Radial oscillations of hybrid stars in general relativity

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We review the equations governing adiabatic, small radial oscillations of compact stars within the framework of general relativity. We apply these equations to modern realistic equations of state and compute oscillation frequencies for hadronic stars and hybrid stars. The results indicate that the 'static stability criterion' or 'turning point criterion' [1] for dynamical stability is not applicable to hybrid stars.

General relativistic framework

We start with the spherically symmetric line element $ds^2 = -e^{2\Phi}dt^2 + e^{2\Lambda}dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2)$ and the energy-momentum tensor for a perfect fluid $T^{\mu\nu} = (\rho + P)u^\mu u^\nu + Pg^{\mu\nu}$. To derive the oscillation equation Einstein's equation $G^{\mu\nu} = 8\pi T^{\mu\nu}$ is perturbed in such a way, as to preserve spherical symmetry. This is done by expressing all time tependent quantities as a sum of a time independent part and a time dependent perturbation. Omitting all nonlinear terms in the velocity one can derive the oscillation equation [2][4].

$$0 = \frac{d}{dr}\left[P\frac{d\zeta}{dr}\right] + \left[Q + \omega^2 W\right]\zeta$$

with

$$\begin{aligned} r^2 W &= (\rho + P)\, e^{3\Lambda+\Phi} \\ r^2 P &= \gamma p e^{\Lambda+3\Phi} \\ r^2 Q &= e^{\Lambda+3\Phi}\,(\rho + P)\left(\Phi'^2 + 4\frac{\Phi'}{r} - 8\pi e^{2\Lambda}p\right) \end{aligned}$$

Here ζ denotes the renormalized Lagrangian displacement of a fluid element due to the oscillation. All perturbed quantities are assumed to have a harmonic time dependence.

Model for dense matter

For the hadronic matter we employ the relativistic mean field model with the parameter set TM1, which is fitted to the properties of heavy nuclei [7]. The quark matter phase is modelled using an effective MIT bag like model [6].

Results

We compare purely hadronic stars with hybrid stars containg a quark core. In Figure 1 the density discontinuity at the center of hybrid stars is shown. The softening of the matter associated with the phase transition to deconfined quark matter leads to an initial decrease in the gravitational mass. According to the turning point criterion this leads to instability with respect to small perturbations. Our calculation of the eigenmode frequencies is shown in Figure 2. We conclude that the turning point criterion is inapplicable to stars with a sharp internal density discontinuity.

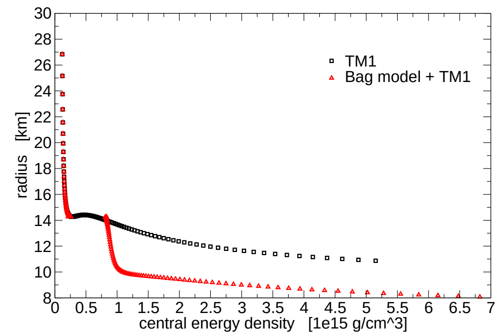

Figure 1: Radius as function of central energy density

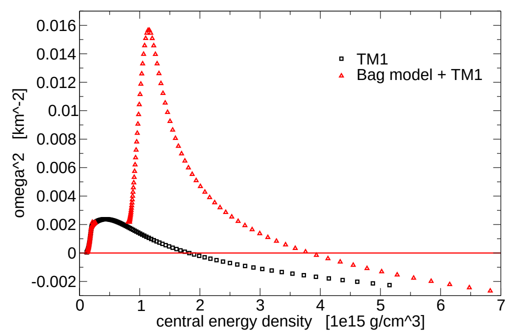

Figure 2: Squared oscillation frequency of fundamental mode as function of central energy density

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