Simulation of Resistive Cooling in cylindrical Penning Traps*

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The simulation of ion clouds in traps taking ion-ion interaction into account is associated with large computational effort. This applies especially for the simulation of cooling methods like e.g. resistive cooling, where the cooling force has a complex structure. In [1] a simulation method for the resistive cooling was presented, using a particle in cell code. There the ion-ion interaction was treated by a mean-field approximation which, however, neglects the influence of ion-ion correlations on the dynamics. We thus developed an algorithm for the simulation of resistive cooling including the complete ion-ion interactions.

Charged particles moving between two conductive electrodes induce a surface charge Q on both of them and cause a potential imbalance u. Connecting the electrodes to an external RLC-circuit leads to an electrical current i. Energy dissipation in the resistor of the circuit then leads to changes in the induced charges on the electrodes and the related electric field which acts back on the ions as a cooling force $\vec{F}_D$. This non-trivial coupling between the circuit and the ion motion is described by the equations:

$$\frac{d^2u}{dt^2} + \frac{R}{L}\frac{du}{dt} + \frac{1}{LC}u = \frac{1}{C}\left(\frac{R}{L}\frac{di}{dt} + i\right) \quad (1)$$

$$i = \sum_{k=1}^{N} \frac{d}{dt} \cdot \Delta Q(\vec{r}_k)$$

$$\ddot{\vec{r}}_j = \frac{1}{m_j} \cdot \Big(\vec{F}_E(\vec{r}_j) + \vec{F}_B(\dot{\vec{r}}_j) + \cdots \quad (2)$$

$$+ \vec{F}_C(\vec{r}_1, \cdots, \vec{r}_N) + \vec{F}_D(\vec{r}_j, u)\Big)$$

R represent the resistance, C the capacitance and L the inductance of the circuit which is driven by the current i and ΔQ is the difference of the induced charges on the Pick-Up electrodes. $\vec{F}_E$, $\vec{F}_B$, $\vec{F}_C$ and $\vec{F}_D := -q_j \vec{\nabla} V_D$ represents the forces from the electrical field, magnetic field, the Coulomb interaction and the induced Potential. N is the number of ions and $\vec{r}_j$ the position vector of the j-th particle. The induced charge on one of the Pick-Up electrodes is given by the equation:

$$Q = a \cdot \sum_{k=1}^{N} \int_{z_<}^{z_>} \int_0^{2\pi} \epsilon_0 \left.\frac{\partial V(r, \phi, z; r_k, \phi_k, z_k)}{\partial r}\right|_a d\phi dz, \quad (3)$$

where $z_<$, $z_>$ representing the z coordinates of the electrode, V_k the potential of the k-th ion using cylindrical coordinates and a the radius of the trap.

In figure 1, the workflow of the resistive cooling algorithm is shown. The algorithm starts with an inital distribution of the positions and velocities of the ions, while the electrical Potential V_D caused by the induced current is neclegted at first. Next equation (2) is solved numerically to calculate a new set of positions, velocities and accelerations from which the right hand side of equation (1) is evaluated. After solving equation (1), finally the electrical potential V_D is calculated and serves together with the new positions and velocities as a starting point for the subsequent cycle.

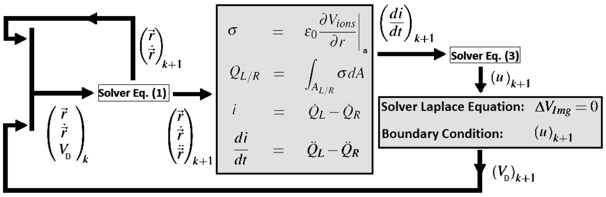

Figure 1: Workflow of the resistive cooling algorithm

Currently we work on the implementation of this algorithm into our existing code [2]. First simulations done with single ions show the expected cooling behavior. Figure 2 shows the simulation result of a single $^{12}C^{5+}$ ion in a cylindrical geometry with radius of $3.5\ mm$. The parameters of the circuit were chosen as $(R,\ L,\ C) \approx (10\ \Omega,\ 1.22\ mH,\ 14.22\ pF)$. The simulation was done on a GPU device using an parallelized Adam-Bashforth method of fourth order.

Next steps are the comparsion of the simulation results with experimental data were the circuit parameters are well known and investigating the cooling behavior for an increasing ion number.

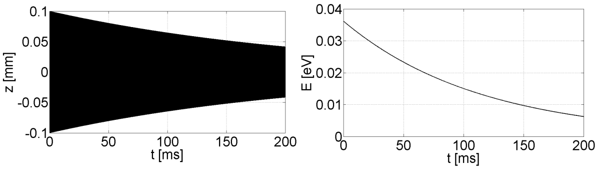

Figure 2: Left: Amplitude of the axial motion, Right: Total axial energy of the ion during resistive cooling.

References

[1] G. Maero , Springer , Applied Physics B , vol. 107 , p. 1087-1096.

[2] J. Steinmann , GSI Report 2009-1 , p. 307.

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