۰	۱۲	۱۴	۸	۹	۷
۳	۸	۶	۵	۶	۴	۱۱	۱۳	۱۴	۸	۹	۷
٤	۸	۸	۵	۶	۵	۱۲	۱۴	۱۶	۱۰	۱۰	۷
٥	۸	۸	۵	۷	۵	۱۳	۱۴	۱۶	۱۰	۱۱	۸
٦	۱۰	۱۰	۶	۷	۵	١٤	۱۵	۱۸	۱۰	۱۱	۸
۷	۱۰	۱۰	۶	۸	۶	١٥	۱۵	۱۸	۱۰	۱۲	۸
۸	۱۰	۱۲	۸	۸	۶						

To solve the problem, due to the randomness of the data, 5 different data from each problem are generated in the same interval and solved by meta-innovative genetic algorithm, whale optimization algorithm and the league champions algorithm. As a result, the average of the results of 5 repetitions of the experiment is considered as the basis of calculations. Table (6) shows the mean values of the objective functions 5 times the repetition of different data for 15 sample problems. Also, the average computational time of solving 15 sample problems in a larger size is presented in Table (6).

Table 6. Average value of objective function obtained from solving large size problems with meta-heuristic algorithms

Sample problem	The objective function			Computational time		
	GA	LCA	WOA	GA	LCA	WOA
۱	۵۲۶۲۷/۴۱	۵۲۶۱۶/۳۲	۵۰۴۸۵/۹۷	۵۳/۰۱	۱۷/۲۰	۳۰/۱۸
۲	۸۰۱۸۲/۴۷	۷۶۰۳۹/۳۹	۷۷۰۱۷/۵۵	۸۶/۵۵	۱۹/۵۴	۶۶/۷۰
۳	۸۷۷۹۵/۶۵	۸۷۳۷۱/۳۶	۸۷۱۷۲/۵۶	۱۰۱/۸۵	۲۵/۸۳	۸۳/۹۳
٤	۱۰۶۰۶۴/۴۲	۱۰۳۷۱۵/۰۹	۱۰۲۷۹۵/۵۹	۱۴۴/۰۸	۴۷/۹۷	۱۰۸/۱۸
٥	۱۶۴۸۰۷/۲۹	۱۶۸۶۸۰/۷۱	۱۶۸۷۷۹/۳۶	۲۳۷/۰۵	۶۵/۶۰	۱۶۳/۰۷
٦	۲۳۵۰۹۶/۱۵	۲۳۶۷۶۵/۰۶	۲۳۴۸۷۲/۹۶	۳۳۵/۹۵	۱۲۰/۲۵	۲۴۶/۰۲
۷	۲۴۸۵۰۴/۰۶	۲۴۹۵۹۰/۵۰	۲۵۲۶۴۲/۷۳	۴۴۷/۳۲	۱۷۴/۵۸	۳۲۰/۹۹
۸	۳۴۲۰۸۰/۰۸	۳۳۹۹۰۹/۷۶	۳۴۰۹۱۸/۹۶	۵۷۰/۶۲	۲۴۷/۳۵	۴۲۹/۴۳
۹	۴۲۹۲۴۲/۰۸	۴۲۷۳۱۹/۱۶	۴۳۰۲۰۷/۶۸	۷۲۱/۱۵	۳۶۱/۵۸	۵۵۱/۶۸
۱۰	۴۸۷۷۰۵/۶۶	۴۸۶۲۰۶/۷۶	۴۸۶۲۹۷/۵۱	۸۶۱/۲۲	۴۹۰/۲۰	۶۸۸/۶۴
۱۱	۵۵۶۱۴۳/۵۷	۵۵۶۹۵۹/۶۹	۵۵۵۹۴۸/۲۲	۱۰۴۱/۶۸	۶۶۴/۳۸	۸۶۵/۱۴
۱۲	۷۵۱۱۳۷/۷۹	۷۵۱۰۸۵/۲۱	۷۵۵۷۲۸/۱۵	۱۲۲۷/۵۵	۹۱۷/۲۸	۱۰۸۹/۴۷
۱۳	۸۹۰۵۰۷/۱۹	۸۹۱۱۷۴/۴۳	۸۸۹۸۲۱/۳۵	۱۵۲۹/۹۲	۱۱۶۸/۶۵	۱۳۶۰/۶۶
۱٤	۱۰۳۷۱۳۶/۳۰	۱۰۳۴۲۳۹/۱۸	۱۰۳۴۶۲۸/۰۲	۱۹۰۳/۸۲	۱۴۹۱/۵۲	۱۷۰۸/۶۹
۱٥	۱۲۶۲۹۲۳/۹۲	۱۲۵۹۹۸۴/۲۵	۱۲۶۱۵۹۰/۶۱	۲۴۴۱/۵۵	۱۹۷۸/۹۵	۲۱۵۰/۴۸

According to the results obtained from Table (6), it can be concluded that the genetic algorithm with the average of the total objective function 448796.94 performed better than the Champions League algorithm with the average of 448110.46 and the whale optimization algorithm with the average of 448593.81. Also, the league champions algorithm has been able to solve 15 sample problems in the average time of 519.39. Genetic algorithms and whale optimization algorithm averaged 780.22 and 657.55 seconds, respectively. As can be seen, with increasing problem size, the average computational time obtained has increased exponentially. Finally, for better analysis of the problem, T-test at 95% confidence level was used to examine the significance of the difference between the means of the objective function and the computational time obtained by solving larger sample problems. Thus, the algorithms have been studied in pairs and if the value of P test statistic is less than 0.05, it will indicate a significant difference between the means of that index. Table (7) summarizes the results of T-test at 95% confidence level to evaluate the significance of the means of the objective function and computational time.

Table 7. Results of T-test for significant evaluation of the difference between the means of the objective function and computational time

Solving method	Mean difference	The lower limit of the confidence interval	The upper limit of the confidence interval	T test statistics	P test statistics
Objective function index					
GA-LCA	۶۸۶	-۲۸۴۲۳۳	۲۸۵۶۰۶	۰	۰/۹۹۶
GA-WOA	۲۰۳	-۲۸۴۹۰۸	۲۸۵۳۱۴	۰	۰/۹۹۹
WOA-LCA	۴۸۳	-۲۸۴۴۳۹	۲۸۵۴۰۶	۰	۰/۹۹۷
Computational time index					
GA-LCA	۱۲۳	-۳۹۶	۶۴۱	۰/۴۹	۰/۶۳۱
GA-WOA	۲۶۱	-۲۴۲	۷۶۴	۱/۰۶	۰/۲۹۷
WOA-LCA	۱۳۸	-۳۳۷	۶۱۳	۰/۶	۰/۵۵۶

The results of Table (7) show that there is no significant difference between the means of the objective function index and computational time between meta-heuristic algorithms. Therefore, in this section, to select the most efficient algorithm in terms of obtaining two indices of the average objective function and computational time, the TOPSIS multi-criteria decision-making method has been used. Also, the weight of the application to the two indicators with the opinion of experts is considered as 50% for each index. Table

(8) shows the means of the computational indices as well as the weight of the utility obtained from the TOPSIS method for the meta-heuristic algorithms used.

Table 8- Summary of the results of TOPSIS multi-criteria decision making method

Indicator	GA	LCA	WOA
Average objective function	۴۴۸۷۹۶/۹۴	۴۴۸۱۱۰/۴۶	۴۴۸۵۹۳/۸۱
Average computational time	۷۸۰/۲۲	۵۱۹/۳۹	۶۵۷/۵۵
Weight of utility	۰/۷۳۴	۰/۹۹۸	۰/۱۴۳

According to the weight of utility obtained from TOPSIS method according to Table (8), it can be concluded that the Champions League algorithm with a weight of 0.998 has a higher efficiency in obtaining near-optimal results in a shorter period of time.

5. Conclusion

In this paper, a problem of inventory management by the vendor in the production-distribution supply chain is presented based on the scenario. Therefore, a one-objective model of the problem was presented by considering the maximization of producer profit. Due to the nonlinearity and NP-Hardness of the problem, meta-heuristic algorithms (genetic algorithm, whale optimization algorithm and league champions algorithm) were used. Analysis of various sample problems showed that the performance of GAMS software in solving large sample problems is not appropriate. The results of statistical test showed that there was no significant difference between the means of the objective function and the computational time obtained. Therefore, the TOPSIS multi-criteria decision-making method was used to select the most efficient algorithm, the results of which showed the selection of the Champions League algorithm with a useful weight of 0.998. The use of fuzzy solid method to control the uncertain model and also the development of hybrid algorithms is proposed by the researcher.

Consent for publication

Publication is permitted by Mashhad University of Medical Science

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