Observation and simulation of transverse BTFs of high energy bunched beams

P. Görgen[1] *and O. Boine-Frankenheim*[1]
[1]TEMF, TU-Darmstadt, Germany

Transverse Beam Transfer Functions (BTFs) are widely used in synchrotrons and storage rings to determine machine and beam properties (for example tune, tune spread, error resonances). In the projected SIS100, BTFs can potentially be used to measure the tune spread during proton operation. The transverse BTF $R(\omega)$ is the fraction of the complex beam response amplitude $A_{\text{resp}}(\omega)$ and the excitation amplitude $A_{\text{drive}}(\omega)$ at a excitation frequeny ω.

$$R(\omega) = \frac{A_{\text{resp}}(\omega)}{A_{\text{drive}}(\omega)} \quad (1)$$

In simple cases the BTF can be calculated analytically. For instance in the presence of tune spread caused by chromaticity, the imaginary part of the BTF is proportional to the transverse tune distribution in the corresponding plane [1].

The situation becomes more complex when the particle tune depends on the transverse amplitude of the particle as in the case of a nonlinear transverse element such as space charge or nonlinear fields. One example is a so-called electron lens, wherein an electron beam of the same profile as the ion beam is guided in parallel to the ion beam [2]. By adjusting the electron beam current and shape, a nonlinear lens can be set up which can be used to reduce transverse tune spread due to space charge (as could be envisioned for SIS100) or the beam-beam effect (for the Relativistic Heavy Ion Collider (RHIC)). We show how BTFs can be used to diagnose the proper operation of such a device.

Analytic calculation

In absence of SIS100 we focused on the case of an electron lens like one recently installed at RHIC. We make the assumption we are in the limit of coasting beams which we justify by the synchrotron frequency of the order of magnitude of the measurement time for a BTF sample. The conditions in SIS100 for high energy proton operation are similar in terms of synchrotron frequency. We calculate the BTF in the presence of a nonlinear lens analytically following [3] and obtain the result:

$$R_i(\omega) \propto \iint_0^\infty \frac{1}{\omega - \omega_i\left(J_x, J_y\right)} \frac{J_i \mathrm{d}\psi}{\mathrm{d}J_i} \mathrm{d}J_x \mathrm{d}J_y \quad (2)$$

With J_x, J_y the particle amplitude in action-angle variables, ω the frequency of the BTF, ω_i the amplitude dependent tune of the particles in the i direction ($i \in \{x, y\}$) and ψ the density of particles in J_x, J_y space. The presence of the derivative of the phase space density makes the BTF sensitive to fluctuations in the phase space density. The presence of J_i means the contribution of the particles increases with their amplitude. Particle-in-cell (PIC) simulations agree with eq. 2. Unfortunately unlike in the afforementioned case of chromaticity for typical ψ and ω_i we can only solve the integral in eq. 2 numerically. Thus, our best method for reconstruction of the tune distribution is fitting against eq. 2 for a known shape of ψ and ω_i.

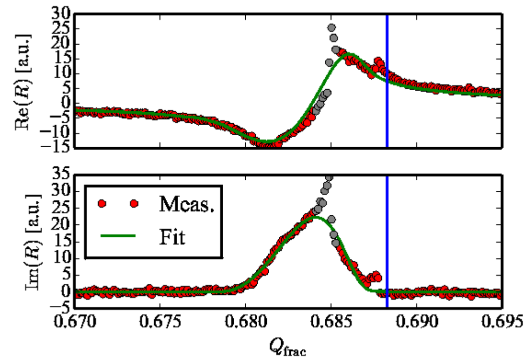

Figure 1: Exemplary BTF measurement of a beam undergoing a weak-strong beam-beam interaction as a substitute for an electron lens. Points are the measurement, the green line represents the fit. The fitting parameters are amplitude and the strength of the interaction ξ which is proportional to the tune spread.

After verifying the fit method in simulation [4] we validated the method in measurement with the beam-beam effect as a substitute for an electron lens in a machine experiment at RHIC. The interaction strenght recovered by the fit was in agreement with the expectation from measurements of the beam current and the emittance. The result of a BTF measurement is shown in Fig. 1 together with the fit.

The agreement between analytic results, simulations and measurement gives confidence that a similar method can be used to directly diagnose space charge tune spread of top energy protons in SIS100. The analytic equations follow the same approach. In application to SIS100 we plan to use our simulation model of the BTF that we successfully applied to RHIC to investigate electron lenses as a possible cure for space charge in SIS100.

References

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doi:10.15120/GR-2014-1-FG-SIS18-05