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# SERVER SELECTION ON BASE OF Z-NUMBERS

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#### ABSTRACT

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#### KEYWORDS

linguistic uncertainty, decision making, membership function, fuzzy z-numbers, generalized fuzzy numbers. The paper is devoted to the problem of multi criteria decision making under linguistic uncertainty. Information of different approaches for modeling linguistic uncertainty have been analyzed. The concept of z-numbers proposed by L. Zadeh have been presented. Z-number is presented as cortege of two fuzzy number A and B, where A is analyzed factor, B is reliability of A assessment. The method of conversion z-numbers into generalized fuzzy numbers have been applied. As test have been used server selection problem. As decision making model have been used weighted average method. All calculations and results are presented.

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**Introduction.** The problem of decision-making under conditions of uncertainty and particular linguistic uncertainty is one of the important problem. Very often, when solving problems of decision making, we are forced to be content only with information supplied by experts in the form of linguistic assessments. Under these conditions, the development of decision-making models requires adequate consideration of the uncertainty.

One of the effective methods for modeling linguistic uncertainty is the approach based on fuzzy sets. There are a large number of works devoted to using classical theory of fuzzy sets and received the name of the fuzzy type -1. This approach was proposed by L. Zadeh in 1965 in the work [1]. In 1975, L. Zadeh proposed a more general approach, called fuzzy type -2 and allowing to take into account the inaccuracy of our knowledge about the membership function [2]. S. Chen in 1985 proposed the concept alternative to fuzzy type -2 and which get the name generalized fuzzy numbers, and which also allows to take in account our knowledge of the inaccurate membership functions [3]. In 2011, L. Zadeh proposed the concept of fuzzy Z-numbers (Z-numbers), which also allows us to take into account the inaccuracy of our knowledge of the membership function using a joint approach from the standpoint of the theory of probability and the theory of possibility [4]. The concept of Z-numbers is associated with the fact that very often the degree of an expert's confidence in assessing the values of various factors is expressed in terms of the theory of probability ("unlikely", "very likely", "incredible"). L. Zadeh proposed to use for this a new type of fuzzy data Z-numbers, consisting of two fuzzy numbers, the first of them is a fuzzy assessment of the factor, the second is the degree of confidence in this estimation, expressed in terms of probability theory. L. Zadeh proposed an algorithm for performing operations with Z-numbers based on the extension principle [4]. All of these approaches are widely used in the development of decisionmaking models under conditions of linguistic uncertainty [5-17]. In this paper, an approach based on the use of fuzzy Z-numbers is considered.

**Basic concepts.** This paper discusses the use of fuzzy z- numbers (z - numbers) for modeling linguistic uncertainty in problems of multi criteria decision (MCDM) [6]. Consider the basic concepts of the theory of fuzzy Z-numbers.

**Definition**: **Z-number**. Z-number represents a tuple consisting of two fuzzy numbers  $Z = (\tilde{A}, \tilde{B})$ ,  $\tilde{A}$  - fuzzy number represented by an expert evaluation of factor;

 $\widetilde{B}$  - fuzzy number which describe the degree of confidence of the expert's assessment;

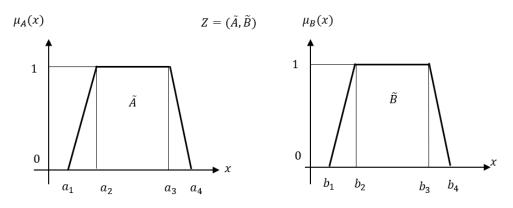


Fig. 1. Graphical representation of a fuzzy z-number

**Definition: Generalized fuzzy number**. Fuzzy set, defined on a universal set of real numbers R, called generalized fuzzy number if its membership function has the following properties:

(i)  $\mu_{\tilde{A}}: R \to [0,1]$  is continuous

(ii)  $\mu_{\tilde{A}}(x) = 0$  for all  $x (-\infty, a] \cup [d, \infty)$ 

(iii)  $\mu_{\tilde{A}}(x)$  strictly increases in [a, b] and strictly decreases in [c, d]

(iv)  $\mu_{\tilde{A}}(x) = w$ , for all  $x \in [b, c]$ , where  $0 < w \le 1$ 

The generalized fuzzy trapezoidal number  $\tilde{A} = (a, b, c, d, w)$  has a membership function in the form

$$\mu_{x}(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \le x \le b \\ w & b \le x \le c \\ \frac{x-c}{d-c} & c \le x \le d \\ 0 & x > d \end{cases}$$

Here w the level of reliability of our knowledge about the membership function.

Consider the order of implementation arithmetic operations on generalized fuzzy trapezoidal numbers. Given two GTFN numbers  $\tilde{A}_1$  and  $\tilde{A}_2$ :

 $\tilde{A}_{1} = (a_{1}, b_{1}, c_{1}, d_{1}, w_{1}) \qquad \tilde{A}_{2} = (a_{2}, b_{2}, c_{2}, d_{2}, w_{2})$ Addition.  $\tilde{A}_{1} \bigoplus \tilde{A}_{2} = (a_{1} + a_{2}, b_{1} + b_{2}, c_{1} + c_{2}, d_{1} + d_{2}; \min(w_{1}, w_{2}))$ Subtraction  $\tilde{A}_{1} \bigoplus \tilde{A}_{2} = (a_{1} - a_{2}, b_{1} - b_{2}, c_{1} - c_{2}, d_{1} - d_{2}; \min(w_{1}, w_{2}))$ Scalar multiplication.

$$\lambda A = \begin{cases} (\lambda a, \lambda b, \lambda c, \lambda d; w) \ \lambda > 0\\ (\lambda d, \lambda c, \lambda b, \lambda a; w) \ \lambda < 0 \end{cases}$$

**Ranking function.** 

To rank alternatives, the centroid method was used which proposed in the work [6]

$$(\tilde{x}_0, \tilde{y}_0) = \left(a + b + c + d - \frac{ac - ab}{(dc) - (a + b)}, \frac{w}{3}\left(1 + \frac{c - b}{(d + c) - (a + b)}\right)\right)$$
  
Ranking function  $R(\tilde{A}) = \sqrt{\tilde{x}^2 + \tilde{y}^2}$ 

If there are also two GTFN numbers, then we have

If  $\tilde{A}_i$  and  $\tilde{A}_i$  two GTFN numbers, then we have

- (i)  $R(\tilde{A}_i) > R(\tilde{A}_i)$  then  $\tilde{A}_i > \tilde{A}_i$
- (ii)  $R(\tilde{A}_i) < R(\tilde{A}_i)$  then  $\tilde{A}_i < \tilde{A}_i$
- (iii)  $R(\tilde{A}_i) = R(\tilde{A}_i)$  then  $\tilde{A}_i = \tilde{A}_i$

## Definition: The probability of a fuzzy event.

The probability of a fuzzy event is a quantity defined as

$$P(A) = \int_{A} p_A(x)\mu_A(x)dx$$

With this concept, a relationship between components A and B is established.

Calculations with z - numbers can be realized by directly using Zadeh extension principle, which requires very large computations and is extremely difficult when solving complex applied problems. An alternative way is the conversion of z - numbers into other formats that support the uncertainty of the membership function and for which algorithms for calculations of moderate complexity have been developed. In work [7] a method for converting Z-numbers into generalized fuzzy numbers is proposed. In fact, the degree of confidence translates into a level of fuzziness.

$$\alpha = \frac{\int x\mu_{\tilde{B}}(x)dx}{\int \mu_{\tilde{B}}(x)dx}$$

Further Z-number of transformed in generalized fuzzy number  $x, \mu_{\alpha}(x)$ )

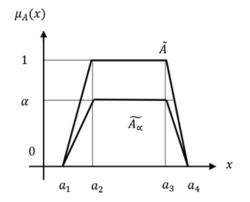


Fig. 2. Graphical representation of a generalized fuzzy number.

## Problem statement and solution method.

Application of fuzzy Z-numbers, consider the example of a server selection problem. This problem is formalized as an MCDM problem. The server is described by 4 attributes.

 $C_1$ - reliability,  $C_2$  - scalability,  $C_3$ - security,  $C_4$  – quality of services.

For all attributes  $C_i$  (i = 1,2,3,4) and weigh coefficients are determined.

 $\omega = (0.3 \ 0.15 \ 0.2 \ 0.35)$ 

Table 1 shows the linguistic terms that are used to assess the alternatives of solutions -Very low " (VL), "Low" (L), "Medium" (M), "High" (H), " Very high (VH).

Table 1.		
Linguistic terms for factor A	Membership function of A	
Very low (VLK)	(0,0.1,0.2.0.3)	
Low (L)	(0.1,0.3,0.45,0.7)	
Medium (M)	(0.4, 0.5, 0.7, 0.8)	
High (H)	(0.5,0.6,0.75,0.85)	
Very high (VH)	(0.6,0.7,0.8,0.9)	

The linguistic terms used to assess the degree of confidence are presented in Table 2.

Table 2.		
Linguistic terms of degree of confidence	Membership function values	
Impossible (IM)	(0.0, 0.01, 0.02, 0.05)	
Very unlikely (VU)	(0.02, 0.15, 0.20, 0.25)	
Likely ( <b>LK</b> )	(0.20, 0.30, 0.50, 0.65)	
Very likely (VLK)	(0.50, 0.60, 0.70, 0.80)	
Certain (C)	(0.70, 0.85, 0.95, 0.99)	

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Decision maker using these terms estimate all potential servers by all attributes, and results are shown in Tables 3.

Table 3.				
	<i>C</i> <sub>1</sub>	<i>C</i> <sub>2</sub>	<i>C</i> <sub>3</sub>	<i>C</i> <sub>4</sub>
<i>A</i> <sub>1</sub>	(M,VU)	(H,LK)	(H,VU)	(H,VLK)
A <sub>2</sub>	(H,VLK)	(VH,LK)	(VH,VLK)	(H,VU)
A <sub>3</sub>	(M,LK)	(H,LK)	(M, VU)	(VH,VLK)

Table 2

Calculate the value of  $\alpha$  for various values of the degree of confidence

 $\alpha(VU) = 0.1$  $\alpha(LK) = 0.6$  $\alpha(VLK) = 0.86$ Present all linguistic terms in GFTN:

 $\widetilde{A}_{11} = (0.4, 0.5, 0.7, 0, 8; 0, 1)$  $\widetilde{A}_{12} = (0.5, 0.6, 0.7, 0.8; 0.6)$  $\widetilde{A}_{13} = (0.5, 0.6, 0.7, 0.8; 0.1)$  $\widetilde{A}_{14} = (0.5, 0.6, 0.75, 0.85; 0.86)$  $\widetilde{A}_{21} = (0.5, 0.6, 0.75, 0.85; 0.86)$  $\widetilde{A}_{22} = (0.6, 0.7, 0.8, 0.0.9; 0.6)$  $\widetilde{A}_{23} = (0.6, 0.7, 0.8, 0.9; 0.86)$  $\widetilde{A}_{24} = (0.5, 0.6, 0.75, 0.85; 0.1)$  $\widetilde{A}_{31} = (0.4, 0.5, 0.7, 0.8; 0.6)$  $\widetilde{A}_{32} = (0.5, 0.6, 0.75, 0.85; 0.6)$  $\widetilde{A}_{33} = (0.4, 0.50, 0.70, 0.80; 0.1)$  $\widetilde{A}_{34} = (0.6, 0.7, 0.8, 0.9; 0.86)$ 

The next step is to aggregate assessments of all alternatives by criteria using the formula  $A_i = \bigoplus_{i=1}^4 (\omega_i \tilde{A}_{ii})$ 

As a result, we obtain global assessment of alternatives (table 4)

Table 4.	
Alternatives	GTFN values
A <sub>1</sub>	(0.52,0.63,0.78,0.89;0,1)
A <sub>2</sub>	(0.6,0.705,0.86,0.96;0.1)
<i>A</i> <sub>3</sub>	(0.53,0.64,0.81,0.93;0,1)

To compare alternatives we use the ranking function and we have:

 $Rank(A_3)=3.91> Rank(A_1)=3.69> Rank(A_2)=3.45$ 

The supplier  $A_3$  is the best.

Conclusions. This article investigated the problem of constructing multicriteria model of decision- making model (MCDM) in terms of linguistic uncertainty. As a decision-making model, weighed average model is used, and fuzzy z- numbers as a model of uncertainty. The problem of server selection was used as a test problem.

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