CONTRIBUTIONS TO THE SIZING OF SANITARY PROTECTION AREAS FOR GROUNDWATER CATCHMENTS

Esmeralda CHIORESCU[1], Feodor FILIPOV[1], Sergiu JITĂREANU[2]

esmeralda_chiorescu@yahoo.com

Abstract

Following adequate hydro-geological conditions, water supply from local groundwater resources represents a viable solution for farms located far away from surface-water sources or from areal water supply systems, providing major economical and social advantages.

To exploit these resources safely care in order groundwater contamination with elements, substances, products and / or pathogenic microorganisms character, especially when abstraction is required to provide drinking water to be taken even in the design of water cachment, specific technical measures, one of the most important areas consisting of correct size of the protection perimeters with a strict diet and restriction diet.

This paper improves the analytical models of sizing the sanitary protection areas for water catchments through perfect wells or drains from unconfined aquifers, aiming at finding numerical solutions with a higher degree of accuracy.

The mathematical sizing models are differentiated according to the main characteristics of the aquifer and water catchment, such as container rock's type (with interstitial porosity or with cracks and/or cavities); aquifer's type (unconfined or confined) and feeding method (with or without initial dynamics); tapping with different wells and drains according to the water bed's opening (perfect/imperfect).

For a better understanding of the mathematical models mentioned above, and the technique for numerically solving, two examples there are presented in which all details items are considered in the sizing of sanitary protection zones with strict diet regime and with restrictions regime

Key words sanitary protection area, unconfined aquifer, initial dynamics, perfect well, perfect drain

One of the most important protective measures for the quality of the underground waters that are to be tapped for drinking is setting out sanitary protection perimeters with severe regime and restriction regime **(**Cojocaru D. et. al., 2013**)**. These areas are delimited on the surface, around the well or the catchment drain and serve preventing the contamination and pollution of the groundwater with elements, substances, products and/or pathogenic microorganisms.

Nowadays in our country, for the setting of the sanitary protection areas with severe and restriction regimes, it is generally accepted the hypothesis that there is a self-cleaning process in the aquifer. Hence, the sanitary protection perimeter has been conceived based on a so called "transit time" defined as being (Hotărârea nr. 930/11.08.2005) and (Ordin nr.1278, 2011): the amount of time needed to start, develop and end the physical, chemical and biological processes of de-polluting the possible elements, substances and/or polluting products in the underground water. Considering the hydrodynamic regime of the groundwater in the influence area of the catchments, a certain distance corresponds to the "transit time", named "sanitary protection distance with severe/restriction regime", covered by a drop of pollutant in aquifer, in the flowing direction of the underground stream, upstream the catchment point.

The difference between the "severe regime" size and the "restriction regime" size is given by the amount of time of the "transit time": $t_S = 20$ days in severe regime and respectively, $t_S = 50$ days in restriction regime.

For the analytical sizing models of the sanitary protection areas of higher complexity, only graphic-analytical solutions are given, which are difficult to use in practice due to the great computational effort as well as to the poor accuracy of results. Moreover, some of the calculus relations presented are erroneous.

From the authors' analyses, there has been noticed that some sizing relations of the sanitary protection areas for water catchments through perfect wells or drains from unconfined aquifers in container rocks with interstitial porosity without initial dynamics from study (Ordin nr.1278, 2011) are erroneous or insufficiently precise. Thus, we state:

- the equation of depression curve for the perfect catchment wells, in unconfined aquifers,

$$\left(h-h_0\right)^2=\left(H-h_0\right)^2\frac{\ln\left(l/r_0\right)}{\ln\left(R/r_0\right)} \qquad (1)$$

- the equation of sanitary protection distance for perfect drain in the aquifer level, without the initial growth rate,

$$D = \frac{k}{q} \cdot \left[\sqrt[3]{\left(\frac{3 \cdot q^2 \cdot t}{4 \cdot n_e \cdot k} + h_0^3 \right)^2} - h_0^2 \right] \quad (2)$$

- the equation of variation of relative depth η of the curve of depression depending on the time of transit, t for *perfect* drain in the aquifer free level, with the initial growth rate,

$$\frac{k \cdot i_0^2}{n_e \cdot H} \cdot t = \frac{\eta_0{}^2 - \eta^2}{2} - \eta_0 - \eta + \ln \frac{1 - \eta_0}{1 - \eta} \quad (3)$$

Imprecise relation for the Hoffman-Lillich method (rewritten for coherent measurements units for time t and filtration coefficient k) .

$$D = \frac{k \cdot i_m}{n_e} \cdot t \quad (4)$$

In this paper, the analytical models of sizing the sanitary protection areas for water catchments through perfect wells or drain from unconfined aquifers in container rocks with interstitial porosity without initial dynamics, are improved in order to find numerical solutions with a higher degree of accuracy.

MATERIAL AND METHOD

The fact that the formulas (*1*) and (*4*), for a perfect well in unconfined aquifer, and (*2*) and (*3*), for perfect drain in unconfined aquifer, are erroneous, has been emphasized first by covering in detail the steps of deducing the calculus relations for the sanitary protection distances.

The sanitary protection distance for the perfect well in unconfined aquifer

It has been considered the case in which the aquifer has no initial dynamics, which in practice is acceptable for slopes of the impermeable bed that satisfy the condition $i_0 < 0.003$.

The apparent velocity v, in an unconfined aquifer is given by the Darcy equation:

$$v = k \cdot i_h \quad (5)$$

where, in the case of a well, the hydraulic slope i_h has the following differential expression:

$$i_h = \frac{dh}{dx} \quad (6)$$

where h is the height of the water free level at distance x from the well's axis.

Out of the relations (*5*) and (*6*) the following differential expression results for the velocity v:

$$v = k \cdot \frac{dh}{dx} \quad (7)$$

Replacing in the continuity equation of well (*fig. 1*)

$$Q = 2\pi \cdot x \cdot h \cdot v \quad (8)$$

the expression (*8*) for velocity, we get:

$$Q = 2\pi \cdot k \cdot x \cdot h \cdot \frac{dh}{dx} \quad (9)$$

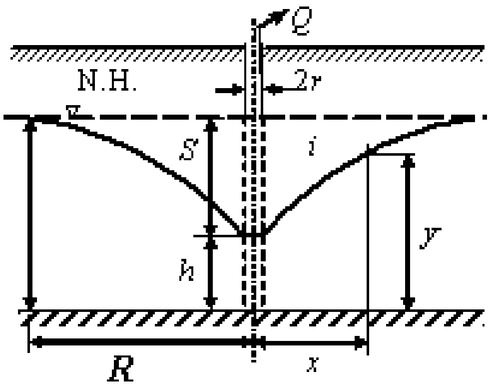

Fig. 1. Calculation chart for a perfect catchment well in unconfined aquifer without initial dynamics

From the relation above, we get the expression of the hydraulic slope dependent on the flow rate:

$$\frac{dh}{dx} = \frac{Q}{2\pi \cdot k \cdot x \cdot h} \quad (10)$$

Equation (*8*) may be rewritten in the form of a differential equation with separable variables x and h ,

$$Q \cdot \frac{dx}{x} = 2\pi \cdot k \cdot h \cdot dh \quad (11)$$

which by integration between the limits r_0, x and respectively, h_0, h,

$$Q \cdot \int_{r0}^{x} \frac{\mathrm{d}\xi}{\xi} = 2\pi \cdot k \cdot \int_{h0}^{h} \eta \cdot \mathrm{d}\eta \quad (12)$$

leads to the depression curve equation in the implicit form shown below:

$$\pi \cdot k \cdot \left(h^2 - h_0^2\right) - Q \cdot \left(\ln x - \ln r_0\right) = 0 \quad (13)$$

From (*10*) the depth h may be made explicit in this way:

$$h = \frac{1}{\sqrt{\pi \cdot k}} \sqrt{\pi \cdot k \cdot h_0^2 + Q \cdot \left(\ln x - \ln r_0\right)} \quad (14)$$

At the superior limit, the depth h is bounded by the depth corresponding to the hydrostatic level of the aquifer, H, and the coordinate x – by the influence radius of the well, R; inserting into the equation (*10*) these limits, that is h=H and x=R, the height h_0 and/or the flow rate Q may be developed in this way:

$$h_0 = \sqrt{H^2 - \frac{Q}{\pi \cdot k} \cdot \left(\ln R - \ln r_0\right)},$$

$$Q = \pi \cdot k \cdot \frac{H^2 - h_0{}^2}{\ln R - \ln r_0} \quad (15)$$

Replacing the expression (*15*) for the flow rate Q in equation (*11*) it gives:

$$h^2 - h_0^2 = \left(H^2 - h_0^2\right) \cdot \frac{\ln\left(x/r_0\right)}{\ln\left(R/r_0\right)} \qquad (16)$$

That is the correct relation which, for $x=l$, must replace the erroneous relation (*1*).

For the radius R, for unconfined aquifers, the empirical formula of Kusaikin is recommended (Luca O., 2000):

$$R = 575 \cdot \left(H - h_0\right) \cdot \sqrt{k \cdot H} \qquad (17)$$

Eliminating the depth h_0 from relations (8) and (9), the following equation has occurred with unknown R:

$$575\left[H - \sqrt{H^2 - \frac{Q}{\pi k}\ln\frac{R}{r_0}}\right]\sqrt{kH} - R = 0 \qquad (18)$$

equation which, usually is solved numerically.

Replacing expression (*15*) in eq. (*18*), it gives the following explicit form for the depression curve equation:

$$h = \frac{1}{\sqrt{\pi \cdot k}}\sqrt{\pi \cdot k \cdot H^2 - Q \cdot \left(\ln R - \ln x\right)} \qquad (19)$$

Considering the equation above, the equation of the hydraulic slope (9) becomes:

$$\frac{dh}{dx} = \frac{1}{2\sqrt{\pi \cdot k}} \cdot \frac{Q}{x\sqrt{\pi \cdot k \cdot H^2 - Q \cdot \left(\ln R - \ln x\right)}} \qquad (20)$$

In evaluating the distance covered by the underground stream in a period of time t, the actual speed of the stream is considered, v_e, which can be defined, either dependent on the apparent speed v, given by the relation *(7*), and, considering the actual porosity, n_e

$$v_e = \frac{v}{n_e}, \quad v_e = \frac{k}{n_e} \cdot \frac{dh}{dx} \qquad (21)$$

either dependent on the space x and time t,

$$v_e = dx/dt \qquad (22)$$

Eliminating speed v_e from the two relations above, the following differential relation occurs:

$$\frac{dx}{dt} = \frac{k}{n_e} \cdot \frac{dh}{dx} \qquad (23)$$

Inserting the relation (*21*) into the relation above, there results the following linear differential equation of the first degree:

$$\frac{dx}{dt} = \frac{\sqrt{k}}{2\sqrt{\pi} n_e} \cdot \frac{Q}{x\sqrt{\pi \cdot k \cdot H^2 - Q \cdot \ln R/x}} \qquad (24)$$

The sanitary protection distance D, corresponding to time t_S, is the particular solution of equation (*24*) corresponding the following initial and, respectively final, conditions:

$t = 0, x = r_0$ and $t = t_S, x = D$ (*25*)

Equation (24) with conditions (25) may be usually achieved only by numerical methods, for instance, through Runge-Kutta method of the fourth order.

The average slope i_m from the formula of the Hoffmann-Lillich method (4) is evaluated with the relation:

$$i_m = \frac{h_D - h_0}{D - r_0} \qquad (26)$$

where h_D represents the height of the water level at distance D from the well axis.

In study (Ordin nr.1278/20.04.2011) there is a graphic-analytical method of applying the Hoffmann-Lillich method, which is laborious and less precise. To improve this method, we have suggested the assessment of the heights h_0 şi h_D with the relations (*12*) and (*18*) and then, by eliminating the slope i_m from relations (*4*) and (*23*), the following equation has been deduced, which, if numerically solved, allows the direct determination of the distance D:

$$\begin{aligned}&\frac{\sqrt{\pi}}{t_S}\frac{n_e}{\sqrt{k}}D\left(D - r_0\right) - \sqrt{\pi k H^2 - Q\ln R/D} \\ &+ \sqrt{\pi k H^2 - Q\ln R/r_0} = 0\end{aligned} \qquad (27)$$

The sanitary protection distance for the perfect drain in unconfined aquifer without initial dynamics

The deduction of the analytical expression for the sanitary protection distance D, follows mostly the same steps as for the equation (*21*).

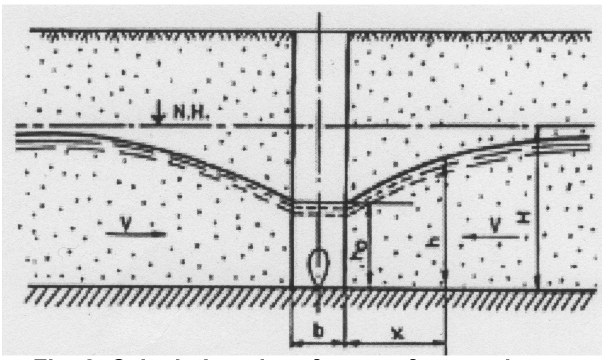

Fig. 2 Calculation chart for a perfect catchment drain in unconfined aquifer without initial dynamics.

Adopting the calculus diagram from (*fig. 2)*, for the velocity v the differential expression (*7*) is valid.

Replacing in the continuity equation:

$$q = h \cdot v$$

the expression (*7*), we get:

$$q = k \cdot y \cdot \frac{dh}{dx} \qquad (28)$$

where q is the specific flow rate.

From the relation above we get the expression of the slope dependant on the flow rate:

$$\frac{dh}{dx} = \frac{q}{k \cdot h} \qquad (29)$$

Equation (*29*) may be rewritten in the form of a differential equation with separable variables *x* and *h* ,

$$q \cdot dx = k \cdot h \cdot dh$$

which by integration between limits *B/2*, *x* and, respectively, h_0, *h*,

$$q \cdot \int_{B/2}^{x} d\xi = k \cdot \int_{h_0}^{h} \eta \cdot d\eta$$

leads to the depression curve equation in the implicit form shown below:

$$q \cdot \left(x - \frac{B}{2} \right) = \frac{k}{2}\left(y^2 - h_0^2 \right) \tag{30}$$

From equation (*27*), the depth *h*, may be developed as follows:

$$y = \sqrt{h_0^2 + \frac{q}{k} \cdot \left(2x - B \right)}\,, \tag{31}$$

Considering the equation above, the slope expression (*26*) becomes:

$$\frac{dh}{dx} = \frac{1}{\sqrt{h_0^2 + \frac{q}{k} \cdot \left(2x - B \right)}} \frac{q}{k} \tag{32}$$

Inserting the expression (*32*) into relation (*23*), for the slope dy/dx , the following linear differential equation of the first order occurs:

$$\frac{dx}{dt} = \frac{q}{n_e} \cdot \frac{1}{\sqrt{h_0^2 + \frac{q}{k} \cdot \left(2x - B \right)}} \tag{33}$$

The sanitary protection distance *D*, corresponding to time t_S , is the particular solution to equation (*33*) corresponding the following initial and, respectively final, conditions:

t=0, x=$B/2$ and t=t_S, x=D (34)

The differential equation (*33*) is with variables *t* and *x* separable and may be rewritten in the form:

$$\sqrt{h_0^2 + \frac{q}{k} \cdot \left(2x - B \right)} dx = \frac{q}{n_e} \cdot dt \tag{35}$$

which must be integrated with limits (*34*):

$$\int_{B/2}^{D} \sqrt{h_0^2 + \frac{q}{k} \cdot \left(2\xi - B \right)} d\xi = \frac{q}{n_e} \cdot \int_0^{t_S} d\tau \tag{36}$$

Solving the integrals above through exact analytical methods, the protection distance *D* may be developed as follows:

$$D = \frac{k}{2q} \left[\sqrt[3]{\left(\frac{3q^2}{kn_e} \cdot t_S + h_0^3 \right)^2} - h_0^2 \right] + \frac{B}{2} \tag{37}$$

It has been noticed that, even when neglecting the width *B*, the relation (*2*) is incorrect, leading to smaller values of the distance *D* of about 20%.

The sanitary protection distance for the perfect drain in unconfined aquifer with initial dynamics

In the case of a catchment drain in an aquifer with initial dynamics ((i_0> 0.003), The hydraulic slope *I,* has the following differential expression:

$$i_h = i_0 + \frac{dh}{dx} \tag{38}$$

Thus, considering relations (*5*) and (*28*), the following differential expressions arise for the speed *v* si and the specific flow rate *q* :

$$v = k \cdot \left(i_0 + dh/dx \right) \tag{39}$$

$$q = k \cdot h \cdot \left(i_0 + dh/dx \right) \tag{40}$$

Integrating equation (*40*) taking into account the relative depth η :

$$\eta = h/H \tag{41}$$

the equation of depression curve results (Ordin no.1278/20.04.2011) :

$$x = \frac{H}{i_0} \cdot \left(\eta_0 - \eta + \ln \frac{1 - \eta_0}{1 - \eta} \right) \tag{42}$$

Following the deduction process of equations (*20 ÷24*), we get the differential equation with separable variables *t* and *h* :

$$\frac{q}{k \cdot n_e} \cdot dt = \frac{h^2}{q - i_0 k \cdot h} dh \tag{43}$$

Integrating the above equation between the limits *0*, *t* and h_0, *h* through exact analytical methods, it follows the variation law of depth *h* depending on transit time *t*, developed in relation with variable *t:*

$$\begin{aligned} \frac{q}{n_e k} t = {} & \frac{h_0^2}{2 i_0 k} + \frac{q \cdot h_0}{\left(i_0 k \right)^2} + \frac{q^2}{\left(i_0 k \right)^3} \ln\left(q - i_0 k \cdot h_0 \right) \\ & - \left[\frac{h^2}{2 i_0 k} + \frac{q \cdot h}{\left(i_0 k \right)^2} + \frac{q^2}{\left(i_0 k \right)^3} \ln\left(q - i_0 k \cdot h \right) \right] \end{aligned} \tag{44}$$

Taking into account the relative depth (*41*), the above equation may be rephrased in this way:

$$\frac{k \cdot i_0^2}{n_e \cdot H} \cdot t = \frac{{\eta_0}^2 - \eta^2}{2} + \eta_0 - \eta + \ln \frac{1 - \eta_0}{1 - \eta} \tag{45}$$

so in eq. (*3*) the sign "-" in front of linear element η_0 is wrong.

RESULTS AND DISCUSSION

The mathematical models presented in the previous section have been applied in the complex ground water catchment system with both drains and wells from Timisesti – Zvoranesti.

Case study no.1: *well F4 from Zvoranesti-Timisesti catchment front*

The catchment front with small depth wells from Zvoranesti-Timisesti has been designed for the flow rate Q_{cap} = 250 l/s, and is situated in the bottom land from the right bank of the Moldova River, parallel to Timisesti drains, at about 1 km. downstream from them; it has the length , L= 2580 m, and consists in 30 drilled exploiting wells (F1, F2, …, F30 – 26 of them working), with depths between 11 and 12 meters.

The catchment well F4 is built by drilling until the confining bed (perfect well), with slope i_0= 1.5 ‰ < 3 ‰ (aquifer without initial dynamics).

The building-functional parameters of well F4 – r_0, Q and h_0 - as well as the hydrological parameters of the exploited aquifer – H, k, n_e – are written directly on the diagram from (*fig. 3*).

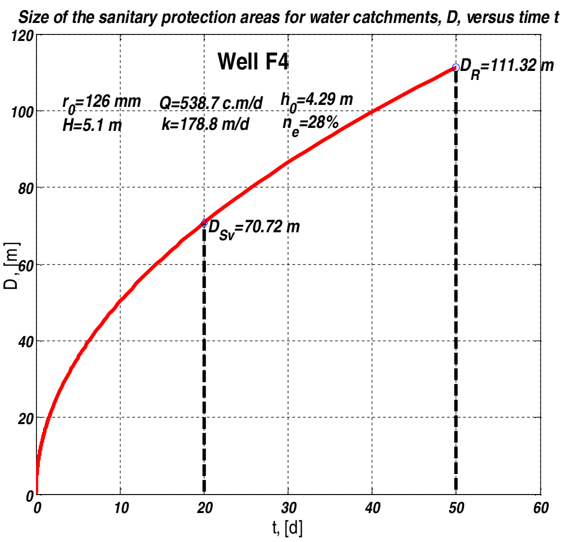

Fig. 3. Diagram of determination of sanitary protection distances for well F4

Numerically solving equation (*24*) with conditions (*25*) – successive for:

t=t_S=t_{Sv}=20 days and t=t_S=t_R=50 days (46)

the following values resulted:

D_{Sv}=7.52 m and D_R=12.53 m (47)

Thus, formula (*4*) has lead to the following under-evaluations of the sanitary protection radiuses: for the severe regime, 9.40 times; for the restriction regime, 8.88 times – so, evaluations eq. (*4*) may be considered to be inadmissibly small.

Case study no.2: *New Drain – Timisesti*

The New Drain of *Timisesti* catchment has been designed for flow rate Q_{DN} = 1200 l/s; it is built as an accessible drain gallery (*fig. 4*); it has a length of 4078 m, with 15 visiting chimneys (C1, C2, ..., C15) and a collecting well at the downstream end from which two water transport pipes lead to Iasi.

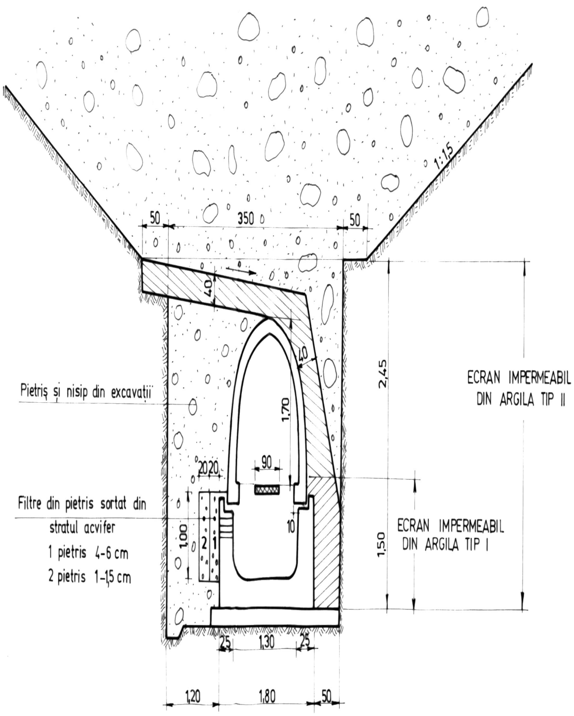

Fig 4. New Drain Timisesti. Transversal section through catchment drain (gallery)

In the section of visiting chimney *C14*, the drain is located on the impermeable base layer (perfect drain), which has the slope i_0= 1.65 ‰ < 3 ‰ (aquifer without initial dynamics); the following values have been determined for the geometrical, hydraulic and hydro-geological parameters, used as input data for the assessment of the sanitary protection distances (*2*) and (*37*):

B=1.3 m, h_0=0.304 m, q=3.07e-5 c.m/s·m

k=9.1e-4 m/s, n_e=30%, H=6.01 m

Considering successively conditions (*46*), the results are:

- with equation no (2):
  D_{Sv} = 78.08 m, D_R = 145.76 m; (*48*)
- with equation no (37):
  D_{Sv1} = 100.79 m, D_{R1} = 186.15 m.

So, formula (2) has lead to the following under-evaluations of the distances of sanitary protection: for the severe regime, 2.53%, for the restrictions regime, 21.69%.

CONCLUSION

- The depression curve equation of the perfect catchment wells in unconfined aquifers (*1*) is completely erroneous and must be substituted by relation (*13*).

- Coefficients of equation (*2*) for the determination of the sanitary protection distances for perfect drains in unconfined aquifers, with slope i_0 = 1.5

‰ < 3‰ are erroneous, this leading to under-evaluations of about 20%, so equation (*2*) must be replaced by equation (*37*).

- Equation (4) corresponding to Hoffman-Lillich method is very inaccurate, since in *Case study no.1*, comparing solutions (47) and (*48*), lead to under-evaluations of about 9 times: so, we suggest using instead the differential equation (*24*) with initial conditions (*25*).

- The sign "-" in front of the linear element η_0 from the variation law of the relative depth η dependent on transit time t is erroneous, therefore equation (*3*) must be replaced by the correct relation (*45*).

- The mathematical models to determine the sanitary protection distances for perfect drains and wells are suitable for accurate numerical solving, possible with the help of adequate automatic calculus programs. As a general conclusion, we recommend all engineers not to use calculus formulae, even if these are presented in official documents, without thoroughly checking them first, because, otherwise, designing errors may occur leading to major economic effects.

Notations:

B = width of drain on water surface corresponding the depth h_0, [L];

D_{Sv} = radius of sanitary protection area with severe regime, [L];

D_R = radius of sanitary protection area with restriction regime, [L];

h_0 = water depth (height) in the well for extracting the flow rate Q (or water depth in the drain for extracting the specific flow rate q), [L];

h_D = height of the water level at distance D from the well axis, [L];

H = thickness of unconfined aquifer, [L];

i_0 = hydraulic slope (hydraulic gradient) of the aquifer in natural regime, [-];

i_h = hydraulic slope of the depression curve,[-];

i_m = average slope of the depression curve on the interval [x_0, D],[-];

k = hydraulic conductivity, [LT^{-1}];

n_e = effective porosity, [-];

N.H. = hydrostatic level of the aquifer, [L];

r_0 = well's radius, [L];=influence radius of the well [L];

q = specific flow rate, [L^2T^{-1}];

Q = catchment flow rate, [L^3T^{-1}];

s = denivelarea in put, [L];

t = time, [T];

t_S = transit time left to cover the saturated area, [T];

t_{Sv} = transit time left to cover the saturated area, needed to size the sanitary protection area with severe regime, [T];

t_R = transit time left to cover the saturated area, needed to size the sanitary protection area with restriction regime, [T];

v = apparent velocity, [LT^{-1}];

v_e= actual (average) velocity of the underground stream, [LT^{-1}];

x = distance from the well's axis, [L];

η = relative depth for curve depression [-];

η_0 = relative depth corresponding to h_0 [-].

REFERENCES

Cojocaru D., Brânzilă M., Cojocaru P, **2013** - *Contributions to hydrogeological protection perimeter size calculation under severe and restraining regime*. Bul. Inst.Politehnic din Iaşi, Tomul LIX (LXIII), Fasc. 3, pag. 81-90, 2013.

Luca O., **2000 -** *Hidraulica mişcărilor permanente*. Eh. H.G.A., Bucuresti, 2000.

* * **Norme speciale privind caracterul şi mărimea zonelor de protecţie sanitară şi hidrogeologică*. Hotărârea nr. 930/11.08.2005.

* * **ORDIN nr.1278/20.04.2011 pentru aprobarea Instructiunilor privind delimitarea zonelor de protectie sanitara si a perimetrelor de protectie hidrogeologica*. Monitorul Oficial din 13.05.2011.