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Some Uses of Mathematical Techniques in Accounting

H. G. Trentin

Although figures have always been the language of accountants, they have been derived only from the simplest mathematical methods in the past. Now, however, more sophisticated techniques are both available and essential in management services. Here is a review of —

SOME USES OF MATHEMATICAL TECHNIQUES IN ACCOUNTING

by H. G. Trentin

Arthur Andersen & Co.

THERE WAS a time when the use of mathematics in accounting was confined to computing interest, discounts, annuities, returns on investments, partnership interests, sinking funds, and a few other accounting applications requiring a bit more than simple addition, subtraction, multiplication, and division. As a matter of fact, most accountants got along quite well on just these four basic arithmetical functions.

Now, however, the role of the accountant has broadened to meet the needs of business management. The new horizons of accounting make it necessary to deal with probability statistics, correlation analysis, network diagraming (including PERT), linear programing, and other more advanced mathematical methods.

In this article each of the methods mentioned will be described in general terms, and one of them (networking) will be illustrated by a case study.

Probability statistics

The accountant is often called upon to provide an analysis of past occurrences and to offer suggestions regarding future trends. For example, based on forecasts of sales and analyses of alternative options, decisions are made regard-

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ing additions to plant, borrowing requirements, inventory commitments, and other matters of substantial financial consequence. Statistical analysis of historical data often permits presentation of sales forecast information in terms of probabilities of achieving different levels of sales.

To take a highly simplified example, consider the problem of a baker who decides how many loaves of bread to produce on a given day of the week, say, Friday, on the basis of the number sold on Fridays in the past. An analysis of historical sales may result in the probabilities shown in Exhibit 1 on this page.

The average number of loaves sold on Fridays was 1,000. The baker also knows that it costs him 32 cents to bake a loaf and that he can sell it for 40 cents as a fresh loaf on the same day but has to reduce the price to 20 cents if it is to be sold as one-day-old bread on the next day. With this information, the baker can now determine what his best course of action would be.

Mathematical analysis would show that he would do best by baking about 975 loaves of bread. At this level he would have a 60 per cent probability of making the sale of an additional loaf with a profit of 8 cents and a 40 per cent probability of finding himself left with one loaf to be sold the next day at a loss of 12 cents. The "expected value" of the gain (8¢ × 60% = 4.8¢) would equal the expected value of the loss (12¢ × 40% = 4.8¢).

Capital budgeting and investment decisions can also be improved if similar probability alternatives are prepared to illustrate the financial consequences. Profits or savings and the present value of cash flows involved in the acquisition of plant and the recovery of costs are not certain figures. Most current financial analyses prepared for these decisions by accountants show the consequences if the assumptions are realized. Introducing the effect of various probability variations from the assumptions and—perhaps most important—how likely they are to occur may change the decision.

Correlation analysis

One of the more difficult problems in cost accounting is to separate variable elements of cost from those that are fixed and independent of production level. Several approaches are possible. The most conventional would be to classify individual elements of expense as either fixed or variable (for example, maintenance labor as fixed and maintenance materials as variable). This procedure, however, fails to reflect the fact that many elements of expense may be partly fixed and partly variable and cannot be completely classified as one or the other.

Where historical data exist on both expenditures and units of production for each of a number of periods, a scatter diagram can be prepared plotting, for example, labor dollars or hours against production level. By drawing a line that seems best to fit the points in this graph, a relationship between fixed and variable portions is established, the fixed being represented by the intercept of the line and the variable by its slope.

Such a line is referred to mathematically as a regression line. It is possible to calculate the intercept and slope from the basic data without actually plotting the line—by the "least squares" method.

Value increases with factors

This mathematical approach becomes increasingly useful when costs are to be analyzed or broken down into not just two elements, namely, fixed and variable, but three or more. For example, such an analysis may permit identification of costs that are fixed and independent of both production level and the number of machine setups; costs that are variable with production level but not with machine setups; and those that are variable depending on the number of machine setups but not on the production level.

Probability	Number of Fresh Loaves
1 out of 10	1,500 or more
5 out of 10	1,000 or more
8 out of 10	900 or more
10 out of 10	700 or more

Probabilities of Achieving Various Levels of Sales

EXHIBIT I

So far, however, such correlation methods have found only limited application both because they often require more historical data than are available and because the same information can usually be arrived at through engineering approximations.

Network diagraming

Network diagraming is the mathematical term applied to various techniques for planning, scheduling, and control, of which critical path methods and PERT are perhaps the most widely used in accounting applications. These are means of analysis based on the division of a project into small steps of short duration in such a way that each step can be separately planned, scheduled, budgeted, and, upon completion, promptly accounted for.

The approach basically consists of (1) the construction of a network diagram representing the dependencies and interrelationships of the steps comprising the total project, (2) the provision of additional data for project control, and

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(3) the introduction of time and cost variations. The last named permits an evaluation of the scheduling of steps to permit selection of a schedule that produces the least direct cost for each step and/or the total project.

These methods are frequently used in production operations, research and development, and construction to schedule large and complex projects. They also have been used by accountants for simpler but important operations like developing schedules for earlier closing of the books in multi-divisional companies.

CPM in construction

The purposes of the construction industry accountant, for example, are well served by the specific objective of the critical path method: to set up an efficient information system that will operate continuously throughout a construction job, producing early, detailed, and accurate estimates of the following:

(a) Variable costs
(b) Final completion date
(c) Potential bottlenecks
(d) Requirements dates for labor, material, and subcontractors
(e) Cash flow.

Divide and conquer

To begin with, critical path analysis of a project requires that it be described in terms of a number of separate activities, or "steps." Whenever possible a step should consist of some physically distinguishable part of the work, the completion of which can be determined by visual inspection. In a process plant, for example, the installation of a pump would be a typical step. In a concrete structure, each pour might be considered a step. The description should state which steps may be done concurrently and which steps must be done in sequence. Steps to be done in sequence must be carefully defined so as to indicate clearly why the completion of one affects the start date of the next.

Cost variances will be accounted for step by step. Therefore, each step should represent only a small percentage of total project cost. Schedule progress will also be measured step by step, so the working time for each step should be short, relative to total project time. As an example, the concrete work for an eight-story office building was analyzed in terms of 42 steps, with step costs ranging from $4,000 to $22,000 and step durations from five to eighteen working days.

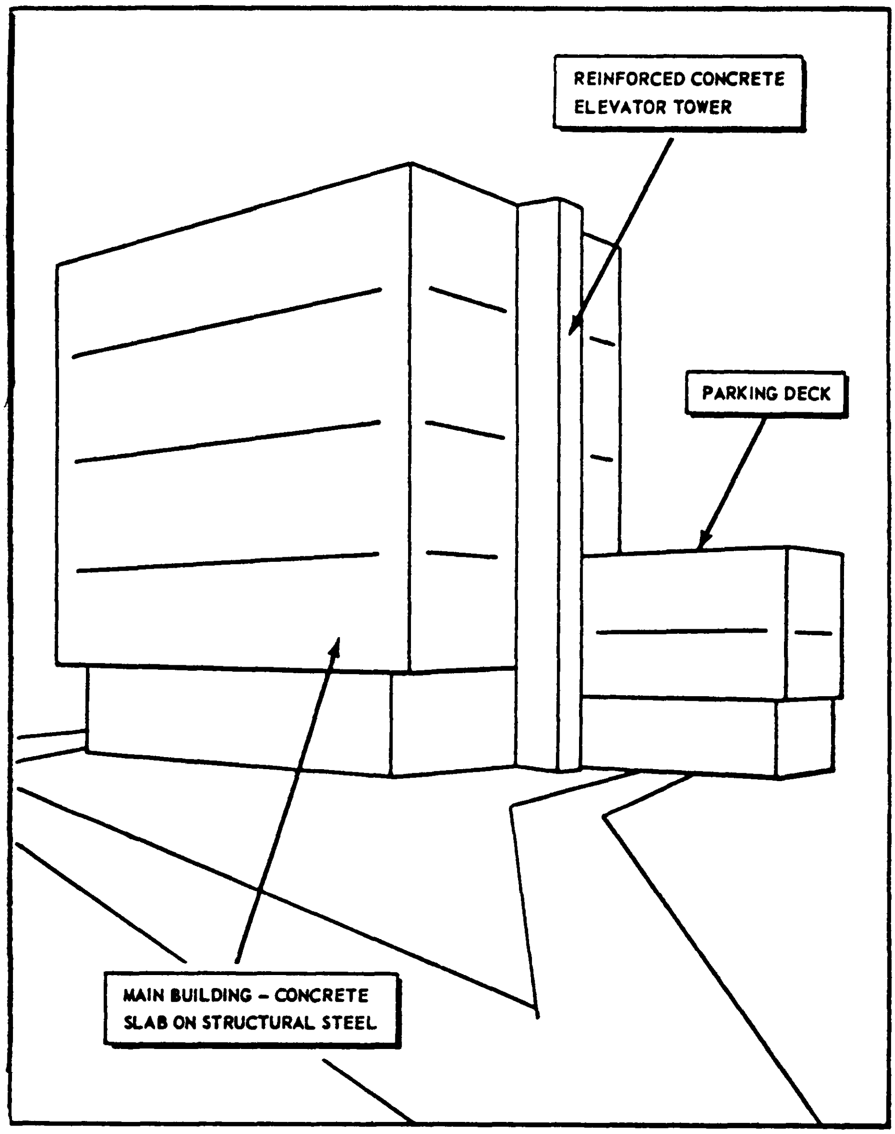
When the steps have been described in this manner, two controls may be established as follows:

Control of direct costs

Critical path permits extremely prompt and precise comparison of budgeted versus actual direct cost

EXHIBIT 2

Sketch of Proposed Building



. . . be described in terms of a number of activities or "steps."

for each step as soon as it is completed. To set up this control, the accountant must first obtain an estimate of the direct cost for each step, broken down as desired into the usual cost classifications (labor of various trades, material of various kinds, etc.). He must then arrange for the reporting of actual cost by step, and, when a step is complete, he should compare actual with budgeted cost. The results should immediately be passed back through management channels to the job site.

When this routine is performed separately for each step, variances are revealed early. In the office building mentioned an overrun was indicated in formwork labor after only six of the 42 concrete steps were completed. Without critical path this overrun would not have been discovered until much later in the project, probably too late for corrective action.

Schedule control is achieved by similar means. In the planning stage estimates of the working time for each step are obtained, and a schedule in the critical path format is prepared.

Critical "bottlenecks" isolated

The network calculation identifies the potential bottlenecks, or "critical" steps. In most projects only a few steps are critical. For example, in a chemical plant maintenance project involving over 200 steps, only 25 were found to be critical. Total project time was reduced 37 per cent by focusing management attention on these 25 steps alone.

Furthermore, the network routine clearly points out which steps may be delayed, and how much, without prolonging the final completion date. This permits the scheduling of efficient crew sizes, convenient arrival dates for material and subcontractors, and low interest on construction funds with no delay to the project.

Networking example

Here is a relatively simple example of the use of the critical path method in the construction industry:

Suppose that you are managing an investment in rental property. You engage a contractor to build, at cost plus ten per cent, the five-story apartment building shown in Exhibit 2 on page 28.

Before beginning construction, he submits to you the following budget and schedule:

APARTMENT BUILDING
$1,200,000
45 weeks required to complete

You believe that a more detailed analysis would yield greater precision. The contractor submits the schedule shown in Exhibit 3 on this page.

Bar chart scheduling

The contractor explains that 66, which seems to be the total number of weeks, has no meaning here since, for example, some of the concrete work can be started before the excavation is entirely finished. Also, the interior work can begin as soon as the service elevator can be manually operated to provide access for workmen. Reflecting these facts, he submits a chart showing completion in 45 weeks (Exhibit 4 on this page).

Excavation	$150,000	18 weeks
Concrete	260,000	21 weeks
Interior work	550,000	11 weeks
Elevator	320,000	16 weeks
Total	$1,280,000	

Construction Budget and Schedule

EXHIBIT 3

This is an example of the traditional construction bar chart. Although it is usually presented on a wide piece of paper, the information it contains is sparse. It really shows only two numbers for each classification of work—a start date and a finish date. It gives no indication of what goes on between these two dates and does not tell why the activities cannot be started and/or finished earlier.

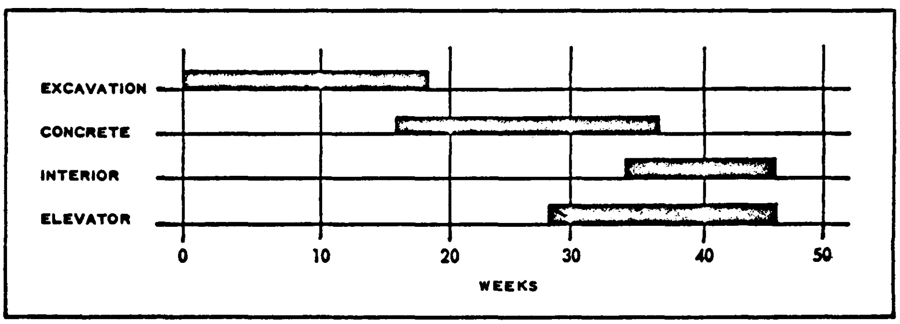
To obtain a better understanding and closer control of the work, you decide to use critical path scheduling on the project. Your first step is to sit down with the contractor and discuss the details of his plan for construction. An afternoon in his office reveals that the following facts appear to govern the sequencing of the work:

Governing sequence

1. It would be very costly to begin the interior work before the

EXHIBIT 4

Apartment House—Schedule



service elevator is operating manually because of the expense of hoisting workmen and material.

2. After the service elevator is operating manually, interior work can proceed concurrently with the installation and testing of automatic elevator controls.

3. The elevator work cannot be started until the reinforced concrete elevator tower is complete.

4. The concrete elevator tower cannot be started until the basement area of the building lot is excavated and shored.

5. Certain concrete foundation work must be in place before the structural steel framework can begin.

6. Structural steel supporting the main building concrete cannot be raised until the elevator tower is complete, for the steel depends on the tower for support.

7. Concrete for the parking deck cannot be started until the main building concrete work is complete since the parking deck area is needed for working space.

Nine main steps

After considering these facts, you decide to divide the project for scheduling purposes into nine steps as follows:

Excavation
- Main building area
- Parking deck area

Concrete
- Foundation (includes all concrete work in main building area and below street level)
- Elevator tower
- Main building (including supporting structural steel)
- Parking deck (both above and below grade)

Interior work, all floors

Elevator
- Manually operated, temporary equipment
- Fully automatic equipment

Instead of four bars on the chart, you now have nine. You indicate the necessary sequencing of the bars by arrows, making a critical path schedule diagram as shown in Exhibit 5 on this page.

Next you ask the contractor for his best estimate of the number of weeks that will be required for the completion of each step. These estimates are entered on the diagram (see Exhibit 6 on page 31).

Now consider the question: "If these estimates are correct, what is the shortest possible time in which the entire job can be completed?" This is answered by computing an earliest possible completion time (which is called "EPC") for each of the steps.

The EPC can be computed for any step if, and only if, it already has been computed for all immediately preceding steps. Therefore, you must begin with a step which is not preceded by any other step (in this case, begin with "excavate basement") and work forward through the schedule.

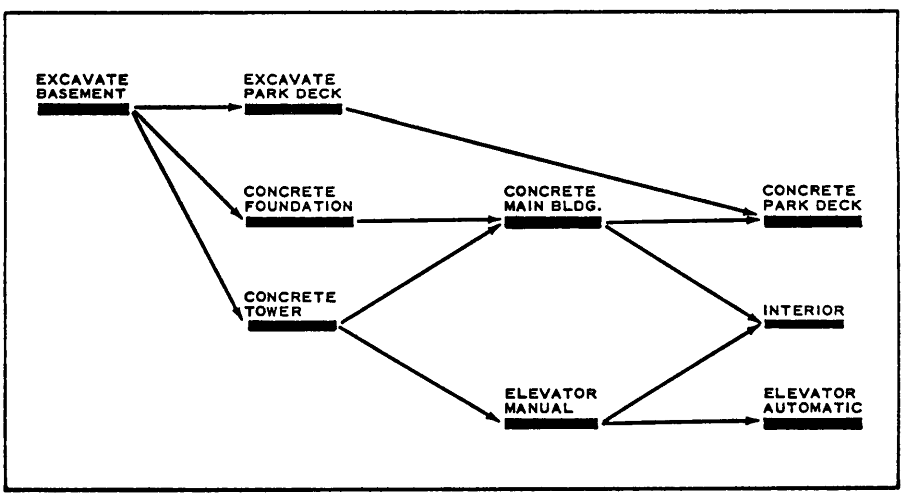
Calculating the EPC

The EPC for a step is the sum of (a) its duration and (b) the largest of the EPC's for the immediately preceding steps. In Exhibit 7 on page 31 the EPC's have been correctly computed for all steps. Note, for example, that the EPC for "Concrete park deck" is the sum of (a) the duration of six weeks and (b) the larger of the EPC's for the "Excavate park deck" (18) and "Concrete main bldg" (23). This amounts to $6 + 23 = 29$. That is, "Concrete park deck" cannot be completed earlier than the 29th week.

The earliest possible completion time for the entire building is the largest of the EPC's. In this case,

EXHIBIT 5

Apartment House—Critical Path Schedule Diagram, Sequencing



the conclusion is that, if the estimates of step duration are correct, your building can be completed in 36 weeks instead of 45. Since the 36-week estimate is more detailed than the 45-week, it is likely to be more precise. You calculate your gain from earlier use of the building and find that, at the net rental value of $700 per day, the financial outlook is better. Without the careful analysis required by critical path, this opportunity for earlier completion might have been overlooked.

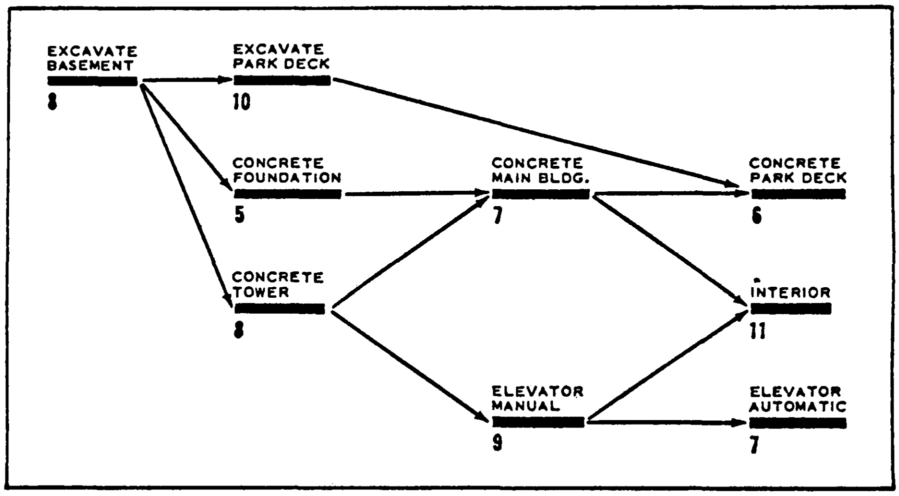
Last possible dates

As you study the new schedule, you notice an interesting difference between the step "Concrete Park Deck" and the step "Interior." One can be delayed past its EPC without affecting final completion date, while the other cannot. In this sense, "Concrete Park Deck" is seven weeks less "urgent" than "Interior." It occurs to you that there may be a way to calculate the degree of urgency for every step in the schedule.

As a matter of fact, there is. It is done by computing a latest possible completion time (called "LPC") for every step. LPC represents the latest possible time the step can be completed without prolonging the entire project.

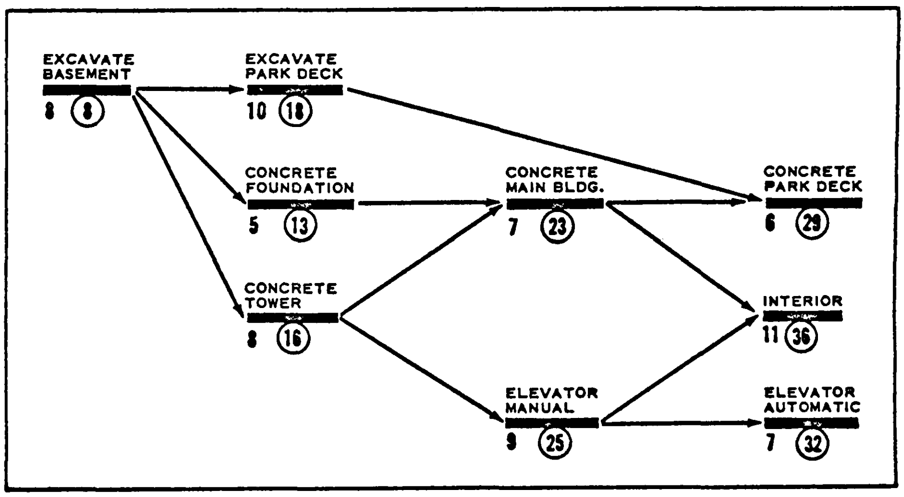
In general, the LPC can be calculated for any step if, and only if, it already has been calculated for all immediately following steps. Therefore, you must begin with a step which has none following and work backward through the schedule.

For any step with none following, the LPC, of course, is equal to the largest EPC, that is, to the project duration. In the example, the LPC for "Interior" would be 36, and so would the LPC's for "Elevator automatic" and "Concrete park deck."



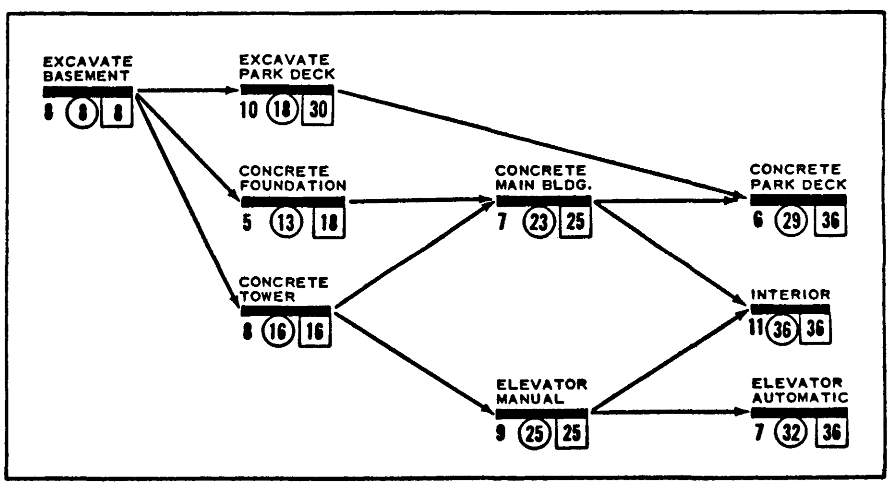
Apartment House—Critical Path Schedule Diagram, Time Estimates

EXHIBIT 6



Apartment House—Critical Path Schedule Diagram, EPC's

EXHIBIT 7



Apartment House—Critical Path Schedule Diagram, EPC's and LPC's

EXHIBIT 8

Excavation	
Basement	$ 60,000
Park deck	90,000
Concrete	
Foundation	50,000
Tower	30,000
Main building	110,000
Park deck	70,000
Interior	550,000
Elevator	
Manual	80,000
Automatic	240,000
	$1,280,000

Cost Breakdown by Critical Path Steps

EXHIBIT 9

The value of critical path analysis arises from treatment of a large, complicated project in terms of its many smaller, simpler parts. Working days are now budgets, like dollars. The estimate of total construction time is the sum of the estimated construction times for certain steps—those on the critical path.

For any other step the LPC is calculated as follows: For each immediately following step, subtract the duration from the LPC. The smallest of these differences is the required LPC.

This is shown in Exhibit 8 on page 31. The LPC's have been calculated correctly. For example, the LPC for "Concrete main bldg" is the smaller of the two numbers (36-6) and (36-11), that is, the smaller of 30 and 25, or 25. Stated in words, "Concrete main bldg" must be finished no later than the 25th week, or the entire project will be delayed beyond the 36th week.

Bottleneck steps

The urgency of each step may now be measured by subtracting its EPC from its LPC. The result is called the "slack" for the step. It represents the number of weeks by which the completion of the step may be delayed beyond its EPC without prolonging the project. For example, "Concrete park deck" has a slack of (36-29) or seven weeks.

Notice that the steps "Excavate basement," "Concrete tower," "Elevator manual," and "Interior" all have zero slacks. These four steps constitute a path, that is, a succession of steps from the beginning to the end of the project. In any critical path schedule diagram there is always at least one path on which all steps have zero slacks. This path is called the "critical path." If a step is on a critical path, any delay in that step will delay the entire project.

In this nine-step schedule four of the steps are critical. In applications involving hundreds or thousands of steps, it is often found that only 10 to 20 per cent of the steps are critical. Management can effectively expedite the entire project by concentrating on these few bottleneck steps, which are precisely identified by critical path analysis.

Cost control

Since your arrangement with the contractor is cost plus ten per cent, you will want to measure, at frequent intervals, how much the actual cost is likely to differ from the estimate. This objective can be met by using the critical path steps as budget subdivisions. You ask the contractor to break down his estimate of cost by critical path steps, and he replies as shown in Exhibit 9 on this page.

The contractor reports that the interior and elevator steps have been subcontracted at the amounts shown, so that the uncertain elements of cost are excavation and concrete work, which the contractor is performing with his own men and equipment.

With the estimate broken down as shown, you are now in a position to monitor the contractor's costs immediately on completion of each step. As soon as he finishes, say, "Concrete foundation," you will know exactly how much work has been done, how much money was budgeted for it, and what it actually cost. There will be no need for an "estimate of percentage complete." This method of cost control contrasts with the traditional one, in which the accountant is required to obtain estimates of percentage completion for various categories of work (concrete, plumbing, etc.) without being certain of how these categories are physically distributed in the structure. (Note: In an actual situation the estimate of cost would probably be broken down not only by step but also by the usual functional classifications as well; for example, formwork, cement finishing, etc.)

Suppose that at the end of the 15th week the concrete tower is complete. A job progress report for your building would contain information like that shown in Exhibit 10 on this page.

Date: November 18, 1967
Working week: 15th

	Cost		Completion Week	
	Original Estimate	Actual	Original Estimate	Actual
Steps completed:				
Excavation, basement	$ 60,000	$ 61,400	8th	6th*
Concrete foundation	50,000	48,700	13th	14th
Concrete tower	30,000	26,200	16th	15th*
Subtotals	$ 140,000	$ 136,300		
		Current Estimate		Current Estimate
Steps not yet completed:				
Excavation, park deck	$ 90,000		18th	19th
Concrete, main building	110,000		23rd	22nd
Concrete, park deck	70,000		29th	28th
Elevator, manual	80,000		25th	24th*
Elevator, automatic	240,000		32nd	31st
Interior	550,000		36th	35th*
Subtotal	$1,140,000	$1,140,000		
Totals:				
Per original estimates	$1,280,000		36th	
Adjusted for results to date		$1,276,300		35th

Estimated Final Completion:
35th week (April 6, 1968)

*Critical steps

Job Progress Report

EXHIBIT 10

Cost control precise

As this example illustrates, the value of critical path analysis arises from treatment of a large complicated project in terms of its many smaller, simpler parts. Working days are now budgeted like dollars. The estimate of total construction time is the sum of the estimated construction times for certain steps—those on the critical path. Similarly, cost control is precise because the budget is broken down by time-oriented steps that are consistent with the physical progress of the work.

Linear programing

Linear programing is an analytical or computational technique for solving a general class of optimization problems involving many variables related in a complex way. The solution of these problems involves the attainment of a measure of effectiveness, such as profits, costs, or quantities produced, for a given set of restraining conditions, e. g., material availability, production capacity, government regulations, etc.

Using the linear programing technique, an analyst can systematically search through unit cost and quantity tables of hundreds of alternatives for making products at various plants of a national company, shipping to and storing at various warehouses, and ultimately shipping to customers in order to arrive at an overall minimum cost solution. With five factories and one hundred depots or stores, you would have to consider at least five hundred shipping routes to get the product to the customer at lowest cost. If the pertinent data as to unit costs of manufacture and shipping and capacities and demands are accurately arranged, the solution can be readily obtained in a rather routine—and mathematically quite simple—operation.

Another example

Another example of the application of this mathematical method to the solution of a budgeting or cost control problem to which the answer is not intuitively evident is the mix problem. In oil refineries, where many choices as to end products are usually available, linear programing techniques will indicate the best selection based on the demands for and prices of end products (gasoline, kerosene, motor oil, paraffin, etc.) and the cost and types of crude oil available. Comparable solutions are available to the mixer of animal feeds, where the requirements are specified in terms of vitamin content, bulk, and other general criteria that could be met by a variety of combinations of different ingredients such as corn, wheat, barley, cotton seed, fish meal, etc., selling at constantly changing prices. The realization of budgeted profits very often depends on probing in a scientific fashion all of the possible alternatives.

Conclusion

Actually, accountants have barely scratched the surface in applying appropriate mathematical methods. Every accountant should do more reading in this exciting and rewarding field. Many excellent publications are readily available. Furthermore, the universities are beginning to provide appropriate mathematical courses for accounting students so that accountants of the future will start their careers fully equipped for the challenges of modern business.