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Single phase second order sliding mode controller for complex interconnected systems with extended disturbances and unknown time-varying delays

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ABSTRACT

Novel results on complex interconnected time-delay systems with single phase second order sliding mode control is investigated. First, a reaching phase in traditional sliding mode control (TSMC) is removed by using a novel single phase switching manifold function. Next, a novel reduced order sliding mode observer (ROSMO) with lower dimension is suggested to estimate the unmeasurable variables of the plant. Then, a new single phase second order sliding mode controller (SPSOSMC) is established based on ROSMO tool to drive the state variables into the specified switching manifold from beginning of the motion and reduce the chattering in control input. Then, a stability condition is suggested based on the well-known linear matrix inequality (LMI) method to ensure the asymptotical stability of the whole plant. Finally, an illustrated example is simulated to validate the feasible application of the suggested technique.

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1. INTRODUCTION

In recent years, there has a growing the investigation attention in sliding mode control (SMC) theory and application for control systems. The outstanding features of SMC include strong robustness against uncertainties and disturbances, computational simplicity, and finite-time convergence. Owing to these advantages, SMC has been applied to solve many practical control plants, such as wind turbines, robotic manipulator, induction motor drives, and photovoltaic [1]–[3]. Unfortunately, the SMC suffers from undesired chattering phenomenon [4], which is very dangerous for actuators in practical models [4], especially for electro-mechanical systems. To reduce this issue, one of the widely used techniques is the second order sliding mode control (SOSMC) method, which was first introduced in the 1980s by [5]. This method is constructed by treating the derivative of the discontinuous fist order sliding mode controller in the traditional SMC (TSMC). Thus, the unwanted high frequency fluctuation phenomenon can be alleviated [6]. Although the SMC has the significant attainments [7]–[9], in overall, there are still three missions that should be settled for SMC design. These comprise: i) chattering-free: a new SMC is designed by eliminating the reaching phase in TSMC so that the complex interconnected systems not only guarantee robustness enhancement but also remove the unwanted chattering in control input; ii) unknown exogenous perturbations: an upper limit of the exogenous perturbations in practical control systems are difficult to get advance information; and iii) output feedback: a disadvantage of the existing researches is that all variables of the plants have to be available. This is invalid in many practical control plants.

For the above first mission which will weaken the undesired high frequency fluctuation in control signal, many studies based on SOSMC approach have been reported in the papers [7], [9]–[11]. A novel second-order sliding mode controller with a saturation level was built in [10] for the nonlinear SISO plant by using the adding a power integrator technique. In [7], a novel controller was designed by using a fractional switching surface for the nonlinear fractional-order systems. By means of the Lyapunov technique, an adaptive SOSMC law was investigated in [9] for the nonlinear systems. In [11], a state feedback controller with time-varying output constraint and mismatched term for the nonlinear dynamic system was constructed by employing power integrator technique. However, these researches did not cogitate the exogenous perturbations affected on the plants. This problem is also the second task which will be solved in this paper.

For the above second mission which will consider the unknown exogenous perturbations, this issue has been examined by researchers [8], [12]–[14]. In [12], a second order sliding mode algorithm was established to solve the nonlinear plants with disturbances limited by positive functions via a backstepping-like technique. Also, this approach, Wu *et al.* [14] suggested a new controller for systems subject to external disturbance. Based on super-twisting, twisting, or homogeneous procedures, Shi *et al.* [13] proposed a new variable-gain fixed-time SOSMC signal for a general class of uncertain nonlinear plants. Liu *et al.* [8] investigated a state feedback controller to solve the asymmetric output constraint problem by employing a power integrator and barrier Lyapunov function. Nevertheless, authors in these works assumed that the limit of external perturbations is positive function or constant. In addition, most of them require the availability of the states of the system, which cannot be ensured in practice because several system's states may be challenging/costly to measure. The limitations will crack in third mission.

For the above third mission which will perform the output feedback controller design, numerous publications of SOSMC laws with output information have been investigated in the research [15]–[17]. Koo *et al.* [15] explored an output-feedback controller based on an observer to stabilize a class of nonlinear large-scale plants with an unknown interconnection by employing a Takagi–Sugeno fuzzy typical. In [16], a new SMC methodology is stretched to a class of mismatched uncertain large-scale systems with the mismatching interconnections and unknown perturbations. However, the norm of system variables is bounded by constant. This condition may be difficult to satisfy for many practical systems. In [17], a novel direct adaptive fuzzy controller based on an observer was constructed for a certain class of high order unknown nonlinear dynamical plants with unmeasurable states. Regrettably, these researches have been showed the SOSMC law based on full order observer with large dimension, which rises the calculation of burden because of the associated closed-loop plants. Further, these researches only ruminate the small systems. This motivated our study to develop decentralized output feedback control scheme for the complex interconnected plants.

Inspired by the above observations, to the best of our knowledge, little devotion has been paid to getting the chattering removal and stabilization control problems for the complex interconnected plants with unknown time-varying delays and external perturbations, which is still open in the literature. In this work, we suggest a reduced order sliding mode observer (ROSMO) based single phase second order sliding mode controller for complex interconnected plants which guarantees the robustness enhancement of the plant and eliminate the unwanted high frequency vacillation in control input. Besides, in the sliding mode, a sufficient condition to asymptotically stabilize the closed-loop systems is given by using well-known linear matrix inequality (LMI) method. Finally, by numerical illustration, the rationality of the suggested ideas, methods, and measures are displayed.

2. MODEL DESCRIPTION OF THE PLANT AND PROBLEM FORMULATION

In this study, we cogitate a class of the complex interconnected plants with unknown time-varying delays consisting of L interconnected subsystems modelled as (1),

$$\begin{aligned} \dot{x}_i(t) &= [A_i + \Delta A_i(t)]x_i(t) + [A_{id} + \Delta A_{id}(t)]x_{id}(t) + B_i[u_i(t) + \zeta_i(x_i(t), x_{id}, t)] + \\ &\quad \textstyle\sum_{j=1, j\neq i}^{L} F_{ij} x_{jd}(t), \\ y_i(t) &= C_i x_i(t) \text{ with } \mathrm{x}_{id} = x_i(t - d_i(t)) \text{ and } \mathrm{x}_i(t) = \chi_i(t) \;\; for \;\; -\bar{d}_i \le t < 0, \end{aligned} \tag{1}$$

where $x_i(t) \in R^{n_i}, y_i(t) \in R^{p_i}, u_i(t) \in R^{m_i}$ are the system variables vector, the output vector, and the control signal, respectively. x_{id} is a delayed state where $d_i := d_i(t)$ is the time-varying delay which is assumed to be unknown, nonnegative and constrained in $\Re^+$; that is, $\bar{d}_i := sup_{t\in\Re^+}[\, d(t)] < \infty$. The constant matrices A_i, A_{id}, B_i, C_i and F_{ij} have appropriate dimensions. A matched nonlinearity of the system is indicated by $\zeta_i(x_i(t), x_{id}, t)$. The symbol $\chi_i(t)$ is a differentiable vector-valued initial function on $[-\bar{d}_i, 0]$.

The mismatched parameter uncertainties of the plant which include $\Delta A_{ii}(t)$ and $\Delta A_{id}(t)$ gratify the form $[\Delta A_i(t)\ \Delta A_{id}(t)] = [D_i\Sigma_i(x_i(t),t)E_i\ D_{id}\Sigma_{id}(x_i(t),x_{id},t)E_{id}]$, where D_i, E_i, D_{id}, E_{id} are known constant matrices and $\Sigma_i(x_i(t),t), \Sigma_{id}(x_i(t),x_{id},t)$ are unknown functions but limited as $\|\Sigma_i(x_i(t),t)\| \le 1$ and $\|\Sigma_{id}(x_i(t),x_{id},t)\| \le 1$. To build a novel chattering-free sliding mode controller, we will convert the original scheme (1) into a new regular form. Based on the attained results in paper [18], we have $\Gamma_i = I_i - E_{ii}^g E_{ii}$, where Γ_i is symmetric matrix with $n_i \times n_i$ dimension and the Moore-Penrose inverse of the E_{ii} is achieved as E_{ii}^g. In mismatching condition, $B_i^{\perp T} D_{ii} \neq 0$, where $B_i^{\perp}$ is an null space of the matrix B_i. Now, we design a single phase switching manifold function for the complex interconnected plants (1) as (2):

$$s_i(y_i(t),t) = \dot{\sigma}_i(y_i(t),t) + \bar{R}_i\sigma_i(y_i(t),t), \tag{2}$$

where $\sigma_i(y_i(t),t) = \bar{\sigma}_i(y_i,t) - \bar{\sigma}_i(y_i,0)\,exp(-\alpha_i t)$, $\bar{\sigma}_i(y_i(t),t) = S_i x_i(t) = F_i y_i(t)$, a diagonal matrix $\bar{R}_i \in R^{m_i \times m_i}$, $\alpha_i > 0$, and $\dot{\sigma}_i(y_i(t),t)$ is the differentiating of $\sigma_i(y_i(t),t)$. In addition, a matrix $S_i = K_i B_i^T(\Gamma_i P_i \Gamma_i + B_i Q_i B_i^T)^{-1} = K_i B_i^T T_i^{-1}$, where K_i is any nonsingular matrix and the symmetric matrices P_i, Q_i are answers of the LMIs: $\Gamma_i P_i \Gamma_i + B_i Q_i B_i^T > 0$ and $B_i^{\perp T}(A_i\,\Gamma_i P_i \Gamma_i + \Gamma_i Q_i \Gamma_i A_i^T)B_i^{\perp} < 0$. The matrix F_i is chosen matrix such that $S_i = F_i C_i$ is resolvable. Now, the state transformation matrix $[\vartheta_i\ \sigma_i]^T = M_i x_i$ and $[\vartheta_{id}\ \sigma_{id}]^T = M_i x_{id}$ will be used to convert the complex interconnected systems (1) into a regular form. The transformation matrix $M_i = [B_i^{\perp T}\ K_i B_i^T T_i^{-1}]$ and its inverse matrix $M_i^{-1} = [T_i B_i^{\perp}(B_i^{\perp T} T_i B_i^{\perp})^{-1}\ B_i(S_i B_i)^{-1}]$. We replace the transformation matrix into the system (1) as (3):

$$\begin{aligned}
\dot{\vartheta}_i(t) &= [\bar{A}_{ii11} + \Delta\bar{A}_{ii11}]\vartheta_i(t) + [\bar{A}_{ii12} + \Delta\bar{A}_{ii12}]\sigma_i(t) + [\bar{A}_{ii11d} + \Delta\bar{A}_{ii11d}]\vartheta_{id}(t) \\
&\quad + [\bar{A}_{ii12d} + \Delta\bar{A}_{ii12d}]\sigma_{id}(t) + \textstyle\sum_{j=1,j\neq i}^{L}\big[\bar{F}_{ij11}\vartheta_{jd}(t) + \bar{F}_{ij12}\sigma_{jd}(t)\big], \\
\dot{\sigma}_i(t) &= [\bar{A}_{ii21} + \Delta\bar{A}_{ii21}]\vartheta_i(t) + [\bar{A}_{ii22} + \Delta\bar{A}_{ii22}]\sigma_i(t) + [\bar{A}_{ii21d} + \Delta\bar{A}_{ii21d}]\vartheta_{id}(t) \\
&\quad + [\bar{A}_{ii22d} + \Delta\bar{A}_{ii22d}]\sigma_{id}(t) + (S_i B_i)[u_i(t) + \zeta_i(x_i(t),x_{id},t)] \\
&\quad + \textstyle\sum_{j=1,j\neq i}^{L}\big[\bar{F}_{ij21}\vartheta_{jd} + \bar{F}_{ij22}\sigma_{jd}\big],
\end{aligned} \tag{3}$$

where

$$\begin{aligned}
&\bar{A}_{ii11} + \Delta\bar{A}_{ii11} = B_i^{\perp T}[A_i + D_i\Sigma_i E_i]T_i B_i^{\perp}(B_i^{\perp T} T_i B_i^{\perp})^{-1}, \\
&\bar{A}_{ii12} + \Delta\bar{A}_{ii12} = B_i^{\perp T}[A_i + D_i\Sigma_i E_i]B_i(S_i B_i)^{-1}, \\
&\bar{A}_{ii21} + \Delta\bar{A}_{ii21} = K_i B_i^T T_i^{-1}[A_i + D_i\Sigma_i E_i]T_i B_i^{\perp}(B_i^{\perp T} T_i B_i^{\perp})^{-1}, \\
&\bar{A}_{ii22} + \Delta\bar{A}_{ii22} = K_i B_i^T T_i^{-1}[A_i + D_i\Sigma_i E_i]B_i(S_i B_i)^{-1}, \\
&\bar{A}_{ii11d} + \Delta\bar{A}_{ii11d} = B_i^{\perp T}[A_{id} + D_{id}\Sigma_{id}E_{id}]T_i B_i^{\perp}(B_i^{\perp T} T_i B_i^{\perp})^{-1}, \\
&\bar{A}_{ii12d} + \Delta\bar{A}_{ii12d} = B_i^{\perp T}[A_{id} + D_{id}\Sigma_{id}E_{id}]B_i(S_i B_i)^{-1}, \\
&\bar{A}_{ii21d} + \Delta\bar{A}_{ii21d} = K_i B_i^T T_i^{-1}[A_{id} + D_{id}\Sigma_{id}E_{id}]T_i B_i^{\perp}(B_i^{\perp T} T_i B_i^{\perp})^{-1}, \\
&\bar{A}_{ii22d} + \Delta\bar{A}_{ii22d} = K_i B_i^T T_i^{-1}[A_{id} + D_{id}\Sigma_{id}E_{id}]B_i(S_i B_i)^{-1}, \\
&\bar{F}_{ij11} = B_i^{\perp T} K_{ij} T_i B_i^{\perp}(B_i^{\perp T} T_i B_i^{\perp})^{-1}, \bar{F}_{ij12} = B_i^{\perp T} K_{ij} B_i(S_i B_i)^{-1}, \\
&\bar{F}_{ij21} = S_i K_{ij} T_i B_i^{\perp}(B_i^{\perp T} T_i B_i^{\perp})^{-1}, \bar{F}_{ij22} = S_i K_{ij} B_i(S_i B_i)^{-1}, \vartheta_i = B_i^{\perp T} x_i, \vartheta_{id} = B_i^{\perp T} x_{id}.
\end{aligned}$$

Following the results in paper [18], we get $\Delta\bar{A}_{ii11}(t) = \Delta\bar{A}_{ii21}(t) = \Delta\bar{A}_{ii11d}(t) = \Delta\bar{A}_{ii21d}(t) = 0$. Thus, we can rewrite as (4).

$$\begin{aligned}
\dot{\vartheta}_i(t) &= \bar{A}_{ii11}\vartheta_i + [\bar{A}_{ii12} + \Delta\bar{A}_{ii12}]\sigma_i + \bar{A}_{ii11d}\vartheta_{id} + [\bar{A}_{ii12d} + \Delta\bar{A}_{ii12d}]\sigma_{id} \\
&\quad + \textstyle\sum_{j=1,j\neq i}^{L}\big[\bar{F}_{ij11}\vartheta_{jd} + \bar{F}_{ij12}\sigma_{jd}\big] \\
\dot{\sigma}_i(t) &= \bar{A}_{ii21}\vartheta_i + [\bar{A}_{ii22} + \Delta\bar{A}_{ii22}]\sigma_i + \bar{A}_{ii21d}\vartheta_{id} + [\bar{A}_{ii22d} + \Delta\bar{A}_{ii22d}]\sigma_{id} \\
&\quad + (S_i B_i)[u_i(t) + \zeta_i(x_i(t),x_{id},t)] + \textstyle\sum_{j=1,j\neq i}^{L}\big[\bar{F}_{ij21}\vartheta_{jd} + \bar{F}_{ij22}\sigma_{jd}\big].
\end{aligned} \tag{4}$$

3. MAIN RESULTS

3.1. A novel reduced-order sliding mode observer construction

To construct a new chattering-free output feedback SOSMC for complex interconnected systems (1), a novel ROSMO will be designed to guess the unmeasurable states of the plant as (5):

$$\dot{\hat{\vartheta}}_i(t) = \bar{A}_{ii11}\hat{\vartheta}_i(t) + \bar{A}_{ii12}\sigma_i(t) + \bar{A}_{ii11d}\hat{\vartheta}_{id}(t) + \bar{A}_{ii12d}\sigma_{id}(t), \tag{5}$$

where $\hat{\vartheta}_i(t)$ and $\hat{\vartheta}_{id}(t)$ are estimation vectors of $\vartheta_i(t)$ and $\vartheta_{id}(t)$, respectively. The dynamics of its errors $\tilde{\vartheta}_i(t) = \hat{\vartheta}_i(t) - \vartheta_i(t)$ and $\tilde{\vartheta}_{id}(t) = \hat{\vartheta}_{id}(t) - \vartheta_{id}(t)$ are governed by the equation.

$$\begin{aligned}\dot{\tilde{\vartheta}}(t) = {} & \bar{A}_{ii11}\tilde{\vartheta}_i + \bar{A}_{ii11d}\tilde{\vartheta}_{id} - \Delta\bar{A}_{ii12}\sigma_i - \Delta\bar{A}_{ii12d}\sigma_{id} + \textstyle\sum_{j=1,j\neq i}^{L}\big[\bar{F}_{ij11}\tilde{\vartheta}_{jd}\big] \\ & - \textstyle\sum_{j=1,j\neq i}^{L}\big[\bar{F}_{ij11}\hat{\vartheta}_{jd} + \bar{F}_{ij12}\sigma_{jd}\big] \end{aligned} \tag{6}$$

By using (3) and $\sum_{j=1,j\neq i}^{L}\left[\bar{F}_{ij11}\tilde{\vartheta}_{jd} - \bar{F}_{ij11}\hat{\vartheta}_{jd} - \bar{F}_{ij12}\sigma_{jd}\right] = \sum_{j=1,j\neq i}^{L}\left[\bar{F}_{ji11}\tilde{\vartheta}_{1d} - \bar{F}_{ji11}\hat{\vartheta}_{id} - \bar{F}_{ji12}\sigma_{id}\right]$, we have (7).

$$\begin{aligned}\dot{\tilde{\vartheta}}(t) = {} & B_i^{\perp T}A_iT_iB_i^{\perp}(B_i^{\perp T}T_iB_i^{\perp})^{-1}\tilde{\vartheta}_i + B_i^{\perp T}A_{id}T_iB_i^{\perp}(B_i^{\perp T}T_iB_i^{\perp})^{-1}\tilde{\vartheta}_{id} \\ & -B_i^{\perp T}D_i\Sigma_iE_iB_i(S_iB_i)^{-1}\sigma_i - B_i^{\perp T}D_{id}\Sigma_{id}(x_{id},t)E_{id}B_i(S_iB_i)^{-1}\sigma_{id} \\ & + \textstyle\sum_{j=1,j\neq i}^{L}\big[\bar{F}_{ij11}\tilde{\vartheta}_{jd}\big] - \sum_{j=1,j\neq i}^{L}\big[\bar{F}_{ij11}\hat{\vartheta}_{jd} + \bar{F}_{ij12}\sigma_{jd}\big]. \end{aligned} \tag{7}$$

Now, we will propose a new theorem which introduces the upper limit of the error dynamics as:
Theorem 1. Let $\tilde{\varpi}_i(t)$ be an upper limit of the error $\left\|\tilde{\vartheta}_i(t)\right\|$. And $\tilde{\varpi}_i(t)$ is answer of the form

$$\begin{aligned}\dot{\tilde{\varpi}}_i(t) = {} & \delta_i\tilde{\varpi}_i(t) + \tilde{\varepsilon}_i\tilde{\eta}_{1i}\{[\|B_i^{\perp T}D_i\|\|E_iB_i(S_iB_i)^{-1}\| + \tilde{\eta}_{2i}\|B_i^{\perp T}D_{id}\|\|E_{id}B_i(S_iB_i)^{-1}\|]\|\sigma_i(t)\| \\ & + \textstyle\sum_{j=1,j\neq i}^{L}\big[\tilde{\eta}_{3i}\|\bar{F}_{ji11}\|\|\hat{\vartheta}_i(t)\| + \tilde{\eta}_{2i}\|\bar{F}_{ji12}\|\|\sigma_i(t)\|\big]\}, \end{aligned} \tag{8}$$

where $\delta_i = \lambda_{max} + \tilde{\varepsilon}_i\tilde{\eta}_{1i}[\|\bar{A}_{ii11d}\| + \|\bar{F}_{ji11}\|] < 0$, λ_{max} is maximum eigenvalue of $\bar{A}_{ii11}$, and $\tilde{\varepsilon}_i > 0$, $\tilde{\eta}_{1i} > 1$, $\tilde{\eta}_{2i} > 1, \tilde{\eta}_{3i} > 1$. The initial situation $\tilde{\varpi}_i(0) \geq \tilde{\varepsilon}_i\|\tilde{\vartheta}_i(0)\| > 0$, where $\tilde{\vartheta}_i(0)$ is an initial condition of the error.
Proof of Theorem 1. Surveyed the paper [19], $\bar{A}_{ii11}$ and $\bar{A}_{ii11d}$ are stable matrices. Thus, we have $\|exp[\,\bar{A}_{ii11}\,t]\| \leq \tilde{\varepsilon}_i\,exp(\,\lambda_{max}$where $\tilde{\varepsilon}_i > 0$. By expressing the (7) to produces:

$$\begin{aligned}\|\tilde{\vartheta}_i(t)\| \leq {} & \|exp(\bar{A}_{ii11}\,t)\|\|\tilde{\vartheta}_i(0)\| + \int_0^t \|exp[\bar{A}_{ii11}\,(t-\tau)]\|\,\big[\|\bar{A}_{ii11d}\|\|\tilde{\vartheta}_{id}(\tau)\| \\ & +\|B_i^{\perp T}D_i\Sigma_i(x_i,t)E_iB_i(S_iB_i)^{-1}\|\|\sigma_i(\tau)\| + \|B_i^{\perp T}D_{id}\Sigma_{id}(x_{id},t)E_{id}B_i(S_iB_i)^{-1}\|\|\sigma_{id}(\tau)\| \\ & + \textstyle\sum_{j=1,j\neq i}^{L}\big(\|\bar{F}_{ij11}\|\|\tilde{\vartheta}_{jd}\|\big) + \sum_{j=1,j\neq i}^{L}\big(\|\bar{F}_{ij11}\|\|\hat{\vartheta}_{jd}\| + \|\bar{F}_{ij12}\|\|\sigma_{jd}\|\big)\big]d\tau, \end{aligned} \tag{9}$$

Following the Lemma 3 in the study [20], we get $\|\tilde{\vartheta}_{id}\| \leq \tilde{\eta}_{1i}\|\tilde{\vartheta}_i\|, \|\sigma_{id}\| \leq \tilde{\eta}_{2i}\|\sigma_i\|$, and $\|\hat{\vartheta}_{id}\| \leq \tilde{\eta}_{3i}\|\hat{\vartheta}_i\|$ with $\tilde{\eta}_{1i} > 1, \tilde{\eta}_{2i} > 1, \tilde{\eta}_{3i} > 1$. Next, the two sides of the inequality (9) are multiplied by $exp(-\lambda_{max_i}\ \ t)$ and then shift them to the right-hand side term, (9) can be denoted as (10):

$$\begin{aligned}& \|\tilde{\vartheta}_i(t)\| \leq \tilde{\varpi}_i(0)\,exp\big[\big(\lambda_{max_i} + \tilde{\varepsilon}_i\tilde{\eta}_1\|\bar{A}_{ii11d}\|\big)t\big] + \\ & \textstyle\int_0^t \tilde{\varepsilon}_i\tilde{\eta}_1\,exp\big[\big(\lambda_{max_i} + \tilde{\varepsilon}_i\tilde{\eta}_1\|\bar{A}_{ii11d}\|\big)(t-\tau)\big] \\ & \times\{[\|B_i^{\perp T}D_i\|\|E_iB_i(S_iB_i)^{-1}\| + \tilde{\eta}_{2i}\|B_i^{\perp T}D_{id}\|\|E_{id}B_i(S_iB_i)^{-1}\|]\|\sigma_i(\tau)\| \\ & + \textstyle\sum_{j=1,j\neq i}^{L}\big[\tilde{\eta}_{3i}\|\bar{F}_{ji11}\|\|\hat{\vartheta}_i\| + \tilde{\eta}_{2i}\|\bar{F}_{ji12}\|\|\sigma_i\|\big]\}d\tau = \tilde{\varpi}_i(t), \end{aligned} \tag{10}$$

where $\tilde{\varpi}_i(t)$ is solution of (8). Therefore, we can determine that $\|\tilde{\vartheta}_i(t)\| \leq \tilde{\varpi}_i(t)$ for all time. Theorem 1 is proved entirely.

3.2. Design of a single phase second order sliding mode controller

In this part, a new proposed controller is created based on the suggested observer in the above part. Firstly, by using (1) and the transformation matrix M_i, differentiating $\dot{\sigma}_i(y_i(t),t)$ can be described as (11):

$$\begin{aligned}\dot{\sigma}_i(t) = {} & S_iA_iT_iB_i^{\perp}(B_i^{\perp T}T_iB_i^{\perp})^{-1}\vartheta_i + S_iA_iB_i(S_iB_i)^{-1}\sigma_i + S_iA_{id}T_iB_i^{\perp}(B_i^{\perp T}T_iB_i^{\perp})^{-1}\vartheta_{id} \\ & + S_iA_{id}B_i \times (S_iB_i)^{-1}\sigma_{id} + (S_iB_i)u_i(t) + \textstyle\sum_{j=1,j\neq i}^{L}S_iF_{ji}\Big[T_jB_j^{\perp}\big(B_j^{\perp T}T_jB_j^{\perp}\big)^{-1}\vartheta_{id} + \\ & + B_j\big(S_jB_j\big)^{-1}\sigma_{id}\Big] + \psi_i(t) + \alpha_i\bar{\sigma}_i(y_i,0)\,exp(-\alpha_it), \end{aligned} \tag{11}$$

where $\psi_i(t) = S_i\Delta A_{ii}(t)T_iB_i^{\perp}(B_i^{\perp T}T_iB_i^{\perp})^{-1}\vartheta_i(t) + S_i\Delta A_{id}(t)T_iB_i^{\perp}(B_i^{\perp T}T_iB_i^{\perp})^{-1}\vartheta_{id}(t) +$
$S_i\Delta A_{ii}(t)B_i(S_iB_i)^{-1}\sigma_i(t) + S_i\Delta A_{id}(t)B_i(S_iB_i)^{-1}\sigma_{id}(t) + (S_iB_i)\zeta_i(x_i(t),x_{id},t)$.

The second order derivative of $\sigma_i(y_i(t),t)$ is specified as (12).

$$\ddot{\sigma}_i(t) = S_iA_iT_iB_i^{\perp}(B_i^{\perp T}T_iB_i^{\perp})^{-1}\dot{\vartheta}_i + S_iA_iB_i(S_iB_i)^{-1}\dot{\sigma}_i + S_iA_{id}T_iB_i^{\perp}(B_i^{\perp T}T_iB_i^{\perp})^{-1}\dot{\vartheta}_{id} + S_iA_{id}B_i \times (S_iB_i)^{-1}\dot{\sigma}_{id} + \sum_{j=1,j\neq i}^{L} S_iF_{ji}\left[T_jB_j^{\perp}\left(B_j^{\perp T}T_jB_j^{\perp}\right)^{-1}\dot{\vartheta}_{id} + B_j\left(S_jB_j\right)^{-1}\dot{\sigma}_{id}\right] + (S_iB_i)\dot{u}_i(t) + \dot{\psi}_i(t) - \alpha_i^2\bar{\sigma}_i(y_i,0)\, e\,xp(-\alpha_i t). \tag{12}$$

Differentiating the switching manifold function (2) and combining the (11), (12), we obtain:

$$\dot{s}_i(t) = S_iA_iT_iB_i^{\perp}(B_i^{\perp T}T_iB_i^{\perp})^{-1}\dot{\vartheta}_i(t) + S_iA_iB_i(S_iB_i)^{-1}\dot{\sigma}_i(t) + S_iA_{id}T_iB_i^{\perp}(B_i^{\perp T}T_iB_i^{\perp})^{-1}\dot{\vartheta}_{id}(t) + S_iA_{id}B_i(S_iB_i)^{-1}\dot{\sigma}_{id}(t) + \sum_{j=1,j\neq i}^{L} S_iF_{ji}\left[T_jB_j^{\perp}\left(B_j^{\perp T}T_jB_j^{\perp}\right)^{-1}\dot{\vartheta}_{id}(t) + B_j\left(S_jB_j\right)^{-1}\dot{\sigma}_{id}(t)\right] + (S_iB_i)\dot{u}_i(t) + \dot{\psi}_i(t) + \bar{R}_i\dot{\bar{\sigma}}_i(y_i,t) + [\bar{R}_i\alpha_i - \alpha_i^2]\bar{\sigma}_i(y_i,0)\, exp(-\alpha_i t). \tag{13}$$

Secondly, an exogenous perturbation $\dot{\psi}_i(t)$ in (13) is assumed to satisfy the constraint $\|\dot{\psi}_i(t)\| \leq \sum_{r=0}^{r}[\upsilon_{ri}(\|x_i\|)^r]$, where r is the perturbation's order and $\upsilon_{ri} > 0$. In the real control applications, the disturbance order r and the constants υ_{ri} in the plant are unknown due to complication of the plant configuration. Now, we are in position to show that the system's trajectories hit the sliding manifold (2) from beginning of the motion. With these aims, a new control signal is established as (14):

$$\dot{u}_i(t) = (S_iB_i)^{-1}\{\tilde{\rho}_{1i}[\|\hat{\vartheta}_i\| + \tilde{\varpi}_i] + \tilde{\rho}_{2i}\|\sigma_i(t)\| + \tilde{\rho}_{3i}\|\dot{\sigma}_i\| + \tilde{\alpha}_i\|s_i(t)\| + \bar{\kappa}_1\sum_{j=1,j\neq i}^{L}[\tilde{\eta}_{4i}\|\bar{F}_{ji11}\|\left(\|\hat{\vartheta}_i\| + \tilde{\varpi}_i\right) + \tilde{\eta}_{2i}\|\bar{F}_{ji12}\|\|\sigma_i\|] + \sum_{j=1,j\neq i}^{L}[\tilde{\rho}_{4i}\left(\|\hat{\vartheta}_i(t)\| + \tilde{\varpi}_i(t)\right) + \tilde{\rho}_{5i}\|\sigma_i(t)\|] + \bar{\kappa}_2\sum_{j=1,j\neq i}^{L}[\tilde{\eta}_{4i}\|\bar{F}_{ji11}\|\left(\|\hat{\vartheta}_i\| + \tilde{\varpi}_i\right) + \tilde{\eta}_{2i}\|\bar{F}_{ji12}\|\|\sigma_i\|] + \tilde{\rho}_{6i}\|\dot{\sigma}_i\| + \tilde{\rho}_{7i} + \|\bar{R}_i\|\|\dot{\bar{\sigma}}_i\| + \|[\bar{R}_i\alpha_i - \alpha_i^2]\|\|\bar{\sigma}_i(y_i,0)\|\, exp(-\alpha_i t)\}sign(s_i(t)), \tag{14}$$

where $\tilde{\alpha}_i > 0$ and $\tilde{\rho}_{1i}, \tilde{\rho}_{2i}, \tilde{\rho}_{3i}, \tilde{\rho}_{4i}, \tilde{\rho}_{5i}, \tilde{\rho}_{6i}, \tilde{\rho}_{7i}$ are gains of control which will be found later.

Theorem 2. Study the interconnected system (1) with the proposed controller (14) where the switching manifold is demarcated by (2). Then, the state variables of (1) forward to the switching manifold (2) in finite time and stay on sliding mode under the controller (14) when scalar gains filling the following settings:

$$\tilde{\rho}_{1i} \geq [\|S_iA_iT_iB_i^{\perp}(B_i^{\perp T}T_iB_i^{\perp})^{-1}\| + \tilde{\eta}_{4i}\|S_iA_{id}T_iB_i^{\perp}(B_i^{\perp T}T_iB_i^{\perp})^{-1}\|][\|\bar{A}_{ii11}\| + \tilde{\eta}_{4i}\|\bar{A}_{ii11d}\|],$$
$$\tilde{\rho}_{2i} \geq [\|S_iA_iT_iB_i^{\perp}(B_i^{\perp T}T_iB_i^{\perp})^{-1}\| + \tilde{\eta}_{4i}\|S_iA_{id}T_iB_i^{\perp}(B_i^{\perp T}T_iB_i^{\perp})^{-1}\|][\|\bar{A}_{ii12}\| + \|B_i^{\perp T}D_i\| \times \|E_iB_i(S_iB_i)^{-1}\| + \tilde{\eta}_{2i}\|\bar{A}_{ii12d}\| + \tilde{\eta}_{2i}\|B_i^{\perp T}D_{id}\|\|E_{id}B_i(S_iB_i)^{-1}\|], \tilde{\rho}_{3i} \geq \|S_iA_iB_i(S_iB_i)^{-1}\| + \tilde{\eta}_{2i}\|S_iA_{id}B_i(S_iB_i)^{-1}\|, \tilde{\rho}_{4i} \geq \tilde{\eta}_{4i}\|S_i\|\|F_{ji}\|\left\|T_jB_j^{\perp}\left(B_j^{\perp T}T_jB_j^{\perp}\right)^{-1}\right\|[\|\bar{A}_{ii11}\| + \|\bar{A}_{ii11d}\|],$$
$$\tilde{\rho}_{5i} \geq \tilde{\eta}_{4i}\|S_i\|\|F_{ji}\|\left\|T_jB_j^{\perp}\left(B_j^{\perp T}T_jB_j^{\perp}\right)^{-1}\right\|[\|\bar{A}_{ii12}\| + \|B_i^{\perp T}D_i\|\|E_iB_i(S_iB_i)^{-1}\| + \tilde{\eta}_{2i}\|\bar{A}_{ii12d}\| + \tilde{\eta}_{2i}\|B_i^{\perp T}D_{id}\|\|E_{id}B_i(S_iB_i)^{-1}\|],$$
$$\tilde{\rho}_{6i} \geq \tilde{\eta}_{2i}\|S_i\|\|F_{ji}\|\left\|B_j\left(S_jB_j\right)^{-1}\right\|, \tilde{\rho}_{7i} \geq \|\dot{\psi}_i(t)\| \tag{15}$$

Proof of Theorem 2. We study the positive definite function as $V_i(s_i) = \sum_{i=1}^{L}\|s_i(y_i(t),t)\|$, where $s_i(y_i(t),t)$ is switching manifold as (2). By differentiating $V_i(s_i)$ and using the (13), we get (16).

$$\dot{V}_i(s_i) = \sum_{i=1}^{L}\frac{s_i(t)}{\|s_i(t)\|}\{S_iA_iT_iB_i^{\perp}(B_i^{\perp T}T_iB_i^{\perp})^{-1}\dot{\vartheta}_i(t) + S_iA_iB_i(S_iB_i)^{-1}\dot{\sigma}_i(t) + S_iA_{id}T_iB_i^{\perp}(B_i^{\perp T}T_iB_i^{\perp})^{-1} \times \dot{\vartheta}_{id}(t) + S_iA_{id}B_i(S_iB_i)^{-1}\dot{\sigma}_{id}(t) + \sum_{j=1,j\neq i}^{L} S_iF_{ij}[T_iB_i^{\perp}(B_i^{\perp T}T_iB_i^{\perp})^{-1}\dot{\vartheta}_{jd}(t) + B_i(S_iB_i)^{-1} \times \dot{\sigma}_{jd}(t)] + (S_iB_i)\dot{u}_i(t) + \dot{\psi}_i(t) + \bar{R}_i\dot{\bar{\sigma}}_i(y_i,t) + [\bar{R}_i\alpha_i - \alpha_i^2]\bar{\sigma}_i(y_i,0)\, exp(-\alpha_i t)\} \tag{16}$$

Rendering to the Lemma 3 of the work [20], we attain $\|\sigma_{id}(t)\| \leq \tilde{\eta}_{i2}\|\sigma_i(t)\|$ and $\|\vartheta_{id}(t)\| \leq \tilde{\eta}_{4i}\|\vartheta_i(t)\|$, where $\tilde{\eta}_{i2} > 1, \tilde{\eta}_{4i} > 1$. Besides, following the property of the norm, we achieve $\|\vartheta_i\| \leq \|\hat{\vartheta}_i(t)\| + \tilde{\varpi}_i(t)$. Consequently, the first (4) can be modified as (17).

$$\left\|\dot{\vartheta}(t)\right\| \le \|\bar{A}_{ii11}\|\|\vartheta_i\| + [\|\bar{A}_{ii12}\| + \|B_i^{\perp T} D_i\|\|E_i B_i (S_i B_i)^{-1}\|]\|\sigma_i\| + \|\bar{A}_{ii11d}\|\|\vartheta_{id}\|$$
$$+[\|\bar{A}_{ii12d}\| + \|B_i^{\perp T} D_{id}\|\|E_{id} B_i (S_i B_i)^{-1}\|]\|\sigma_{id}\| + \textstyle\sum_{j=1, j\neq i}^{L}\left[\left\|\bar{F}_{ij11}\right\|\left\|\vartheta_{jd}\right\| + \left\|\bar{F}_{ij12}\right\|\left\|\sigma_{jd}\right\|\right]. \quad (17)$$

By substituting the suggested controller (14), control gains (15), and (17) into (16), it is obvious that $\dot{V}_i \le -\sum_{i=1}^{L}\tilde{\alpha}_i\|s_i(t)\|$, where $\tilde{\alpha}_i > 0$. Thus, the state trajectories of the plant (1) reach the sliding manifold (2) in finite time and stay on it. Theorem 2 is demonstrated completely.

4. STABILITY ANALYSIS OF SLIDING MOTION

In this part, we are in situation to give a satisfactoriness constraint in LMI such that the closed-loop plant (3) in sliding mode is asymptotically stable. Let us create with seeing the LMI:

$$\begin{bmatrix} \Omega_i & \bar{E}_i^T & \bar{E}_{id}^T & \bar{X}_i\bar{D}_i & \bar{X}_i\bar{D}_{id} \\ \bar{E}_i & -\gamma_i^{-1}I_i & 0 & 0 & 0 \\ \bar{E}_{id} & 0 & -(\tilde{\eta}_{4bi}\gamma_{1id})^{-1}I_i & 0 & 0 \\ \bar{D}_i^T\bar{X} & 0 & 0 & -\gamma_i^{-1}I_i & 0 \\ \bar{D}_{id}^T\bar{X}_i & 0 & 0 & 0 & -\gamma_{i1d}^{-1}I_i \end{bmatrix} < 0, i = 1, 2, \ldots, L \quad (18)$$

where $\Omega_i = \bar{A}_i^T\bar{X}_i + \bar{X}_i\bar{A}_i + \gamma_{2id}^{-1}\bar{X}_i + \tilde{\eta}_{4ai}\gamma_{2id}\bar{A}_{id}^T\bar{X}_i\bar{A}_{id} + \sum_{j=1,j\neq i}^{L}\left(\tilde{\eta}_{4i}\bar{F}_{ji11}^T\bar{X}_i + \tilde{\eta}_{4i}\bar{X}_i\bar{F}_{ji11}\right)$, $\bar{A}_i = B_i^{\perp T}A_iT_iB_i^{\perp}(B_i^{\perp T}T_iB_i^{\perp})^{-1}$, $\bar{D}_i = B_i^{\perp T}D_i$, $\bar{E}_i = E_iT_iB_i^{\perp}(B_i^{\perp T}T_iB_i^{\perp})^{-1}$, $\bar{A}_{id} = B_i^{\perp T}A_{id}T_iB_i^{\perp}(B_i^{\perp T}T_iB_i^{\perp})^{-1}$, $\bar{D}_{id} = B_i^{\perp T}D_{id}$, $\bar{E}_{id} = E_{id}T_iB_i^{\perp} \times (B_i^{\perp T}T_iB_i^{\perp})^{-1}$, $\bar{X}_i \in R^{(n_i-m_i)\times(n_i-m_i)}$ is any positive definite matrix, and $\gamma_i > 0, \gamma_{1id} > 0, \gamma_{2id} > 0$, $\tilde{\eta}_{4ai} > 1, \tilde{\eta}_{4bi} > 1$. Then, we can construct the following theorem.

Theorem 3. Suppose that the sufficiency constraint (18) has answer $\hat{X}_i > 0$, the positive constants $\gamma_i > 0, \gamma_{1id} > 0, \gamma_{2id} > 0$, and $\tilde{\eta}_{4ai} > 1, \tilde{\eta}_{4bi} > 1$. The manifold function is premeditated by (1). Then, the complex interconnected time delay systems (4) restricted to the sliding manifold surface $s_i(y_i(t), t) = 0$ is asymptotically stable.

Proof of Theorem 3. The motion dynamics of the complex interconnected systems (3) in the sliding mode can be represented as (19):

$$\dot{\vartheta}_i = [\bar{A}_i + \bar{D}_i\Sigma_i(x_i, t)\bar{E}_i]\vartheta_i + [\bar{A}_{id} + \bar{D}_{id}\Sigma_{id}(x_{id}, t)\bar{E}_{id}]\vartheta_{id} + \textstyle\sum_{j=1,j\neq i}^{L}\left[\bar{F}_{ij11}\vartheta_{jd}\right], \quad (19)$$

where the constant matrices $\bar{A}_i, \bar{D}_i, \bar{E}_i, \bar{A}_{id}, \bar{D}_{id}$, and $\bar{E}_{id}$ are defined in (18). Now, we define a Lyapunov function as $V = \sum_{i=1}^{L}\vartheta_i^T\bar{X}_i\vartheta_i$, where the positive definite matrix $\bar{X}_i$ is demarcated in (18). Derivative of V with respect to the motion dynamics (19), we get (20).

$$\dot{V} \le \textstyle\sum_{i=1}^{L}\{\vartheta_i^T[\Omega_i + \gamma_i\bar{E}_i^T\bar{E}_i + \tilde{\eta}_{4bi}\gamma_{1id}\bar{E}_{id}^T\bar{E}_{id} + \gamma_i^{-1}\bar{X}_i\bar{D}_i\bar{D}_i^T\bar{X}_i + \gamma_{i1d}\bar{X}_i\bar{D}_{id}\bar{D}_{id}^T\bar{X}_i]\vartheta_i\}. \quad (20)$$

By using the Lemmas in published researches [20]–[23], the LMI (18) is equivalent to the inequality (21).

$$\Omega_i + \gamma_i\bar{E}_i^T\bar{E}_i + \tilde{\eta}_{4bi}\gamma_{1id}\bar{E}_{id}^T\bar{E}_{id} + \gamma_i^{-1}\bar{X}_i\bar{D}_i\bar{D}_i^T\bar{X}_i + \gamma_{i1d}\bar{X}_i\bar{D}_{id}\bar{D}_{id}^T\bar{X}_i < 0 \quad (21)$$

From (20) and (21), we obtain $\dot{V} \le 0$. That is, the sufficiency constraint (18) is gratified, the plant (3) is asymptotically stable in the sliding mode. The proof of Theorem 3 is finished.

5. NUMERICAL SIMULATION

In this part, we present numerical examples which adjusted from the research [24] to demonstrate the advantage of the control structures suggested in this paper. Consider the mathematical model of the complex interconnected plants with unknown perturbations composed of two subsystems:

Subsystem I: $n_1 = 3, m_1 = 2, i = 1, j = 2$, and the dynamics are specified as (22):

$$\dot{x}_1(t) = \left\{\begin{bmatrix} -2 & 0 & -1 \\ 1 & 0 & 0 \\ 2 & 1 & 0 \end{bmatrix} + \Delta A_1(t)\right\}x_1(t) + \left\{\begin{bmatrix} -2 & 0 & 2 \\ -1 & -1 & 1 \\ -2 & 2 & 0 \end{bmatrix} + \Delta A_{1d}(t)\right\}x_{1d}(t) + \begin{bmatrix} 0 \\ 1 \\ -0.5 \end{bmatrix}[u_1(t)$$
$$+\zeta_1(x_1(t), x_{1d}, t)] + \begin{bmatrix} -0.2 & 0 & -0.1 \\ 0.1 & 0 & 0 \\ 0.2 & 0.1 & 0 \end{bmatrix}x_{2d}(t), y_1(t) = C_1x_1(t) = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}x_1(t), \quad (22)$$

where the mismatched uncertainties in state matrix and delayed state matrix are respectively $\Delta A_1(t) = [0\ \ 0\ \ 1]^T \Sigma_1(x_1(t), t)[1\ \ 1\ \ 0]$ with $\Sigma_1(x_1(t), t) = 0.14\, sin(0.1t)$ and $\Delta A_{1d}(t) = [0\ \ 1\ \ 0]^T \Sigma_{1d}(x_1(t), x_{1d}, t) \times [1\ \ 1\ \ 0]$ with $\Sigma_{1d}(x_1(t), x_{1d}, t) = 0.22\, sin(0.1t)$. The external perturbation input is assumed to be $\|\dot{\psi}_1(t)\| \leq \upsilon_{01} + \upsilon_{11}(\|x_1\|) + \upsilon_{21}(\|x_1\|)^2$ with $\upsilon_{01} = 0.12, \upsilon_{11} = 0.32,$ and $\upsilon_{21} = 0.55$.

Subsystem II: $n_2 = 3, m_2 = 2, i = 2, j = 1$, and the dynamics are specified as (23):

$$\dot{x}_2(t) = \left\{\begin{bmatrix} 0 & 2 & 0 \\ 1 & 2 & 0 \\ 2 & 1 & -2 \end{bmatrix} + \Delta A_2(t)\right\} x_2(t) + \left\{\begin{bmatrix} 0 & 2 & -1 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} + \Delta A_{2d}(t)\right\} x_{2d}(t) + \begin{bmatrix} 0 \\ 1 \\ -0.5 \end{bmatrix} [u_2(t)$$
$$+\zeta_2(x_1(t), x_{2d}, t)] + \begin{bmatrix} -0.2 & 0 & -0.1 \\ 0.1 & 0 & 0 \\ 0.2 & 0.1 & 0 \end{bmatrix} x_{1d}(t), y_2(t) = C_2 x_2(t) = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} x_2(t), \quad (23)$$

where the mismatched uncertainties in state matrix and delayed state matrix are respectively $\Delta A_2(t) = [0\ 0\ 1]^T \Sigma_2(x_2(t), t)[1\ \ 1\ \ 0]$ with $\Sigma_2(x_2(t), t) = 0.28\, sin(0.1t)$ and $\Delta A_{2d}(t) = [0\ 1\ 0]^T \Sigma_{2d}(x_2(t), x_{2d}, t) \times [1\ \ 1\ \ 0]$with $\Sigma_{2d}(x_2(t), x_{2d}, t) = 0.41\, sin(0.1t)$. The external disturbance input is assumed to be $\|\dot{\psi}_2(t)\| \leq \upsilon_{02} + \upsilon_{12}(\|x_2\|) + \upsilon_{22}(\|x_2\|)^2$ with $\upsilon_{02} = 0.12, \upsilon_{12} = 0.22,$ and $\upsilon_{22} = 0.34$. For simulation, the initial condition of the two subsystems are $x_1(0) = x_2(0) = [1\ \ -1\ \ 2]^T$ and the unknown time-varying delay is $d(t) = 0.15(1 + sin\, 0.5t)$ [25]. By using MATLAB software, we have obtained the significant results. In particular, Figure 1 shows the time history of the subsystems states in Figure 1(a), and the manifold function in Figure 1(b), and the ROSMO in Figure 1(c). Figure 2 displays the time response of the observer errors in Figure 2(a), the upper bound of errors in Figure 2(b), and the proposed single phase second order sliding mode controller (SPSOSMC) in Figure 2(c).

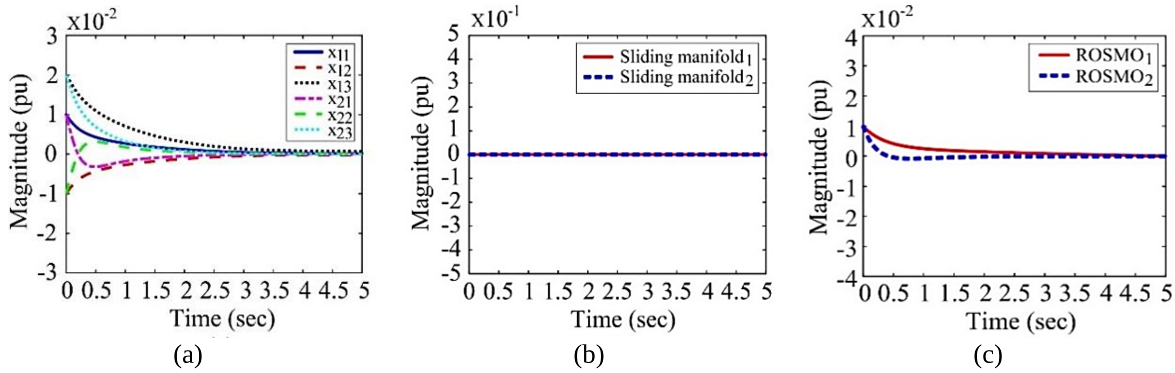

Figure 1. Time history of (a) the system variables, (b) the manifold functions, and (c) the ROSMO of two subsystems

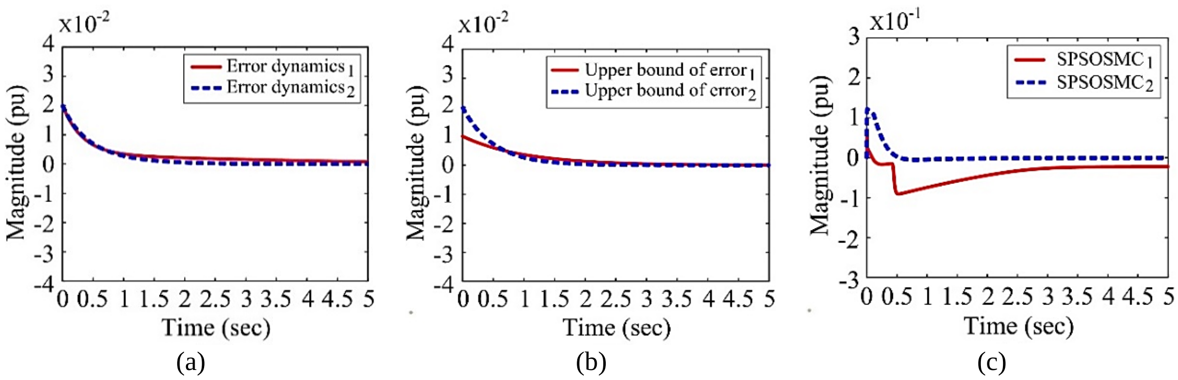

Figure 2. Time response of (a) the observer errors, (b) the upper bound of errors, and (c) the proposed SPSOSMCs

6. CONCLUSION

This paper has presented the novel single phase SOSMC scheme, which removes completely the reaching phase and uses output variables only, for the complex interconnected plants with unknown perturbations. A new single phase sliding manifold function has been proposed such that the robustness enhancement of the closed-loop plants is guaranteed, and the desired dynamic response is obtained. The ROSMO has been suggested to estimate the unmeasurable state variables for supporting the controller design. Based on this ROSMO and Moore-Penrose inverse approach, the new SPSOSMC has been considered to cancel the undesired high frequency fluctuation and stabilize the complex interconnected systems. Further, by employing the well-known LMI technique, the sufficient condition in the sliding mode has been constructed such that the property of asymptotical stability is ensured. Finally, a mathematical simulation is executed to validate the effectiveness of the offered method.

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