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Use of Simulation to Solve a Queueing Problem

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Queueing or waiting line theory can involve complex and time-consuming mathematics. However, this is not always essential. Here's an example of a quick, effective method of solving a queueing problem —

THE USE OF SIMULATION TO SOLVE A QUEUEING PROBLEM

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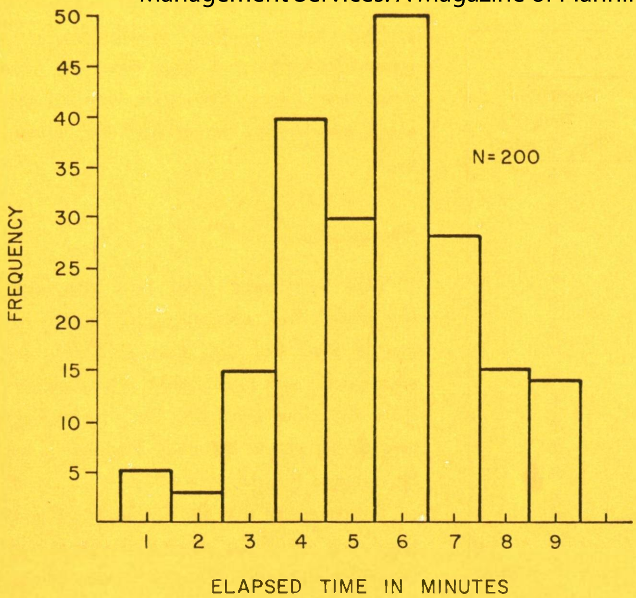
MATHEMATICAL waiting-line (queueing) theory is being successfully applied to the solution of many business problems involving arrivals (requirements for service) and service times (accomplishing the service). Some common examples: machinists standing in line for tools from a tool crib, cars waiting at a toll booth, or production parts being held up waiting for inspection.

Bottlenecks (waiting for service) and idle capacity (waiting to supply service) both incur costs. The staffing of service facilities for the minimum-cost combination requires forecasting the probable combined costs of waiting time and of service availability before service is actually rendered. These costs, however, frequently are difficult to forecast.

The simplest queueing problem would involve constant arrivals—say, one every five minutes—and uniform service time—say, ten minutes per "customer." The answer is readily apparent: Two "servers" per "customer" will result in no waiting for the customers, no idleness for the servers, thus minimum cost. Unfortunately, this situation is extremely rare.

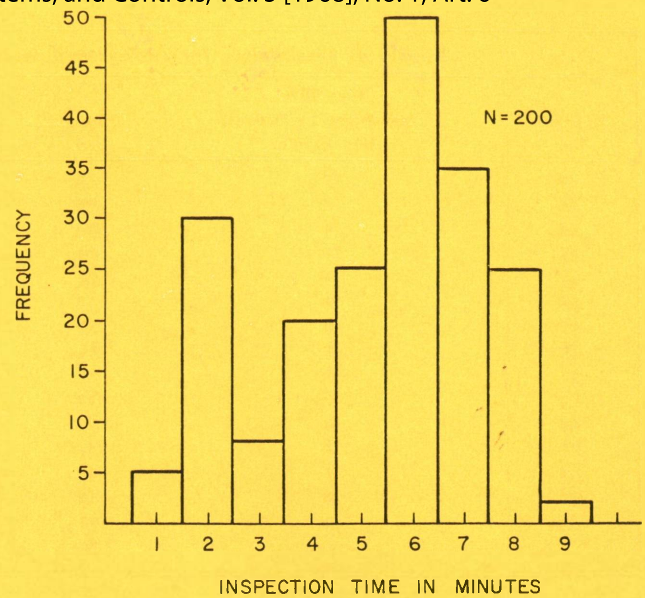
More commonly arrivals and service times are both variable. Sometimes the patterns of distribution of arrivals and service times fit certain standard statistical distributions, for example, Poisson arrivals and negative exponential service times. When this is found to be the case, existing equations and tables can be used to determine the pertinent data.

There is still a third category of queueing problem, the situation wherein arrivals and service rates



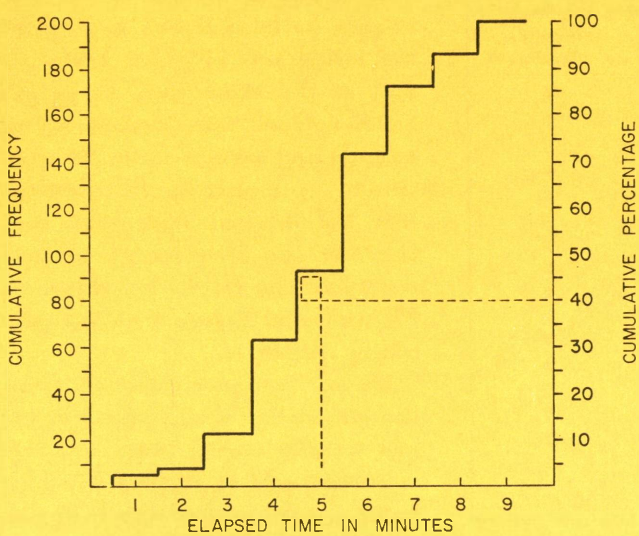
Distribution of Time Between Arrivals of Operators

FIGURE 1



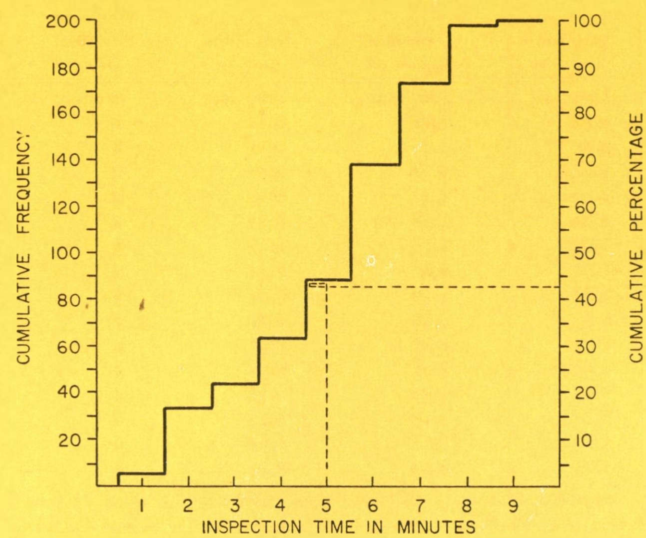
Distribution of Inspection Times

FIGURE 2



Cumulative Distribution of Time Between Arrivals of Operators

FIGURE 3



Cumulative Distribution of Inspection Times

FIGURE 4

are neither uniform nor in conformity with standard distributions. A valuable tool in the examination of this type of problem is simulation,[1] a method of duplicating a complicated operation by a set of rules and computations so that alternative decisions may be observed in action. This article presents an example of the application of simulation via the Monte Carlo method,[2] a form of simulation in which randomly chosen numbers determine the course of the computation.

The XYZ Company is a company engaged in intermittent manufacture to stock. Material in process is checked at centralized inspection stations strategically located throughout the plant. Rather widely varied inspection operations are performed by the inspectors manning the inspection stations. Because of the relatively long distances traveled by the operators to the inspection stations and the relatively short time usually required for the inspections, it is the practice of the operators to wait for the work to be checked. The operators arrive at the station and are attended to on a first-come-first-served basis.

The quality manager notices that operators frequently wait in line at one of the inspection stations.

[1] For other examples of the use of simulation see "Simulation in Financial Planning" by E. N. Khoury and H. Wayne Nelson, M/S, March-April '65, p. 13, and "Using Simulation to Design a Management Information System" by Adolph F. Moravec, M/S, May-June '66, p. 50.

[2] The Monte Carlo method is discussed in detail in "Setting Inventory Reorder Points" by Felix A. McCameron, M/S, May-June '65, p. 25.

TABLE 1

Sample of Time Values for Arrivals and Inspections			
Random No.	Time Since Last Operator Arrived (Fig. 3) min.	Random No.	Inspection Time (Fig. 4) min.
40	5	43	5
16	4	55	6
54	6	94	8
46	5	74	7
17	4	23	4
56	6	12	2
09	3	43	5
10	3	81	7
81	7	47	6
73	7	95	8
64	6	23	4
49	6	49	6
74	7	59	6
36	5	18	3
12	4	71	7
		50	6

TABLE 2

Simulation of Conditions at Inspection Station—1 Inspector					
Operators Arrive at	Inspection Begins at	Inspection Ends at	Operator Waiting Time	Inspector Idle Time	Number of Operators Waiting
8:00 a.m.	8:00 a.m.	8:05 a.m.	0 min.	0 min.	0
8:05	8:05	8:11	0	0	0
8:09	8:11	8:19	2	0	1
8:15	8:19	8:26	4	0	2
8:20	8:26	8:30	6	0	2
8:24	8:30	8:32	6	0	3
8:30	8:32	8:37	2	0	2
8:33	8:37	8:44	4	0	2
8:36	8:44	8:50	8	0	3
8:43	8:50	8:58	7	0	3
8:50	8:58	9:02	8	0	2
8:56	9:02	9:08	6	0	3
9:02	9:08	9:14	6	0	2
9:09	9:14	9:17	5	0	2
9:14	9:17	9:24	3	0	2
9:18	9:24	9:30	6	0	2

Duration of sample: 90 min.; Operators' waiting time: 73 min.

TABLE 3

Sample of Time Values for Arrivals and Inspections—2 Inspectors					
Random No.	Time Since Last Oper. Arrived* min.	Random No.	Inspection Time Inspector 1** min.	Random No.	Inspection Time Inspector 2** min.
78	7	46	6		
33	5			68	7
58	6	26	4		
27	4			96	8
54	6	98	8		
82	7			87	7
85	8	72	7		
52	6			65	6
48	6	96	8		
24	4			82	7
03	1	33	5		
95	9			82	7
95	9	83	7		
55	6			01	1
03	1	21	3		
				12	2

*From Fig. 3; **From Fig. 4

He thinks an additional inspector might relieve the condition and consequently cut the cost of idle operator time. He asks one of his staff engineers to study the situation.

Simulation

The engineer first collects data showing the characteristics of arrivals and service times. Both, he observes, are randomly distributed. The distributions are shown in Figure 1 on page 59 and Figure 2 on the same page.

Testing for fit, he finds that neither set of data approximates any of the standard distributions (Poisson, exponential, etc.). Consequently, the use of Monte Carlo simulation is indicated.

In order to adhere to the frequency patterns shown to exist by the histograms (Figures 1 and 2) and at the same time to permit simulation of the randomness of arrivals and service times, he constructs cumulative distributions fom the original histograms and converts the frequencies to percentages. The results are shown in Figure 3 and Figure 4 on the preceding page.

He now samples randomly from the cumulative distributions to select specific arrival times and service responses to employ in simulating the inspection station operation. The dotted lines in Figures 3 and 4 illustrate how this is done. A sample of resulting time values is presented in Table 1, this page.

With the information derived in Table 1, the engineer can now simulate the operation of the inspection station. The results are shown in Table 2 on this page. Operator waiting time over an elapsed period of 90 minutes totals 73 minutes. (Waiting time does not include the time required for the inspection after the inspector is "waited on.")

The operators' average hourly rate is $4. On the basis of this rate and the results of the simulation, he makes the following calculations:

Simulation of Conditions at Inspection Station—2 Inspectors

Operators Arrive at:	Inspection Begins at:	Inspection Ends at:	Performed by Inspector No.:	Operator Waiting Time min.	Inspector 1 Idle Time min.	Inspector 2 Idle Time min.
8:00 a.m.	8:00 a.m.	8:06 a.m.	1	0	0	6
8:07	8:07	8:14	2	0	6	1
8:12	8:12	8:16	1	0	0	2
8:18	8:18	8:26	2	0	6	2
8:22	8:22	8:30	1	0	0	2
8:28	8:28	8:35	2	0	5	0
8:35	8:35	8:42	1	0	0	7
8:43	8:43	8:49	2	0	7	1
8:49	8:49	8:57	1	0	0	6
8:55	8:55	9:02	2	0	2	0
8:59	8:59	9:04	1	0	0	0
9:00	9:02	9:09	2	2	5	0
9:09	9:09	9:16	1	0	0	7
9:18	9:18	9:19	2	0	3	2
9:24	9:24	9:27	1	0	5	6
9:25	9:25	9:27	2	0	0	0

Duration of sample: 87 min.

Operators' waiting time: 2 min.

TABLE 4

Operator waiting time per 8-hour day $= \frac{480}{90} \times 73 = 389$ minutes.

Cost of waiting time per day $= \frac{389}{60} \times \$4 = \$25.93$.

Two-inspector simulation

The cost of waiting time (nearly $130 a week) appears excessive. Perhaps an additional inspector is needed. To determine whether this expenditure is justified, the engineer performs a second simulation, using the original data, with two inspectors manning the inspection station. Now the total operator waiting time over an elapsed period of 87 minutes is cut to two minutes. The results of this second simulation are shown in Table 3 on page 60 and Table 4 on this page.

The inspectors' average hourly rate is $3.85. On the basis of this rate and the results of the simulation, the engineer makes the following calculations:

Operator waiting time per 8-hour day $= \frac{480}{87} \times 2 = 11$ minutes.

Cost of waiting time per day $= \frac{11}{60} \times \$4 = \$0.73$.

Cost of added inspector per day $= 8 \times \$3.85 = \$30.80$.

Therefore, the total daily cost with two inspectors is $0.73 + $30.80, or $31.53. Comparison of this cost with the previous one ($25.93) indicates that the existing arrangement, with one inspector, is more economical and that the addition of another inspector is not justified.

Application

Naturally, two simulations based on runs totaling less than two hours are far from adequate to produce results yielding a reasonable degree of confidence. In actual practice, longer runs (say, a full eight hours) would be simulated, and at least fifty iterations would be conducted for each of the situations—one inspector and two inspectors.

This would be a formidable task to perform by hand. Fortunately, appropriate computer programs are available. Once the original data have been assembled, a computer can make short work of the many iterations required.

Where waiting lines with irregular arrivals are concerned, casual observation can often be misleading. In the example described in this article it seemed to show the desirability of adding another inspector; testing this action by simulation kept the quality manager from making a mistake that could have cost the company about $1,500 a year at one inspection station alone. The technique is equally applicable to many other problems, ranging all the way from machine maintenance and truck terminal design to timing of traffic signals and scheduling of patients in hospital clinics.

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